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PHASE CONJUGATE MIRRORS

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Phase conjugate mirrors are attractive devices to improve the beam quality of lasers. At first, the fundamental properties of phase conjugate mirrors (PCMs) are summarized and a basic mathematical description is given. Degenerate four-wave mixing (DFWM) facilitates phase conjugation of weak signals and their amplification at the same time, but requires additional pumping beams. In contrast, self pumped phase conjugation is achieved by stimulated Brillouin scattering (SBS) in appropriate media. Liquids and gases of high purity as well as solid materials like silica fibers can be used with high efficiency. Solid-state laser systems with average output powers up to 500 W have been equipped with SBS phase conjugate mirrors to compensate for phase distortions in active media.

1. Introduction

Phase conjugation is a nonlinear optical process which generates a light beam having the same wave fronts as an incoming light beam but opposite propagation direction, see Fig. 1. Therefore phase conjugation is called also wavefront reversal. A nonlinear optical device generating a phase conjugated wave is called a *phase conjugator* or *phase conjugate mirror*.

In Fig. 2, we consider the conjugation property of a PCM on a probe wave emanating from a point source. A diverging beam, after “reflection” from an ideal

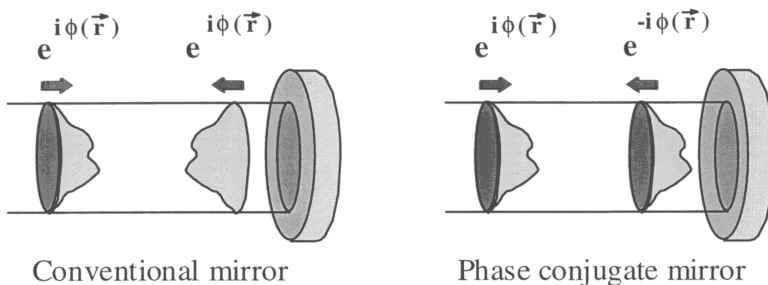


Fig. 1. Wave front reflection at a conventional and a phase conjugate mirror.

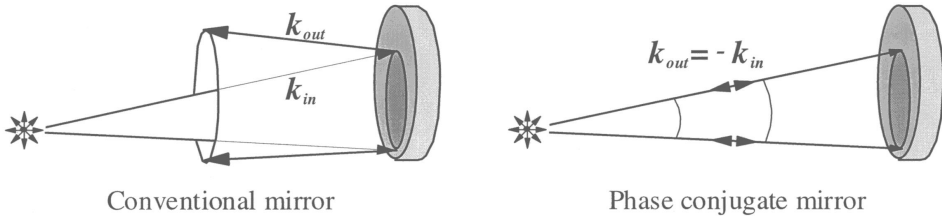


Fig. 2. Beam propagation after reflection at a conventional mirror and a PCM, both illuminated by a point source.

PCM, gives rise to a converging conjugate wave that precisely retraces the path of the incident probe wave, and therefore propagates in a time-reversed sense back to the same initial point source.

A phase conjugator reflects light, mostly laser beams, if the incident power is high enough (*self-pumped phase conjugator*) or if the nonlinear material in the phase conjugator is pumped by additional laser beams, e.g. two additional beams in a *degenerate four-wave mixing* arrangement. In principle phase conjugation could be achieved also by a *deformable mirror* which is controlled by a wave front sensor adapting the local mirror curvature to the incoming wave front. Instead of a deformable mirror also a *2-dimensional phase modulator* could be used. However, deformable mirrors and other phase modulators are not applied up to now to solve practical problems requiring phase conjugation.

2. Basic Mathematical Description

The incoming wave E_{in} is given by Eq. (1) with frequency f , where the amplitude E_0 and phase Φ are combined to the complex amplitude A . The complex conjugate is denoted by *c.c.*

$$E_{in}(x, y, z, t) = \frac{1}{2} E_0(x, y, z) e^{2\pi i(ft + \Phi(x, y, z))} + c.c. = A(x, y, z) e^{i\omega t} + c.c. \quad (1)$$

$$A(x, y, z) = \frac{1}{2} E_0(x, y, z) e^{2\pi i\Phi(x, y, z)}. \quad (2)$$

The phase conjugated wave exhibits the same wave fronts, however the sign of the phase Φ is inverted due to the inverted propagation direction. Thus, the phase conjugated wave E_{pc} can be written as Eq. (3).

$$E_{pc}(x, y, z, t) = \frac{1}{2} E_0(x, y, z) e^{2\pi i(ft - \Phi(x, y, z))} + c.c. = A_{pc}(x, y, z) e^{i\omega t} + c.c. \quad (3)$$

$$A_{pc}(x, y, z) = \frac{1}{2} E_0(x, y, z) e^{-2\pi i\Phi(x, y, z)} = A^*(x, y, z). \quad (4)$$

As can be seen A_{pc} equals the complex conjugated A^* , what explains the term *phase conjugation*. From Eqs. (1) and (3) we derive that the incident and phase conjugated wave are also related to each other by

$$E_{in}(x, y, z, -t) = E_{pc}(x, y, z, t). \quad (5)$$

Thus, the phase conjugate wave E_{pc} propagates as if one would reverse the temporal evolution of the incident wave E_{in} . Therefore the term “*time reversed replica*” is sometimes used to describe the phase conjugate wave.

An ideal PCM also maintains the polarization state of an incident wave after phase conjugation. As an example, a probe wave that is right-handed circularly polarized (RHCP) will result in a RHCP reflected wave, after conjugation. This is in contrast to a conventional mirror, which reflects an incident RHCP field to yield a left-handed circularly polarized (LHCP) wave.¹

One should realize that an ideal phase conjugated wave exhibits the same frequency f as the incident wave and reveals the same polarization state. Often, real phase conjugators do not have these properties. However, if a PCM maintains the polarization state it is called a “*vector phase conjugator*”.

The nonlinear optical process which comes closest to yielding an ideal phase conjugate wave is the backward-going, degenerate four-wave mixing interaction. Other classes of interaction (e.g. stimulated effects) result in nonideal conjugate waves due to frequency shifts, nonconjugated field polarization, etc. Although the application of stimulated effects, especially stimulated Brillouin scattering (SBS), yields to nonideal phase conjugate mirrors they are used the most to solve practical problems requiring phase conjugation (e.g. compensation of phase distortions in high average power laser systems²).

3. Phase Conjugation by Degenerate Four-Wave Mixing

Four-wave mixing can be understood as a real time holographic process, which facilitates phase conjugation. If the frequencies of the incoming wave, the two additionally required pump waves and the phase conjugated or reflected wave are equal the process is called degenerate four-wave mixing (DFWM), see Fig. 3.

Interference of the incoming wave $E_{in}(x, y, z, t)$ with the pump wave $P_1(x, y, z, t)$ results in a spatially periodic intensity pattern which modulates the absorption coefficient or refractive index of the optical material resulting in a dynamic or transient amplitude or phase grating. The other pump $P_2(x, y, z, t)$ is diffracted at this grating producing the phase conjugated wave. This corresponds to the conventional holographic process where the read-out wave is replaced by the second pump wave counterpropagating to the first pump or reference wave.

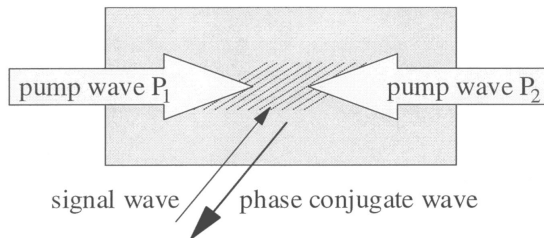


Fig. 3. Setup for phase conjugation by four-wave mixing.

Recording of a hologram is the first step in phase conjugation and leads to a transmission function t in the hologram plane (variables will not be noted further more to simplify the readability):

$$t \propto |P_1 + E_{in}|^2 = \dots = |P_1|^2 + P_1 E_{in}^* + P_1^* E_{in} + |E_{in}|^2. \quad (6)$$

During the read-out the phase conjugate wave can be generated. Therefore the hologram is illuminated with a second pump wave P_2 , propagating in the opposite direction to P_1 . This is in contrast to standard holography. Since P_2 precisely retraces the path of P_1 in the opposite propagation direction, P_2 equals P_1^* . This means, that the two pump beams should be phase conjugated to each other, so that their spatial phases cancel and do not influence the phases of the reflected beam.

In the hologram plane we obtain a field strength distribution as follows:

$$P_2 t = P_1^* t \propto P_1^* |P_1|^2 + |P_1|^2 E_{in}^* + (P_1^*)^2 E_{in} + P_1^* |E_{in}|^2. \quad (7)$$

The second term $|P_1|^2 E_{in}^*$ corresponds to the phase conjugate wave of E_{in} . The other expressions lead to three additional waves which are not of interest here. They can be suppressed in thick nonlinear media in case of Bragg diffraction.

Common dynamic grating materials for phase conjugation are:

- photorefractive crystals (BaTiO₃, LiNbO₃, ...)
- liquid crystals (molecular reorientation effects)
- laser crystals (spatial hole-burning, excited state absorption)
- saturable absorbers
- absorbing gases and liquids (thermal gratings)
- semiconductors (Si, Ge, GaAs, ...).

The disadvantage of phase conjugation by four-wave mixing is the requirement of two additional pump waves for the nonlinear medium. However, this facilitates amplification of the phase conjugate wave in the nonlinear medium at the same time. Vector phase conjugation is not achieved by this simple DFWM scheme but requires polarization dependant interactions.

4. Self Pumped Phase Conjugation

Self pumped phase conjugation of continuous wave laser beams in the lower power range (mW to W) can be realized in FWM loop arrangements using photorefractive media (not discussed here). For pulsed lasers, self pumped phase conjugation is achieved by stimulated scattering. For practical application, stimulated Brillouin scattering³ in

- gases (SF₆, Xe, C₂F₆, CH₄, N₂, ...) under high pressure,
- liquids (CS₂, CCl₄, Freon, GeCl₄, acetone, methanol ...), and
- solids (bulk quartz glass, silica glass fibers) is used.

Table 1 shows the Brillouin gain coefficient g , the phonon lifetime τ for different gaseous, liquid and solid state SBS media.

Table 1. Brillouin gain coefficient g and phonon lifetime τ for different SBS media.

SBS Medium	Brillouin gain coefficient g [cm/GW]	Phonon lifetime τ [ns]
SF ₆ (20 bar)	25	15
Xe (50 bar)	90	33
C ₂ F ₆ (30 bar)	60	10
CS ₂	130	5.2
CCl ₄	6	0.6
Acetone	20	2.1
Quartz	2.4	5

A phase conjugate mirror consists simply of a gas or liquid cell or a fiber piece. The incoming wave is focused into the material where an oppositely traveling wave is generated initially by spontaneous scattering. This wave interferes with the incoming wave and induces a sound wave or another type of phase grating reflecting the incoming beam similarly as a dielectric multilayer mirror. The induced density variations have the frequency of the initial sound wave, which is amplified therefore and reinforces the backscattering.

The amplification depends strongly on the extension of the interference area. Therefore the phase conjugated backscattered part dominates, leading to an exponential rise of the reflected phase conjugated signal. The wave fronts of the sound wave grating match the wave fronts of the incoming beam. Any disturbance of the incident wavefront will result in a self adapted mirror curvature with response times in the ns range.

For applications the “threshold”, reflectivity and conjugation fidelity are the most important parameters that characterize the performance of a Brillouin scattering phase conjugate mirror. A sharply defined threshold does not exist for the nonlinear SBS process. However, after exceeding a certain input energy a steep increase of reflectivity can be observed. Often this is called the energy threshold of the phase conjugate medium. For long pulses as compared to the phonon lifetime (typically several ns) the SBS is expected to become stationary. In this case the energy threshold can be substituted by a power threshold. Well above this threshold, the reflectivity is not stationary but exhibits statistical fluctuations because SBS starts from noise.

It is important to emphasize that the power and not the intensity determines the “threshold”, in case of strongly monochromatic input waves. Slight focusing leads to lower intensity, but also to a longer Rayleigh length and a larger interaction area. Stronger focusing reduces the interaction length, but results in stronger refractive index modulation. Both effects compensate each other if the interaction length is not limited by the coherence length.

Practically, for most laser sources the coherence length is rather short. Here, the interaction length should be short compared to the coherence length. This requires adequate focusing of the beam into the SBS medium. Focal length and scattering material have to be chosen suitable to achieve a high SBS reflectivity and a good reproduction of the wave front. Side effects in the material like absorption, optical breakdown or other scattering processes have to be avoided. Figure 4 shows the energy reflectivity of carbon disulfide CS_2 as a function of the input power at $1 \mu\text{m}$ wavelength. Carbon disulfide shows one of the smallest power thresholds for liquids of about 18 kW. Applying gases as SBS media, the power thresholds are about one order of magnitude higher. A saturation of energy reflectivity close to 80% is a typical value for liquid SBS media, although reflectivities up to 96% had been demonstrated.⁴

For high power input pulses bulk solid state media like quartz are investigated as SBS media, too.⁵ To reduce the power threshold of SBS a waveguide geometry can be applied.⁶ The beam intensity inside the waveguide is high within a long interaction length resulting in low power thresholds. To avoid toxic liquids and gases under high pressure multimode silica fibers can be used.⁷ The lower Brillouin gain of quartz glass compared to suitable SBS gases and liquids can be overcome using fibers with lengths of several meters resulting in SBS thresholds down to 200 W peak power.⁸

The power threshold can be estimated from Eq. (8), where A_{eff} is the effective mode field area inside the fiber core, L_{eff} the effective interaction length, which depends on the coherence length, and g is the Brillouin gain coefficient; for quartz g is about 2.4 cm/GW .⁹

$$P_{th} = \frac{21A_{\text{eff}}}{L_{\text{eff}} g}. \quad (8)$$

Table 2 shows the power threshold, the maximum energy reflectivity, the far field fidelity, the M^2 -limit and an approximated power limit of fiber phase conjugators

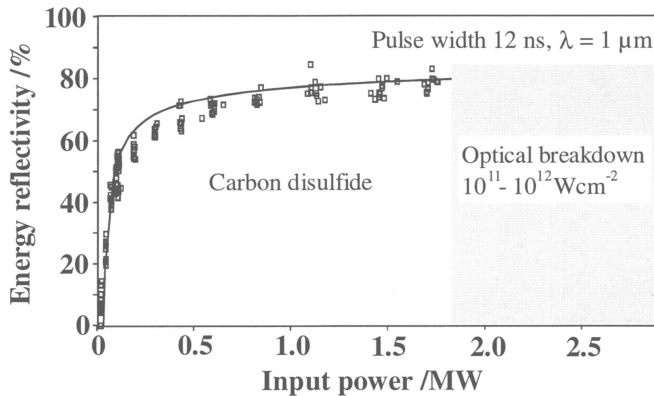


Fig. 4. Commonly used carbon disulfide (CS_2) shows an SBS threshold of about 18 kW (pulse peak power) under stationary conditions.

Table 2. SBS threshold, max. reflectivity, far field fidelity, M^2 -limit and power limit for different fiber phase conjugators, coherence length 1.5 m. The reflectivity is corrected with respect to Fresnel and coupling losses.

LCore diameter (μm)	SBS threshold (kW)	Maximum reflectivity (%)	Far field fidelity (%)	M^2 -limit	Power limit (kW)
200	17	80	93	63	160
100	6.4	80	91	31	40
50	2.0	88	70	16	10
25	0.3	86	—	8	2.5

with different core diameters. The used quartz-quartz fibers had a step-index geometry and a numerical aperture of 0.22. They were investigated with a Nd:YAG oscillator amplifier system generating pulses of 30 ns (FWHM) pulses at 1.06 μm wavelength. Regarding applications it is important to couple also spatially aberrated beams into the fiber. The upper limit for the beam parameter product is given due to the finite numerical aperture and the core diameter of the fiber. This can be expressed by a times diffraction limit value M^2 . The upper power limit is approximated assuming a damage threshold above 500 MW/cm² for ns pulses.

An important feature of a fiber phase conjugator is the threshold behavior for different M^2 -values of the incoming beam. In case of a fiber the SBS threshold is nearly independent of the incoming beam quality. This is caused by mode conversion inside the fiber resulting in homogeneous illumination and therefore in constant SBS reflectivity. In case of a Brillouin cell the reflectivity depends on the far field distribution of the incoming beam. Here phase distortions result in amplitude fluctuations in the focal region. A comparison between a diffraction limited beam ($M^2 = 1.0$) and a highly distorted beam ($M^2 = 10$) showed an increase of the SBS threshold of 300% in case of the Brillouin cell. For the fiber phase conjugator no remarkable changes of the power threshold were observed.¹⁰

Practically, the reproduction of the initial wave front is not perfect after phase conjugation. To characterize the deviation with respect to the reference wave the term *fidelity* is introduced.¹¹

$$F = \frac{|\int E_s E_p^* d^2r|^2}{\int |E_s|^2 d^2r \cdot \int |E_p|^2 d^2r}. \quad (9)$$

The fidelity equals unity in case of perfect wave front reproduction and is smaller than unity for practical cases. To calculate the fidelity, the electric field distribution of the incident signal E_s and the phase conjugate wave E_p has to be known. The determination requires sophisticated measurement equipment. In contrast, the *far field fidelity* can be determined with less effort and is therefore often used. The transmission through an aperture of the phase conjugate signal is compared with the transmission of the input signal. The ratio is called far field fidelity, because the aperture is placed in the focal plane of a focusing lens.

5. Applications of SBS Phase Conjugation

Phase conjugation generates a wave which retraces the incoming wave in a time-reversed way. Thereby it is possible to eliminate phase distortions in optical systems. For example, in a solid state laser amplifier, the incoming beam is not only amplified but suffers also from phase distortions due to thermal refractive index changes in the laser crystal. After passing this amplifier crystal, the beam is reflected by a phase conjugator and passes the crystal a second time. Because the wave fronts are inverted with respect to the propagation direction, the refractive index changes reduce the phase distortions and after the second passage, these distortions disappear so that the beam quality of the incoming wave is reproduced.

Typically, phase conjugators are applied in Master Oscillator Power Amplifier (MOPA) setups, where a nearly diffraction limited master oscillator beam is increased in power within an amplifier arrangement, see Fig. 5. After the first amplification pass the beam quality is reduced due to thermally induced phase distortions. The spatially distorted beam enters the SBS mirror and becomes phase conjugated. The initial beam quality of the master oscillator can be roughly reproduced after the second amplification pass. The amplified beam is extracted with an optical isolation, which consists in this case of a Faraday rotator and a polarizer.

Figure 6 shows a MOPA system producing up to 210 W average output power at 2 kHz average repetition rate ($1.08 \mu\text{m}$ wavelength). The system is part of an advanced setup yielding to 520 W average output power.² The oscillator beam has a nearly diffraction limited beam quality ($M^2 < 1.2$) which is already reduced in front of the first amplifier ($M^2 \cong 1.5$). This results from optical components between oscillator and amplifier which introduce phase distortions. After single pass amplification the beam quality decreases to $M^2 \cong 5$ due to phase distortions introduced by both pumped amplifier rods at 6.5 kW pumping power for each amplifier. After phase conjugation and double pass amplification the initial beam quality can be nearly reproduced ($M^2 < 1.9$). Differences between the initial and final beam quality are caused by a fidelity smaller than unity and diffraction at several apertures in the amplifier chain.

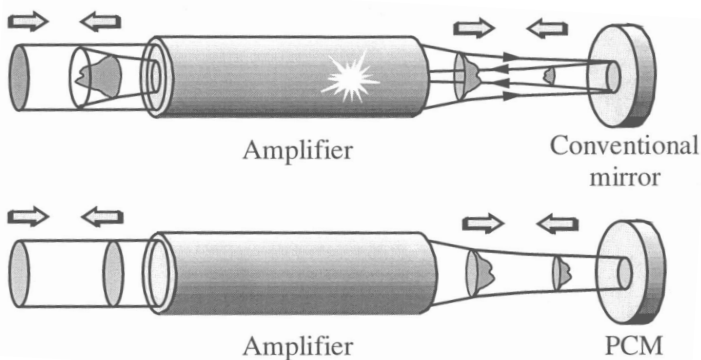


Fig. 5. Double pass scheme with phase conjugate mirror to compensate for phase distortions.

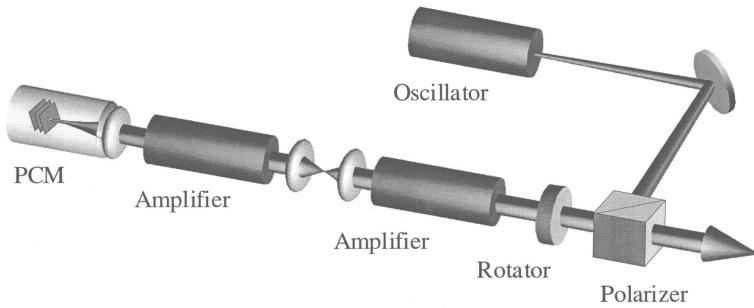


Fig. 6. Master oscillator power amplifier setup with phase conjugate mirror.

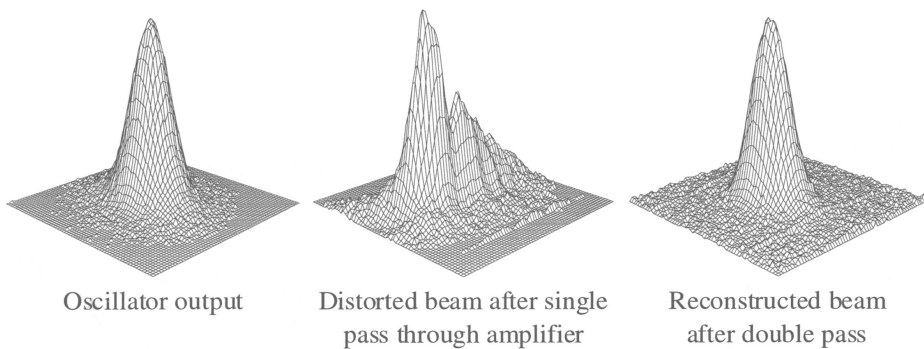


Fig. 7. Far field intensity distributions of the oscillator beam, the distorted beam after single pass amplification and the highly amplified beam after double pass amplification with phase conjugation.

The performance of the phase conjugate mirror can be illustrated by far field intensity profiles recorded at different positions in the setup. In Fig. 7, the oscillator output beam exhibits a smooth Gaussian profile corresponding to the nearly diffraction limited beam quality. After single pass amplification the reduction of beam quality is confirmed by a strongly aberrated far field profile. After phase conjugation and double pass amplification the initial intensity distribution can be nearly reproduced. In this example the average power of the master oscillator beam (approx. 1 W) was increased to 130 W after double pass amplification.

Presently, this is the most often frequent application of phase conjugation. In addition phase conjugators are useful as mirrors in laser oscillators replacing one of the conventional mirrors. Again, the phase conjugator eliminates phase distortions in the laser medium induced by optical or discharge pumping.

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