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GPS INFERRED GEOCENTRIC REFERENCE FRAME FOR SATELLITE POSITIONING AND NAVIGATION

Abstract

Accurate geocentric three dimensional positioning is of great importance for various geodetic and oceanographic applications. While relative positioning accuracy of a few centimeters has become a reality using Very Long Baseline Interferometry (VLBI), *the uncertainty in the offset of the adopted coordinate system origin from the geocenter is still believed to be of the order of one meter. Satellite Laser Ranging* (SLR) *is capable of determining this offset to better than* 10 cm, *though, because of the limited number of satellites, this requires a long arc of data. The Global Positioning System* (GPS) *measurements provide a powerful alternative for an accurate determination of this origin offset in relatively short period of time. Two strategies are discussed, the first utilizes the precise relative positions predetermined by* VLBI, *where as the second establishes a reference frame by holding only one of the tracking sites longitude fixed. Covariance analysis studies indicate that geocentric positioning to an accuracy of a few centimeters can be achieved with just one day of precise* GPS *pseudorange and carrier phase data.*

Introduction

The fully operational Global Positioning System (GPS) will consist of at least eighteen satellites distributed in six orbital planes [Parkinson and Gilbert, 1983]. This system will allow a user, anywhere on earth or in a low earth orbiting satellite/space station, to view at least five satellites most of the time. Two precision data types can be derived from the GPS transmitted signals : P-code pseudorange and carrier phase at two L-band frequencies [Milliken and Zoller, 1978]. These precision data types provide the opportunity to produce geodetic measurements accurate to the centimeter level and orbit determination of low earth orbiters to the sub-decimeter level [Yunck et al., 1986]. The ephemerides for the GPS satellites, as distributed by Naval Surface Warfare Center (NSWC), are based upon the World Geodetic System (WGS 84) [DMA, 1987] and their accuracy is of the order of ten meters [Swift, 1985]. In applications where high precision is essential, the GPS satellite orbits need to be adjusted to a much higher precision along with other parameters in the network [Yunck et al., 1986]. The GPS satellites can be simultaneously observed from several sites in a geodetic network. Within such a network

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a few fiducial tracking sites are included [Davidson et al., 1985]. The relative positions between these fiducial sites are known to a higher level of precision, typically a few centimeters, as a result of repeated measurements of the baselines using VLBI (Very Long Baseline Interferometry) [Sovers et al., 1984]. Based upon these highly precise relative positions of the fiducial sites, filter strategies can be designed to adjust the satellite orbits to enhance their accuracy to far better than ten meters [Bertiger and Lichten, 1988]. The ephemerides thus adjusted now refer to the same coordinate frame in which the fiducial baselines are known. It is generally believed that the best VLBI coordinate system origin approximates the geocenter to about one meter. The Satellite Laser Ranging (SLR) technique is capable of realizing the geocenter offset to better than 10 cm; but this is possible only after a long period of observations.

Although absolute positioning is of less interest for geodynamic applications, it can be an important factor when tracking deep space vehicles and it is essential for orbit determination of earth observing satellites, such as NASA's Ocean Topography Experiment, Topex/Poseidon [Born et al., 1985], to be launched in mid 1992. This paper investigates two strategies for precise determination of the geocenter, thus establishing a geocentric coordinate frame for satellite positioning and navigation. In the first strategy, GPS P-code pseudorange and carrier phase measurements are made from a set of globally distributed tracking stations. A network consisting of six stations were selected. Of these, three are the fiducial sites whose relative location has been well determined by VLBI. Since it is the relative location, rather than the absolute location, of the fiducial sites that is well determined by VLBI, only relative baseline coordinates should be fixed to define the orientation and absolute scaling of the reference frame. The geocenter position and the coordinates of other, non-fiducial sites are to be adjusted together with the GPS orbits. The coordinate frame thus defined is consistent with the VLBI frame, with improved geocenter offset.

An alternate strategy is to simultaneously adjust the GPS orbits and geodetic station coordinates with respect to one reference site in the network whose longitude is held fixed. The absolute scaling is determined by the adopted gravitational constant GM of Earth; the geocentric radius at the stations are inferred from the adjusted periods of GPS orbits and the pseudorange measurements; and the latitude is inferred from the time signature of earth rotation in the GPS measurements. The coordinate system thus defined will be an earth centered, earth fixed (ECEF) coordinate frame. The solution is free from any a priori uncertainty of site positions and the inferred reference frame is strictly self contained. This type of technique has been adopted by the Satellite Laser Ranging (SLR) and Lunar Laser Ranging (LLR) communities [Dow and Agrotis, 1985]. The coordinate origin offset from the geocenter is given by the weighted mean coordinates offsets of all stations in the network.

A precise knowledge of absolute position of the coordinate system origin is essential to various geodetic geodynamic applications, for example, the orbit determination of Topex/Poseidon, which is seeking an altitude accuracy of 13 cm or better, will require a very precise geocenter location.

Datum Definition and Coordinate Reference Frame

A rectangular coordinate system, such as, the World Geodetic Reference System (WGS 84) is defined with the Z-axis parallel to the direction of the Conventional Terrestrial Pole as defined by BIH on the basis of the BIH station coordinates, the X-axis along the line of intersection between the Conventional Terrestrial Pole implied equatorial plane and the WGS 84 reference plane which is parallel to the BIH defined zero meridian, and, the Y-axis on the equatorial plane to complete the right handed earth fixed Cartesian system. The origin of the coordinate system is defined to be at the Earth's center of mass. But the knowledge of the geocenter location limits the precise location of this origin.

The almanac and the ephemerides of GPS satellites are given in the WGS 84 coordinate system [Swift, 1984; Decker, 1986]. The coordinates of the ground stations derived by observing the GPS measurements will therefore be with respect to the WGS 84 reference frame. But it should be noted that the absolute accuracy of any geocentric position determination depends upon the knowledge of the location of the geocenter relative to the assumed origin. The coordinate system thus defined is an ECEF coordinate system which rotates around a mean astronomic pole. Such a system, although based on sound scientific principles, in order to allow for any imperfection or arbitrariness, is called the Conventional Terrestrial System (CTS) [Mueller, 1985]. However, events occur in an instantaneous real world, based upon an interim true equator and equinox frame of date, which is in a coordinate system different from CTS. This system is referred as Interim "True" Celestial System (ITS). The relationship between CTS and ITS is a transformation through a rotation [S] and a wobble [W] :

$$\mathbf{X}_{CTS} = [\mathbf{W}]\,[\mathbf{S}]\,\mathbf{X}_{ITS} \tag{1}$$

where the X's are position vectors. The wobble [W] is given by

$$[\mathbf{W}] = \mathbf{R}_y(-x_p)\,\mathbf{R}_x(-y_p) \tag{2}$$

where $R_r(p)$ denotes a matrix of rotation, by an amount p about the r-axis; x_p and y_p define the polar motion which is the angular separation between the CTS pole and ITS pole. The rotation [S] is given by

$$[\mathbf{S}] = \mathbf{R}_z(\mathrm{GAST}) \tag{3}$$

where GAST is the Greenwich Apparent Sideral Time given by

$$\mathrm{GAST} = \mathrm{GMST}^{0h\,UT} + \bar{\omega}(t_{df} + \mathrm{UT1} - \mathrm{UTC}) + \Delta\psi \cos\epsilon \tag{4}$$

$\mathrm{GMST}^{0h\,UT}$ is the Greenwich Mean Sidereal Time at 0 hour UT adjusted with respect to J2000 [Kaplan, 1981], $\bar{\omega}$ is the mean rate of advance of the GMST per day and t_{df} is the day fraction in UTC of time of observation. The last term in Equation (4) is commonly known as the equation of the equinox, where $\Delta\psi$ is the nutation in longitude and ϵ is the true obliquity of the ecliptic of date.

In general, celestial bodies are expressed in the Conventional Inertial System (CIS). Position vectors in this system are related to ITS through a nutation [N] and a precession [P] [Mueller, 1969] :

$$\mathbf{X}_{ITS} = [\mathbf{N}]\,[\mathbf{P}]\,\mathbf{X}_{CIS} \tag{5}$$

The nutation [N] is given by

$$[\mathbf{N}] = \mathbf{R}_x(-(\epsilon + \Delta\epsilon))\,\mathbf{R}_z(-\Delta\psi)\,\mathbf{R}_x(\epsilon) \tag{6}$$

where ϵ is the mean obliquity of date; the nutation angles $\Delta\psi$ and $\Delta\epsilon$ are computed from IAU 1980 (Wahr) Nutation Series corrected for their Long-Period Terms [Mueller, 1988] expressed with respect to J2000. These corrections to the nutation terms in longitude ($\delta\Delta\psi$) and in obliquity ($\delta\Delta\epsilon$) will in theory change the polar motion components and the GAST as shown by [Zhu and Mueller, 1983]. The precession [P] is given by

$$[\mathbf{P}] = \mathbf{R}_z(-z)\,\mathbf{R}_y(\theta)\,\mathbf{R}_z(-\zeta) \tag{7}$$

where ζ, θ and z are the standard precession rotation angles based on the IAU 1976 precession constants [Melbourne et al., 1983]. Therefore, the position vectors in WGS 84, which is one of the CTS, can be expressed with respect to CIS using the above transformations.

Strategie to Determine the Origin Offset from the Geocenter

In the past several years the fundamental concept behind accurate GPS orbital adjustment has been that of the fiducial network [Thornton et al., 1986]. A fiducial network consist of three or more tracking stations whose (relative) positions have been determined in an earth fixed coordinate frame to a very high accuracy, usually by VLBI. Several receivers at other, less accurately known, stations also observe the GPS satellites along with the fiducial network. The data are then brought together to simultaneously adjust the GPS satellite orbits and the positions of the non-fiducial sites. Thus the fiducial stations established by VLBI provide a self-consistent earth fixed coordinate system with respect to which the improved GPS satellite orbits and the non-fiducial stations can be expressed to a greater accuracy. At the same time the coordinate frame origin offset from the geocenter can also be estimated using the same set of data. Past experience in this area has indicated that an over-constrained network, where more baselines than necessary are fixed, can in fact produce a degraded solution. This is because in an over-constrained network the a priori uncertainty on the fixed parameters, which are more than necessary, will result in a suboptimal filter weighting. The solution will then be highly influenced by this mismodeling of these parameters.

In the first strategy proposed, the fiducial baselines are treated in three different ways as listed below :

Case A. Fix two fiducial baselines

Case B. Constrain two fiducial baselines by a priori weighting

Case C. Fix only one fiducial baseline.

The baselines define the orientation and the scale of the adopted coordinate frame. The absolute scaling can also be fixed by the Earth's gravitational constant, GM. Both are known to an accuracy of about one part in 10^8. The baseline length is used to define the absolute scaling so that the resulting coordinate frame will be consistent with the VLBI frame defined by the fiducial baselines. For the case with two baselines fixed, it is convenient to select one of the fiducial stations common to both fixed baselines as the reference site. The filter process is so designed that the baselines between the reference site and all other non-fiducial sites are adjusted along with the Earth Orientation Parameters (EOP), namely polar motion (x_p, y_p) and (UT1 − UTC) rate, the GPS satellite orbits and the absolute coordinates of the reference site, which in turn infers the adjustment of the geocenter position coordinates. The Earth's GM is also adjusted, although the data strength may not be great enough to improve the value of GM appreciably.

In the second strategy, the same GPS tracking network of globally distributed stations is used. However, only the longitude of a reference site is held fixed; all other site coordinates are adjusted simultaneously with the GPS orbits. Here, the GM of Earth provides the absolute scaling. The geocentric radius at a station can be derived from the adjusted periods of GPS orbits and pseudorange measurements. The time signature of the measurements defines the latitude. ***Figure 1*** graphically demonstrates the time signature of the measurements for two hypothetical cases. The first graph shows the periodic signature generated by the pseudorange (ρ) measurements to an orbiting GPS from a stationary receiver. The period is equal to the GPS orbit period which is nearly 12 hours; the amplitude is proportional to the geocentric position vector of the receiver projected on to the orbital plane. The second graph shows the case when a stationary GPS satellite is above the equator of a spinning earth. The period is now 24 hours; the amplitude is proportional to the cosine of the receiver latitude. The variation of the signature with respect to the receiver latitude is depicted in the sketch. Because of the difference in period, the effects due to rotating receiver can be separated from GPS orbiting signature and the latitude can be unambiguously recovered.

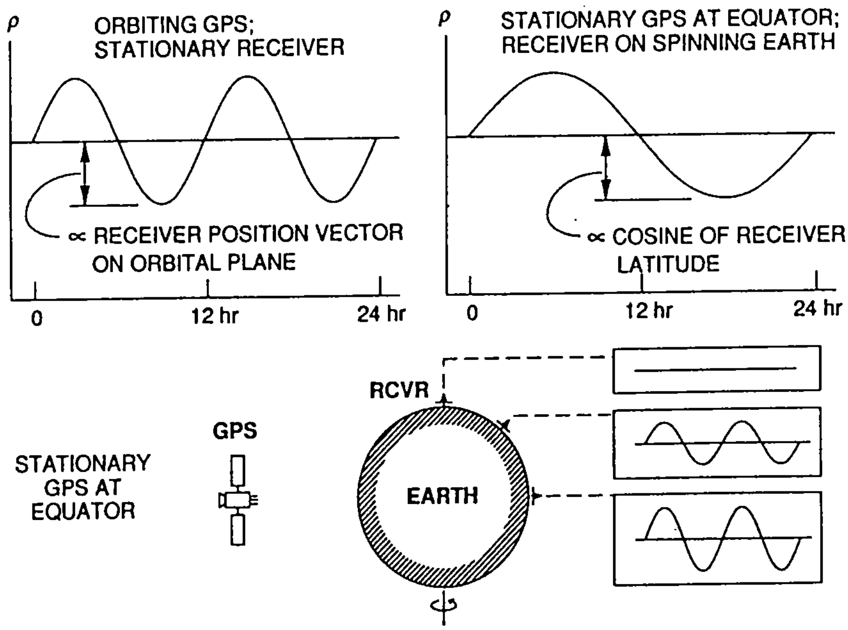

Fig. 1 — Time signature of GPS *measurements*

A simple mathematical model can be written out for the estimate of geocenter offset. This offset is expressed as the weighted mean of the position offsets of all stations. The equations corresponding to the geocenter offset $\Delta\mathbf{G}$ are represented as

$$\Delta \mathbf{G} + \Delta \mathbf{x}_i + v_i = 0 , \qquad i = 1 . 2 , \ldots , n \tag{8}$$

where $\Delta \mathbf{x}_i$ is the i^{th} geocentric station position vector offset and v_i is the residual vector associated with $\Delta \mathbf{x}_i$. The corresponding error covariance matrix of the geocenter offset can be expressed as

$$\underset{3 \times 3}{\Sigma_{\Delta G}} = [\mathbf{A}^T \mathbf{W} \mathbf{A}]^{-1} \tag{9}$$

where

$$\underset{3 \times 3n}{\mathbf{A}^T} = [- \mathbf{I} - \mathbf{I} \ldots - \mathbf{I}]$$

and **W** is a (3n x 3n) weight matrix which is the inverted covariance matrix of the station position estimates.

Covariance Analysis

A covariance analysis was carried out to assess the accuracy with which the geocenter offset from the origin of the adopted coordinate frame can be determined with each of the approaches proposed in the previous section. A full constellation of 18 GPS satellites distributed in six orbital planes was assumed. A data arc spanning over 34 hours from a network of six globally distributed tracking stations was also assumed. The three fiducial sites are the three NASA Deep Space Network (DSN) tracking sites *(Figure 2)* at Goldstone, CA; Canberra, Australia and Madrid, Spain. The remaining sites at Japan, Brazil and South Africa are non-fiducial sites. Simultaneous

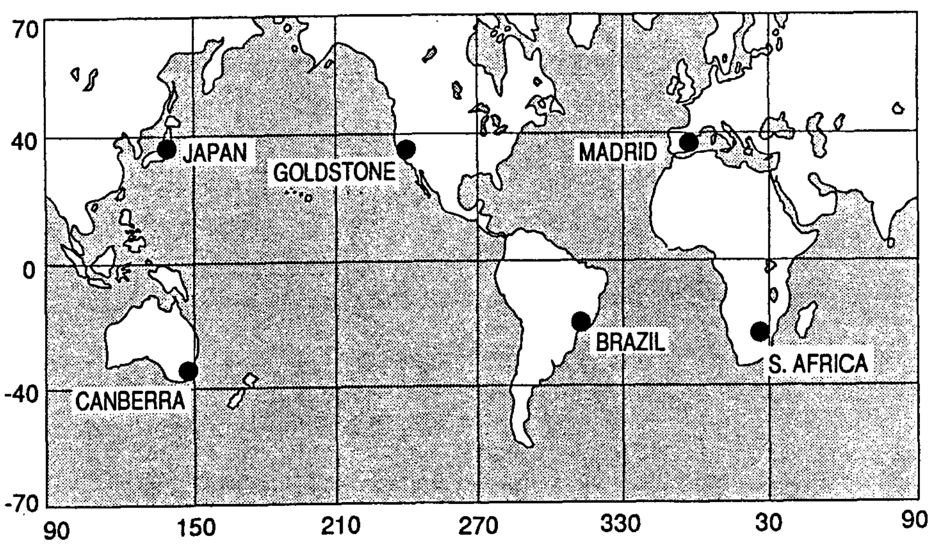

Fig. 2 — A global GPS tracking network

GPS P-code pseudorange and carrier phase measurements are made at all of these stations. The relative positions of the three DSN sites have been measured repeatedly by VLBI over many years and are known to an accuracy about 3 cm . Goldstone was selected to be the reference site because of its common VLBI visibility with the other two DSN sites at Canberra and Madrid. P-code pseudorange and carrier phase data noise were assumed to be 5 cm and 0.5 cm respectively when integrated over 30 minutes and corrected for ionospheric effects by dual-frequency combination. Carrier phase biases were adjusted with a large a priori uncertainty. *Table 1* lists the error sources

Table 1

Error Sources and Other Assumptions for Strategy 1 (Fixing Baselines)

Reference Site:	Goldstone
Other Fiducial Sites:	Canberra, Madrid
Non-fiducial Sites:	Brazil, Japan, South Africa
GPS Constellation:	18 Satellites in 6 Orbital Planes
Cut-off Elevation:	10 degrees
Data Type:	P-code pseudorange;
	carrier phase
Data Span:	6 – 34 hours
Data Interval:	30 minutes
Data Noise:	5 cm — pseudorange;
	0.5 cm — carrier phase
Carrier Phase Bias:	10 km (adjusted)
Clock Bias:	3 µsec — white noise (adjusted)
GPS Epoch State:	10 m; 1 mm/sec (adjusted)
Geocenter Position:	10 m each comp. (adjusted)
Baseline Coordinates:	3 cm each component — fiducial;
	10 cm each component — non-fiducial (adjusted)
Zenith Troposphere:	random walk parameter (adjusted):
	20 cm bias; 1.3 cm batch to batch
Earth's GM:	one part in 10^8 (adjusted)
Solar Pressure:	10%
(UT1–UTC) Rate:	10 m/day (adjusted)
Polar Motion (x_p, y_p):	10 m (adjusted)

assumed for the first strategy. The abundance and broad distribution of the GPS measurements allow all the GPS and station clocks to be treated as white-noise processes and adjusted to remove their effects on the solutions. Also adjusted are the zenith tropospheric delays at all ground sites, which were treated as random-walk parameters to model the temporal change. Such models have been proved to be effective in removing their errors without heavily depleting the data strength [Bertiger and Lichten, 1988].

The Orbit Analysis and Simulation Software, OASIS [Wu and Thornton, 1985], developed by JPL, was used to carry out the study. In OASIS, partial derivatives with respect to Cartesian components of site locations and the geocenter are readily produced. It is shown in the Appendix that baseline partials are related to site location partials as follows.

1. The partial derivative with respect to a Cartesian component of the reference sites is the sum of all partial derivatives with respect to the same component of all sites forming the baselines. Note that this is also the partial derivative with respect to the same component of the geocenter position.

2. The partial derivative with respect to a baseline Cartesian component is the same as the partial derivative with respect to the same component of the non-reference site forming the baseline.

Hence, the site location coordinate partials can readily be used in place of the baseline coordinate partials, and the geocenter offset coordinate partials in place of the reference site absolute coordinate partials.

The second strategy assumes the same network of six tracking sites. The estimated quantities are the coordinates of all six sites except the longitude of the reference site (Goldstone), together with the GPS satellite states, white-noise clocks, random-walk troposphere parameters and carrier phase biases. Because the longitude of Goldstone is held fixed, the position components need to be given in an ellipsoidal coordinate system, viz., longitude, latitude and height. *Table 2* lists the assumption variations that apply to this strategy. Other assumptions are kept the same as in *Table 1.* With this strategy, the error covariance matrix of geocenter offset is given by Equation (9) in the previous section.

Table 2

Variations of Assumptions from Table 1 for Strategy 2

(Fixing Only One Longitude)

Reference Site:	Goldstone
Reference Site Coordinates (adjusted):	10 m (Latitude)
	0 m (Longitude)
	10 m (Height)
Other Site Coordinates (adjusted):	10 m each component

Results of Covariance Analysis

In the covariance analyses for both strategies, data arcs of various lengths were used to study the solution convergence. In all cases the station at Goldstone was considered to be the reference site although in the second strategy any of the ground sites can be a reference site where the only fixed component is the longitude.

Table 3 tabulates the a priori error associated with the fiducial baselines, Goldstone–Canberra and Goldstone–Madrid, in all three cases of Strategy 1 . The value of GM was adjusted although it was found that the data strength of the GPS measurements is not great enough to improve on its a priori value. It should be noted that adjusting Earth's GM makes GPS satellite states consistent with the absolute scaling as implied by the baselines.

Table 3

Fiducial baselines in Strategy 1

Case	Baselines	Adjusted	a priori σ
A	Goldstone-Canberra	no	3 cm
	Goldstone-Madrid	no	3 cm
B	Goldstone-Canberra	yes	3 cm
	Goldstone-Madrid	yes	3 cm
C	Goldstone-Canberra	yes	10 cm
	Goldstone-Madrid	no	3 cm

Figure 3 shows the total error of the origin offset as the length of the data span increases from 6 hours to 34 hours for Case A of Strategy 1 , where two baselines are fixed. At the end of 34 hours the origin offset error is 4.0 cm (RMS of all three components). The graph shows a rapid reduction of error in origin offset between 6 and 12 hours . The result continues to improve after 12 hours but not at a very high rate. The reason for this can be seen in ***Table 4.*** Tabulated here are the results from Strategy 1 (all three cases) and Strategy 2 . The effects on geocenter due to data noise and the baselines are demonstrated. In Case A the origin offset error has come down to the level of the baseline error after 12 hours; data gathered thereafter only gradually reduced the effects of data noise. At the end of 34 hours the effect of data noise is reduced to 3.4 cm and it would continue to reduce as the arc length increases. The contribution of the baseline error, however, converged to about 2.5 cm after 12 hours and remained unchanged thereafter. This indicates that the geocenter can be determined only up to the a priori accuracy of the fiducial baselines. Therefore, with this strategy, any improvement

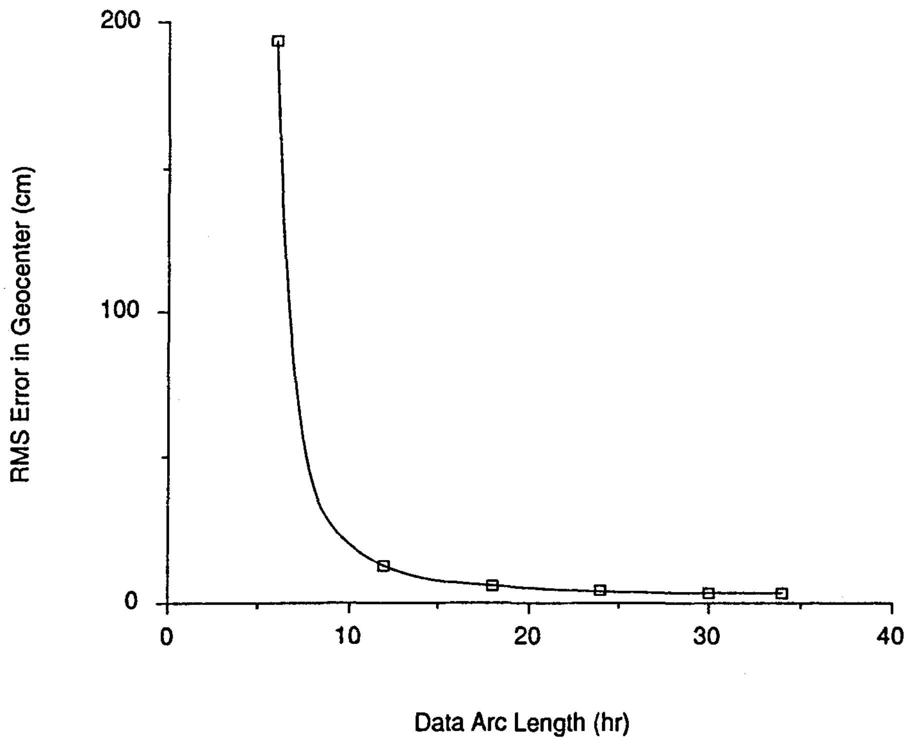

Fig. 3 — Geocenter offset with two baselines fixed (Case A)

on the baseline accuracy can improve the ultimate accuracy of the origin offset from the geocenter. For instance, it is customary to find baselines reported with a higher accuracy in length than in the other two components. When a smaller error of 1 cm is assumed for the fiducial baseline length, along with 3 cm for each of the transverse and vertical components, the RMS error on the origin offset from the geocenter reduces to 3.5 cm with a 34-hour arc of GPS measurements. In Case B, the baseline vectors constrained to their a priori error, are also estimated. The geocenter offset error after 34 hours reduces to 3.8 cm. Note that the error involved here is mainly due to data noise alone. The geocenter offset error in Case C is 4.4 cm after 34 hours which is slightly worse than the previous cases. However, the effect due to one fixed baseline reduces to 2 cm after 12 hours and converges to 1.7 cm after 18 hours. The effect due to the data noise will continue to decrease for longer data arc; but the baseline effect will remain unchanged. When perfect EOP are assumed, the geocenter offset error after 34 hours is found to be 4.1 cm. This slight improvement is due to reduced data noise effect when fewer parameters are estimated. The analysis of Strategy 1 has indicated that a constant bias for polar motion and (UT1–UTC) rate can be included in the filter as additional adjusted parameter without significantly degrading the performance. However, GPS measurements are insensitive to any constant (UT1–UTC) bias error.

In Strategy 2, no tracking site coordinates, except the longitude of the site at Goldstone, were held fixed. Here, as before, simultaneous adjustment of all GPS satellite states, tracking site coordinates, carrier phase biases and zenith tropospheric corrections

Table 4

Covariance analysis results on the geocenter offset error

	Case		Error due to	Arc Lengths (hrs.) 6	12	18	24	30	34
Strategy 1	A	2 Baselines Fixed	Data Noise (cm.)	193.3	12.4	6.0	4.2	3.7	3.4
			Baselines (cm.)	8.0	2.5	2.0	2.2	2.2	2.2
			RSS (cm.)	193.4	12.6	6.3	4.7	4.3	4.0
	B	2 Baselines Constrained	Data Noise* (cm.)	193.4	12.6	6.2	4.6	4.0	3.8
	C	1 Baseline Fixed	Data Noise (cm.)	193.5	12.9	6.5	4.8	4.2	4.0
			Baselines (cm.)	7.6	2.0	1.6	1.8	1.7	1.7
			RSS (cm.)	193.5	13.1	6.7	5.1	4.6	4.4
Strategy 2			Data Noise* (cm.)	143.7	8.0	4.0	2.7	2.3	2.1

* The only error involved in these cases are those due to Data Noise

were carried out for various arc lengths ranging between 6 and 34 hours. The errors affecting the origin offset from the geocenter in this strategy are the data noise and the GM of Earth, which defines the absolute scaling. At the end of 6 hours *(Table 4)* the RMS error of the origin offset is 143.7 cm which reduces to 8 cm at the end of 12 hours. This indicates that the control on the absolute scaling and the orientation in latitude is greatly improved after all the GPS satellites have been tracked by the globally distributed sites for a complete orbit cycle. At the end of 34 hours the RMS error reduces to 2.1 cm. The results here show a strong trend of decreasing RMS error as the data arc length increases, because the origin offset determination is limited only by the data noise for this strategy. This result can be compared with Case C of Strategy 1 when EOP are not estimated; there is about 50 % improvement in the geocenter offset error with this method. The Earth's GM is known accurately enough so that its effect is of the order of 0.2 cm after 12 hours and is 0.1 cm at the end of 34 hours.

Effect of Coordinate Frame Origin Offset on Orbit Determination of A Low Earth Orbitting Satellite

To gain further insight into the significance of an accurate definition of the geocenter, the effect on the radial position of a low earth orbiting satellite, in particular Topex/Poseidon, was studied. The error assumptions used are the same as given in *Table 1* except for those parameters listed in *Table 5.* The result presented by Case A of Strategy 1 shows 4 cm error in geocenter offset *(Figure 3)* after 34 hours. Therefore, the origin offset was assumed to have an error of 4 cm in each component and left unadjusted. A reduced dynamic tracking technique [Wu et al., 1987] was

Table 5

Variations of Assumptions from Table 1

for Topex/Poseidon Orbit Determination

Data Span:	2 hours
Data Interval:	5 minutes
Topex Epoch State:	1 km; 1 m/sec (adjusted)
3-D Force on Topex:	process-noise (adjusted):
	0.50 $\mu m/s^2$ bias;
	0.35 $\mu m/s^2$ batch to batch
Gravity:	50% of current uncertainty (20x20 Lumped)
Geocenter:	4 cm each component

implemented in the study where a fictitious 3–D force on Topex was adjusted as process noise with constrained a priori uncertainty. *Table 6* shows the error in the radial component of Topex caused by various sources. The total error in Topex altitude over the two-hour arc has an RSS value of 9.7 cm *Figure 4* shows the altitude error variation with time, along with the part contributed by a 4 cm geocenter uncertainty, over the two-hour arc. Without the refinement with GPS measurements, the geocenter position uncertainty would be greater than 10 cm, and Topex altitude determination error would be greater than 14 cm.

Table 6

Breakdown of Topex altitude determination error

Error Sources	Topex Altitude Error (cm)
Data Noise	6.1
Geocenter	3.8
Fiducial Baselines	1.6
Non-fiducial Baselines	2.9
Gravity	5.6
RSS	9.7

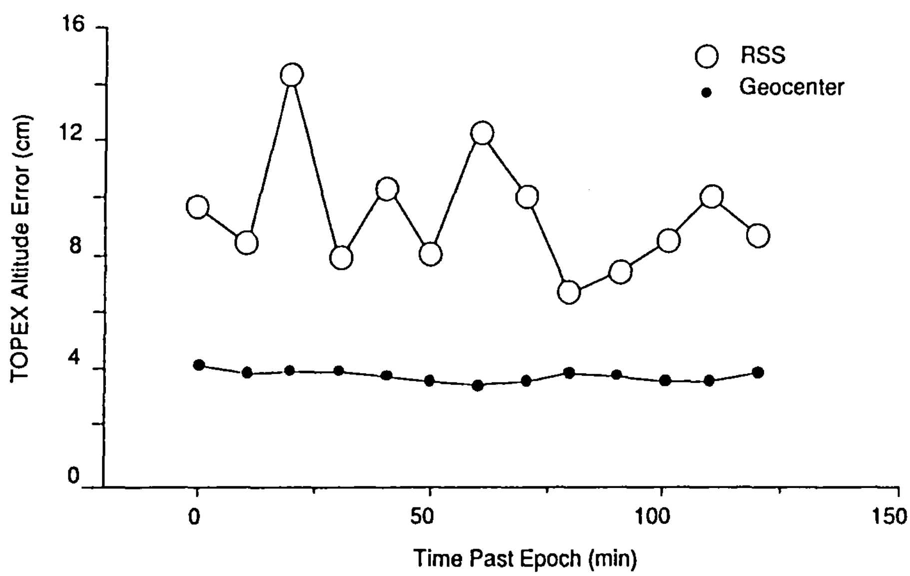

Fig. 4 — Total Topex altitude error and effects of 4-cm *geocenter error over a* 2-hr *period.*

Summary and Conclusions

A geocentric coordinate frame provides a physically meaningful and unambiguous definition of the coordinate origin. Two basic strategies for establishing a geocentric coordinate frame by GPS measurements have been investigated. All three cases of the first strategy make use of the precise relative positions which have been predetermined by VLBI to fix the frame orientation and the absolute scaling, while the offset from the geocenter is determined from GPS measurements. The reference frame thus adopted is consistent with the VLBI coordinate system. The second strategy establishes a reference frame by holding only the longitude of one of the tracking sites fixed. The absolute scaling is inferred from the adopted gravitational constant (GM) of Earth; the orientation in latitude is inferred from the time signature of earth rotation in the GPS measurements. The coordinate system thus defined is a geocentric earth fixed coordinate system. The covariance analysis has shown that geocentric positioning to an accuracy of a few centimeters can be achieved with just a one-day arc of precise GPS pseudorange and carrier phase data.

Each of the two strategies has its advantages in different applications. The first strategy should always be adopted in applications requiring a coordinate frame consistent with the VLBI reference frame. Among these applications are the monitoring of crustal motions in areas which have been investigated by VLBI observations and the determination of the earth rotation parameters, namely, polar motion and variation of (UT1−UTC). The second strategy, which holds the longitude at a reference site fixed, strictly limits itself in an ECEF frame established by the adopted values for the fixed longitude and the GM of Earth, and by GPS measurements. This method provides a superior result as long as the precise applications are within the same ECEF frame. Applications in which such an ECEF coordinate frame can be adopted include datum definition and network densification in an area where ECEF coordinates are appropriate. Various topographic and oceanographic surveys, and prospecting surveys can benefit from its simplicity. In Topex/Poseidon orbit determination this method can also be very convenient if a CTS frame such as WGS 84 is adopted.

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APPENDIX

Measurement Partial Derivatives with Respect to Baseline Components

Let the Cartesian coordinates of the set of N tracking sites be (x_1, y_1, z_1), $(x_2, y_2, z_2), \ldots, (x_N, y_N, z_N)$. We can form the following baseline vector

$$\mathbf{B}_i = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad ; \; i = 2, 3, \ldots, N \tag{A.1}$$

where site 1 has been selected as the reference site with which all baselines are formed. For completeness, we also define

$$\mathbf{B}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \tag{A.2}$$

for the reference site. Re-arranging the above equations

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{cases} \mathbf{B}_i & ; \; i = 1 \\ \mathbf{B}_i + \mathbf{B}_1 & ; \; i = 2, 3, \ldots, N \end{cases} \tag{A.3}$$

For simplicity but without loss of generality, partial derivatives with respect to only the x-component of baselines will be derived. The relation for the other two components follows directly.

$$\frac{\partial x_i}{\partial b_{x,j}} = \begin{cases} 1 & ; \; j = 1 \\ \delta_{ij} & ; \; j = 2, 3, \ldots, N \end{cases} \tag{A.4}$$

where δ_{ij} is the Kronecker's delta. The partial derivative of a measurement R with respect to the baseline components b_j can be expressed in terms of those with respect to the site coordinates x_j by the following chain rule :

$$\frac{\partial R}{\partial b_{x,j}} = \frac{\partial R}{\partial x_1}\frac{\partial x_1}{\partial b_{x,j}} + \frac{\partial R}{\partial x_2}\frac{\partial x_2}{\partial b_{x,j}} + \ldots + \frac{\partial R}{\partial x_N}\frac{\partial x_N}{\partial b_{x,j}} \tag{A.5}$$

which, with the substitution of (A.4), becomes

$$\frac{\partial R}{\partial b_{x,j}} = \begin{cases} \sum\limits_{n=1}^{N} \dfrac{\partial R}{\partial x_n}, & j = 1 \\ \\ \dfrac{\partial R}{\partial x_j}, & j = 2, 3, \ldots, N \end{cases} \tag{A.6}$$

Hence, the partial derivative of the measurement with respect to a Cartesian component of a baseline is the same as that with respect to the same component of the non-reference site forming the baseline; and the partial derivative with respect to a component of the reference site is the sum of all partial derivatives with respect to the same component of all sites forming the baselines.

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