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FEATURE

The invisibility of length contraction

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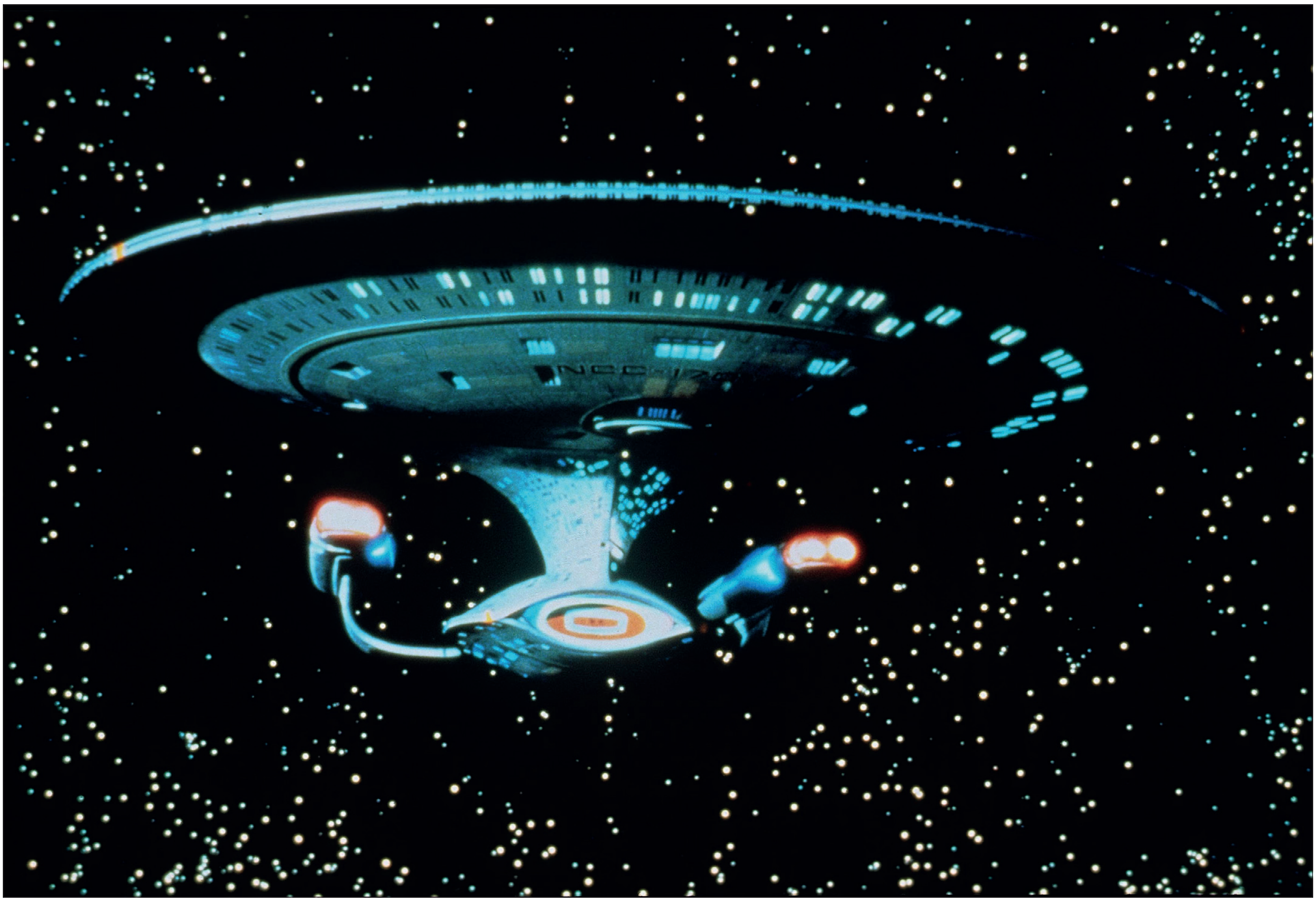

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The invisibility of length contraction

The idea that objects contract in length when they travel near the speed of light is a widely accepted consequence of Einstein's special relativity. But if you could observe such an object, it wouldn't look shorter at all – bizarrely, it would seem to have been rotated, as **David Appell** explains

If the Starship *Enterprise* dipped into the Earth's atmosphere at a sub-warp speed, would we see it? And if the craft were visible, would it look like the object we're familiar with from TV, with its saucer section and two nacelles? Well, if the *Enterprise* were travelling fast enough, then – bright physicists that we are – we'd expect the craft to experience the length contraction dictated by special relativity.

According to this famous principle, a body moving relative to an observer will appear slightly shorter in the direction the body's travelling in. Specifically, its observed length will have been reduced by the Lorentz factor $(1-v^2/c^2)^{1/2}$, where v is the relative velocity of the moving object and c is the speed of light in a vacuum. However, the *Enterprise* won't be seen as shorter despite zipping along so fast. In fact, it will appear to be the same length, but rotated.

You might not have heard of this phenomenon before, but it's often called the "Terrell effect" or "Terrell rotation". It's named after James Terrell – a physicist at the Los Alamos National Laboratory in the US, who first came up with the idea in 1957. The apparent rotation of an object moving near the speed of light is, in essence, a consequence of the time it takes light rays to travel from various points on the moving body to an observer's eyes.

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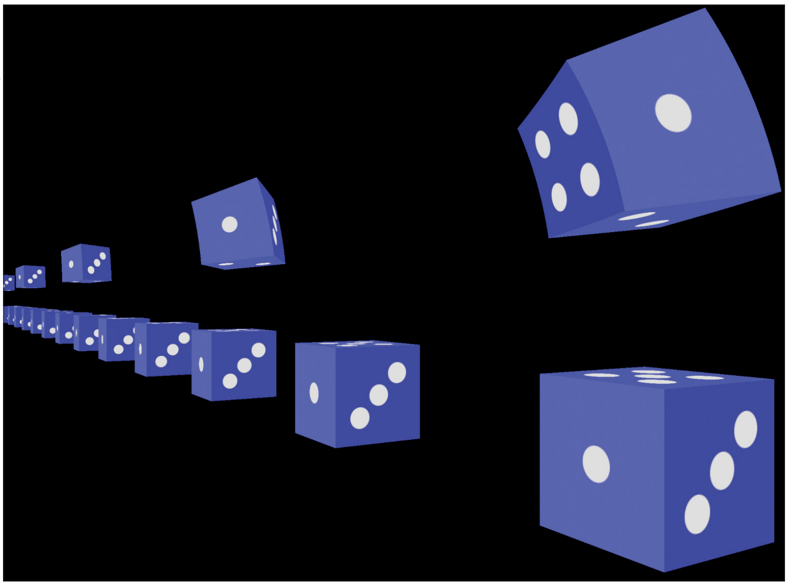


Curiouser and curiouser A row of stationary dice (bottom), with other dice moving from left to right (top) at 90% of the speed of light. All cubes, whether moving or at rest, have the same orientation. However, we cannot see the Lorentz contraction of the upper cubes, which instead are rotated. Indeed, due to the fact the speed of light is finite, we can actually see the "rear" sides of the upper cubes.

Terrell's insight was that even if those rays leave different parts of the moving body at the same time, they won't reach the observer's eyes at the same time. The net result is a distorted view. Amazingly, for an object that subtends a small solid angle, the Terrell effect will cancel out the Lorentz contraction, making the object appear to have been rotated. Indeed, it allows you to see partly around the object and observe its reverse side.

Of course, the rapidly moving *Enterprise* would also be subject to the Doppler effect, which would render it a different colour. Indeed, the frequency shift might even make the craft totally invisible to human eyes. The *Enterprise* would also appear brighter or dimmer due to relativity's "headlight effect" (which I'll come back to later). All in all, these effects will make a rapidly moving object, such as the *Enterprise*, appear nothing like it does at rest.

Now, if you think that's confusing – don't worry, you're not alone.

Length contraction: the short story

The idea of length contraction was postulated by the Irish theoretical physicist George FitzGerald in 1889 and by the Dutch theorist Hendrik Lorentz in 1892. Also known as Lorentz–Fitzgerald contraction or just Lorentz contraction, it was invoked to account for the negative outcome of Albert Michelson and Edward Morley's memorable experiment of 1887. The two Americans had famously tried – and failed – to detect the "aether", a hypothetical stationary medium that was supposed to carry electromagnetic waves, much as the surface of water carries water waves.

Keen to rescue the hypothesis of a stationary aether, Lorentz and Fitzgerald's solution to Michelson and Morley's null result was to suggest that objects travelling at a substantial fraction of the speed of light will – when viewed by another observer – appear shortened in the direction of their travel. Based on a calculation by Oliver Heaviside of how the magnetic vector potential in Maxwell's equation transformed between reference frames, their idea was, in truth, just one of a few proposed "solutions". None was completely satisfactory, however, and for many years, length contraction remained an *ad hoc* hypothesis.

It was only in 1905 that Albert Einstein cut through the confusion when he published his special theory of relativity. It did away with the aether altogether and proposed two postulates. The first is that the laws of nature are the same in all "inertial" reference frames – i.e. those frames moving at a constant velocity with respect to one another, in which any object with no net forces on it will appear at rest. The second postulate states that an observer in any inertial reference frame will measure the speed of light in a vacuum to be the same.

From his theory, Einstein derived length contraction as well as time dilation, the equivalence of mass and energy, and more. In particular, he realized that an observer at rest on a high-speed train will measure the length of a passing high-speed train as shortened in the direction of its motion, by the factor $(1–v^2/c^2)^{1/2}$ where v is the relative velocity between the observer and the other train. (Paradoxically, an observer on that train will measure the first as shortened by the same factor. Isn't special relativity fun?)

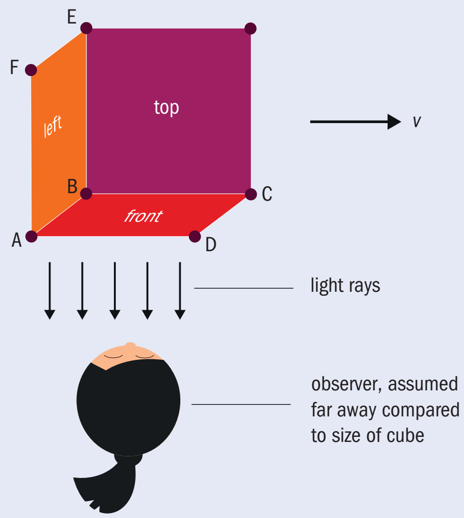
Length contraction has never been directly measured. But its effects show up in the magnetic force that acts between parallel, current-carrying wires. Bizarrely, this force, which is purely magnetostatic, appears in one wire due to length contraction as experienced by the charge carriers in the other wire's frame. (It's complicated. You can find more information in a paper by Paul van Kampen from Trinity College Dublin – *Eur. J. Phys* **29** 879). Current-carrying wires aside, there is little doubt that Lorentz contraction occurs if length is measured – that is, when different points on a moving object are all measured at the same instant of time in the observer's stationary frame of reference.

But when you view an object with your eyes or a camera, it's a different story. You're then recording the photons the object emits – and these photons arrive at your eyeball or camera lens at the same time. What both Einstein and Lorentz overlooked, however, is the fact that the photons may have been emitted by the object at a *different* time, especially if the object is large. In fact, in 1922 Lorentz erroneously claimed, in the Dutch version of his *Lectures of Theoretical Physics*, that the contraction could be photographed.

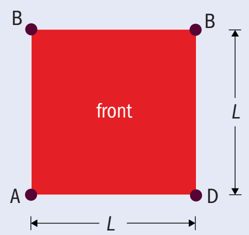
Lost in history

Over the years, few people paid much attention to observing Lorentz contraction. Anton Lampa, an Austrian physicist who had once helped Einstein get his first university professorship, did publish a paper on this topic in 1924 (*Zeitschrift für Physik* **27** 138). Concerning the appearance of a moving rod to an observer, his work was unfortunately largely overlooked. Indeed, in his famous 1940 children's book

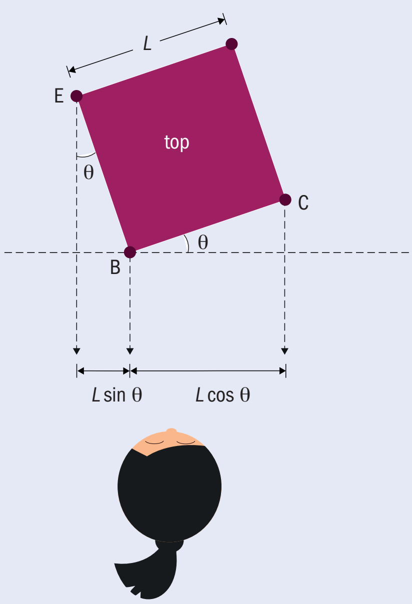
1 Why Lorentz contraction disappears for a cube moving at near the speed of light

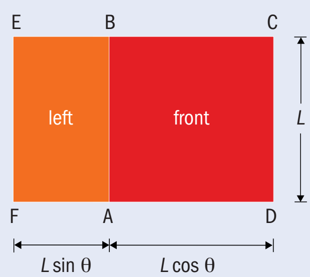


(*a*) To understand why an object appears rotated, not contracted, when moving at relativistic speeds, imagine you're looking at a cube of length *L* moving from left to right. You, the observer, are standing far enough away, compared to the cube's dimensions, that the solid angle subtended by the cube is small and light rays from it are essentially parallel.

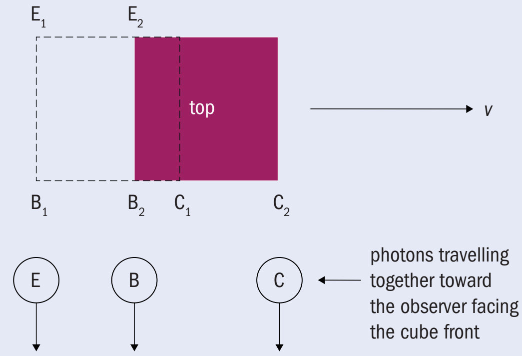


(*b*) If the cube is moving much more slowly than the speed of light, all you'd see – when the object's directly in front of you – would be the cube's "front" side.

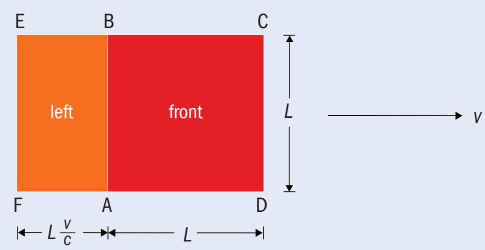




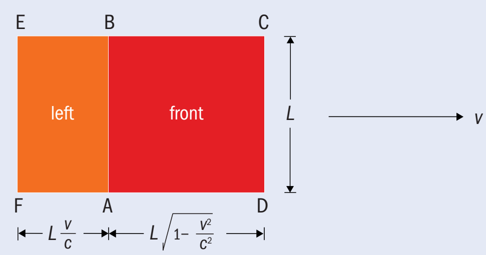
(*c*) Now consider a cube at rest or moving slowly, but rotated around an axis through the top and bottom by some angle relative to the line of sight. For an observer looking head-on, two sides would now be visible but not seen to be of equal area.



(*d*) If an unrotated cube is moving at a substantial fraction of the speed of light, but without considering special relativity, there is no length contraction in any direction. Shown here is the top of the cube at successive times, with the dashed square showing the position at an initial time and the solid square its position after travelling from B_1 to B_2. As the speed of light is finite, photons emitted from E_2 will arrive after those emitted from B_2; the eye will not see them as simultaneous. But "retarded" photons from E_1 *will* be seen at the same time as those from B_2 and C_2 when the time for a photon to travel from E_1 to B_1 is the same as the time for the cube to travel from B_1 to B_2 – that is, when $L/c = b/v$, where *b* is the distance from B_1 to B_2. (Again, we're assuming the observer is far enough away so that the light rays are essentially parallel.)

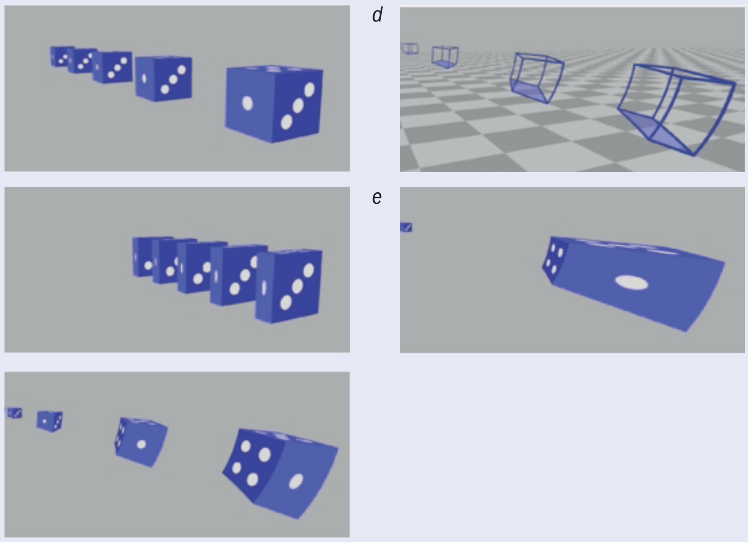


(*e*) Continuing from above, but still not considering special relativity, the observer will not only see the "front" of the cube but, due to the "retarded" photons, also its "left" side, with its length *L* reduced to $L(v/c)$. The cube will appear distorted – though not rotated, since the "front" is not foreshortened and still has a length *L*.



(*f*) Finally, let's consider both photon retardation and relativistic effects, as we must in the real world. Moving at near the speed of light, the "front" of the cube is foreshortened and appears to have a length $L(1-v^2/c^2)^{1/2}$. Compared to part *b*, the cube appears the same as one of length *L* but rotated by an angle θ, where (as in part *c*) $\cos\theta = L(1-v^2/c^2)^{1/2}/L$ i.e. $\theta = \arcsin(v/c)$. The moving cube will thus look the same as an undistorted, rotated, non-relativistic cube of length *L*. The Lorentz contraction has amazingly "disappeared".

2 Length contraction



U Kraus 2008 *Eur. J. Phys.* **29** 1

(*a*) A row of dice at rest moving from left to right in a single file at 95% of the speed of light. (*b*) The moving dice are length contracted, so that one might (wrongly) expect them to look as here. (*c*) If you actually observe the dice, however, they will appear rotated. (*d*) But when some perception in depth is provided, you'd see them as sheared rather than rotated. (*e*) Shown here is the predicted "classical" appearance of the dice, with no length contraction. You can view a short film of part *c* online at bit.ly/2xDc55j.

To fully understand the appearance of a rapidly moving object, you have to calculate the geometry not only for arbitrary viewing angles but also for arbitrary distances, velocities and sizes

Mr Tompkins in Wonderland, the Russian-born physicist George Gamow got length contraction all wrong. He showed bicycles as simply shortened in the direction they are travelling, instead of being distorted and elongated when approaching an observer, or contracting as they receded into the distance.

Researchers only properly started to take notice of the practicalities of observing Lorentz contraction when Terrell published an internal Los Alamos article in 1957 about the effect, followed two years later by a paper in the *Physical Review* (**116** 4). His work was spotted by the theorist Victor Weisskopf, who, while serving as president of the American Physical Society, wrote an article for *Physics Today* presenting Terrell's findings in a simpler form (September 1960 p24).

Like Terrell, Weisskopf also noted an earlier 1959 article by the British mathematical physicist Roger Penrose, in which he analysed the appearance of a relativistically moving sphere. In his 1905 paper on special relativity, Einstein had said that such a sphere would look like an ellipsoid, contracted in the direction of motion. But Penrose reckoned that the object would still be spherical, albeit rotated. Indeed, thanks to Penrose's insights, the Terrell effect is sometimes referred to as the "Penrose–Terrell effect" (though the two worked independently).

Terrell's and Penrose's simple insight – overlooked by nearly every physicist before them – was that light rays that simultaneously leave a moving object do not necessarily strike the eye or camera film at the same time. The eye or lens sees images from photons that strike it simultaneously, but these photons – especially for a rapidly moving object – do not leave the object simultaneously. Surprisingly enough, in certain circumstances this difference exactly cancels the Lorentz length contraction and the moving object appears rotated.

Rotation, not contraction

To appreciate why Lorentz contraction disappears for an object moving at relativistic speeds, Peter Signell from Michigan State University has considered the simple case of a cube moving left to right when viewed head-on (figure 1). Amazingly, the "front" of the cube nearly disappears – and the observer can see almost the entire "left" side. Indeed, it's easy to extend this thinking to other shapes (such as Penrose did with spheres) and to other viewing angles.

Of course, to fully understand the appearance of a rapidly moving object, you have to calculate the geometry not only for arbitrary viewing angles but also for arbitrary distances, velocities and sizes (i.e. subtended angles). You also have to include two purely relativistic effects: the relativistic Doppler effect and relativistic aberration – the "headlight" effect mentioned earlier whereby light emitted isotropically by a moving object is seen by a stationary viewer as bunched in the direction of motion.

Do all that and a "small" object moving from left to right will appear to change colour from high-frequency blues to low-frequency reds – and may even be invisible if its apparent colours are Doppler-shifted outside the visual range of your eye or recording medium. The object will also become dimmer, while its apparent angle of rotation will change from near zero far away to nearly 90°. And because the left-hand side of the object becomes visible, the object will appear to be receding before it has even reached the observer.

As for a sphere, it will – as Penrose showed – retain a circular outline no matter what its speed and subtended solid angle, albeit rotated and partly distorted. It will also undergo all the colour and intensity changes noted above. The bottom line is that the observed shape of any object will not depend on its Lorentz transformation. Yes, length contraction occurs – and can be detected by careful measurement. But if you try to observe length contraction, you can't because your view of it is compensated for by the finite speed of light.

Cool findings

Over the years, only a few hardy researchers have studied the geometrical appearance of large objects moving at relativistic speeds. In 1961 Mary Boas of DePaul University in Chicago showed that straight lines will appear curved (*Am. J. Phys.* **29** 283), while four years later David Scott and M R Viner of the University of Toronto showed that extended objects – those not meeting Terrell's assumption of a small subtended angle – are distorted in shape too (*Am. J. Phys.* **33** 534).

"Their grossly altered appearance," Scott and Viner wrote, "might be described as a non-uniform shear deformation in each of the two perpendicular planes through the line of motion." Weirdly, the non-central parts of such bigger objects will, at larger subtended angles, appear rotated and behind the centre along a hyperbola.

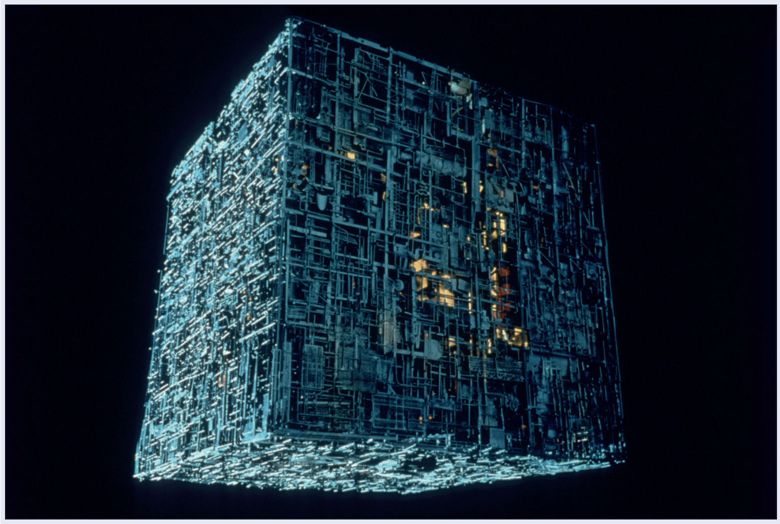
Strange things also happen to the apparent speed of a relativistically moving object. In 2005 Robert Deissler, who is now at Case Western Reserve University in Ohio, showed that an object with a measured speed v will appear to be travelling faster than it actually is (*Am. J. Phys.* **73** 663). That's because the object's apparent position is always behind the actual position as light takes a finite amount of time to travel to the eye or lens.

According to Deissler's calculations, the apparent velocity of a distant object approaching an observer is $v/(1-v/c)$, which means that if $v \geq c/2$ the object will appear to be moving faster than the speed of light (you can do the sum to check). And if the object's speed starts getting close to c, it'll seem to be travelling infinitely fast. Indeed, this effect, which is purely geometrical, has been observed in some stars and galaxies that seem to be moving faster than the speed of light.

Considering all these effects – geometrical plus relativistic – is a challenge, but one rendered easier by computer graphics, as described by Zachary Sherin, Ryan Cheu and Philip Tan from the Game Lab at the Massachusetts Institute of Technology and Gerd Kortemeyer from Michigan State University in 2016 (*Am. J. Phys.* **84** 369). Many such graphics and videos have also been collected at the website spacetimetravel.org, a translation of a German website created by Ute Kraus and Corvin Zahn from the University of Hildesheim in Germany (figure 2). (Kraus has written more about her work in "First-person visualizations of the special and general theories of relativity" *Eur. J. Phys.* **29** 1).

Some six decades after Penrose and Terrell's publications, the Terrell effect is still not widely known – Terrell died in 2009 after a lifetime at Los Alamos – and many textbooks and science presenters still get length contraction wrong. But when you realize what Penrose and Terrell had to say, you'll surely wonder why you never thought about Lorentz contraction in this way before. As the German philosopher Arthur Schopenhauer once wrote, in a remark that's often erroneously attributed to Erwin Schrödinger: "The task is…not so much to see what no-one has yet seen; but to think what nobody has yet thought, about that which everybody sees."

High-speed vision



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What's the fastest speed we can see with the human eye? One way to answer this question has been suggested by Jack Singal, a physicist at the University of Richmond in Virginia. He imagined an object – a Borg spaceship from *Star Trek* if you like (shown above) – moving at speed v, passing linearly across one's field of vision, visible if it were at rest, and directly overhead. For simplicity, he ignored any potential distortion from the atmosphere and assumed the ship is bright enough to see.

Now if A is the angle (in radians) over which the human eye can focus, T is the neurological "refresh rate" of human visual perception, and D is the closest distance between you and the object, then according to simple geometry, and ignoring any unfocused peripheral vision, $v = DA/T$. Given that $A \approx 1°$ (0.017 radians) and $T \approx 0.001$ s, then if the Borg ship is 100 km away, skimming the very top of the Earth's atmosphere, the fastest visible speed will be 1750 km/s or roughly 0.6% of the speed of light. (Such an object would take only four minutes to get from Earth to the Moon, and its purely relativistic length contraction would be, for a 1000 m-long ship, only 2 cm.)

However, that limiting observational speed may be too high. Jordan DeLong, a psychologist who is a data-science director of the Los Angeles market-research firm Research Narrative, thinks the situation is more complex. "The eye isn't uniformly sensitive like a digital camera," says DeLong, who has carried out research on vision especially as it relates to film. "The periphery is better at detecting motion, but not as sensitive to detail and colour." And brighter visual stimuli are transmitted more quickly to the brain.

DeLong points to a classic study from 1985, in which a team led by Craig Meyer, now of the University of Virginia, immobilized the heads of five male subjects, who were asked to track a moving spot as closely as possible. Using a device to monitor their eye movements, Meyer found that – in this near-perfect set-up – their eyes could track a spot to an upper limit of about 90° per second (*Vision Research* **25** 561). "That could be a small object really close to the eye that's moving slowly, or a large thing in the distance moving more quickly," says DeLong. The equation above yields a maximum detectable velocity at a distance of 100 km of only about 160 000 km/s, or 0.05% of the speed of light.