TRANSACTIONS

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

VOL. IX. NEW YORK CITY, JANUARY, 1892. No. 1.

REGULAR MEETING, JAN. 19th, 1892.

The meeting was called to order at 8.15 P. M. by Vice-President Thomas D. Lockwood.

The Secretary read the following list of Associate Members elected by Council, January 19th :

Name.	Address.	Endorsed by
BARBERIE, E. T.	Electrician, Safety Insulated Wire Co., 234 W. 29th St., New York City.	Wm. Maver, Jr. Chas. Cuttriss. G. A. Hamilton.
DESMOND, JERE. A.	Supt. and Electrician, Kingston Electric Light and Power Co. Kingston, N. Y.	Chas. J. Bogue. Rob't. J. Sheehy. H. A. Foster.
GRUNOW, WILLIAM JR.	Expert Mechanician and Manufacturer of Special Machinery and Instruments, 204 and 206 East 43d St., New York.	M. I. Pupin. Francis B. Crocker. W. H. Freedman.
INRIG, ALEC GAVAN	Rue St. Gommaire, 23, Antwerp Belgium.	T. C. Martin. Joseph Wetzler. Ralph W. Pope.
MCCARTHY, LAWRENCE A.	Western Union Telegraph Co., New York City, 1053 Bedford Ave., Brooklyn, N. Y.	Alfred S. Brown. Geo. H. Stockbridge. Wm. Maver, Jr.
MACFARLANE, ALEXANDER	Professor of Physics, University of Texas, Austin, Texas.	E. L. Nichols. H. J. Ryan. Ernest Merritt.
MOLERA, E. J.	Civil Engineer, 40 California St., San Francisco, Cal.	T. C. Martin. Joseph Wetzler. Ralph W. Pope.
PAGE, A. D.	Assistant Manager, Edison General Electric Co. Lamp Works, Harrison, N. J.	F. R. Upton. John W. Howell. H. Ward Leonard
READ, ROBERT H.	Patent Attorney, with Electrical Review, 13 Park Row, New York City.	S. S. Wheeler. Chas. S. Bradley. Ralph W. Pope.

WEBSTER, DR. ARTHUR G.	Docent in Physics, Clark University, Worcester, Mass.	M. I. Pupin. F. B. Crocker. Louis Bell.
WILLIAMS, WILLIAM PLUMB	Electrical Engineer, Nicholson Electric Hoisting Company, Box 147, Cleveland, Ohio.	T. C. Martin. G. M. Phelps. Franklin L. Pope.
WILSON, HARRY C.	Supt. of P. O. Telegraph, with the Government, Kingston, Jamaica, West Indies.	T. C. Martin. Nikola Tesla. Thos. D. Lockwood.

Total, 12.

THE CHAIRMAN:—[Vice-President Lockwood.] The Institute has every reason to congratulate itself on the accessions to its membership which it is now receiving. It is a matter to be lamented that the weather, which may be properly characterized by the same description that Shakespeare gave to the late lamented Cleopatra, namely that "Age cannot wither, nor custom stale, its infinite variety," has prevented a large audience at the beginning of our proceedings. But what we lack in quantity we must make up in intensity of hearing—if you will pardon the use of the old terms. The subject that we have to-night before us, and which you will find so ably dealt with by Mr. Steinmetz, relates to that phenomenon of molecular friction, which Mr. Ewing has denominated "hysteresis." Mr. Ewing, as we all know, has made the subject so peculiarly his own, that one might at first suppose there was nothing new to be known about it; but I am confident that after this paper is read, those of us who read it with Mr. Steinmetz will find that there is something new under the sun. We will now hear Mr. Steinmetz's paper.

A paper read at the sixty-third meeting of the American Institute of Electrical Engineers, New York, January 19th, 1892. Vice-President Lockwood in the Chair.

ON THE LAW OF HYSTERESIS.

BY CHAS. PROTEUS STEINMETZ.

In the number 137, of December 17th, 1890, of the *Electrical Engineer* I published a short article under the title "Note on the Law of Hysteresis," where I showed that in a set of determinations of the loss of energy due to hysteresis by reversals of magnetism, for different magnetizations, made by Ewing, this loss of energy due to hysteresis can fairly well be expressed by the equation:

$$H = \eta B^{1.6},$$

where H is the energy consumed by hysteresis during one magnetic cycle, in ergs per cubic centimetre, B the magnetization in lines of magnetic force per square centimetre, and η [1] a numerical coefficient, in this case $= .002$.

Considering that even the simple law of magnetism—that is, the dependence of the magnetization B upon the magneto-motive force F (for instance, in ampere turns per centimetre length of the magnetic circuit) has until now defied all attempts of mathematical formulation, it appeared a strange feature that the apparently much more intricate phenomenon of hysteresis, or rather of the consumption of energy by hysteresis, should yield to analyti-

1. If any quantity has a right to be called "magnetic resistance," it is this coefficient η; for η is the *coefficient of conversion of magnetic energy into heat*, while as "electric resistance" we define the *coefficient of conversion of electric energy into heat.*

The term generally denoted "magnetic resistance"—that is, the inverse value of magnetic conductivity, does not deserve this name at all, but is more properly called "reluctance."

cal formulation in such a simple way, to be directly proportional to the 1.6th power of the magnetization. At the same time the coincidence of Ewing's tests with the curve of the 1.6th power was near enough to be considered as something more than a mere incident, but at least as a clue to a law of hysteresis, the more as this law held not only for low and medium magnetization, but even for very high saturation, without showing any kink at that point where the magnetic characteristic goes over the bend or "knee" and thereby entirely changes its shape, nor any marked tendency of deviation of the extremest observed values from the calculated curve.

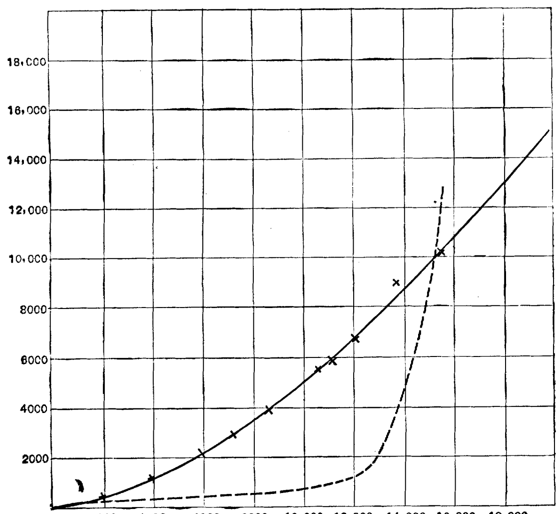


Fig. 1.

In Fig. 1 and Table I, I give from the article referred to, the calculated curve of hysteretic loss, as a drawn line, with Ewing's tests marked as crosses, and in dotted line the curve of magnetomotive force *F*, corresponding to the different magnetizations, as abscissæ.

In the table, I:

F = the M. M. F., in absolute units,

B = the magnetization, in lines of magnetic force per square centimetre,

H_{obs} = the observed values, and

H_{calc} = the calculated values of hysteretic loss, in ergs per cubic centimetre,

$H_{calc} - H_{obs}$ = the difference between both, in ergs and in percentages.

TABLE I.

F:	B:	H_{obs}:	H_{calc}:	$H_{calc} - H_{obs}$: =	%:
1.50	1,974	410	375	+ 35	+ 8.5
1.95	3,830	1160	1082	+ 58	+ 5.0
2.56	5,950	2190	2190	......	......
3.01	7,180	2940	2956	− 16	− .5
3.76	8,790	3990	4080	− 90	− 2.3
4.96	10,590	5560	5510	+ 50	+ .9
6.62	11,480	6160	6260	−100	− 1.7
7.04	11,960	6590	6690	−100	− 1.5
26.5	13,700	8690	8310	+380	+ 4.4
75.2	15,560	10,040	10,190	−150	− 1.5
				Av: ± 98 =	± 2.6

To study more completely this phenomenon of hysteresis and of the energy consumption caused thereby, I endeavored to make a number of determinations with different magnetic circuits and at different magnetizations.

To be enabled to carry out these experiments, I am highly obliged to Mr. Rudolph Eickemeyer, of Yonkers, N. Y., who, being greatly interested in the laws of the magnetic circuit and having done considerable work himself in this branch of electrical science, not only put the large facilities of his well-known factory at my disposal, but also guided the experiments with his valuable advice. A part of the instruments used in the tests are of Mr. Eickemeyer's invention and covered by his patents.

To be able to deal not only with the small amounts of energy which the reversal of magnetism in a tiny bit of iron wire sends through the ballistic galvanometer, but to reduce the determinations to readings of considerable power-values, and where a much greater exactness can be reached, and at the same time to determine the dependence of the hysteretic loss of energy upon the velocity of the magnetic cycles, I decided to use alternating currents, at least as far as this could be done, whereby the determination of the energy consumed by hysteresis is reduced to a simultaneous wattmeter, voltmeter, ammeter and speed reading.

At the same time this electro-dynamometer method has the advantage that the magnetic cycle is completed in a steady, continuous motion, while in the ballistic method the magnetic cycle is completed by sudden changes in the magnetization, which jumps from point to point, to enable the production of the induced current. This feature introduces an error into the ballistic method, for if a magnetic cycle is gone through by sudden changes, a larger amount of energy may be consumed than if the magnetization varies steadily in harmonic vibration.

Suppose, around a magnetic circuit, an alternating current of N complete periods per second is sent in n convolutions.

Let $C =$ the effective strength of the current,

$E =$ the effective E. M. F. induced in the circuit by self-induction, after subtracting the E. M. F.'s induced by the self-induction of the instruments,

$W =$ the energy consumed in the circuit, after subtracting the energy consumed by the electric resistance,

Then, l being the length and s the cross-section of the magnetic circuit, all in centimetres, amperes, volts, watts, etc.,

Let $B =$ the maximum magnetization in lines of magnetic force per square centimetre,

$H =$ the loss of energy by hysteresis, in ergs per cycle and cubic centimetre; it is

$$W = l\, s\, N\, H \times 10^{-7}$$

hence

$$H = \frac{W}{l\, s\, N} \times 10^{+7}$$

the hysteretic loss of energy, and

$$E = \sqrt{2}\, \pi\, s\, B\, N\, n \times 10^{-8}$$

hence

$$B = \frac{E \times 10^{+8}}{\sqrt{2}\, \pi\, s\, N\, n} \quad (^1)$$

the maximum magnetism.

For higher frequencies, 80 to 200 periods per second, the alternating current was derived from a 1 H. P. 50 volt Westinghouse dynamo. This was driven by a 3 H. P. Eickemeyer continuous current motor. By varying the excitation of the motor field and

1. This formula holds rigidly only for the sine-wave, but as shown in the following, the currents used in the tests were at least very near sine-waves. Besides, a deviation from the sine shape would not alter the results at all, but only slightly change the coefficient η.

varying the E. M. F. supplied to the motor, the speed and therefore the frequency of the alternating current could be varied in wide limits. At the same time, supplied with constant E. M. F. and like all the Eickemeyer motors of unusually small armature reaction, this electromotor kept almost absolutely constant speed under varying load, the more as it never ran with full load.

For low frequencies, this bipolar continuous current motor was used as a bipolar alternating dynamo, as shown in a patent of Mr. Stephen D. Field. On the continuous current commutator two sliding rings were mounted and connected with opposite commutator bars. In the ordinary continuous current brushes a continuous current was sent in, which set the machine in motion as an electromotor, while from the sliding rings by two separate brushes, alternating currents were taken off. By varying the E. M. F. supplied to the motor, the E. M. F. of the alternating current was varied, while a variation of the motor field gave the variations of the frequency. The curve of E. M. F. was very nearly a sine-wave, the ratio of maximum E. M. F. to effective E. M. F. found = 1.415, while the sine-wave requires 1.414—that is, essentially the same.

To determine whether the change of the shape of the alternating current by varying load and varying excitation had any influence upon the readings, the variations of the alternating E. M. F. were produced:

1. By varying the excitation of the field of the Westinghouse dynamo.
2. By running the Westinghouse dynamo fully excited, feeding the secondaries of a bank of converters, feeding from the fine wire coils of these converters the fine wire coils of another bank of converters, and taking current off from the secondaries of these converters, connected from one to six in series.
3. By changing the E. M. F. by means of a Westinghouse converter of variable ratio of transformation.
4. By loading the dynamo when small currents were used for the tests.

But after having found that all these different ways of varying the alternating E. M. F. gave no perceptible difference whatever in the readings, I afterwards used the most convenient way to vary the excitation of the dynamo field and, where higher E. M.

F's were needed, to increase the E. M. F. by an interchangeable converter, which gave the ratios: 1: 1, 2, 3, 4, 5.

For the determination of the frequency, a direct-reading speed indicator (horizontal ball governor, acting upon a spring) was used, which was carefully calibrated.

For the electric readings, instruments of the electro-dynamometer type were used, zero-reading—that is, the movable coil was carried back by the torsion of a steel spring to zero position.

These instruments were specially built for alternating currents, with very low self-induction and low internal resistance, using bifilar german silver wire as additional resistance.

In the ammeter the range of readings was from 3 to 40 amperes, the internal resistance = .011 ω.

The normal inductance (that is, E. M. F. of self-induction induced by one ampere alternating current, flowing through the instrument with a frequency of 100 complete periods per second): = .045 ω.

In the voltmeter the range of readings was from .5 volts upwards, but to avoid the necessity of corrections for self-induction sufficient additional resistance was used to decrease the correction under 1 per cent., and then the lowest readings were from 3 to 6 volts.

The internal resistance of the voltmeter is = 2.5 ω, its normal inductanee = 4.12 ω.

In the wattmeter the resistance of the coarse wire coil (fixed coil) was = .026 ω, its normal inductance = .073 ω.

The internal resistance of the fine wire coil was = .25 ω, its normal inductance = .33 ω.

In most of the readings sufficient additional resistance was used to make the correction for self-induction of the fine wire coil negligible. Only in a few readings where it exceeded 1 per cent. it was taken in account.

For small currents an Eickemeyer ammeter was used, which, while reading from .7 to 3 amperes, though built originally for continuous currents, had already been used by me for alternating currents and had been checked for its constancy of readings several times, and always found to give no perceptible difference in its readings for continuous currents and for alternating currents up to over 200 complete periods per second, the highest frequency I could reach.

Its internal resistance is $= 1.1\ \omega$, its normal inductance $= 2.03\ \omega$.

Several sets of readings for different frequencies were taken on an old Westinghouse voltmeter converter. The fine wire coil and one of the 50 volt coils were left open. Into the other coarse wire coil an alternating current was sent, in series to ammeter and coarse wire coil of wattmeter, while the voltmeter and the fine wire coil of the wattmeter were connected in shunt around the whole circuit.

Hence a correction had to be applied for the self-induction of ammeter and coarse wire coil of the wattmeter and for the resistance of the circuit. Only in very few readings this correction amounted to somewhat more than 10 per cent. Generally it was much smaller.

The instruments were calibrated several times and their constants found to remain constant.

The speed indicator was calibrated carefully and its corrections added.

Each reading consisted of an ammeter reading, a voltmeter reading, a wattmeter reading and a speed reading.

Before and after each set of readings the zero positions of the instruments were determined, and only those sets of readings used where the zero position had remained constant.

Before and after each set of alternating current readings a continuous current was sent into the circuit and a few readings for different currents taken. Voltmeter and ammeter readings combined gave the resistance of the circuit, and both combined with the wattmeter reading gave a check for the instruments, here being watts = volts × amperes. Only those sets were used again where an entire agreement was found, and with the alternating current first readings with small currents, then with large currents, and then again with small currents taken, so that I believe every possible care was exercised to avoid any errors in the tests.

As before said, the first sets of tests were made on the magnetic circuit of a small Westinghouse converter.

The constants of this converter, so far as they are of interest here, are :

Mean length of magnetic circuit, 21 cm.

Mean cross-section of magnetic circuit, $= 43.67\ \text{cm.}^2$

Hence volume of iron, $= 917.\ \text{cm}^3$.

Resistance of secondary coil, $= .2\ \omega$.

Further sets of readings were taken on a magnetic circuit, built up of very thin sheets of iron, alternately 8 in. × 1 in. and 3 in. × 1 in., in rectangular shape, very carefully insulated against eddy currents with layers of thin paper between the sheets. On the two long sides two coils of each 50 turns, very coarse wire (3 No. 10 in parallel), were wound and connected in series, thereby giving $n = 100$ turns of an internal resistance of .048 ω.

Here the mean length of the magnetic circuit was $l = 41$ cm.

The cross-section, $s = 3.784$ cm.^{2}

The circuit consisted of 58 layers of sheet-iron of the thickness $s = .02577$ (1) and the width $w = 2.579$.

The whole volume of iron was = 155 cm.^{3}

The sheet-iron pieces were first freed from scales by dipping into dilute sulphuric acid.

In one set of tests an open magnetic circuit was used, by leaving the short end pieces (3 in. × 1 in.) off, and using two piles each of 66 pieces (8 in. × 1 in.) of the same iron, the same pieces as used in the former closed circuit tests.

In these readings, for the determination of the hysteretic loss, only voltmeter and wattmeter, but no ammeter, were used, and the conductivity curve determined separately by voltmeter and ammeter.

The calculation of the readings was done in the following way:

After applying the corrections for self-induction of instruments, resistance and speed, the readings were reduced to lines of magnetic force per square centimetre B and consumption of energy by hysteresis per magnetic cycle H, in ergs.

Then the results were plotted on cross-section paper and if any value was found to be very much out of the curve connecting the other values, it was stricken out as evidently erroneous, not considering it worth while to determine whether it was a wrong reading of any one of the instruments or a mistake in the calculation.

Then from the other values of B and H, under the supposition that H were proportional to any power x of B:

$$H = \eta B^x$$

this exponent x was determined.

1. Calculated from the weight.

This value x will be seen always to be so near to 1.6 that 1.6 can be considered at least as first approximation to x.

Then, under the assumption

$$x = 1.6$$

hence

$$H = \eta \ B^{1.6}$$

the coefficient η was calculated, and now the equation

$$H = \eta \ B^{1.6}$$

plotted in a curve, as given in the figures, and the observed values of H drawn in and marked.

From the curve were taken the calculated values of H, corresponding to the observed values of B, the difference $H_{calc} - B_{obs}$ determined, and expressed in per cents. of H_{calc}.

These values are given in the tables and shown in the curves.

I. Magnetic Circuit of the Westinghouse Converter.

Fig. 2; Table II.

MAGNETIC CHARACTERISTIC.

$F.$ = M. M. F., in ampere turns per centimetre length of magnetic circuit.

$B.$ = Magnetization, in lines of magnetic force per square centimetre.

TABLE II. (1)

F.	*B.*	*F.*	*B.*	*F.*	*B.*
2	1500	12	14,750	45	18,150
3	3400	14	15,080	50	18,500
4	6800	16	15,370	55	18,820
5	9600	18	15,630	60	19,140
6	11,750	20	15,880	65	19,440
7	12,850	25	16,450	70	19,740
8	13,600	30	16,950	75	20,020
9	14,100	35	17,370	80	20,300
10	14,350	40	17,780	85	20,560
				90	20,820

HYSTERESIS.

$B.$ = Magnetization, in lines of magnetic force per square centimetre.

$H.$ = Loss of energy by hysteresis, in ergs per cycle, and cubic centimetre, $= 10^{-7}$ watt-second.

TABLE II. (2)

Frequency: $N. = 28$ complete periods per second.

B.	H. obs.	H.[1]) calc.	H.−H = calc. obs.	%.
3510	1178	1160	−18	−1.6
10,560	6286	6610	+324	+4.9
13,800	10,286	10,180	−106	−1.0
17,940	15,357	15,600	+243	+1.6
		av:	±173 =	±2.3

Exponent of power, derived from tests:

$$x = 1.6111 \sim 1.6$$

Coefficient of hysteresis:

$$\eta = .002410$$

hence, theoretical curve:

$$H = .00241\ B^{1.6}$$

TABLE II. (3)

Frequency: $N = 36$ complete periods per second.

B.	H. obs.	H. calc.	H.—H.= calc. obs.	%.
7090	3333	3500	+167	+4.8
10,250	5667	6310	+643	+10.2
13,410	9694	9700	+6	+.1
17,080	14,417	14,400	+17	+.1
19,340	16,111	17,600	+1489	+8.4
		av:	±464 =	±4.4

Exponent of power, derived from tests:

$$x = 1.6476 \sim 1.6$$

Coefficient of hysteresis:

$$\eta = .002315$$

hence, theoretical curve:

$$H = .002315\ B^{1.6}$$

TABLE II. (4)

Frequency: $N = 137$ complete periods per second:

B.	H. obs.	H. calc.	H. — H. = calc. obs.	%
4000	1490	1410	− 80	−5.7
4670	1818	1800	− 18	−1.0
5510	2358	2350	− 8	− .3
5760	2482	2520	+ 38	−1.5
5840	2540	2580	+ 40	+1.6
6690	3285	3180	− 105	−3.3
6800	3358	3290	− 68	−2.1
6860	3374	3370	− 4	− .1
12,430	8336	8610	+ 274	+3.6
13,750	10,000	10,100	+ 100	+1.0
		av:	±73.5 =	±2.0

Exponent of power, derived from tests:

$$x = 1.5887 \sim 1.6$$

Coefficient of hysteresis:

$$\eta = .002438$$

hence, theoretical curve.

$$H = .002438\ B^{1.6}$$

TABLE II. (5)

Frequency. $N = 205$ complete periods per second.

B.	*H.* obs.	*H.* calc.	*H—H.* calc—obs.	%
1790	376	400	+24	+6.0
1990	463	460	— 3	— .7
2380	585	610	+35	+5.7
2620	735	720	—15	—2.1
3060	893	920	+27	+2.9
3390	1054	1100	+46	+4.2
3660	1297	1240	—57	—4.6
3710	1288	1250	—38	—3.0
4620	1822	1800	—22	—1.2
5070	2024	2070	+46	+2.2
4990	2034	2010	—24	—1.2
5910	2693	2620	—73	—2.8
6100	2844	2750	—96	—3.5
6550	3039	3080	+41	+1.3
7290	3673	3640	—33	— .9
8050	4341	4300	—41	—1.0
8320	4410	4530	+120	+2.7
8240	4561	4460	—101	—2.2
		av :	± 47	=±2.7

Exponent of power, derived from tests:

$$x = 1.6012 \sim 1.6$$

Coefficient of hysteresis:

$$\eta = .002434$$

hence, theoretical curve.

$$H = .002434\ B^{1.6}$$

From these 4 sets of readings, we get the results:

1.	$N = 28$	4	readings: $x =$	1.6111	$\eta = .002410$
2.	36	5	"	1.6476	.002315
3.	137	10	"	1.5887	.002438
4.	205	18	"	1.6012	.002434

Therefrom we derive the average, by giving to each value as weight the number of readings, where it is based upon:

$$x = 1.60513 \sim 1.6$$

$$\eta = .0024164$$

Hence:

$$H = .0024164\ B^{1.6}$$

This curve is used for calculating the values given as H_{calc}, and is plotted in Fig. 2 in drawn line.

The observed values of H are drawn in Fig. 2:

1. For $N = 28$ with the mark ○
2. " 36 " " *
3. " 137 " " Y
4. " 205 " " +

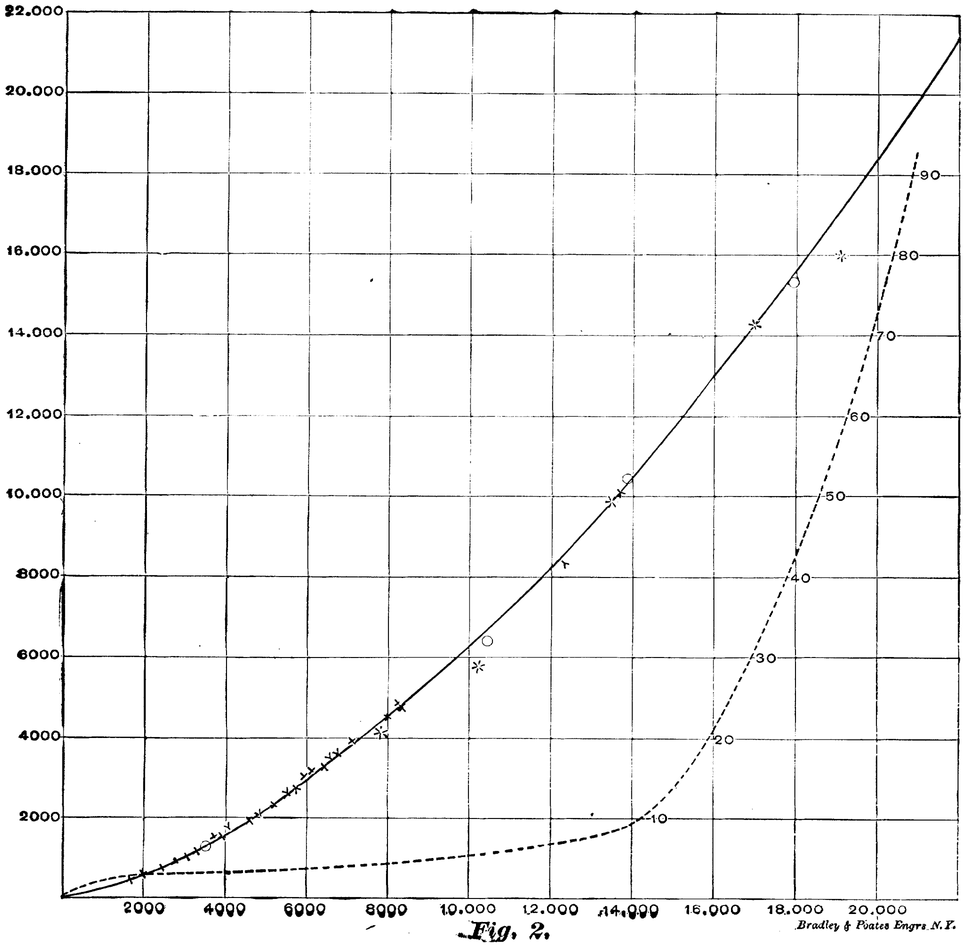


Fig. 2.

The magnetic characteristic is drawn in dotted lines.

From this curve of hysteretic loss

$$H = .0024164\, B^{1.6}$$

we derive the values:

TABLE II. (6.)

B.	H.	B.	H.
1000	152	13,000	9230
2000	462	14,000	10,400
3000	884	15,000	11,610
4000	1400	16,000	12,880
5000	2000	17,000	14,180
6000	2680	18,000	15,550
7000	3430	19,000	16,970
8000	4240	20,000	18,400
9000	5130	25,000	26,290
10,000	6070	30,000	35,210
11,000	7070	35,000	45,060
12,000	8130	40,000	55,800

II.—Magnetic Circuit Built up of Well Insulated Layers of Very Thin Sheet-Iron. Fig. 3; Tables III.

MAGNETIC CHARACTERISTIC.

F = M. M. F. in ampere turns per centimetre length of magnetic circuit.

B = magnetization in lines of magnetic force per square centimetre.

TABLE III. (1.)

F.	B.	F.	B.	F.	B.
2	1700	12	13,750	45	17,500
3	4200	14	14,260	50	17,900
4	7400	16	14,600	55	18,300
5	9200	18	14,900	60	18,650
6	10,400	20	15,200	65	19,030
7	11,160	25	15,700	70	19,380
8	11,850	30	16,200	75	19,730
9	12,470	35	16,680	80	20,080
10	13,070	40	17,050	85	20,400
				90	20,750

HYSTERESIS.

B = magnetization in lines of magnetic force per square centimetre.

H = loss of energy by hysteresis, in ergs per cycle and cubic centimetre, $= 10^{-7}$ watt-seconds.

CLOSED MAGNETIC CIRCUIT.

Frequency: N = 85 complete periods per second.

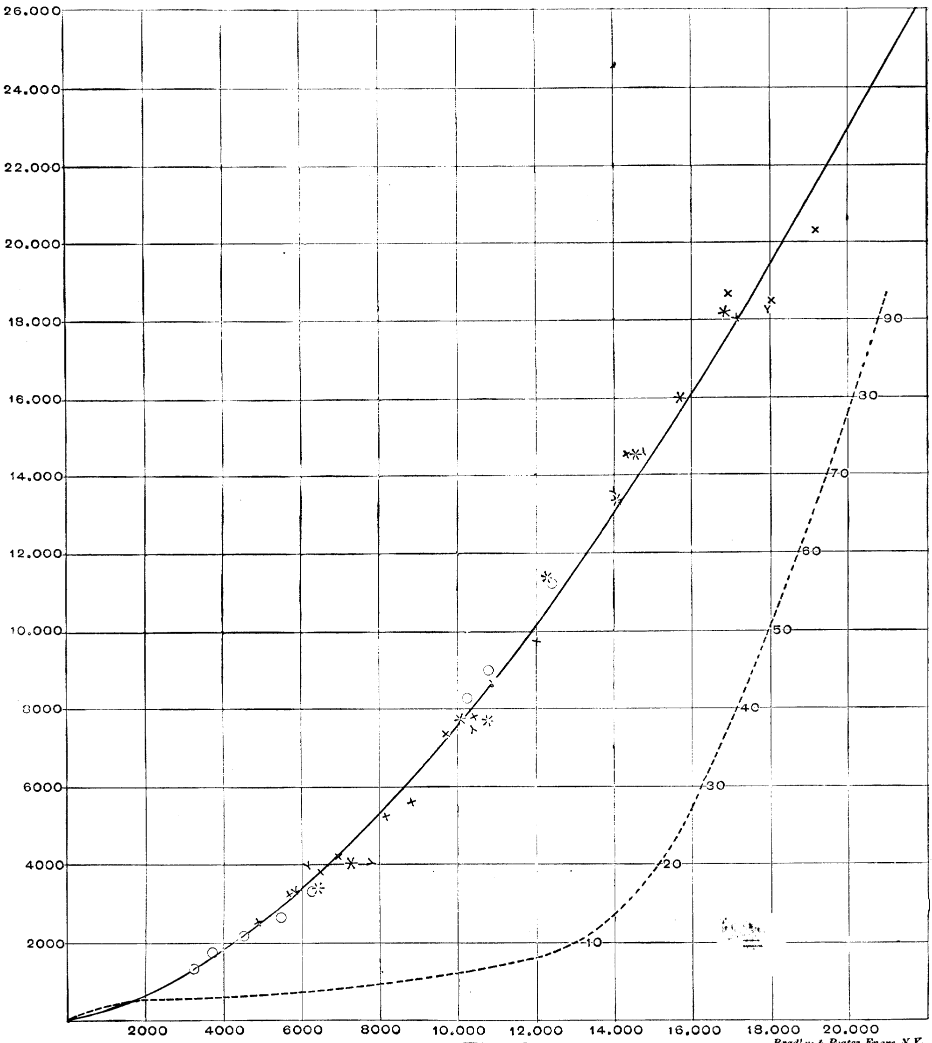

26.000
24.000
22.000
20.000
18.000
16.000
14.000
12.000
10.000
8000
6000
4000
2000
2000
4000
6000
8000
10.000
12.000
14.000
16.000
18.000
20.000
10
20
30
40
50
60
70
80
90
Bradley & Poates Engrs. N. Y.

Fig. 3.

TABLE III. (2.)

B.	*H.* obs.	*H* calc.	*H.* — *H.* = calc. obs.	%
5910	3320	3140	— 180	— 5.7
6200	3690	3420	— 270	— 7.9
7690	4220	4700	+ 480	+ 10.2
10,470	7160	7700	+ 540	+ 7.0
11,110	8370	8464	+ 96	+ 1.1
14,030	12,600	12,280	— 320	— 2.6
14,890	13,730	13,540	— 190	— 1.4
17,190	17,040	17,040	.....	...
17,940	17,570	18,240	+ 670	+ 3.7
		av :	± 315 =	± 4.4

Exponent of power, derived from tests :

$$x = 1.6041 \sim 1.6$$

Coefficient of hysteresis :

$$\eta = .00285$$

hence, theoretical curve :

$$H = .00285\ B^{1.6}$$

TABLE III. (3.)

Frequency, N = 138 complete periods per second.

B.	*H.* obs.	*H.* calc.	*H.* = *H.* = calc. obs.	%
5220	3030	3015	— 15	— .5
5750	3620	3550	— 70	— 1.9
6540	4320	4355	+ 35	+ .8
7070	4830	4890	+ 60	+ 1.2
8210	5950	6160	+ 210	+ 3.4
8520	6090	6530	+ 440	+ 6.7
9570	7850	7840	— 10	— .1
10,450	8780	9040	+ 260	+ 2.9
11,990	11,060	11,230	+ 170	+ 1.5
14,570	15,840	15,340	— 500	— 3.3
14,660	16,160	15,580	— 580	— 3.7
16,770	20,350	19,260	— 1090	— 5.6
17,970	20,620	21,440	+ 820	+ 3.9
19,320	23,180	24,120	+ 940	+ 3.8
		av :	± 371 =	± 2.8

Exponent of power, derived from tests :

$$x = 1.6044 = 1.6$$

Coefficient of hysteresis :

$$\eta = .00335$$

hence theoretical curve :

$$H = .00335\ B^{1.6}$$

TABLE III. (4.)

Frequency, $N = 205$ complete periods per second :

B	H. obs.	H. calc.	H—H. = calc —obs.	%
6360	4440	4660	+220	+4.8
7340	5380	5780	+400	+6.9
10,030	9510	9510	......	......
10,860	9980	10,670	+690	+6.5
12,230	13,700	12,940	—760	—5.9
14,600	17,390	17,160	—230	—1.3
14,700	17,830	17,340	—490	—2.8
15,750	19,700	19,360	—340	—1.7
16,700	21,990	21,300	—690	—3.2
		av:	±425	±3.7

Exponent of power, derived from tests :

$$x = 1.697 = 1.6$$

Coefficient of hysteresis :

$$\eta = .00373$$

hence theoretical curve :

$$H = .00373\ B^{1.6}$$

OPEN MAGNETIC CIRCUIT.

Two gaps of $\sim$ 4 cm. length.

TABLE III. (5.)

Frequency, $N = 138$ complete periods per second.

B.	H. obs.	H. calc.	H. — H. = calc. obs.	%
3150	1570	1560	— 10	— .6
3640	2110	2020	— 90	—4.4
4690	2930	2950	+ 20	+ .7
5490	3510	3780	+ 270	+7.2
6270	4380	4690	+ 310	+6.6
10,250	10,450	10,290	— 160	—1.6
11,000	11,810	11,520	— 290	—2.5
12,280	14,250	13,740	— 510	—3.7
		av :	± 208	±3.4

Exponent of power derived from tests :

$$x = 1.6040 \sim 1.6$$

Coefficient of hysteresis :

$$\eta = .00394$$

hence theoretical curve :

$$H = .00394\ B^{1.6}$$

From these four sets of readings we get the results:

CLOSED MAGNETIC CIRCUIT.

N = 85	9 readings:	x = 1.6041	η = .00285
138	14 "	1.6044	.00335
205	9 "	1 6970	.00373

OPEN MAGNETIC CIRCUIT.

N = 138 8 readings: x = 1.6040 η = .00393

Herefrom it seems that the consumption of energy by hysteresis per magnetic cycle increases with increasing frequency—that is, with increasing velocity of the magnetic change.

The three values of three coefficients of hysteresis for closed circuit in their dependence upon the frequency N, can be expressed by the *empirical* formula:

$$\eta = (0017 + .000016\, N - .00000003\, N^2)$$

To compare the values of hysteretic loss for different frequencies, in Fig. 3 the curve of hysteretic loss for $N = 100$ complete periods per second is plotted, giving:

$$\eta_{100} = .003$$

hence

$$H = .003\, B^{1.6}$$

and the observed values of H are not directly drawn in, but the observed values of H multiplied with the factor:

$$\frac{\eta_{100}}{\eta_{\text{obs.}}}$$

to compare the different frequencies with each other.

These values are plotted for:

N = 85	with the mark	Y	Closed magnetic circuit.
138	" "	+	
205	" "	*	

$N = 138$ with the mark o; Open magnetic circuit.

From this curve of hysteretic loss,

$$H = .003\, B^{1.5}$$

we derive the values, for the frequency of $N = 100$ complete periods per second.

TABLE III. (6.)

B.	*H.*	*B.*	*H.*
1000	190	13,000	11,460
2000	570	14,000	12,900
3000	1100	15,000	14,430
4000	1740	16,000	15,990
5000	2490	17,000	17,610
6000	3330	18,000	19,290
7000	4260	19,000	21,060
8000	5280	20,000	22,830
9000	6360	25,000	32,640
10,000	7530	30,000	43,680
11,000	8790	35,000	55,950
12,000	10,080	40,000	69,270

Especially noteworthy is the last set of readings, on open magnetic circuit, in so far as it proves the fallacy of the general opinion that the hysteretic loss of energy in the iron is smaller in the open magnetic circuit than in the closed circuit.

For the coefficient of hysteresis observed on open magnetic circuit

$$\eta = .00393$$

is even greater than that for closed magnetic circuit,

$$\eta = .00335$$

But this discrepancy is easily explained by the fact that in the closed magnetic circuit the magnetization is nearly uniform throughout the whole iron. But in the open magnetic circuit the magnetic field intensity differs considerably from point to point, being a maximum in the middle of the magnetizing coils, a minimum at the ends of the iron sheets. Now, the values of B given in the table, are the average values of the magnetization, and the values H, the average values of hysteretic loss. But the average value of the 1.6th powers of different quantities B is larger than the 1.6th power of the average value of B.

For instance, in a cubic cm. of iron magnetized to $B = 12{,}000$ is $H = 10{,}080$; in a cubic cm. of iron magnetized to $B = 6000$ is $H = 3330$; hence of these 2 cubic centimetres the average magnetization is

$B = 9000$, and the average $H = 6{,}705$ ergs

but to $B = 9000$ corresponds $H = 6360$ ergs; that is, about 5 per cent. less, and the difference becomes still greater, if the values B differ still more.

Taking this into account, it seems that the loss of energy due to hysteresis depends only upon the intensity of magnetization, and perhaps upon the frequency, but is independent of open or closed magnetic circuit, as is to be expected.

III.—Fig. 4. Tables IV.

A third set of determinations of the hysteretic loss of energy is given in the following:

Again a magnetic circuit was built up of 17 layers of a soft

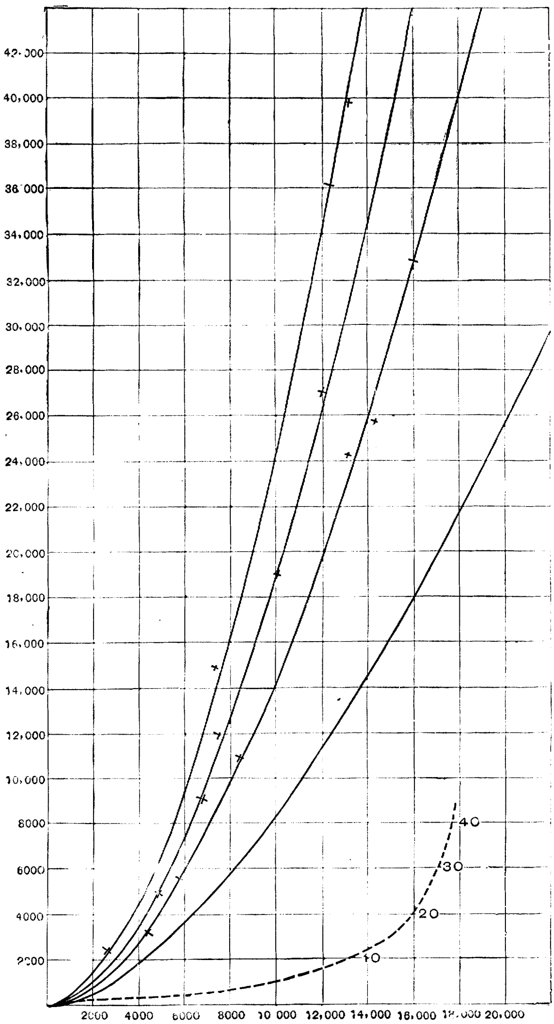


Fig. 4.

kind of sheet-iron, each layer consisting of two pieces of 20 cm. length, 2.54 cm. width, and two pieces of 7.6 cm. length and 2.54 cm. width, of the thickness $\delta = .0686$ cm., that is, of considerably greater thickness than in the former set of tests.

Here evident proof of the induction of eddy-currents in the iron was found. Especially perceptible was a decrease in the watts consumed by the iron, when a larger M. M. F. of high frequency was left acting upon the iron. This decrease must be attributed to the increase of the electric resistance of the iron, caused by its increasing temperature.

To eliminate this source of error as far as possible, before each set of tests an alternating current of high frequency ($N = 200$) and considerable strength was sent through the magnetizing coils and left on for ten to fifteen minutes, and then first readings with low magnetization, then with high, and then again with low magnetization were taken. But, nevertheless, as was to be expected, in these tests the observed values agreed less with each other than in the former readings.

The method of determination, the apparatus, etc., were the same as in the second set of tests, only that ammeter, voltmeter, and wattmeter were used at the same time. In calculating these tests, the law of the 1.6th power was assumed as true, and the loss of energy in the iron expressed by the equation,

$$H = \eta \, B^{1.6} + \varepsilon \, N \, B^2$$

where

$$H_1 = \eta \, B^{1.6}$$

is the true hysteretic loss per cycle and cm³., which is independent of the frequency, and

$$H_2 = \varepsilon \, N \, B^2$$

is the loss of energy by eddy-currents per cycle which is proportional to the frequency N.

From this expression

$$H = H_1 + H_2$$

the coefficients η and ε were calculated and the agreement or disagreement of these coefficients η and ε allow now to check the correctness or incorrectness of the law of the 1.6th power.

These tests gave the following results:

MAGNETIC CHARACTERISTICS.

F = M. M. F., in ampere turns per centimetre length of magnetic circuit.

B = magnetization, in lines of magnetic force per square centimetre.

TABLE IV. (1.)

F.	*B.*	*F.*	*B.*	*F.*	*B.*
1.5	2,700	7	11,700	18	15,450
2	4,350	8	12,200	20	15,800
3	7,100	9	12.700	25	16,400
4	8,850	10	13,100	30	16,800
5	10,000	12	13,900	35	17,200
6	10,800	14	14,500	40	17,500
		16	15,000		

HYSTERESIS.

B = magnetization, in lines of magnetic force per square centimetre.

H = loss of energy by hysteresis, in ergs per cycle and cm³. (= 10^{-7} joules) = $H_1 + H_2$

$H_1 = \eta\, B^{1\cdot 6}$ = loss of energy by hysteresis proper, in ergs per cycle and cm³. (= 10^{-7} joules).

$H_2\, \varepsilon\, N\, B^2$ = loss of energy by eddy-currents, in ergs per cycle and cm³. (= 10^{-7} joules).

TABLE IV. (2.)

Frequency, $N = 78$.

$\eta = .00331$ $\varepsilon = .751 \times 10^{-6}$

B.	H_1	H_2	$H.$(1) calc.	$H.$ obs.	△	= %
4171	2,060	1,080	3,140	3,060	+ 80	+ 2.6
5850	3,540	2,120	5,660	5,640	+ 20	+ .3
9520	7,740	5,600	13,340	13,440	− 100	− .8
13,160	12,960	10,710	23,670	24,540	− 870	− 3.7
14,320	14,880	12,720	27,600	26,460	+1140	+ 4.0
16,050	17,280	15,900	33,180	33,180	......	
				av:	{+ 6.9 / − 4.5}	± 1.9 (+ .4)

TABLE IV. (3.)

Frequency, $N = 140$.

$\eta = .00331$ $\varepsilon = .730 \times 10^{-6}$

B.	H_1	H_2	$H.$(1) calc.	$H.$ obs.	△	= %
4880	2,650	2,720	5,360	5,280	+ 80	+ 1.5
6780	4,490	5,270	9,760	9,420	+ 340	+ 3.4
7720	5,530	6,830	12,360	12,600	− 240	− 1.9
10,200	8,640	11,940	20,580	20,400	+ 180	+ .9
12,080	11,300	16 700	28,000	29,100	−1100	− 4.0
17,200	19,860	33 840	53,700	53,000	+ 700	+ 1.3
				av:	{+ 7.1 / − 5.9}	± 2.2 (+.2)

TABLE IV. (4.)

Frequency, $N = 207$

$\eta = .00336$ $\varepsilon = .757 \times 10^{-6}$

B.	H_1	H_2	H.[1] calc.	H. obs.	$\triangle$	= %
2710	1,030	1,290	2320	2,340	− 20	− .8
4720	2,510	3,910	6,430	6,480	− 50	− .8
7540	5,320	9,970	15,290	15,960	− 670	− 4.4
12,380	11,700	26,800	38,500	38,500	.....	
13,200	13,000	30,400	43,400	42,600	+ 800	+ 1.8
				av:	{+ 1.8 / − 6.0}	± 1.6 (− .8)

Therefrom we get the results:

$N =$ 78,	6 readings,	$\eta =$ .00331	$\varepsilon = .751 \times 10^{-6}$
140,	6 "	.00331	$.730 \times 10^{-6}$
207,	5 "	.00336	$.757 \times 10^{-6}$

The values found for η are so nearly alike that we can consider them as constant, and take their mean value

$$\eta = .00333$$

as the coefficient of hysteresis.

Even the values found for ε are not much different from each other, not more than was to be expected from the unavoidable differences in the temperature of the iron, which because of the high electric temperature coefficient of iron makes ε rather variable.

Taking the average of ε, we derive

$$\varepsilon = .746 \times 10^{-6}$$

and as formula of iron loss,

$$H = .00333\, B^{1.6} + .746 \times 10^{-6}\, N\, B^2$$

In Fig. 4 are drawn the four curves,

1. True hysteretic loss, $H = .00333\, B^{1.6}$
2. Iron loss for $N = 78$ $.00333\, B^{1.6} + .00005856\, B^2$
3. " " 140 $.0001022\, B^2$
4. " " 209 $.0001567\, B^2$

The observed values are plotted by crosses, +

1. $H_{calc.}$ is calculated by using for η the mean value $\eta = .00333$, but for ε the individual values, corresponding to the particular set of observations.

IV.—Figs. 5 and 6; Tables V and VI.

Two other sets of determinations of the hysteretic loss of energy, for the frequency 170 complete periods per second, were made on two laminated horse shoe magnets, with laminated keeper or armature.

The method of observation and of calculation was the same as in III., and the same precautions were taken.

The dimensions of the horse shoe magnets were:

Mean length of magnetic circuit: 38 cm.
" cross-section: 70 cm.^{2}
" volume of iron: 2660 cm.^{3}
" distance of keeper from magnet, in the first case: .15 cm.
" distance of keeper from magnet, in the second case: .08 cm.

each magnet consisting of 300 sheets well insulated iron, of the thickness .0405 cm.

In the first set of readings, considerable eddy-currents were found; in the second set, only a small amount of eddies.

The magnetic conductivity of the iron was not determined, because the reluctance of the magnetic circuit mainly consisted of that of the air gap between magnet and keeper.

The results were,

B = magnetization, in lines per cm.^{2}
$H_{obs.}$ = observed loss of energy in the iron, in ergs per cycle and cm.^{3}, for $N = 170$.
H_1 = true hysteretic loss of energy.
H_2 = loss of energy by eddy-currents.
$H_{calc.}$ = whole calculated loss of energy, $= H_1 + H_2$

TABLE V.

Frequency, $N = 170$.

$\eta = .0045$ $\varepsilon = 1.16 \times 10^{-6}$

$B.$	H_1	H_2	$H.$ calc.	$H.$ obs.	$H.$ — $H.$ calc. obs.	= %
342	51	23	74	70	+ 4	+ 5.3
410	68	34	102	102	......	.....
546	108	59	166	166	+ 1	+ .6
620	132	78	210	219	— 9	— 4.3
670	150	90	240	234	+ 6	+ 2.5
746	178	111	289	300	— 11	— 3.7
830	210	138	348	333	+ 15	+ 4.3
1020	293	208	501	524	— 23	— 4.5
1100	345	234	579	549	+ 30	+ 5.3
1200	392	290	682	695	— 13	— 2.0
1310	436	343	779	795	— 16	— 2.1
1490	539	445	984	985	— 1	— .1
1930	820	742	1562	1547	+ 15	+ 1.0
2600	1310	1280	2590	2670	— 80	— 3.1
					+ 71 — 153	+ 19.0 — 19.8
				av:	± 16	± 2.8

Therefore we get the formula for the loss in the iron,

$$H = .0045\, B^{1.6} + 1.16\, N \times 10^{-6}\, B^2$$

In Fig. 5 are shown

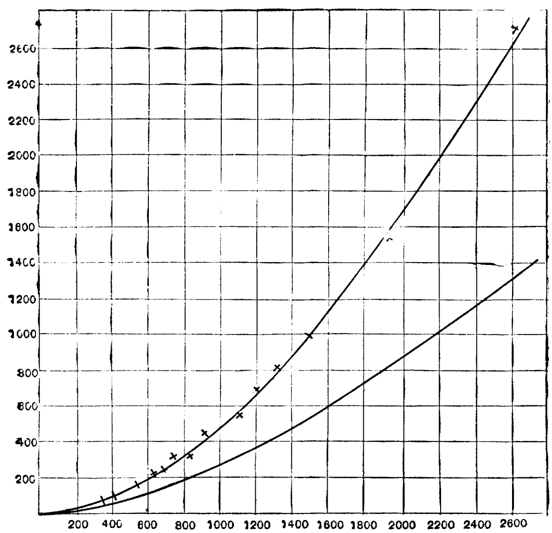


Fig. 5.

1. The curve of true hysteretic loss,

$$H_1 = .0045\, B^{1.6}$$

2. The curve of the whole loss in the iron,

$$H = H_1 + H_2$$

with the observed values marked by crosses +

TABLE VI.

Frequency, $N = 170$

$\eta = .00421$ $\varepsilon = .2083 \times 10^{-6}$

$B.$	H_1	H_2	$H.$ calc.	$H.$ obs.	$H.$ — $H.$ calc. obs.	= %
85	5.2	.3	5.5	5.6	— .1	— 1.8
182	17.3	1.3	18.6	16.9	+ 1.7	+10.0
211	22.0	1.7	23.7	23.5	+ .2	+ .9
560	105	11	116	122	— 6	— 5.0
670	140	15	155	146	+ 9	+ 6.1
685	145	16	161	157	+ 4	+ 2.6
775	176	21	197	202	— 5	— 2.4
800	186	22	208	200	+ 8	+ 4.0
1000	265	35	300	300	......	....
1070	296	41	337	353	— 16	— 4.6
1130	322	47	369	386	— 17	— 4.3
1250	379	56	435	430	+ 5	+ 1.2
1380	445	69	514	514	— 26	— 4.7
2200	940	170	1110	1130	— 20	— 1.8
2420	1090	208	1298	1268	+ 30	+ 2.4
					+ 38	+27.2
					— 90	—24.6
				av:	± 10	± 3.4

Therefore we get the formula for the loss in the iron,

$$H = .00421\ B^{1.6} + .2083 \times 10^{-6}\ N\ B^2$$

In Fig. 6 are shown,

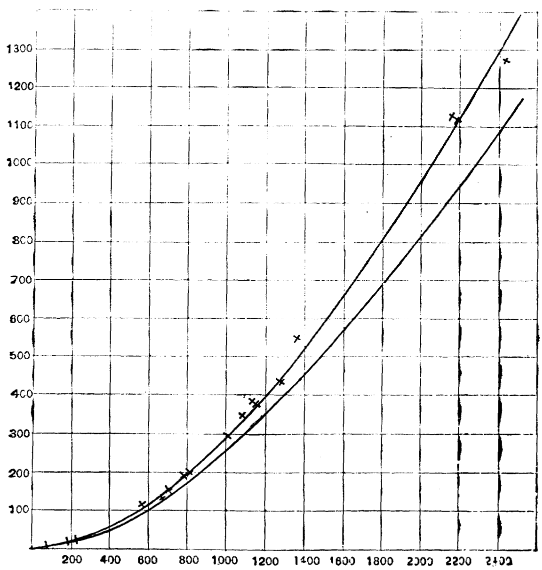


Fig. 6.

1. The curve of true hysteretic loss,

$$H_1 = .00421\ B^{1.6}$$

2. The curve of the whole loss in the iron,

$$H = H_1 + H_2$$

with the observed values marked by crosses +

Especially interesting are these two sets of readings in so far as they cover quite a different range of magnetization as the tests in I. to III.

In I. to III. the tests cover the range from 1790 to 19,340 lines of magnetic force per cm.^{2}, that is, for medium magnetization up to high saturation, while the tests in IV. cover the range from 85 to 2600 lines per cm.^{2}, that is, from medium down to very low magnetization.

The law is found exactly the same,

$$H = \eta\ B^{1.6} + \varepsilon\ N\ B^2$$

and herewith proved for the full range from 85 lines per cm.^{2} up to 19,340 lines, a ratio from 1 ÷ 230.

This seems not to agree with Ewing's theory of the molecular magnets. According to this theory, for very small magnetization the hysteresis should be expected to disappear, or almost disappear, and the cycle be reversible. Then for medium magnetization, where the chains of molecular magnets break up and rearrange, hysteresis should increase very rapidly, and slowly again for saturation. Nothing of this is the case, but hysteresis seems to follow the same law over the whole range of magnetization, and is certainly not zero for even such a low magnetization as 85 lines per cm.^{2}

Magnetometer Tests.

The method used in the foregoing has the great advantage that

1. It allows the taking of a greater number of readings, over a wide range of magnetization, in a short time, by mere simultaneous instrument readings, and thereby reduces the probable error by increasing the number of observations.
2. It allows the use of electro-dynamometers, as the most reliable electric measuring instruments.

3. It deals with larger amounts of energy, counting by watts or even hundreds of watts, whereby a much greater accuracy can be reached than by the ballistic galvanometer.

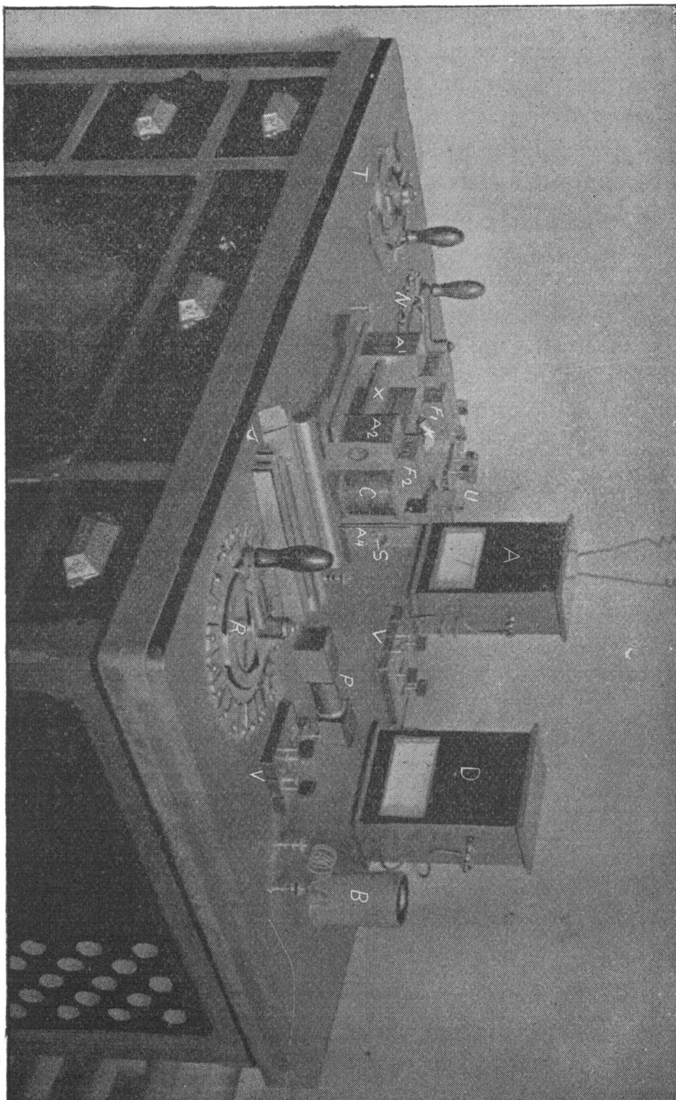


FIG 7.

4. It measures the hysteresis under the influence of an harmonically, and not suddenly varying M. M. F., that is under the

same conditions, where it becomes of importance for practical engineering.

But it has the great disadvantage that it can be used only for testing sheet-iron or other thoroughly laminated iron, where eddies are either inappreciable or can be calculated also. For testing solid iron and steel pieces, this method cannot be used, because of the tremendous amount of eddies which would flow in a solid piece of iron.

To determine the hysteretic loss of energy in steel and cast-iron the Eickemeyer differential magnetometer was used. Complete description of this instrument and its use is to be found in the *Electrical Engineer*, March 25th, 1891, wherefrom is taken a part of the following description. In Fig. 7 is shown this instrument, which I shall be glad to show in our factory to anybody who is interested in it. In Figs. 8 and 9 are diagrams of its action.

The principle of this instrument resembles somewhat the principle of the well-known differential galvanometer, applied to the magnetic circuit. In Fig. 8, suppose F_1 and F_2 were two E. M. F.'s connected in series; for instance, two cells of a battery, x and y the two resistances which we want to compare. Either resistance x and y is shunted respectively by a conductor a and b of equal resistance, which influences a galvanometer needle G in opposite directions but with equal strength.

Then the zero position of the needle G shows that the electric current c_a, flowing in a, is equal to the current c_b in b. But let the current in x be c_x, and in y, c_y; then we must have

$$c_a + c_y = c_b + c_x$$

because the currents c_a and c_y are the two branches of the same integral current as c_b and c_x

Therefore, if $c_a = c_b$, then

$$c_x = c_y$$

But if $c_a = c_b$, and $a = b$, the difference of potential at the ends of a (or, what is the same thing, y) is equal to the difference of potential at the ends of b or x and, therefore, the current in x and y, and the potential differences being the same, it follows that $x = y$.

That is, this method of connection allows us to compare an unknown resistance x with a standard resistance y.

Now, instead of "electric current," say "magnetic current" or "number of lines of magnetic force;" instead of "electromotive force" or "potential difference," say "magneto-motive force;" and instead of "electric resistance," say "reluctance," and we have the principle of this instrument.

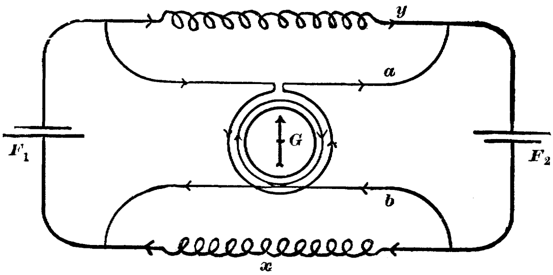


FIG. 8

Its magnetic circuit consists of two pieces of best Norway iron, ⌴⌐ shaped, shown in the illustration of the complete instrument, Fig. 7, and in the diagram Fig. 9, at F_1 and F_2. The middle portion is surrounded by a magnetizing coil C. Therefore if coil C is traversed by an electric current, the front part s_1 of the left iron piece becomes south, and the back part n north polarity.

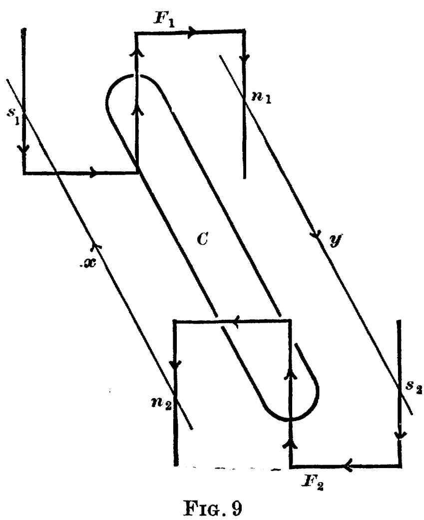


FIG. 9

The front part of the right iron piece n becomes north, and the back part south; and the lines of magnetic force travel in the front from the right to the left, from n_2 to s_1; in the back the opposite way, from the left to the right, or from n_1 to s_2, either through the air or, when n_2 and s_1, or n_1 and s_2, are connected by a piece of magnetizable metal, through this and through the air.

In the middle of the coil C stands a small soft iron needle with an aluminium indicator, which plays over a scale K, and is held in a vertical position by the lines of magnetic force of the coil C itself, deflected to the left by the lines of magnetic force traversing the front part of the instrument from n_2 to s_1, deflected to the right by the lines traversing the back from n_1 to s_2. This needle shows by its zero position that the magnetic flow through the air in front from n_2 to s_1 has the same strength as the magnetic flow in the back from n_1 to s_2 through the air.

Now we put a piece of soft iron x on the front of the instrument. A large number of lines go through x, less through the air from n_2 to s_1, but all these lines go from n_1 to s_2 through the air at the back part of the magnetometer, the front part and back part of the instrument being connected in series in the magnetic circuit. Therefore the needle is deflected to the right by the magnetic flow in the back of the instrument

Now we put another piece of iron, y, on the back part of the instrument. Then equilibrium would be restored as soon as the same number of lines of magnetic force go through x, as through y, because then also the same number of lines go through air in the front as in the back. As will be noted, the air here takes the place of the resistances a and b, influencing the galvanometer needle G, as in the diagram, Fig. 8.

The operation of the instrument is exceedingly simple and is as follows: Into the coil C an electric current is sent which is measured by the ammeter A, and regulated by the resistance-switch R. Then the needle which before had no fixed position, points to zero.

Now the magnetic standard, consisting of a cylindrical piece of Norway iron of 4 cm.^{2} cross-section and 20 cm. length is laid against the back of the instrument, with both ends fitted into holes in large blocks of Norway iron, A_3, A_4, which are laid against the poles S_1 N of the magnetometer, so that the transient resistance from pole-face to iron is eliminated.

The sample of iron that we wish to examine is turned off to exactly the same size, 4 cm.^{2} cross-section and 20 cm. length, and fitted into blocks A_1 A_2 in front of the magnetometer. Then so many fractional standard-pieces of Norway iron are added in front, that the needle of the instrument points to zero. This means that the 4 cm.^{2} Norway iron in the back, carry under the same difference of magnetic potential, the same magnetism as the 4 cm.^{2} of the examined sample plus the x cm.^{2} of fractional standard, added in the front. Hence, 4 cm.^{2} of the examined sample are equal in magnetic conductivity to $(4 - x)$ cm.^{2} of Norway iron, and the magnetic conductivity of this sample is $\frac{4 - x}{4} \times$ 100 per cent of that of Norway iron, for that difference of magnetic potential, viz., magnetization, that corresponds to the magnetometer current.

To get absolute values, the instrument has been calibrated in the following way: In the front and in the back the magnetic circuit of the instrument has been closed by 4 cm.^{2} Norway iron. Then another piece of iron, and of any desired size, has been added in the front. This piece, y, carrying some magnetism also, equilibrium was disturbed. Then through a coil of exactly 110 turns, surrounding this piece y, an electric current i was sent and regulated so that equilibrium was restored. In this case no magnetism passed through y, or in other words, the M. M. F. of the current i 110 i ampere turns, is equal to the differences of magnetic potential between the pole-faces of the instrument. In this way, for any strength of current in the main coil C of the magnetometer, the difference of magnetic potential produced thereby between the pole-faces of the instrument, was determined and plotted in a curve, for convenience in ampere turns per cm. length.

Now, the Norway iron standard was compared on the magnetometer with sheet-iron, of which, from tests with low frequency alternating currents, the magnetization corresponding to any M. M. F. was known, and therefrom derived the magnetic characteristic of the Norway iron standard, and plotted in a curve also.

In the way explained before, the iron sample that was to be determined, was balanced by the magnetometer by Norway iron, thereby giving its magnetic conductivity in per cent. of that of the Norway iron standard, the magnetometer current read, from the curves taking the M. M. F. corresponding thereto—denoted with F— and the magnetization of the Norway iron, corresponding to this M. M. F, F, and from the determined per centage of conductivity of the examined sample, the magnetization B of this sample corresponding to the M. M. F. F.

With this instrument a number of magnetic cycles of different samples of steel and cast-iron were determined.

First, a powerful alternating current was sent through the magnetometer and around all the iron pieces used, to destroy any trace of permanent or remanent magnetism.

Then the examined sample was laid against the front, the standard against the back of the magnetometer, balanced, and a larger number of magnetic cycles completed between given limits, for instance, + 95 and — 95 ampere turns M. M. F. per cm. length. Then readings were taken from maximum M. M. F. + 95 down to zero, and again up to the maximum — 95, down over zero and up to + 95, thereby completing a whole magnetic cycle, and then of a second magnetic cycle, a few readings were taken as check for the first one.

In this way for different M. M. F.'s the curve of hysteresis was found, and by measuring its area the loss by hysteresis determined.

The further calculation was done in a somewhat different way. Generally the number of cycles was not large enough to determine conveniently the exponent by analytical methods.

Therefore the law of the 1.6 M. power:

$$H = \eta B^{1.6}$$

was assumed as true, and for each cycle from the known values of H and B determined the co-efficient η.

If for different cycles the values of η agreed, this would prove the assumption, the correctness of the law of 1.6th power, while a disagreement would disprove it.

In the following for a number of samples the magnetic cycles are given:

$F =$ M. M. F., in ampere turns per cm. length.

B_r and $B_d =$ the intensity of magnetization, in kilolines, corresponding to M. M. F. F, for the rising and the decreasing branch of the magnetic curve.

The area of the looped curve, representing the loss of energy by hysteresis is derived by adding the values of B_d, and subtracting therefrom the sum of the values B_r, B_d and B_r, being given from 5 to 5 ampere turns, or .5 absolute units, the difference of the sums of B_d — B_r just gives the loss by hysteresis, in ergs per cycle.

Cast-Steel, Annealed and Hardened. Fig. 10; Table VII.

Of one kind of steel, two test pieces were cast, at the same casting, turned off to standard size and, by comparing them in the

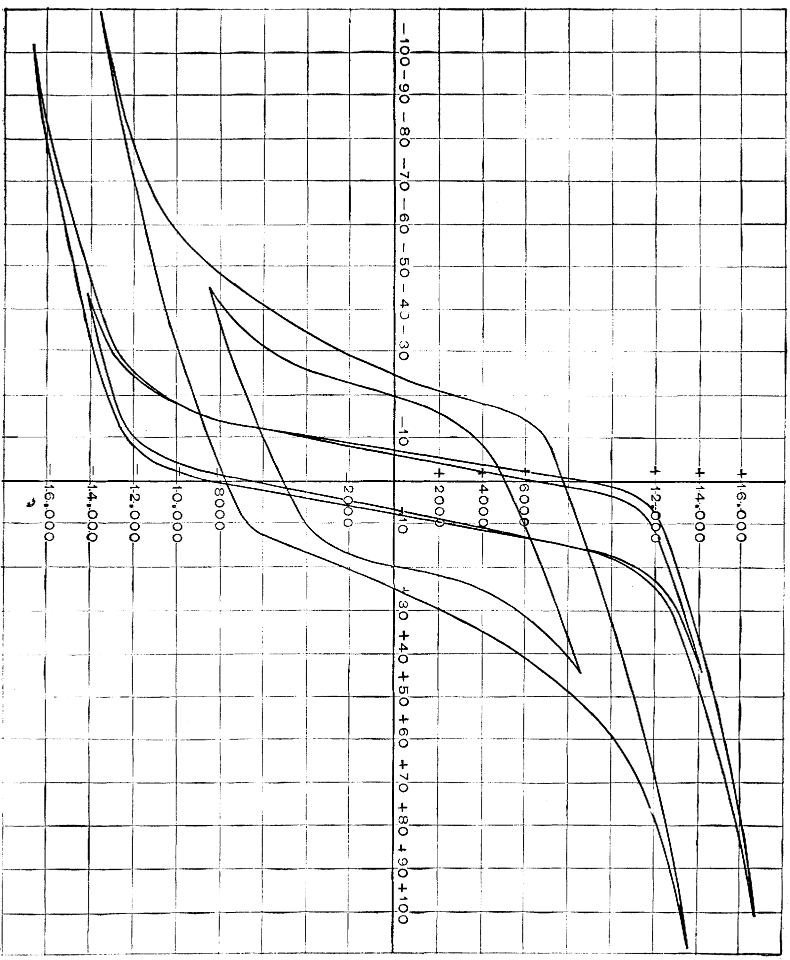


Fig. 10.

magnetometer, found to be exactly alike.

Then the one piece was hardened, the other left annealed.

Magnetometer tests gave the following magnetic cycles :

TABLE VII.

	Hardened.						Annealed.			
$F.$	B_r	B_d	B_r	B_d	B_r	B_d	B_r	B_d	B_r	B_d
0	± 5.0		± 7.0		± 7.8		± 6.6		± 8.6	
5	—4.4	+5.6	— 6.4	+ 7.5	— 7.3	+ 8.2	— 1.4	+10.7	— 2.6	+11.3
10	—3.7	6.1	— 5.6	7.9	— 6.8	8.6	+ 3.4	11.9	+ 3.7	12.3
15	—2.7	6.5	— 4.4	8.2	— 5.6	8.9	8.4	12.5	8.4	12.7
20	0	6.9	— 1.9	8.6	— 2.3	9.2	10.9	12.8	10.8	13.0
25	+3.9	7.3	+ 1.9	9.0	+ .4	9.5	12 2	13.1	12.0	13.3
30	5.5	7.6	4.2	9.3	2.5	9.8	13.0	13.4	12.7	13.6
35	6.7	8.0	6.2	9.6	4.2	10.1	13.5	13.7	13.2	13.9
40	7.7	8.3	7.6	9.9	5.8	10.4	13.9	14.0	13.5	14.2
45	8.5		8.7	10.2	7.2	10.7	14.1		13.8	14.5
50	(44.5.)		9.6	10.5	8.4	11.0	(44.5.)		14.1	14.7
55			10.4	10.8	9.6	11.2			14.4	15.0
60			10.9	11.1	10.4	11.5			14.7	15.2
65			11.4		10.9	11.8			15.0	15.4
70			(64.5.)		11.4	12.0			15.3	15.6
75					11.9	12.3			15.6	15.8
80					12.2	12.5			15.8	16.0
85					12.5	12.7			16.0	16.1
90					12.8	12.9			16.2	16.3
95					13.0	13.1			16.4	16.5
100					13.2	13.3			16.6	
105					13.4	13.4			(101.0.)	
110					13.5					
					(108.0.)					
$H=$ obs.	48,300		77,800		101,100		34,800		45,000	

Herefrom as coefficient of hysteresis, was found

$$\eta = .02494 \mid .02512 \mid .02490 \mid .007997 \mid .007962$$

Average, $\eta = .024987 \sim .025$ $\quad\quad$ $\eta = .007980 \sim .0080$

Hence, when *annealed*, the hysteretic loss is

$$H = .008\ B^{1.6}$$

when *hardened*

$$H = .025\ B^{1.6}$$

and calculated by means of these formulas, we derive

$$H_{\text{calc.}} = 48{,}400 \quad 77{,}500 \quad 101{,}500 \quad 34{,}730 \quad 45{,}100$$

and

$$H_{\text{calc.}} - H_{\text{obs.}} = +100 \quad -300 \quad +400 \quad -70 \quad +100$$

= per cent. of

$$H_{\text{calc.}} \quad +.2 \quad -.4 \quad +.4 \quad -.2 \quad +.2$$

In Fig. 10 are drawn some of the magnetic curves for both samples.

It is especially interesting to note that though the chemical constitution of both samples is exactly the same, their magnetic behavior is entirely different, so that the *magnetic* properties of iron seem to be determined much more by its *physical* than its *chemical* constitution.

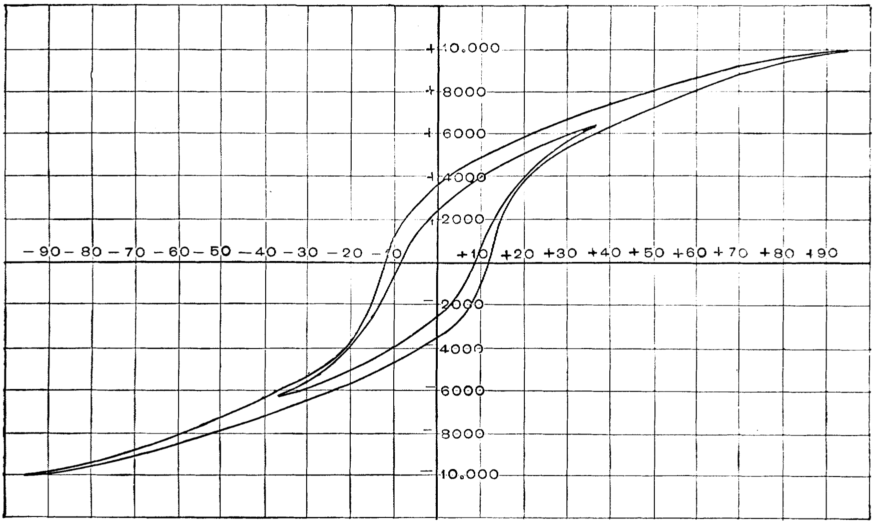


Fig. 11.

ANOTHER SAMPLE OF CAST-STEEL OF LOW MAGNETIC CONDUCTIVITY.

FIG. 11.

TABLE VIII.

$F.$	B_r	B_d	B_r	B_d	B_r	B_d	B_r	B_d
0	± 2.5		± 2.8		± 3.1		± 3.4	
5	−1.5	+3.4	−1.9	+3.6	−2.1	+3.9	−2.7	+4.2
10	+ .6	4.1	− .4	4.3	− .6	4.6	−1.3	4.8
15	2.7	4.6	+2.7	4.9	+2.2	5.2	+2.3	5.4
20	3.9	5.1	4.0	5.5	4.2	5.8	3.8	5.9
25	4.7	5.6	4.9	6.0	5.1	6.2	4.8	6.4
30	5.5	6.0	5.6	6.4	5.7	6.6	5.5	6.7
35	6.2	6.3	6.2	6.7	6.1	6.9	6.0	7.1
40	6.38		6.6	7.0	6.6	7.2	6.5	7.4
45	(37.0.)		7.0	7.3	7.0	7.5	7.0	7.7
50			7.4	7.5	7.4	7.8	7.4	7.9
55			7.64		7.8	8.1	7.8	8.2
60			(52.0.)		8.1	8.4	8.1	8.5
65					8.4	8.6	8.4	8.8
70					8.7	8.8	8.7	9.0
75					8.95		9.0	9.2
80					(75.0.)		9.3	9.5
85							9.5	9.6
90							9.8	9.8
95							10.0	
							(95.0.)	
$H =$ obs.	14,600		19,900		25.000		29,600	
$\eta =$	.0119		.0122		.0119		.0118	

Average, $\eta = .001195$

$\sim .012$

Herefrom,

$$H = .012\ B^{1.6}$$

$H_{calc.}$ =	14,620	19,520	25,140	30,020
$H_{calc.} - H_{obs.}$	+ 20	− 380	+ 140	+ 420
= per cent. of $H_{calc.}$	+ .1	− 1.9	+ .6	+ 1.4

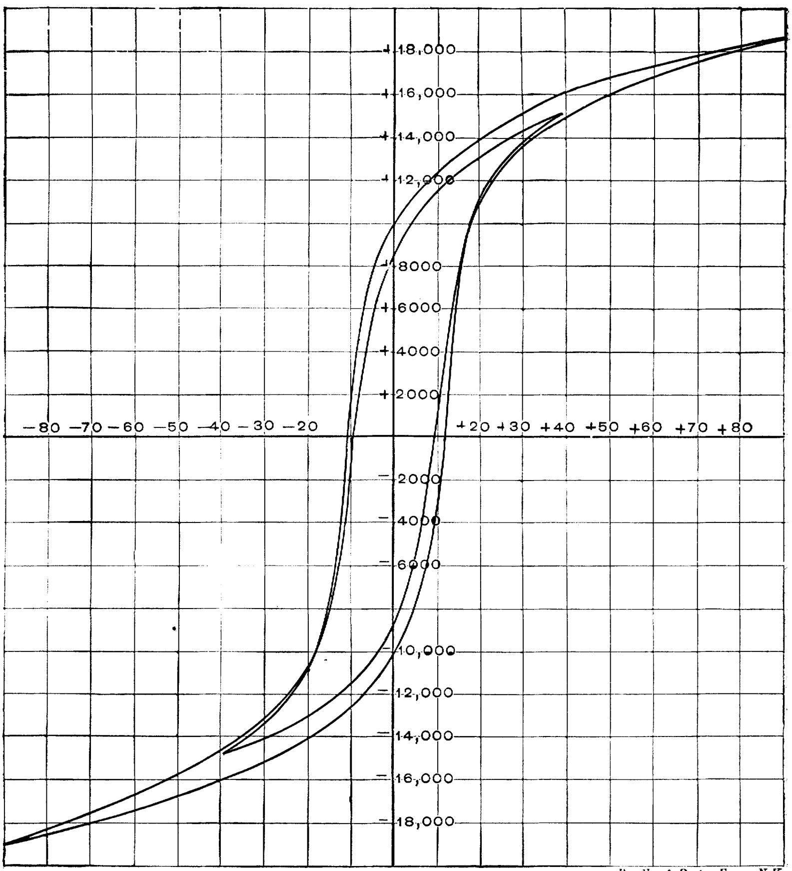


Fig. 12.

With regard to hysteresis, this kind of cast-steel is 50 per cent. worse than the annealed cast-steel No. 1, but still twice as good as the hardened sample. But, magnetically, it is poor—that is, of low conductivity, giving for 40 ampere turns M. M. F. per centimetre length only ∼ 6600 lines of magnetic force per square centimetre, while the annealed steel gives ∼ 14,000—that is, more than twice as many, and even the hardened steel gives more, ∼ 8000.

Soft Machine Steel. Fig. 12.

TABLE IX.

	I.		II.		III.		
F	B_r	B_d	B_r	B_d	F	B_r	B_d
0	± 8.3		± 9.6		50	15.9	16.8
5	− 5.7	+10.2	− 7.5	+11.2	55	16.4	17.0
10	+ 1.2	11 6	− 2 0	12.4	60	16.9	17.4
15	7.4	12.6	+ 7.2	13.5	65	17.3	17.7
20	11.0	13.4	10.9	14.2	70	17.7	18.0
25	12.6	13.8	12.4	14.8	75	18.0	18.2
30	13.5	14.2	13.3	15.3	80	18.3	18.4
35	14.2	14.5	14.0	15.7	85	18.6	18.7
40	14.8		14.7	16.0	90	18.8	
45	(39.0.)		15.3	16.4		(90.0.)	
$H_{obs.}$ =	44,400					64,000	
η =	.00944					.00928	

Average, $\eta =$ .00936

hence

$H_{calc.} =$ 44,000 64,600

$\Delta =$ − 400 + 600

= ± 1.0 per cent.

CAST-IRON. FIG. 13.

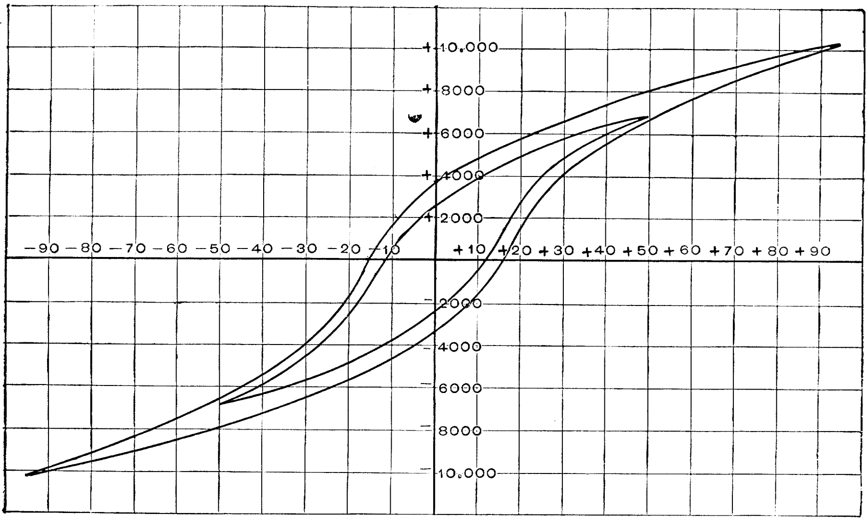


Fig. 13.

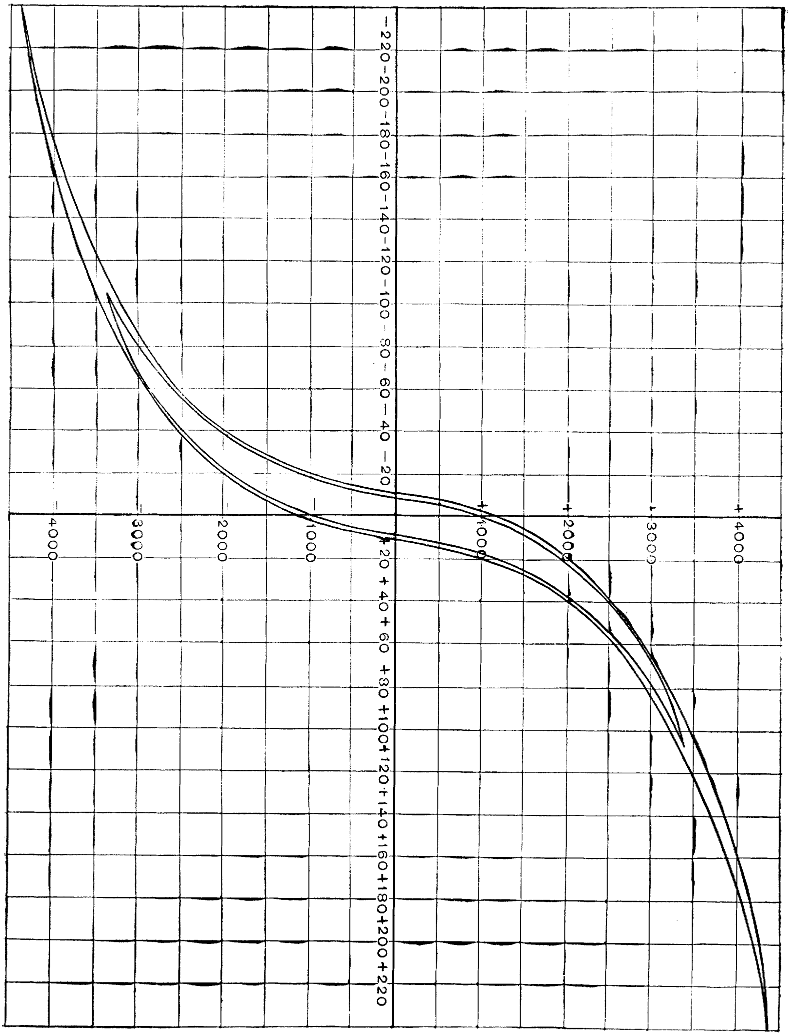
TABLE X.

	I.		II.			II.	
F	B_r	B_d	B_r	B_d	F	B_r	B_d
0	± 2.5		± 3.5		50	6.8	7.9
5	—1.7	+3.2	—2.7	+4.1	55	7.3	8.2
10	— .6	3.9	—1.7	4.7	60	7.8	8.6
15	+ .9	4.4	— .2	5.2	65	8.2	8.9
20	2.6	4.9	+1.6	5.7	70	8.6	9.2
25	3.8	5.4	3.0	6.1	75	9.0	9.4
30	4.6	5.8	4.0	6.5	80	9.4	9.7
35	5.2	6.1	4.9	6.8	85	9.7	9.9
40	5.8	6.4	5.5	7.2	90	10.0	10.1
45	6.3	6.6	6.1	7.6	95	10.3 (95.0.)	
50	6.8 (50.0.)						
$H_{obs.}$ =	22,300 ergs		42,000 ergs				
η =	.01647		.01589				

Average, $\eta = .01616$

$H_{calc.} =$	22,000	42,800
$H_{calc.} - H_{obs.} =$	— 300	+ 800
= per cent.,	— 1.5	+ 1.9

Fig. 14.



MAGNETIC IRON ORE. FIG. 14; TABLE XI.

In the following are given the magnetic curves of a piece of magnetic iron ore, apparently pure Fe_3O_4, of the dimensions, 1 in. × 1 in. × $2\frac{21}{32}$ in.

TABLE XI.

MAGNETIC CHARACTERISTIC.

F = M. M. F., in ampere turns per centimetre length of magnetic circuit.

B = magnetization, in lines of magnetic force per square centimetre.

F	B	F	B	F	B
10	750	70	2930	140	3770
20	1510	80	3080	160	3930
30	2000	90	3220	180	4070
40	2320	100	3350	200	4200
50	2560	110	3470	220	4310
60	2760	120	3580	240	4400

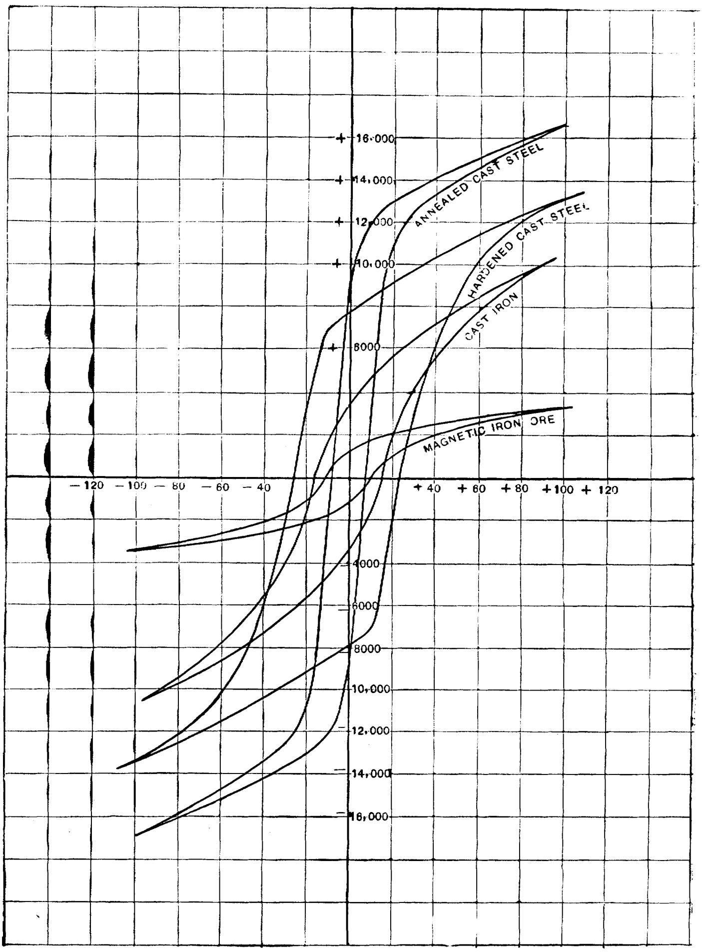


Fig. 15.

TABLE XII.

CYCLIC MAGNETIZATION.

	I.		II.			II.	
F	B_r	B_d	B_r	B_d	F	B_r	B_d
0	± 900		± 1020		130	3640	3740
10	0	+1520	— 200	+1660	140	3730	3820
20	+1200	1920	+1000	2020	150	3820	3900
30	1800	2230	1750	2280	160	3910	3980
40	2160	2500	2150	2520	170	3990	4050
50	2450	2700	2390	2710	180	4050	4110
60	2670	2850	2610	2880	190	4120	4170
70	2850	3000	2800	3020	200	4190	4230
80	3020	3120	2980	3150	210	4250	4280
90	3190	3250	3140	3280	220	4320	4340
100	3340	3360	3280	3410	230	4360	4370
110	3440		3410	3530	240	4400	
120	(106.)		3530	3640		(240.)	

$$H_{\text{obs.}} = 9{,}340 \text{ ergs} \qquad 13{,}780 \text{ ergs}$$

$$\eta = .02049 \qquad .02041$$

$$\text{Average, } \eta = .02045$$

Curve of hysteresis,

$$H = .02045\, B^{1.6}$$

$$H_{\text{calc.}} = 9{,}320 \text{ ergs} \qquad 13{,}810 \text{ ergs}$$

$$H_{\text{calc.}} - H_{\text{obs.}} = -20 \text{ ergs} \qquad +30 \text{ ergs}$$

$$= -.2 \text{ per cent.} \qquad +.2 \text{ per cent.}$$

As seen, the coefficient of hysteresis of magnetic iron ore, $\eta = .020$, ranges between that of cast-iron, $\eta = .016$, and of hardened steel, $\eta = .025$.

The magnetic conductivity is approximately 20 per cent. of that of wrought-iron.

In Fig. 15 is given a comparison of the hysteretic curves of

Hardened steel,
Annealed steel,
Cast-iron,
Magnetic iron ore,

in the same size.

This figure shows well the three characteristic forms of hysteretic curves :

TABLE XI.

$$H = \eta B^{1.6} \; [+ \varepsilon N B^2]$$

H in ergs per cycle and cm.^{3}, B in lines of magnetic force per cm.^{2}, F in ampere turns per cm.

Material.	Hysteretic Coefficient. η	Magnetization at the M. M. F. $F = 10$	$= 40$	$= 90$	Residual Magnetism R. For $F = 40$	$= 90$	Coercitive Force C. For $F = 40$	$= 90$	$\frac{\eta}{C}$ For $F = 40$	$= 90$
Very soft iron wire (Ewing)	.0020	12800	14700	16600			(1.5) ¶	(1.9)		
Westinghouse converter, sheet-iron	.0024	14400	17800	20800			(1.8)	(2.0)		
Very thin sheet-iron, standard	.0030 *	13100	17100	20700			(2.3)	(2.8)		
Thick sheet-iron	.00333 †	13100	17500	....			(2.5)	(3.1)		
Sheet-iron	.00421 ‡	....	....	....			(3.2)	(3.9)		
Sheet-iron	.00450 §	....	....	....			(3.4)	(4.2)		
Soft annealed cast-steel	.0080	7800 ‖	14000	16300	5100	8200	6.0	7.0	.00133	.00114
Soft machine steel	.0094	5800	14800	18800	8300	9600	9.1	11.1	.00103 **	.00085 **
Cast-steel of low magnetic conductivity	.0120	2000	6400	9800	2600	3300	9.1	11.6	.00132	.00104
Cast-iron	.0162	1600	6100	10100	2100	3400	10.4	15.2	.00156 **	.00106
Hardened cast-steel	.0250	1200	8000	12900	4500	7500	19.0	23.5	.00132	.00107
Magnetic iron ore	.02045	750	2320	3220		900		10.0		.00204
							Average		.00132	.00108

*, For $N = 100$.

†, $\varepsilon = .746 \times 10^{-6}$.

‡, $\varepsilon = .2083 \times 10^{-6}$.

§, $\varepsilon = 1.16 \times 10^{-6}$.

‖, This, and the following values of this column are derived as average of rising and decreasing branch of the magnetic characteristic, because at $F = 10$ the magnetism is still very unstable.

¶, Computed by means of the average values of $\frac{\eta}{C} = .00132$ and $= .00108$.

**, Left out by taking the average of $\frac{\eta}{C}$

1. The hardened steel curve, of high coercitive force, has the bend or "knee" on the *negative* side, so that for zero M. M. F. the "remanent" magnetism is still in the saturation part of the curve—that is, in stable equilibrium; therefore permanently magnetizable.
2. The soft iron curve, with the bend on the *positive* side, so that for zero M. M. F. the "remanent" magnetism, though still very high, is already below the range of saturation, on the branch of unstable equilibre Therefore the remanent magnetism is very unstable and easily destroyed, the more as the coercitive force is very small.
3. The cast-iron curve, which has no marked knee at all, but a steady curvature of low remanent magnetization, but with regard to coercitive force ranging between 1 and 2.

The curve of the magnetic iron ore shows all the characteristics of a cast-iron curve.

Having derived, now, a larger number of values of the hysteretic coefficient η for different kinds of iron and other material, we shall put them together for comparison in Table XI.

It is remarkable, in these results, that for several samples of each set the quotient $\frac{\eta}{C}$ gives almost exactly the same value, while other values disagree therefrom. From this average value of $\frac{\eta}{C}$ are calculated the values of the coercitive force C of sheet-iron, given in the brackets.

For convenience, in the following table are given the values W of consumption of energy in watts per cubic inch, for 100 complete periods (magnetic cycles) per second, and for the magnetization of H lines of force per square inch, giving as coefficient of hysteresis the value $\gamma = 8.3 \times 10^{-6} \eta$

In Table XII., I have given a number of experimental values of the consumption of energy by hysteresis and believe to have shown that this consumption of energy can fairly well be expressed by the empirical formula,

$$H = \eta \; B^{x}$$

where the exponent x is equal, or at least very nearly, to 1.6, and the coefficient η a constant of the material, which ranges from .002 up to .025 and more, and may possibly have a slight

TABLE XII.

$$W = \gamma H^{1.6}$$

W in watts per cubic inch and 100 complete periods per second.

H in lines of magnetic force per square inch.

γ	$H =$	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000	110,000	120,000	150,000
Very soft iron wire (Ewing) .... .166 X 10^{-10}	$W =$	.042	.13	.24	.38	.55	.73	.94	1.16	1.40	1.66	1.94	2.23	3.18
Westinghouse converter, sheet-iron .... .201		.051	.15	.29	.46	.66	.89	1.14	1.41	1.70	2.01	2.34	2.69	3.84
Very thin sheet-iron .... .249		.063	.19	.36	.58	.82	1.10	1.41	1.74	2.11	2.49	2.90	3.34	4.77
Thick sheet-iron .... .277		.070	.21	.40	.64	.91	1.22	1.56	1.94	2.34	2.77	3.22	3.71	5.30
Sheet-iron .... .350		.089	.27	.50	.81	1.15	1.55	1.98	2.45	2.96	3.50	4.07	4.69	6.69
Sheet-iron .... .374		.095	.29	.54	.87	1.23	1.66	2.12	2.62	3.17	3.75	4.36	5.02	7.17
Soft annealed cast-steel .... .663		.17	.51	.97	1.53	2.19	2.93	3.75	4.64	5.60	6.63	7.73	8.88	12.7
Soft machine steel .... .778		.20	.59	1.14	1.80	2.57	3.44	4.40	5.45	6 57	7.78	9.06	10.4	14.9
Cast-steel of low magnetic conductivity .... .994		.25	.76	1.45	2.30	3.28	4.39	5.62	6.95	8.39	9.94	11.6	13.3	19.0
Cast-iron 1346		.34	1.02	1.96	3.11	4.44	5.94	7.60	9.41	10.9	13.4	15.7	17.6	25.8
Hardened cast-steel 2077		.52	1.58	3.03	4.80	6.85	9.18	11.7	14.6	17.5	20.7	24.2	27.8	39.8

$H = 25{,}000$; alternate current transformer, American style (high frequency).

$H = 35{,}000$; " " " European " low "

Only the values smaller than .25 *W*, can be of practical use; in those larger than 10 the iron gets at least red hot if in larger quantities.

dependence upon the *velocity* wherewith the magnetic cycle is performed, as the second set of alternate-current readings seems to indicate.

In the following table, I give the values of the hysteretic resistance η for some iron samples, subjected to a magnetic cycle between $F = +190$ and -190 ampere turns per centimetre, calculated from Hopkinson's tests[1] by the assumption of the law of hysteresis.

η = the coefficient of hysteresis.

B = the maximum magnetization in lines of magnetic force per square centimetre.

R = the remanent magnetization in lines of magnetic force per square centimetre.

TABLE XIII.

Material.	Condition.	η	B	R
Wrought-iron	Annealed	.00202	18,250	7,250
Soft Bessemer steel..	.045 per cent. C. annealed	.00262	18,200	7,860
Soft Wittworth steel.	.09 " " "	.00257	19,840	7,080
	.32 " " "	.00598	18,740	9,840
	.89 " " "	.00786	16,120	10,740
	.32 " " oil-hard.	.00954	18,800	11,040
	.89 " " "	.01844	16,120	8,740
Silicon steel	3.44 " Si., wrought	.00937	15,150	11,070
	3.44 " " annealed	.00784	14,700	8,150
	3.44 " " oil-hard.	.01282	14,700	8,080
Manganese steel	4.73 " Mn., wrought	.05963	4,620	220
	8.74 " " "	.....	747	....
	12.36 " " "	.....	310	....
	4.73 " " annealed	.04146	10,580	5,850
	8.74 " " "	.08184	1,985	540
	4.73 " " oil-hard.	.06706	4,770	2,160
	8.74 " " "	.....	733	...
Chrome-steel........	.62 " Cr., wrought	.01179	15,780	9,320
	1.2 " " "	.01851	14,680	7,570
	.62 " " annealed	.00897	14,850	7,570
	1.2 " " "	.01638	13,230	6,490
	.62 " " oil-hard.	.03958	13,960	8,600
	1.2 " " "	.04442	12,870	7,890
Tungsten steel	4.65 " Wo., wrought	.01516	15,720	10,140
	4.65 " " annealed	.01435	16,500	11,010
	3.44 " " oil-hard.	.04776	14,480	8,640
	2.35 " " very hard	.05778	12,130	6,820
Grey cast-iron.......	3.45 p. c. C.; .17 p. c. Mn.	.01826	9,150	3,160
White cast-iron	2.04 " C.; .34 " "	.01616	9,340	5,550
" "	4.5 " C.; 8.0 " "	.....	385	77

These values of the hysteretic resistance vary from .002 up to .082, 41 times the first value.

But especially marked is, that η depends much less upon the chemical constitution of the iron sample, than upon its physical

1. From "Kalender fur Electrotechniker," by Uppenborn, Berlin, Germany.

condition, *annealing decreasing*, and *hardening increasing* the hysteresis very considerably.

So far as the chemical constitution is concerned, the purer the iron the lower is its hysterešis, while any kind of foreign matter increases the hysteresis. Especially manganese increases the hysteretic loss enormously, much less wolfram and chromium, least silicon and carbon. Connected with the increase of hysteresis is always a decrease in magnetic conductivity.

I wish to add a few remarks on two alleged phenomena connected with hysteresis, which have been talked about considerably, without yet being made clear; the decrease of hysteresis for open magnetic circuit, and the decrease of hysteresis of a transformer with increasing load.

With regard to the first, as shown, actual tests do not show a smaller value of hysteresis for open than for closed magnetic circuit.

And it can not be understood how that could be.

For consider an iron molecule of the magnetic circuit exposed to the harmonically varying M. M. F. and performing a magnetic cycle. Evidently it can make no difference for this iron molecule, whether some trillion of molecules distant the magnetic circuit ends in air, or is closed entirely in iron, supposing that the M. M. F. and the magnetism, and therefore also the magnetic reluctivity, are the same in both cases.

Neither can it make any difference whether the M. M. F. is caused only by one sine-wave of electric current, or is the resultant of several M M. F.'s, as in the loaded transformer. It is the same as with the electric current, where the energy converted into heat in each molecule of the conductor does not depend either, whether the material of the conductor on some other point changes, or whether one or more E. M. F.'s are acting upon the circuit.

Hence, until absolutely exact and undoubtable determinations of the hysteretic loss for fully loaded transformers are at hand, the assumption of a decrease of hysteresis with increasing load must be rejected.

That an apparent decrease with increasing load has been observed several times may be conceded, for besides the exceedingly great liability to errors in these tests, where the hysteretic loss comes out as the small difference of two large values, primary energy and secondary energy, and therefore is very much affected by the slightest error in any one of the components, it must be understood that the main possible errors in the determinations on fully loaded transformers all point this way. Neglect of secondary self-induction, decrease of magnetization with increasing load, slowing down of the dynamo-alternator, etc., all cause an apparent decrease in the hysteretic loss for increasing load. At least in one set of tests, those made by Prof. Ryan, at Cornell University, on a small Westinghouse converter, I was able to show in my "Elementary Geometrical Theory of the Alternate Current Transformer"[1] that the observed decrease of the hysteretic loss disappears by reducing the different readings to the same magnetization and the same frequency.[2]

If, indeed, the shape of the wave of M. M. F. varies, then a certain difference in the value of the hysteretic loss can be imagined. Compare it with a mechanical or elastic cycle. A moving pendulum, or an oscillating spring, for instance, continuously converts potential energy into kinetic energy and back; in each oscillation consuming, that is, converting into heat, a part of the energy by internal and external friction. Now, if this motion of spring or pendulum is truly harmonic, less energy is converted into heat than if the motion varies abruptly, is jerking, etc. So, in a magnetic cycle, between the same limits of magnetization the hysteretic loss might be smallest, when the cycle is entirely harmonical, but might be larger if the M. M. F. varies abruptly; for instance, when caused by an intermittent current.

Now, in a transformer with open secondary the M. M. F. acting upon the iron is that of the primary current, and this current is rigidly determined in its shape by the E. M. F. of the dynamo and the E. M. F. of self-induction. But in a fully loaded transformer the secondary current is proportional to the changes of the magnetism, therefore increases very considerably in the moment of a sudden change of magnetism. Hence, if a sudden and abrupt change in the primary current occurs, just as suddenly the secondary current increases in the opposite direction, and thereby makes a sudden change of resulting M. M. F. and magnetism impossible, so that the fully loaded transformer compares with the elastic spring which oscillates freely, while the open-

1. Dec. 1891, *Electrical Engineer*, New York.
2. The latest tests of Ewing prove that, in a fully-loaded transformer the loss by hysteresis is *not* smaller than for open secondary circuit.

circuited transformer compares with a spring, where the motion is determined by a rigidly-acting outside force.

Hence, if the shape of the alternating primary current differs considerably from the sine law, a certain decrease of the hysteretic loss for increasing load can be expected, though certainly not such an enormous decrease as some former tests seemed to point out. These tests must undoubtedly have given erroneous results, perhaps caused by the neglect of the secondary self-induction, which, even if very small and causing only a slight error in the secondary energy, must cause an enormous error in the hysteretic loss, the small difference between the two large values—primary and secondary energy

That an electro-magnet without keeper loses its magnetism quicker than a magnet with keeper, or a closed magnetized iron ring, is a phenomenon. which has nothing whatever to do with this loss of energy by hysteresis, but is merely due to the demagnetizing force of the remanent magnetism. For the remanent magnetism in an open magnetic circuit causes between its poles a certain difference of magnetic potential, which in the moment of breaking the electric circuit acts as demagnetizing M. M. F., and, if the coercitive force is small, as in wrought-iron or annealed steel, almost entirely destroys the remanent magnetism, while in an iron of large coercitive force it affects the permanent magnetism very little. In the closed magnetic circuit the remanent magnetism causes no or very little difference of magnetic potential, and therefore no destruction of the remanent magnetism by its own demagnetizing M. M. F. takes place. But with the hysteretic loss of energy this phenomenon has nothing to do.

To combine the results, what I believe to have proved is that loss of energy in iron caused by reversals of magnetism can be expressed by the analytical formula:

$$H = \eta \, B^{1.6} + \varepsilon \, N \, B^2.$$

where

η = the co-efficient of hysteresis,

ε = the co-efficient of eddy currents,

N = the frequency of the alternations of magnetism,

$\eta \, B^{1.6}$ = the loss of energy by hysteresis proper, or by *molecular friction*, and

$\varepsilon \, N, B^2$ = the loss of energy by eddy currents, per magnetic cycle and per cm.^{3}, proportional to the frequency N.

TABLE XIV.

B	$B^{1.6}$	Increase per 100	B	$B^{1.6}$	Increase per 100	B	$B^{1.6}$	Increase per 100
500	.0208	42	9000	2.122	378	17,000	5.870	55
1000	.0631	85	9500	2.313	389	17,500	6.148	56
1500	.1206	115	10,000	2.511	400	18,000	6.434	57
2000	.1913	142	10,500	2.716	41	18,500	6.722	58
2500	.2732	164	11,000	2.925	42	19.000	7.017	59
3000	.3659	185	11,500	3.141	43	19,500	7.312	59
3500	.4684	205	12,000	3.363	44	20,000	7.613	60
4000	.5800	223	12.500	3.589	45	22.000	8.868	63
4500	.7000	240	13.000	3.821	46	24,000	10.193	66
5000	.8283	258	13,500	4.060	47	26,000	11.59	70
5500	.9662	275	14,000	4.303	48	28,000	13.05	73
6000	1.111	292	14,500	4.580	49	30,000	14.57	76
6500	1.261	308	15,000	4.807	50	35,000	18.65	82
7000	1.420	324	15,500	5.062	51	40,000	23.09	89
7500	1.583	339	16,000	5.329	53	45,000	27.87	96
8000	1.758	353	16.500	5.598	54	50,000	33.00	103
8500	1.936	366						

For convenience, I give in Table XIV., the values of the 1.6th power of the numbers, from 500 to 50,000 with the parts proportional, or the increase of $B^{1.6}$ for 100 lines of magnetic force.

Yonkers, N. Y., December 7th, 1891.

Discussion.[1]

The Chairman:—Gentlemen, the poet has informed us that "better fifty years of Europe than a cycle of Cathay." What he would have done had he met a cycle of magnetism, we can but conjecture. The Institute has therefore good reason, I conceive, to congratulate itself that one of its members does not shrink from such a conflict. I am sure I shall but express the sentiments of every member present, when I say that we are much obliged to Mr. Steinmetz for his very elegant and exhaustive treatment of a subject whose title, to say the least, has a most unpromising and uninteresting sound—a subject dealing with the causes of those indispositions of iron to change its magnetic condition which in our old telegraphic days we were wont to sum up by the unscientific term of "residual magnetism."

Before calling for general discussion, I would like to ask Mr. Steinmetz whether, in his experiments and tests, he had determined whether or not there was any real foundation in fact for the distinction which Professor Ewing has drawn between the molecular friction, which he calls "static hysteresis," and the real time-lag, which he denominated "viscous hysteresis."

Mr. Steinmetz:—I really am not yet prepared to answer the question whether viscous or time hysteresis exists or not. My tests in only one set of determinations gave me an increase of hysteretic loss with increasing frequency, which seems to point to

1. Discussion by Messrs. Bradley, Kennelly, Lockwood and Pupin.

the existence of a viscous hysteresis. For if a viscous hysteresis exists, it would show by an apparent increase of the coefficient of hysteresis, with increasing frequency. But most of the tests do not show this, but give the same coefficient of hysteresis for different frequencies.

At any rate, if there exists such a time-hysteresis—which I shall try to find out—it follows the law of the 1.6th power also.

But I think, only at much higher frequencies than those I have used in my tests, can we hope to meet with viscous hysteresis. I hope to be able at a future meeting to give more detailed information on this and some other phenomena connected with the magnetic hysteresis.

THE CHAIRMAN :—Gentlemen,the subject is before you. While a few of us were in the parlor, prior to the reading of the paper, I heard Mr. Steinmetz condoling with himself in relation to the weather and expressing the hope that there would still be a very considerable discussion. It is therefore to be hoped that any of us who may feel able to grapple with such a subject will not hesitate to do so.

MR. CHARLES S. BRADLEY :—I do not feel able to discuss this paper, but I know it will prove very valuable to us. Our work of late has been upon transformers. I am connected with the Fort Wayne Electric Company, whose transformers now use about 2,000 lines of force to the square centimetre, and we have been trying to increase the lines of force. We encountered the very phenomena treated in this paper, and therefore it is very interesting to me, and I think that we ought to congratulate ourselves upon having a member who can tackle such a subject. It is very seldom that in America, anything of this kind is taken up. We see it very often in Europe, but our commercial age will hardly permit us to devote our time to such experiments and carry them out as they should be.

MR. JOSEPH WETZLER :—A gentleman who is present but who is not a member, has asked me to inquire of the author whether he made any experiments on mitis iron and, if so, what his results were.

MR. STEINMETZ :—I never made any experiments with regard to hysteresis, on mitis iron—only on different kinds of cast-iron.

MR. A. E. KENNELLY :—Mr. President and gentlemen, I think that we have to congratulate ourselves upon a magnetic and physical treat in the paper that we have just listened to. Mr. Steinmetz has been, I think, the first to point out this remarkable law of hysteresis—the variation of the energy consumed per cycle, with the total flux per square centimetre that passes through it. I think that it is perhaps preferable to express the exponent in the equation as a vulgar fraction instead of as a decimal—not that it alters the facts in any way, but merely because it gives us a little more hope of being able to understand what the equation means, if not now, at least let us say in the future. If, instead of writing the energy—Mr. Steinmetz calls it H, as $\eta \; B^{1.6}$, we write it $\eta \; B^{\frac{8}{5}}$, it gives us some hope of being able to transform that in a simple manner, which will give us the fundamental law concerned. I think there is very little doubt that the law Mr. Steinmetz gives is the true one. It is, first of all, as he showed us some time ago, in accordance with the values observed by Professor Ewing, and so far as my own knowledge goes I am able to corroborate it, for I have observed the same law in the case of one sample of wrought-iron taken by a ballistic method, and another sample of wrought-iron taken by wattmeter method, both giving the $\frac{8}{5}$ power, although I do not know what the exact value of the coefficient η was in those particular instances. It is very puzzling to understand what that peculiar fraction $\frac{8}{5}$ means. It is rather too high and unwieldy a fraction to be understood at a glance. But whatever its inner meaning may be, its outward and visible indications are clear enough, because if you double the flux density in a piece of iron you will treble the energy which is consumed in it per cycle, by hysteresis, independent of the energy that is consumed in it by eddy currents. Of course, if you have any curve which starts from the zero point and rises up in that way, and if you take arbitrary distances like this in the form of a, a^2, a^3, and so on, then if you want to find out whether that curve follows any such law as

$$Y = b \; X^n$$

you have only got to mark off the ordinates corresponding to those abscissæ, and to see if with the powers of a along X you have a constant ratio from one to another in the ordinates. If you do, that ratio will be a^n. In this case, if a is 2, a^n is almost exactly 3. For the 1.6th power of 2 is 3.03, which means that if you double the maximum magnetization in a piece of wrought-iron, you will have 3.03 times the hysteresis loss, and this is a simple way of stating the results which Mr. Steinmetz has pointed out.

MR. STEINMETZ:—As pointed out by Mr. Kennelly, this law of hysteresis gives a very simple numerical meaning. It means that by doubling the magnetization you approximately treble the hysteretic loss and quadruple the eddy loss. So if you make but two tests therefrom, you can find out the amount of energy consumed by eddies and the amount consumed by hysteresis for any magnetization.

And, in general, you will see at once whether the ratio of the iron loss for doubled magnetization is nearer to three, or rather 3.031, or to four, that is, whether hysteresis or eddies consume more energy in the iron.

I would like to add a few remarks regarding the results of the tests given in the paper. This law of hysteresis is of interest from another point of view:

We all know, now, that energy is always the same and indestructible, and merely changes its form and appearance, so that a certain quantity of any kind of energy converted into any other kind of energy always gives an exactly determined amount of the other form of energy, which we call the law of conservation of energy.

But this law of conservation of energy needs a certain restriction or, rather, addition, because every conversion of one form of energy into another is not possible, but only those where the value of a certain integral, called by Clausius the "entropy," is positive or more correctly, is *not negative*, though the case, that the integral of entropy equals zero, hardly exists in nature otherwise but as mathematical fiction, or, in plain English, only those conversions whereby the sum of the latent heat of the universe increases.

According to this law of entropy, if the complete conversion of one form of energy into another is possible, the opposite conversion is not completely possible. Or if we convert a certain amount of one form of energy into another form of energy, and this back again into the first form of energy, which we call a cyclic conversion of energy—we do not get back the original amount of energy, but less, and a part of the energy has been lost; that means, converted into and dissipated as heat.

Therefore no complete cyclic conversion of energy exists, but by any such cycle the amount of available energy has decreased by that fraction that was converted into heat.

Now, these cyclic conversions of energy are of great importance in nature.

For instance, a moving pendulum, an oscillating spring, a discharging condenser completes cyclic processes. In the moving pendulum, continuously kinetic mechanical energy is converted into potential mechanical energy, when it moves from the vertical position into its greatest elongation, while when moving from elongation into vertical position its potential energy is reconverted into kinetic energy, thereby completing a cycle, so that in vertical position all the energy is kinetic, in elongation all the energy potential.

In the same way, in the oscillating spring, a cycle is performed between potential energy of elasticity and kinetic energy of motion, in the discharging condenser between electrostatic and electrodynamic energy, and that the pendulum and the spring come to rest, and the condenser discharges, is due to the continuous loss of energy by dissipation as heat, caused by the law of entropy.

Now, in none of these cyclic conversions of energy, so far as I know, was the law known, which determines and analytically formulates the loss of energy by conversion into heat. The electro-magnetic cycle is the first one where in the law of hysteresis, this law of dissipation of energy by heat, finds an analytical formulation.

In the alternating electromagnetism we have such a cyclic conversion of energy from electric into magnetic energy and back. Magnetism represents a certain amount of stored up or potential energy determined by the integral

$$\int F\, d\, B$$

Now, as long as the magnetism increases, electric energy is transferred from the electric current and converted into potential magnetic energy. While the magnetism decreases, potential magnetic energy is reconverted into electric energy, and appears in the electric circuit as E. M. F.

But the full amount of energy is not given back to the electric circuit, but less. Less by that amount that has been converted into heat by hysteresis.

Hence the law of hysteresis is the dependence of the integral of entropy in the electromagnetic cycle, upon the intensity of magnetization, and therefore of interest.

DR M. I. PUPIN:—I agree fully with Mr. Steinmetz's last remarks that no process in nature is perfectly reversible and that the phenomenon of magnetic hysteresis is only a special case of the irreversibility of natural processes. It is only a special case of the general law which was first announced by the late Professor Clausius, the law namely that the entropy of the universe is tending toward a maximum, that is, that there is a certain function of the properties of matter of the universe which increases as the amount of heat energy increases in the universe. Now, as in every process there is a certain amount of energy converted into heat, the amount of heat in the universe is continually increasing. Therefore the entropy is continually increasing and therefore steadily approaching its maximum. Professor Rankine made a guess as to how many years would elapse before the whole energy of the universe will be converted into heat, when there will be no life, no natural phenomena excepting heat vibrations. It is very far off yet

Closely connected with this magnetic hysteresis is, I think, the so called electro-static hysteresis. Of course experimental researches in this field have not been carried on far enough yet, to enable us to speak with any definiteness, but still it is beyond all doubt that if you polarize a dielectric and depolarize it again, a certain amount of heat is developed. I think one of the obstacles to the commercial introduction of the condenser, is its getting hot. Now some think it gets hot on account of the convection currents which are passing between the plates of the condenser by means of the air currents and the dust that is in the air; but if you use paraffine so that it will prevent those convection currents, even then you will observe heat developed in the paraffine which must be attributed to the same cause which develops heat when iron is magnetized and demagnetized; that is hysteresis.

Polarization and depolarization of paraffine, and in fact any other dielectric, is not a perfectly reversible process.

Allow me now to comment upon a few points brought up in Mr. Steinmetz's paper. I always believed thoroughly in Professor Ewing's views with regard to the following experimentally well supported assumption, namely that in very low magnetizations the act of magnetizing and demagnetizing is practically reversible, and that when a high point of saturation, say 24,000 or 25,000 lines per square centimetre is reached, that after that the loss due to hysteresis does not increase. I do not see why it should increase, because after that the iron does not receive any stronger magnetization. The additional lines of force after passing the saturation point are due to the increased magnetization of the air itself, and that magnetization is practically reversible.[1] I see that Mr. Steinmetz has found out an increase, independent of the degree of saturation. There is a discrepancy, and I am inclined to side with Professor Ewing, until I am convinced by Mr. Steinmetz that his method of measurement and observation could not be objected to in any particular whatever. Unfortunately, Mr. Steinmetz has not discussed his method so that one can examine it critically. He has given the general idea, the instruments employed, etc., but there is no discussion of the theory of the method, and also of the probable percentage of his errors of observation. I am sure that Mr. Steinmetz will do that at some future time. It would be very interesting and very important indeed to know whether that disagreement is in favor of Mr. Steinmetz or of Professor Ewing.

There is on page 49 a discussion of the variation of the hysteresis loss with the load. In that discussion Mr. Steinmetz says as long as the secondary current is open, the form of the wave of the primary current may not be a sine curve; but that when the secondary current is started, the wave of the magneto-motive force is forced into the shape of the sine curve on account of the reaction of the secondary current. Now I would beg to disagree with Mr. Steinmetz; I think it is just the opposite. It does not make any difference what the electromotive force is, as long as there is a very large self-induction in the circuit,—as there certainly is in the primary circuit as long as the secondary is open, the wave of the primary circuit is independent of the wave of the impressed electromotive force and is practically a sine wave. But when the secondary circuit is closed, then the impressed electromotive force, being assisted by the electromotive forces in the secondary circuit, asserts itself and gives the primary current its own shape, and the stronger the secondary current, the larger assistance the primary impressed electromotive force gets from it. The secondary current aids the primary impressed

1. A. E. Kennelly, on "Magnetic Reluctance" TRANSACTIONS, vol. viii. No. 11, p. 500.

E. M. F. to assert itself and force the primary current into its shape, that is, the shape of of the impressed E. M. F. That can be proved very easily both from theoretical and practical standpoints. So that I do not see the force of Mr. Steinmetz's argument.

MR. STEINMETZ :—The method used in my tests was the well-known electro-dynamometer method, as explained in the paper, with some slight modifications to insure the greatest possible exactness in the results.

With regard to the difference between open circuited and fully loaded transformers, I think Professor Pupin misunderstood me. I did not say that the wave of the primary *current* in the transformer under *full load* resembles the sine wave more than with *open circuit*, for that would have been wrong. What I said was that the wave of the *magnetism* and of the *resulting* M. M. F. in the transformer under full load resembles more the sine wave than it does in the open circuited transformer.

Suppose the impressed E. M. F. at the terminals of the transformer differs from the sine shape, differs even considerably. Then the primary current, which at open circuit represents the resulting M. M. F., will differ much less from the sine shape than the impressed E. M. F., being smoothed out and rounded off to a very great extent by the heavy self-induction of the open circuit transformer. For in the moment of any sudden rise of the impressed E. M. F., already a small rise of the primary current and, therefore, of the magnetism, will induce sufficient counter E. M. F. to make a rapid increase of the primary current impossible.

Hence, in the open circuited transformer, the wave of the magnetism will resemble the sine wave more than the wave of impressed E. M. F. But, nevertheless, it must differ from the sine wave if the impressed E. M. F. differs from sine shape. For, as before said, the resulting or current producing E. M. F. and, therefore, the current, is rigidly determined by the small difference of impressed and induced E. M. F., and the induced E. M. F. must therefore have a shape very similar to the impressed E. M. F., hence differing from sine shape the more the impressed E. M. F. differs therefrom.

Now, the induced E. M. F. is the differential quotient of the magnetism. Hence, if the magnetism is a sine wave its differential quotient, the induced E. M. F, has to be a sine wave also and, on the other hand, the more the induced E. M. F. differs from sine shape, the more its integral function, the magnetism, is forced to differ. Indeed, the magnetism may apparently differ, in its absolute value, less from sinusoidal form than the impressed E. M. F., for it is not the instantaneous values of the magnetism which are directly influenced by the shape of impressed E. M. F., but the greater steepness or flatness of the curve of magnetism which is directly caused by the impressed E. M. F. But it is just this difference in the *velocity* of change, that is, in the *quickness* of rise or decrease of the magnetism, and not the magnetism itself, which would have to account for an increased loss by hysteresis. Hence, it is really *not* the difference of the curve of magnetism, from sine shape, but that of the curve of induced and, therefore of impressed E. M. F., which may possibly cause an increase in the loss by hysteresis.

Quite different in the transformer at full load. Indeed, its apparent self-induction is essentially decreased and the primary current will therefore resemble the shape of the impressed E. M. F., and differ from the sinusoidal form, much more than for open circuit.

But at full load the wave of magnetism and of resulting M. M. F. is much more independent of that of primary current and primary E. M. F. It is caused by the combined action of the instantaneous values of primary and of secondary current, and the secondary current, again, is induced by the magnetism. Hence the result will be, if a sudden change of impressed E. M. F. occurs and produces a sudden change of primary current, just as suddenly as the opposite change of the secondary currents will take place, so that the resultant M. M. F. of both combined currents will not change perceptibly, but practically independent of either current, will alternate freely in sinusoidal waves, in spite of any difference in the wave shape of primary and secondary current from the sine law.

And, indeed, a glance over the curves of instantaneous values of the electric quantities in the transformer, as they have been determined, for instance, by Professor Ryan, at Cornell University, and communicated to this Institute some time ago[1], shows a considerable discrepancy at open circuit between the primary current and the sine wave, while in the loaded transformer the secondary E. M. F. and, therefore, the magnetism, almost universally resembles sine shape.

With regard to Ewing's theory of the molecular magnets, I do not say that I disbelieve in it, neither that I believe in it. At the first view, this theory did not seem to agree with the results of my tests, as I said in my paper, but I did not take the time to think it over more completely whether this theory could be made to agree with the tests; my aim was to gather *facts*, being convinced that based upon a large number of facts, a theory will be found in due time to explain them. [See appendix, p. 64.]

DR. PUPIN :—Magnetic force is certainly a resultant of the primary and secondary currents. As long as the secondary is open, the primary current will be a sine wave, practically. It does not make any difference what the impressed electromotive force is of the alternator, and therefore the magneto-motive force will be a sine wave and the magnetic induction will vary like a sine wave If you close the secondary circuit, the self-induction

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1. TRANSACTIONS, vol. vii, p. 1 *et seq.*

in the primary is reduced, and therefore the back electromotive force in the primary is smaller and the impressed electromotive force begins to assert itself more and more and gives to the primary current its own shape. The shape of the secondary current, as long as the secondary's resistance is very large and the secondary current is small—that, too, is practically a sine wave, the primary current being also practically a sine wave, the resultant of the two—that is, the magneto-motive force—must also be a sine wave. But now, if you diminish the resistance in the secondary circuit, that is, increase the load, then the shape of the primary current begins to correspond to the shape of the impressed electromotive force, and also the shape of the secondary current begins to correspond to the impressed electromotive force, and the resultant of the two, the magnetizing current, must also begin to correspond more and more in shape to the impressed electromotive force—that is, the magneto-motive force begins to correspond to the shape of the impressed electromotive force. The same is true of the magnetic induction. We are not to forget that the secondary current does not depend on the rate of change of the primary current only. The relation is a little more complicated. There is a difference in phase between the primary and secondary, varying anywhere between 90 degrees and 180 degrees. When the difference in phase is nearly 180 degrees, that is, at full load, then the primary current and the secondary current correspond to each other almost exactly in shape, and have the same shape as the impressed electromotive force.

MR. STEINMETZ:—I can not yet quite agree with Dr. Pupin. The resultant of two M. M. F.'s of equal shape, but different phase, need not have the same shape, but can have an entirely different form. So for instance the resultant of two very ragged-looking waves can be a complete sine wave. Let us come down to numerical values. Take for instance a 1000 volt alternator, feeding into the primary coil of a transformer. The internal resistance of the primary coil is 20 ω. The current flowing through the primary, at open secondary circuit, a small fraction of an ampere. Hence, what I call the "resulting E. M. F.," that is the E. M. F. which sends the current through the resistance, is only a few volts.

But this "resulting E. M. F.," is the difference of this instantaneous values of primary impressed, and primary induced E. M. F. The difference is only a few volts, the primary impressed E. M. F. = 1000 volts, hence the primary induced E. M. F. must be almost like the impressed E. M. F., and must differ from sine-shape, therefore, if the impressed E. M. F. differs; and if the differential quotient of magnetism, the induced E. M. F., is non-sinusoidal, the curve of magnetism is non-sinusoidal also.

In the transformer at full load the current and therefore the difference between induced and impressed E. M. F. is much greater, the induced E. M. F. is therefore much more independent of the impressed E. M. F., the more, the greater the load is, hence the curve of magnetism alternating freer than at open circuit, and therefore more approximating the harmonic vibration of the sine-wave

DR. PUPIN :—It does not by any means follow that at every moment the difference between the impressed E. M. F. and the back E. M. F. is small when average value of the current is small, and that is the point in your argument. And even if it is I do not see how that can prove that the shape of the current and the impressed E. M. F. are the same.

MR. STEINMETZ:—We have seen that the effective value of the current, and therefore the effective or average value of the difference of primary impressed and primary induced E. M. F. must be small. This indeed does not prove that some of the instantaneous values of this difference may not be considerable. But first, this could be only the case with very few values, because, if for any great length of time the current were considerable, this would show in the average or effective value, the more, as this is the average of the squares of instantaneous values.

On the other hand, to make the current considerable only for a moment, while immediately before and after it is small, either the induced E. M. F. must suddenly decrease enormously, and the next moment increase just as suddenly—which is impossible, because it is the differential quotient of magnetism—or the primary E. M. F. had to rise and decrease again very suddenly, and such a sudden rise, and immediately afterwards decrease of primary impressed E. M. F., not only is an electro dynamic alternator unable to produce, but no electric circuit would permit a current of such enormously large value and short duration to pass. Hence we can from the small value of effective primary current, conclude that also its instantaneous values without exception must be small.

DR. PUPIN :—I do not suppose that a wave which is not a sine, must necessarily be a wave that goes up and down with sudden variations. I think that every good commercial machine is constructed in such a way that the electromotive force is a perfectly smooth curve. There may be small corners, but even those corners are very nicely rounded. Generally speaking it is a sign of good construction of the machine when the impressed electromotive force is a smooth curve—certainly not a curve that has kinks in it. Kinks in the current curve are produced by a harmonically varying resistance. It would be almost impossible to construct a machine so badly as to give kinks in the electromotive force curve. The current may run smoothly, but still be very far from a sine wave. A sine wave is not the only smoothly running wave. There are many other waves that are nice and smooth. The only possibility of having such a current as Mr. Steinmetz described, would be simply to introduce into the circuit a harmonically variable resistance. An arc light circuit represents a harmonically variable resistance, and introduces those complications, the kinks. An arc light machine violates most of the well established rules in dynamo construction, but it does the work of the arc light circuit admirably, and it does it because it encourages kinks and other irregularities in the current wave.

MR. STEINMETZ :—I entirely agree with Professor Pupin, that there is really nowadays almost no possibility of getting such sharp pointed waves of alternating E. M. F. that a difference of the hysteretic loss between open circuit and closed circuit could be expected. And I did not believe myself in this cause of the discrepancy of former tests on transformers under full load and with open secondary circuit. I made this remark only to be absolutely just, and not entirely to reject as erroneous, determinations made by others, but at least to point out a cause which might produce, though not at all likely, a slight difference between the values found under full load and with open circuit.

Indeed, all our modern alternators produce waves very much resembling sine curves, and the only way to get from them such rapidly changing E. M. F's is, as Dr. Pupin pointed out, the introduction of variable resistances, as arc lamps, into the circuit.

But some of the older types of alternators, as, for instance, the Klimenko alternator at the Vienna exhibition, 1882,[1] gave evidently sharp pointed E. M. F.'s, as I found by drawing the curve of instantaneous values of E. M. F. of an alternator of a similar type, where induction was produced by making and breaking the magnetic circuit. As you see, this is a very similar case to that referred to by Dr. Pupin, only that in this case a variable magnetic reluctance and not a variable electrical resistance was introduced into the circuit.

MR. KENNELLY :—It is unfair, perhaps, when we have such a good paper, to offer criticisms upon it, but when it is as likely as this is to become classical I think that in self defense we ought to try to keep it as free from all imperfections as possible. I am taking the liberty of making a criticism on one term Mr. Steinmetz has used. He has spoken of the normal inductance of the coil of his ammeter as so many ohms, and I would suggest that it would be preferable to employ the word impedance, instead of inductance, because an inductance is a henry and an impedance is an ohm, and I think it is a pity to confuse the two ideas.

MR. STEINMETZ :—I did not use the term *inductance* as synonymous with coefficient of self-induction, where it would be expressed in henrys, but I used inductance in the very sense that Mr. Kennelly means with *impedance*.

I intentionally used the term *inductance*, following a proposition which I read once, I do not remember where, but which seemed to me so highly commendable, that I should like to see it introduced in practical engineering.

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1. A remarkable feature was that it consumed 4 H. P. when running under full load, but almost 6 H. P. when running fully excited but without taking current off, that is, without load.

Indeed, the "coefficient of self-induction" gives all the information needed for determining the electric phenomena in inductive circuits. But everybody will concede that it is a tedious, cumbersome work, from the "coefficient of self-induction" to calculate, for instance, the instrument corrections for a whole set of tests made with somewhat differing frequencies. Besides, I think it will be some time before the "practical electrician" will handle the "coefficient of self-induction" just as easily as he now does ohms and amperes.

Let us consider somewhat closer the phenomena in an inductive circuit. If a sine wave of alternating current flows through an inductive circuit, a certain E. M. F. is consumed by opposing E. M. F.'s.

First, by the electric resistance of the circuit, an E.M.F. E_1 is consumed, which is proportional to the current C, with a coefficient of proportionality, R, which is called the true or ohmic resistance, or, in short, the *Resistance* of the circuit.

This E. M. F. is of *equal* (but opposite) phase with the current C:

$$E_1 = R\ C$$

Then by the action of the changing magnetic field of the circuit an E. M. F., E_2 is consumed, which lags one-quarter of a phase, or 90 degrees, behind the current, and is proportional to the current C, with a coefficient of proportionality I, which I call the *Inductance* of the circuit:

$$E_2 = I\ C$$

This inductance, I, is of equal dimension with the resistance R, hence measured in ohms also.

This inductance, I, is proportional also to the frequency of the alternating current. Hence, if I call the inductance for 100 complete periods per second the *Normal inductance* I_0, for any other frequency N the inductance is simply

$$I = \frac{N}{100}\ I_0$$

Now, the "normal inductance" is a constant of the circuit just as well as the "resistance" or the "coefficient of self-induction," and only depends upon the latter by the equation,

$$I_0 = 200\ \pi\ L$$

only that "inductance" is measured in ohms also, therefore most easily combined with the resistance.

The combination of the *resistance*—which determines the E M. F. of equal phase with the current—with the inductance, which determines the E. M F. lagging one-quarter phase behind the current, is the "impedance," or "apparent resistance."

Hence,

$$\text{Impedance} = \sqrt{(\text{Resistance})^2 + (\text{Inductance})^2}$$

The quotient of inductance and resistance is the angle of difference of phase between current and impressed E. M. F.

$$\tan \varphi = \frac{\text{Inductance}}{\text{Resistance}}$$

You see, it is easy to make a person understand that he has in an alternating current circuit two kinds of resistances. a "resistance" which consumes energy and an "inductance" which does not consume energy, and make him calculate the apparent resistance or "impedance" as the hypothenuse of a right-angled triangle, with resistance and inductance as catheti; while the coefficient of self-induction will frighten the "practical man" still for quite a while.

On the other hand, "inductance" is more convenient than "coefficient of self-induction," because expressed in the same dimensions as resistance, in ohms.

I used the term "normal inductance," because in reducing the readings I found it much more convenient than the use of the "coefficient of self-induction," and therefore recommend its use.

MR. WETZLER:—Before moving to adjourn, I would like to move a vote of thanks to Mr. Steinmetz for his admirable and interesting paper this evening.

THE CHAIRMAN:—Gentlemen, it is with feelings of peculiar gratification that I put this motion. I was very glad indeed to hear Mr. Bradley, in his initiatory remarks. speak of the marked excellences of the paper we have heard read, and I was pleased also to hear him remark upon the rarity of such papers in Amica. Mr. Bradley, I think, did our sister societies of Europe more than justice, because it is in but few of the societies over there, and I am speaking of English-speaking countries of course, that we find such papers as this—leaving out the Physical Society and that other in which the most distinguished member of our own profession now presides so ably (I mean the Royal Society), there is none in which papers of this character are of high frequency.

[A vote of thanks was carried and the meeting adjourned.]

APPENDIX:

[COMMUNICATED BY MR. STEINMETZ AFTER ADJOURNMENT.]

Having had time in the last few days to consider more deeply the relation of this law of hysteresis to Ewing's theory of magnetism, I found that this law of hysteresis agrees very nicely with Ewing's theory, giving just the phenomena this theory leads us to expect.

According to Ewing's Theory, for very low M. M. F.'s, forces too small to affect the chains of molecular magnets, the magnetic cycle should be almost reversible, that is, the hysteresis very small or almost nil.

For medium M. M. F.'s, that is M. M. F.'s large enough to break up the chains of molecular magnets, the magnetic cycles must become markedly irreversible, and the hysteresis as function of the M. M. F., must rapidly increase.

For high M. M. F.'s, where the chains of molecular magnets are mostly broken by the superior outside M. M. F., the hysteretic loss, as function of the M. M. F., should be expected to increase slower again and always slower.

This is exactly the case, when the hysteretic loss, follows the law of the 1.6th of the *magnetization* B, as shown best by the affixed curve Fig. 16.

In Fig. 16 the *dotted curve* gives the magnetization B, in lines of magnetic force per cm², as function of the M. M. F. F, in ampere turns per cm.

The *drawn curve* gives the hysteretic loss, in ergs per cm.³ and cycle, calculated by the equation:

$$H = .003507\ B^{1.6}\ ^{(1)}$$

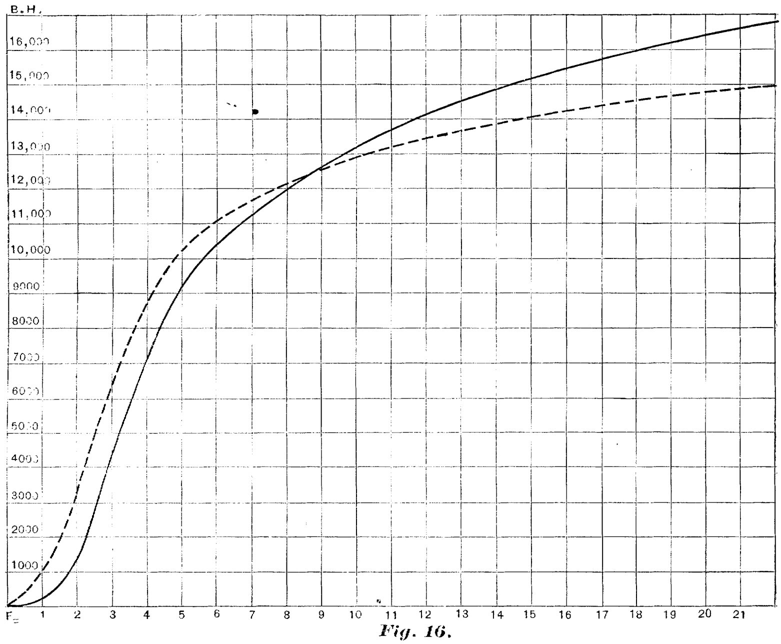

Fig. 16.

but not plotted, as in the former curves, with the magnetizations B as abscissæ, but with the M. M. F.'s: F as abscissæ, that is in the form:

$$H = f(F).$$

As seen, the hysteresis H for low M. M. F.'s, $F = 0 \sim 1$, is very low and almost nil, increases very rapidly for medium M. M. F., $F = 2 \sim 5$, and then increases slower again and always slower, just as Ewing's theory leads us to expect.

Yonkers, N. Y., February 7th, 1892.

1. This curve corresponds to a set of tests not contained in the paper, being made after its completion. I chose this particular set of tests, because it covers a larger range of magnetization than any set of tests given in the paper.