Journal of the
OPTICAL SOCIETY
of AMERICA

VOLUME 27 MAY, 1937 NUMBER 5

Graphical Exposition of the Michelson-Morley Experiment

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(Received January 22, 1937)

THE progress of a light wave or pulse in the Michelson-Morley experiment is most simply comprehended by imagining the interferometer to consist of three intersecting mirrors, two at right angles (end mirrors) and the third (inclined mirror) at 45° to the first two, and considering the manner in which the incident light pulse is broken up into reflected pulses pivoting about the point of intersection of the mirrors.[1] For this purpose we utilize the well-known formula for reflection of light at a moving oblique mirror[2] derived from the Huygens' construction, from which it follows that the reflection deviates from the direction for a stationary mirror by the "aberration angle" $\sin^{-1}(v/c)$, where v is the velocity of the apparatus, and c the velocity of light, plus or minus higher order terms in v/c, of which the second-order term, called by Lodge "the error of reflection," is $\frac{1}{2}v^2/c^2$. As a consequence two pulses leave the point of intersection of the mirrors, inclined to the angle of incidence of the original pulse by 90° plus or minus v/c (if the angle is small), and inclined to each other by the small angle v^2/c^2. If the point of observation is at the end of the inclined mirror away from the point of intersection, and the length of the interferometer is L, the path difference between the sections of the emergent pulses observed is $L(v^2/c^2)$.

In order to exhibit the phenomena the chart (Fig. 1) has been drawn with the arbitrarily high value 0.4 chosen for v/c. In the figure each column presents successive positions in left to right motion of one orientation of the interferometer with respect to the direction of motion. S_0 is the position of the light source when a light pulse is emitted, S its position when the light pulse, after passage through a lens N, enters the apparatus as a plane pulse or wave, represented by a long dashed line. D, R and M are the two end mirrors and the diagonal mirror of the interferometer. The short-dashed and the dotted lines represent the two elements into which the pulse is split after undergoing reflection. W_R and W_D are the emergent pulse fronts coming respectively from end mirrors R and D. The point of observation, occupied by some detector such as a radiometer, is at O. The development of the emergent light pulses can be readily followed in the successive positions taken by the interferometer moving from left to right in each column. Note that for the 90° and 270° orientations the light pulse is *incident at the aberration angle* (due to the motion of the apparatus during the time taken by the light to travel from the source), and leaves with or against the direction of motion; for the 0° and 180° orientations the light enters

[1] Kennedy "Simplified Theory of the Michelson-Morley Experiment," Phys. Rev. **47**, 965 (1935).
[2] Vide Silberstein, *Theory of Relativity*, p. 90.

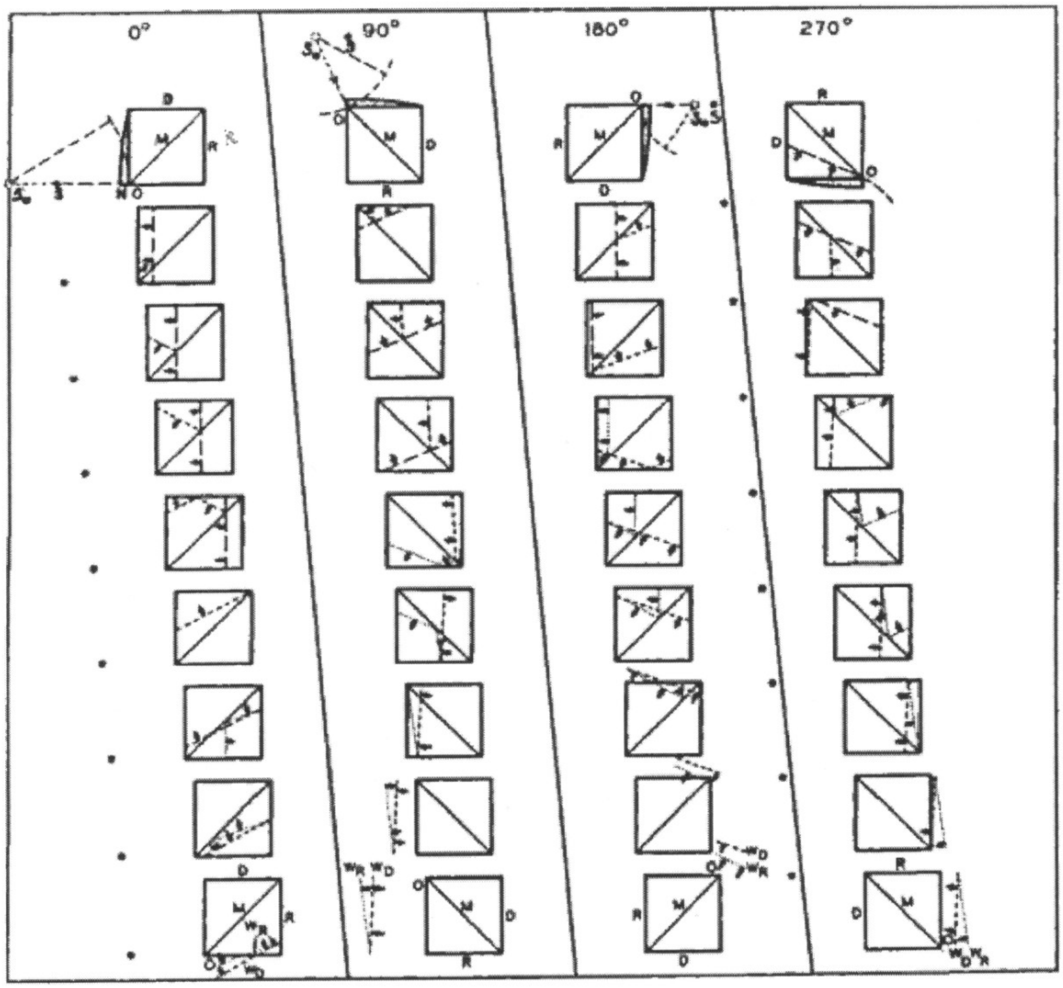

FIG. 1.

with or against the direction of motion, and *leaves at the aberration angle;* in each case leaving with the small inclination v^2/c^2 between the two pulses.

Looking at the final difference of path as the result of the error of reflection, due to motion, the Fitzgerald-Lorentz contraction may be considered as a means of introducing a *tilt* of the inclined mirror just sufficient to correct this error and bring the two emergent pulses into coincidence. The contraction of dimensions necessary to produce this tilt consists of *any* relative contraction in the directions parallel and perpendicular to the direction of motion having the relation $(1-v^2/c^2)^{\frac{1}{2}}:1$. A similar chart to Fig. 1, introducing this tilt, would show the schematic interferometers not as squares, but as rectangles narrow in the horizontal direction. The light pulses, while entering or leaving the apparatus at the aberration angle, as already discussed, would show the two pulses produced by division at the diagonal mirror, leaving in coincidence.[3]

The Fitzgerald-Lorentz contraction is usually understood to be a contraction only in the direction of motion, of value $(1-v^2/c^2)^{\frac{1}{2}}$. By the elementary theory of the experiment the length of path traversed by the pulse traveling parallel to the direction of motion is $2L/(1-v^2/c^2)$, that of the pulse traveling at right angles is

$$2L/(1-v^2/c^2)^{\frac{1}{2}}.$$

As a result of the contraction in the one direction,

[3] See appendix for proof that the Fitzgerald-Lorentz contraction brings the pulse fronts into *exact* coincidence.

the two paths both assume the value

$$2L/(1-v^2/c^2)^{\frac{1}{2}}.$$

They are *equal* for all orientations of the apparatus *at a given velocity*, but both vary together for different velocities.

Another contraction, which produces the same tilt, is a contraction of value $1-v^2/c^2$ in the direction of motion, and of value $(1-v^2/c^2)^{\frac{1}{2}}$ in the direction perpendicular to the motion. This results in both path lengths being *always* $2L$, *irrespective of orientation or velocity*.

The original form of the Michelson-Morley experiment, in which both interferometer arms are of equal length, is equally well satisfied by either kind of contraction. In the experiment of Kennedy and Thorndyke,[4] where the interferometer arms are of unequal length, this is not the case. The null result of that experiment, as the apparatus presumably assumed different velocities with the motion of the earth over extended periods, *requires an actual constancy of both path lengths at all velocities*. This is satisfied by the second form of contraction above considered, as is shown by Table I which gives path lengths and differences for pulses parallel and perpendicular to the direction of motion, for the 0° orientation, assuming interferometer arms of lengths L and KL.

For the unidirectional contraction the path difference ΔP is a function of the velocity, except for the case of $K=1$. The relations shown in the table are embodied in a graphical presentation of the path lengths for all orientations, in Fig. 2. (The radicals of the table have been expanded to the 2nd order for insertion in the diagram.)

Kennedy and Thorndyke actually performed their experiment as a test of the hypothesis of Larmor and Lorentz that a moving light source is reduced in frequency in the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$ whereby the product of distance by measured time would remain constant at all velocities, if the contraction were the unidirectional one. The experiment is, however, equally well explained by assuming an unequal contraction of the apparatus in two directions, and no change in time reckoning.

The line of analysis may be pursued further to yield the following general formulae:

$[(1-v^2/c^2)^{\frac{1}{2}}]^{n+1}$ for contraction in the direction of motion

$[(1-v^2/c^2)^{\frac{1}{2}}]^{n}$ for contraction at right angles to the direction of motion

$[(1-v^2/c^2)^{\frac{1}{2}}]^{1-n}$ for reduction in clock frequency.

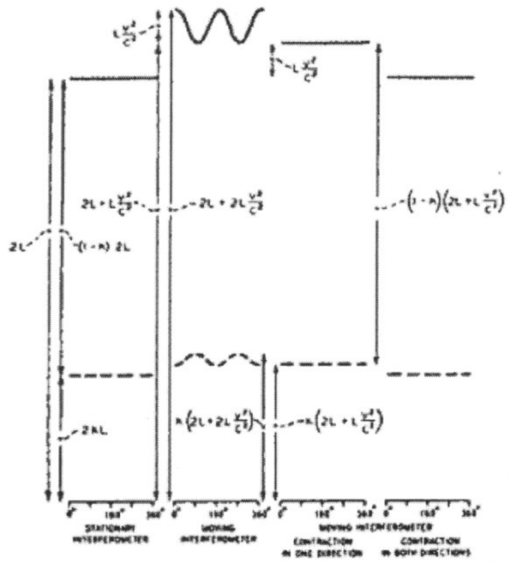

Fig. 2.

[4] Kennedy and Thorndyke, "Experimental Proof of the Relativity of Time," Phys. Rev. **42**, 400 (1932).

TABLE I.

	Path Length in Direction of Motion	Path Length Perpendicular to Motion	Difference of Path
(1) No contraction	$P_1=\frac{2L}{(1-v^2/c^2)}$	$P_2=\frac{2KL}{(1-v^2/c^2)^{\frac{1}{2}}}$	$\Delta P=L\frac{v^2}{c^2}$
(2) Contraction in one direction	$P_1=\frac{2L}{(1-v^2/c^2)^{\frac{1}{2}}}$	$P_2=\frac{2KL}{(1-v^2/c^2)^{\frac{1}{2}}}$	$\Delta P=\frac{2L(1-K)}{(1-v^2/c^2)^{\frac{1}{2}}}$
(3) Contraction in both directions	$P_1=2L$	$P_2=2KL$	$\Delta P=2L(1-K)$

Using these formulae in the Kennedy-Thorndyke experiment we get for the *measured* time interval between arrival of reflected pulses at the origin

$$\Delta T=\left(\frac{L}{c}\frac{[(1-v^2/c^2)^{\frac{1}{2}}]^{n+1}}{1-v^2/c^2}-\frac{kL[(1-v^2/c^2)^{\frac{1}{2}}]^{n}}{c(1-v^2/c^2)^{\frac{1}{2}}}\right)\times[(1-v^2/c^2)^{\frac{1}{2}}]^{1-n}$$

$$=(L/c)(1-k).$$

There is thus always a possible clock rate to make the time interval measure constant, whatever contractions, in the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$ in the two principal directions are assumed. This shows that the experiment of Kennedy and Thorndyke (and any experiment depending on light signals returned to the origin by reflection) is not by itself a test of the Larmor-Lorentz hypothesis; it must be supplemented by independent support for the choice of the exponent n in the above equation.[5]

Attention may be called finally, to the fact that while the contractions of dimensions which have been considered result in equalizing the light *path lengths* in the two arms of the interferometer, the wave fronts still enter or leave the apparatus *at an angle* to the axis, namely the aberration angle. The contraction merely compensates for the deviation of the reflections from the aberration angle. Conditions are not therefore *in all respects* identical in a stationary system and a moving system experiencing a Fitzgerald-Lorentz contraction.

Appendix

Proof that the Fitzgerald-Lorentz contraction exactly compensates the error of reflection

In the schematic interferometer (Fig. 3) let the light be incident from left to right, which is also the direction of motion of the apparatus. The stationary dimensions of the apparatus are $L\times L$; in motion at the velocity v, the length in the direction of motion is $L(1-v^2/c^2)^{\frac{1}{2}}$.

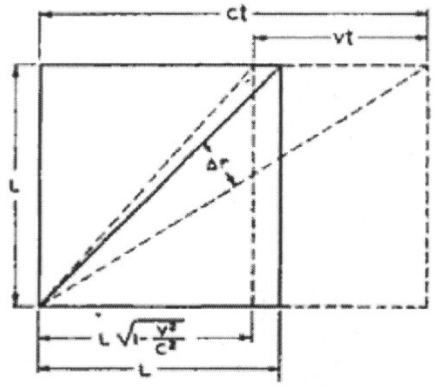

Fig. 3.

Let t be the time taken by the light pulse to travel from its point of first incidence on the diagonal mirror to the point of intersection of the diagonal and plane mirrors, the latter point having moved the distance vt. We then have

$$ct=vt+L(1-v^2/c^2)^{\frac{1}{2}}$$

or

$$L^2=c^2t^2(1-v/c)/(1+v/c).$$

Now the angle Δr, by which the moving diagonal mirror differs from $\pi/4$ is one-half the angle θ, by which the reflected light will deviate in direction from $\pi/2$. We have the trigonometrical relation

$$\frac{\sin\Delta r}{vt-L(1-(1-v^2/c^2)^{\frac{1}{2}})}=\frac{1/\sqrt{2}}{(c^2t^2+L^2)^{\frac{1}{2}}}.$$

Substituting the value of L we get

$$\sin\Delta r=\frac{(1+v/c)^{\frac{1}{2}}-(1-v/c)^{\frac{1}{2}}}{2}=\sin\theta/2,$$

from which

$$\sin^2\theta/2=\frac{1-(1-v^2/c^2)^{\frac{1}{2}}}{2}$$

and

$$\cos^2\theta/2=1-\sin^2\theta/2=\frac{1+(1-v^2/c^2)^{\frac{1}{2}}}{2},$$

$$\sin\theta=4\sin^2(\theta/2)\cos^2(\theta/2)=v/c;$$

that is, the deviation of the reflected pulse front from its direction for stationary apparatus is exactly at the aberration angle. The proof is obviously the same for any contractions of the apparatus dimensions in the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$.

[5] Kennedy and Thorndyke recognize, in their introductory paragraphs, that any contractions, having the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$ are equally valid for explaining the original Michelson-Morley experiment, but restrict their attention to the unidirectional one in interpreting their result as ". . . proof of the relativity of time."