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Eddington's mass-luminosity relation and the laws of thermodynamics

Stephen J. Crothers[1,a)] and Pierre-Marie Robitaille[2,b)]
[1]*PO Box 1546, Sunshine Plaza 4558, Queensland, Australia*
[2]*Department of Radiology and Chemical Physics Program, The Ohio State University, Columbus, Ohio 43210, USA*

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Abstract: Ever since its formulation by A. S. Eddington, the mass-luminosity relation has been viewed as a triumph for theoretical astronomy and astrophysics. The idea that the luminosity of the stars could be controlled solely by their mass was indeed a revolutionary concept. The proof involved two central aspects: (1) the belief that stars could be treated as ideal gases in hydrostatic equilibrium, and (2) that the opacity of Capella could be used as a reference mark applicable to other stars. Yet, when the mass-luminosity relation was advanced, no thought was given to the need for thermodynamic balance. Within thermodynamic expressions, not only must the dimensions (hence units) be consistent on each side of the equals sign, but the extensive nature of the properties must also balance. Namely, thermodynamic expressions must be balanced by properties which are extensive to the same degree. In this regard, mass is an extensive thermodynamic property and can be represented by a homogenous function of degree 1. Conversely, the luminosity of a star is neither extensive nor intensive, but rather can be represented by a homogenous function of degree 2/3. Consequently, the mass-luminosity expression is thermodynamically unbalanced and stands in violation of the laws of thermodynamics. © *2019 Physics Essays Publication.*
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Résumé: Depuis sa formulation par A. S. Eddington, la relation masse-luminosité a été considérée comme un triomphe pour l'astronomie théorique et l'astrophysique. L'idée que la luminosité des étoiles puisse être contrôlée uniquement par leur masse était en effet un concept révolutionnaire. La preuve impliquait deux aspects centraux: 1) la conviction que les étoiles pouvaient être traitées comme des gaz idéaux en équilibre hydrostatique, et 2) que l'opacité de Capella pouvait être utilisée comme un repère applicable à d'autres étoiles. Pourtant, lorsque la relation masse-luminosité a été mise au point, la nécessité d'un équilibre thermodynamique n'a pas été prise en compte. Dans les expressions thermodynamiques, non seulement les dimensions (donc les unités) doivent être cohérentes de chaque côté du signe égal, mais la nature extensive des propriétés doit également être équilibrée. À savoir, les expressions thermodynamiques doivent être équilibrée par des propriétés extensives au même degré. À cet égard, la masse est une propriété thermodynamique extensive et peut être représentée par une fonction homogène de degré 1. Inversement, la luminosité d'une étoile n'est ni extensive ni intensive, mais peut plutôt être représentée par une fonction homogène de degré 2/3. En conséquence, l'expression masse-luminosité est thermodynamiquement déséquilibrée et constitue une violation des lois de la thermodynamique.

Key words: Mass-Luminosity Relation; Astronomy; Astrophysics; Thermodynamics.

I. INTRODUCTION

Pondering the nature of the stars in 1911, Halm was the first to advance that "intrinsic brightness and mass are in direct relationship."[1] Ejnar Hertzsprung eventually echoed Halm,[2] also noting that the brightness of a star could depend on its mass.[3,4] However, it was Arthur Stanley Eddington[5–7] who first derived a mathematical relationship between the mass and luminosity of the stars based on ideal gases in hydrostatic equilibrium. There was only one additional requirement; namely, that the resulting line must pass through mass and luminosity values for the star Capella.[5–7] Eddington required that this star's internal opacity be shared by all others on the main sequence.[5–7] The apparent success of the derivation seemed to offer proof that Eddington's entire approach was valid—the stars could be treated as ideal gases. But in fact, he initially believed that only the giants could be considered in such fashion.[7]

Since that time, Eddington's ideas have come to guide progress in stellar structure and evolution, his mass-luminosity relationship[2] playing a central role in astronomy and astrophysics. Yet, despite the wide modern acceptance of Eddington's mass-luminosity relationship, James Jeans strongly disputed the idea. His objections and ensuing battle with Eddington were based on a fundamentally different approach to how science should be developed.[4] Jeans argued that "…there is no general relation between the masses and

[a)]sjcrothers@plasmaresources.com
[b)]robitaille.1@osu.edu

luminosities of stars…."[8] Milne summarized the situation in these words: "No words are needed to praise Eddington's achievement in calculating the state of equilibrium of a given mass of gas, and in calculating the rate of radiation from its surface. What was wrong was Eddington's failure to realize exactly his achievements: he had found a condition for a star to be gaseous throughout; by comparison with the star, Capella, he had evaluated the opacity in the boundary layers; and he had made it appear unlikely that the stars in nature were gaseous throughout. His claims were the contrary; he claimed to have calculated the luminosity of the existing stars; he claimed to show that they were gaseous throughout; and he claimed to have evaluated the internal opacity of the stars. Jeans deserves great credit for being the first critic to be sceptical about these claims of Eddington's theory, in spite of the attractive plausibility with which the theory was expounded. I think that even today there is much misconception amongst astronomers about the status of Eddington's theory."[9]

Various characteristics of stars are currently derived from the mass-luminosity relation, such as stellar lifetimes and stellar masses.[10,11] However, careful examination of the mass-luminosity equation reveals that it constitutes a violation of the laws of thermodynamics.

II. EDDINGTON'S MASS-LUMINOSITY EQUATION AND THERMODYNAMICS

The mass-luminosity relation is an empirical plot correlating stellar luminosity with mass for main sequence stars. In 1924, Eddington, in advocating the theory of gaseous stars, derived this expression, finding to his surprise[5,6] that this expression could be made to fit the empirical plot by forcing the line derived to pass through the star Capella. This was accomplished by inferring that all stars shared Capella's opacity.[5–7]

However, the resulting expression did not fit the entire main sequence. Consequently, today, Eddington's work has been modified using certain power relations that are restricted to corresponding portions of the main sequence. Setting L for luminosity and M for mass, astronomy maintains that for the lower main sequence, $L \propto M^{1.6}$ and for the upper main sequence $L \propto M^{5.4}$.[12] In general,[13] $L \propto M^n$, $1 < n < 6$, with the value of n chosen *ad hoc* to fit the desired portion of the main sequence mass-luminosity plot. For Sun-like stars, $n = 4$, and for Eddington's overarching curve, $n = 3$.

Still, Eddington's approach to this problem is worth reviewing. First, one can begin by denoting the total pressure, P, within a gaseous star. Then

$$P = p_G + p_R, \tag{1}$$

where p_G is gas pressure and p_R is radiation pressure.[6] Next, the Stefan-Boltzmann law relating luminosity of a thermal emitter to its temperature and surface area can be invoked

$$L = \varepsilon\sigma A T^4, \tag{2}$$

where ε is emissivity ($0 \le \varepsilon \le 1$, a unitless property of the emitter), σ is the Stefan-Boltzmann constant, A is the surface area, and T is the temperature. For a blackbody $\varepsilon = 1$, in which case,

$$L = \sigma A T^4. \tag{3}$$

Note that in this expression, temperature is an intensive thermodynamic property.[14] However, area and luminosity are neither intensive nor extensive.[14] But the quotient of luminosity with area (L/A) is intensive. Equations (2) and (3) are therefore thermodynamically balanced, since T (hence also T^4) is intensive. By combining Eq. (3) with gravitational acceleration,

$$g = \frac{GM}{r^2}, \tag{4}$$

employing the hydrostatic equation

$$\frac{dP}{dr} = -g\rho, \tag{5}$$

and the equation of radiative equilibrium

$$\frac{dp_R}{dr} = -\frac{k\rho H}{c}, \tag{6}$$

in spherical symmetry, $A = 4\pi r^2$, with the radiation power per unit area $H = L/A$, Eddington[6] arrives at the mass-luminosity relation

$$L = \frac{4\pi c G M(1-\beta)}{k_0}, \tag{7}$$

where c is the speed of light in vacuum, k_0 is a constant stellar opacity, G is the constant of gravitation, and M is the stellar mass. The β term is a constant pure number related to total pressure P, gas pressure p_G, and radiation pressure p_R, by

$$\begin{aligned} p_R &= (1-\beta)P, \\ p_G &= \beta P. \end{aligned} \tag{8}$$

Equation (7) is not restricted to "a perfect gas."[6] Even so, it is already clear that Eq. (7) stands in violation of the laws of thermodynamics because luminosity L is neither intensive nor extensive, whereas mass M is extensive. Luminosity is a homogeneous function of degree 2/3 whereas mass is a homogeneous function of degree 1.[14] Since the stellar opacity is intensive and all the other terms on the right side of Eq. (7) are constants, Eq. (7) is not thermodynamically balanced.

To relate Eq. (7) to the gaseous theory of stars, Eddington[6] invoked, outside its proper setting, the equation for an ideal gas (which is thermodynamically balanced), in the form

$$p_G = \frac{\Re}{\mu}\rho T, \tag{9}$$

where p_G is gas pressure, $\Re$ is the universal gas constant, μ is molecular weight in terms of the hydrogen atom, ρ is density, and T is absolute temperature. The equation for radiation pressure is[6]

$$p_R = \frac{1}{3} a T^4, \tag{10}$$

where a is a constant (the Stefan-Boltzmann constant). Combining Eqs. (8)–(10) Eddington[6] obtained

$$P = \frac{aT^4}{3(1-\beta)}. \tag{11}$$

Elimination of T then yields

$$P = \left[\frac{3\Re^4(1-\beta)}{a\mu^4\beta^4}\right]^{\frac{1}{3}} \rho^{\frac{4}{3}}, \tag{12}$$

which is a polytropic form. From this Eddington[6] deduced that[c)]

$$1 - \beta = \frac{0.00309 M^2 \mu^4 \beta^4}{M_\Theta^2}, \tag{13}$$

where M_Θ is the mass of the Sun. Thus, for any given gaseous star of mass M and molecular weight μ, the related value for β can be found from the corresponding solution to Eq. (13), then substituted into Eq. (7) for luminosity.

Putting Eq. (13) into Eq. (7) gives

$$L = \frac{0.01236\pi c G \mu^4 \beta^4 M^3}{k_0 M_\Theta^2}. \tag{14}$$

This equation is not thermodynamically balanced either, which can be easily seen by dividing through by stellar surface area $A = 4\pi r^2$ to yield

$$\frac{L}{A} = \frac{0.00309 c G \mu^4 \beta^4 M^3}{k_0 M_\Theta^2 r^2} = \varepsilon\sigma T^4. \tag{15}$$

Luminosity per unit area is intensive, by the Stefan-Boltzmann law, since temperature is intensive, but $M^3/M_\Theta^2 r^2$ is neither intensive nor extensive because mass is extensive (a homogeneous function of degree 1) and radius is not extensive (a homogeneous function of degree 1/3).[14] The right side of Eq. (15) equates temperature, which is intensive, to a combination of terms that is not intensive. Hence, Eqs. (7) and (14) are invalid.

The thermodynamic inconsistency arises by the incorporation of luminosity with gravitational acceleration via Eqs. (5) and (6). The assumption that luminosity, a surface phenomenon, can be combined with the acceleration due to gravity, a bulk phenomenon, although the bulk is required to facilitate both cases, is false, as it leads directly to insurmountable violations of the laws of thermodynamics by producing nonintensive temperature. In similar fashion, the assumption that gravitational potential energy can be combined with the kinetic energy of an ideal gas to determine temperature stands in violation of the kinetic theory of gases and the laws of thermodynamics, producing perpetual motion machines of both the first and second kind,[15,16] which are forbidden by the laws of thermodynamics. A gas cannot compress itself (i.e., "gravitationally collapse") to do work on itself and raise its own temperature. It is for this reason that the temperature equations for stars are thermodynamically unbalanced,[15–17] and therefore inadmissible. In quite similar fashion, the equations advanced for black hole temperature and black hole entropy violate the laws of thermodynamics.[15–26]

III. GENERALIZATION OF EDDINGTON'S MASS-LUMINOSITY EQUATION

In hydrostatic equilibrium, the maximum luminosity a star can have is given by the Eddington Limit[27]

$$L_{Ed} = \frac{4\pi G c}{\bar{\kappa}} M, \tag{16}$$

where $\bar{\kappa}$ is the Rosseland mean opacity. Mass M is extensive but luminosity L_{Ed} is not. The left side is homogeneous degree 2/3 but the right side is homogeneous degree 1. The Eddington Limit is therefore invalid. Denoting the luminosity of the Sun by L_Θ and its mass by M_Θ, Eq. (16) can be written as

$$L_{Ed} = \left(\frac{4\pi G c M_\Theta}{\bar{\kappa} L_\Theta}\right) L_\Theta \frac{M}{M_\Theta}. \tag{17}$$

The generalized mass-luminosity relation is[28–30]

$$L = \alpha L_\Theta \left(\frac{M}{M_\Theta}\right)^n \quad 1 < n < 6, \tag{18}$$

wherein "… n is about 4 for Sun-like stars, 3 for more massive stars and 2.5 for dim red main sequence stars."[28] The constant α is adjustable *ad libitum* according to the mass M in order to obtain a desired fit between plot and curve.[30] The case of $n = 1$, where radiation pressure is said to dominate, is the Eddington Limit. However, the case $n = 1$ is ruled out by astronomy, not by violations of the laws of thermodynamics, but on the basis that such stars are unstable [hence the lower inequality for Eq. (18)]. Comparing Eqs. (17) and (18),

$$\alpha = \left(\frac{4\pi G c M_\Theta}{\bar{\kappa} L_\Theta}\right). \tag{19}$$

Thus, α is altered by adjustment of $\bar{\kappa}$. Furthermore, α is not just an adjustable constant; it has thermodynamic character but is neither intensive nor extensive since $\bar{\kappa}$ is intensive. Consequently, Eq. (18) is thermodynamically unbalanced and therefore invalid because it equates luminosity (homogeneous degree 2/3) on the left, to a combination of terms on the right having homogeneous degree 1/3, easily seen by substituting Eq. (19) into Eq. (18),

$$L = \left(\frac{4\pi G c M_\Theta}{\bar{\kappa} L_\Theta}\right) L_\Theta \left(\frac{M}{M_\Theta}\right)^n \quad 1 < n < 6. \tag{20}$$

[c)]The details of the derivation of Eq. (13) are unimportant for the argument herein.

If R is the radius of the star of mass M, Eq. (20) can be written

$$L = \left(\frac{4\pi G c M_\Theta}{\bar{\kappa} L_\Theta}\right)\left(\frac{3 L_\Theta R M}{3 R M}\right)\left(\frac{M}{M_\Theta}\right)^n \quad 1 < n < 6. \tag{21}$$

Dividing through by the stellar surface area $A = 4\pi R^2$ gives

$$\frac{L}{A} = \left(\frac{4\pi G c M_\Theta}{\bar{\kappa} L_\Theta}\right)\left(\frac{L_\Theta R \bar{\rho}}{3M}\right)\left(\frac{M}{M_\Theta}\right)^n = \varepsilon\sigma T^4 \quad 1 < n < 6, \tag{22}$$

where $\bar{\rho}$ is the mean stellar density. This can be simplified as follows:

$$\frac{L}{A} = \left(\frac{4\pi G c}{\bar{\kappa}}\right)\left(\frac{R\bar{\rho}}{3}\right)\left(\frac{M}{M_\Theta}\right)^{n-1} = \varepsilon\sigma T^4 \quad 1 < n < 6. \tag{23}$$

The mean stellar density $\bar{\rho}$ is intensive but the radius R is not. The right side equates intensive temperature to a combination of terms that is not intensive. Hence, Eq. (21) is invalid for all values of n. Consequently Eq. (18) is also invalid for all values of n. Setting $n = 1$ in Eq. (23) and multiplying through by area A recovers the Eddington Limit.

Even more extreme *ad hoc* adjustments to Eddington's theoretical mass-luminosity equation have been proposed; for example, that due to Cuntz and Wang[31] for nearby late-K and M dwarf stars on data sampled by Mann *et al.* [32] as calibration for distant late-type stars

$$\begin{aligned} L &= L_\Theta\left(\frac{M}{M_\Theta}\right)^{n(M)}, \\ n(M) &= -141.7M^4 + 232.4M^3 - 129.1M^2 \\ &\quad + 33.29M + 0.215, \\ &\quad 0.20M_\Theta < M < 0.75M_\Theta, \end{aligned} \tag{24}$$

in stark revelation of the mere curve fitting nature of the whole exercise, with complete disregard for the laws of thermodynamics *a priori*. Mann *et al.*[32] first advanced the following empirical equations for radius, luminosity, and mass, respectively, (in solar units), against effective temperature T_{eff}:

$$\begin{aligned} R_* &= -16.883 + 1.18\cdot 10^{-2}T_{\text{eff}} - 2.709\cdot 10^{-6}T_{\text{eff}}^2 \\ &\quad + 2.105\cdot 10^{-10}T_{\text{eff}}^3, \\ L_* &= -0.781 + 7.40\cdot 10^{-4}T_{\text{eff}} - 2.49\cdot 10^{-7}T_{\text{eff}}^2 \\ &\quad + 2.95\cdot 10^{-11}T_{\text{eff}}^3, \\ M_* &= -22.297 + 1.544\cdot 10^{-2}T_{\text{eff}} - 3.488\cdot 10^{-6}T_{\text{eff}}^2 \\ &\quad + 2.650\cdot 10^{-10}T_{\text{eff}}^3. \end{aligned} \tag{25}$$

Using the expressions for L_* and M_* in Eqs. (25), Cuntz and Wang[31] then derived Eqs. (24).

IV. CONCLUSIONS

The mass-luminosity relation violates the laws of thermodynamics in the very same way as the equations for temperatures of gaseous stars and black holes,[15–17] and for black hole entropy.[15–26]

Perplexed by oddities in the theory of gaseous stars, Eddington[6] remarked

> "*We must conclude either that we have been misled altogether in the theory of the mass-luminosity relation or that in dense stars like the sun the material behaves as a perfect gas.*"

Eddington did not consider the third possibility: Astronomy has been misled in the theory of the mass-luminosity relation, as the laws of thermodynamics reveal, and the Sun and stars are not gaseous (perfect or otherwise).[33] Ultimately, the luminosity of a star depends upon its surface area and its vibrational lattice structure.[33] Only condensed matter has a lattice structure.

Confronted with numerous violations of the laws of thermodynamics, astronomy and cosmology seek to move the laws of thermodynamics[34,35] in order to allow gaseous stars and black holes. They justify their actions through the presumed existence of these objects from mere hypothesis. Yet, the laws of thermodynamics are revealing that the theories of gaseous stars and black holes are invalid. The laws of thermodynamics exist, ascertained from experimental physics, and cannot be disregarded or arbitrary altered in order to permit theories that violate them; for otherwise any and all violations of thermodynamics can be circumvented by simply changing the laws of thermodynamics at will. Astronomy and cosmology have built themselves upon thermodynamically invalid foundations and must therefore be entirely reassessed.[33,36–38]

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