Do we need the nuclear force hypothesis?

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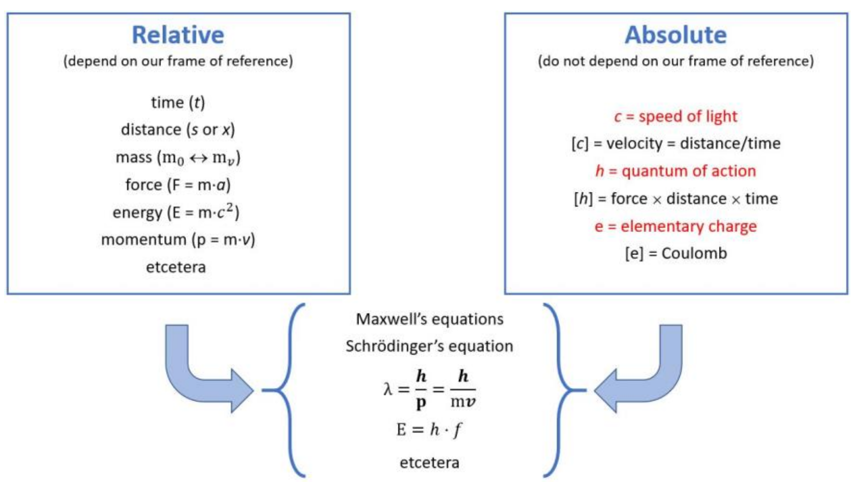
Note to the September 2022 revision: This version is a substantial revision to the first version of this paper (April 2018). It has been expanded substantially because of an additional introduction, which we refer to as *prolegomena*. We also added, removed, or rephrased more text here. Indeed, re-reading it more than a year after we put it on ResearchGate, we were tempted to take it offline because of its poor flow. We hope the added text improves the readability of the paper. The main ideas and the rather mundane conclusions remain unchanged, however. When looking at the revised paper, we now see it has become quite verbose. We recommend the reader to skip the first 😊 smiley. That is where the first version of the paper started. 😊

For the record, this paper was triggered by various exchanges (some of which referenced in the text) on deep electron orbitals, which were relevant in the context of a discussion on hydrino research (which, frankly, we think of as a scam), and also in research related to cold fusion and other presumed low-energy nuclear reactions (probably less of a delusion, although we remain very skeptical on that too). To be very precise here: *we were and remain skeptical on deep (electron) orbitals being possible at all*.

Again, this paper basically summarizes our take on these topics and, as such, this paper offers no new reflections. The revision does not make us think that the paper has improved much. If the reader has to make a choice and wonders where to begin, we recommend our paper on de Broglie's matter-wave. That has all it has to have, including annexes with rather elaborate reflections on the usefulness of wave equations. Those reflections basically boil down to this conclusion: Schrödinger's wave equation is useful (even if it does not incorporate the all-important property of spin), and all others are not. It is, perhaps, useful to note that we also wrote out those reflections as part of the same search for some kind of equation for deep electron orbitals which, as mentioned above, yielded nothing much.

Finally, we note that some research papers start off with a comprehensive list of equations and constants. We thought it might be useful to insert one single diagram to make complicated discussions on what is what easier to digest. It shows the basic distinction between what is *relative* and *absolute* in physics, and how that combines into the equations that we all know so well.



Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.
(Wittgenstein, TLP, 1.1)

Abstract

We discuss the nuclear force hypothesis and explore a (modified) Yukawa potential for it. Our exploration highlights the key issue with Yukawa's proposal and the various models which followed it: Yukawa-style potentials imply a non-conservative force. Such models, therefore, breach the bedrock of classical mechanics: the conservation of (potential and kinetic) energy, linear and angular momentum. These principles can be easily related to the least action principle, which bridges bridges classical and quantum mechanics.[1] As such, we feel no classical or quantum-mechanical model should breach it.

We discuss two obvious solutions to solve this embarrassing issue with Yukawa-style potentials. The first solution would be to introduce a spatially asymmetric potential. The second solution would be to introduce dynamics: if a Yukawa-like nuclear scalar potential would exist, we should probably invent some nuclear vector potential too, and both could and should then combine to provide a picture that would restore the energy, momentum, and physical action conservation principles – both above and below the physical boundary between classical and quantum mechanics, which is set by Planck's quantum of action.

The latter solution (introducing dynamics) would require the elaboration of an equivalent of Maxwell's equations for nuclear force fields, which may or may not be productive in terms of approach but which, in any case, looks rather complicated (read: too complicated for the kind of short exploration we do here).

Going back to the first possible solution (the introduction of a spatial asymmetric potential), we argue that, if there is such thing as a proper *nuclear* potential, and such potential is effectively spatially asymmetric, then it is probably nothing but an electromagnetic dipole field. The nuclear binding energy between the proton and neutron in a deuteron nucleus, for example, is of the order of 2.2 MeV, which can effectively be explained by a dipole field from a neutronic combination within the nucleus. The nuclear potential may, therefore, effectively be nothing but a combination of an electric dipole and the magnetic fields of the (overall neutral) current resulting from the motion of the positive and negative charges and/or the (charged) currents from the motion of the two positive charges.

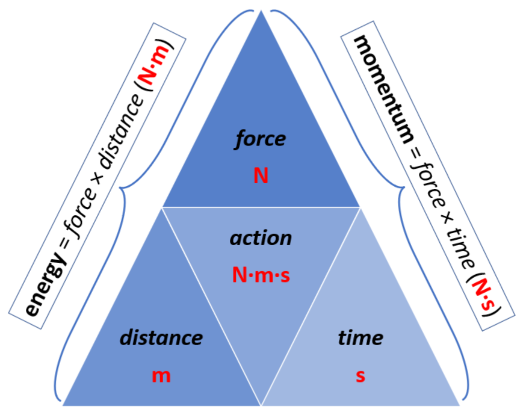
Such potential is, typically, spherically *non*-symmetric but conservative and, as Paolo Di Sia (2018) rightly notes, the *order of magnitude* of the presumed nuclear range parameter is the same as that of the distance which separates the charges (femtometer scale). Hence, *prima facie* measurements do corroborate the hypothesis.

[1] We warmly recommend Feynman's treatment of it. We feel it is no coincidence it is the middle chapter of his three volumes of *Lectures on Physics*. Its style is very engaging because the lecture is (almost) verbatim. Also, while, in an added footnote, he states that "later chapters do not depend on the material of this special lecture" and that it is intended to be "for entertainment only", we will feel it holds the key to understanding many problems, both in classical as well as in quantum mechanics.

To put it simply, the least action principle in quantum mechanics is the quantized version of the classical principle. It establishes Planck's quantum of physical action $h = 6.62607015 \times 10^{-34}$ N·m·s (which, when divided by the ratio of a circle's circumference and its radius (2π), also doubles up as the unit of angular momentum) as *the* physical limit to a mathematical analysis in terms of continuity or infinitesimally small quantities in pretty much the same way as the speed of light sets an upper limit to the velocity any *real* object can have.

However, we still feel rather dissatisfied with the simplicity of the approach. As such, this is and remains an exploratory paper which we will probably use to dig deeper into the issue when time and inspiration would permit us to do so.

Note: As physical dimensions may be confusing (Yukawa himself was very sloppy about them when introducing his model), we think it is useful to present – at the very least – the most basic dimensions of physical action, energy, and momentum. It is a simple diagram, but it shows the *complementarity* between the most basic concepts in both classical as well as quantum physics and may, therefore, help to understand the three basic conservation laws: energy, angular, and linear momentum. It also shows the relativity of space and time once more: space and time are, effectively, not just constructs or categories of the mind. That is what the great philosopher Immanuel Kant told us once, but he is wrong: they are part of *measurable* reality – or reality, *tout court*. We must just be aware different reference frames are different reference frames.



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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Do we need the nuclear force hypothesis?

Prolegomena

The objective of this paper is to sum up our thoughts on what we (and many others, of course) rather vaguely refer to as the nuclear force. The reader who is familiar with our papers will already know that our analysis is not phrased in terms of the Standard Model: the conventional analysis in terms of a weak and a strong force – and the associated bosons (hypothetical virtual particles that are supposed to carry those forces) – does not appeal to us. We think, for example, that there is absolutely no problem to think of fields being quantized and continuous at the same time.[2]

We will say no more about the Standard Model. However, we do want to insert a few thoughts so as to make sure the reader can easily walk through the main body of this paper. We do so in this pre-introduction which, for lack of a better name, we refer to as *prolegomena*.[3] The reader who has read other papers of ours can simply skip it and go straight to the actual introduction of this paper: we just repeat some ideas and insert the references to the papers who have the detail on them. With this warning and advice to the reader, we will now start this pre-introduction.

Our world view is scattered over many papers now. However, we think all of these show that our rather straightforward models of matter-particles are precise and consistent with recent measurements, such as those that come out of JLab's PRad experiment.[4] They also answer questions which cannot be answered by the Standard Model, such as the anomaly in the magnetic moment (which is not an anomaly because the assumption of zero-dimensional charges does not make sense), or the nature of Compton scattering. As such, we think our models are rather solid.

As for our modeling of fields, we developed, among other things, an analogy between (1) the quantized fields that come with superconducting currents and (2) the sub-Planck field oscillations that, in our models, must keep the charge within an electron or a proton in place. We will not summarize the papers in which we do that here.[5] We just want to mention the central question that comes out of such

[2] We did the required effort of at least *trying* to start working ourselves through quantum field theory and gauge theories. Aitchison and Hey (2013, in its fourth edition now) is one of many standard references here but, again, we think such theories – despite them being the standard approach – get off on the wrong foot, and so we quickly tired of reading them.

[3] That is, effectively, a reference to Immanuel Kant's *Prolegomena to Any Future Metaphysics That Will Be Able to Present Itself as a Science*, which was published in 1783. These *prolegomena* are, basically, a summer of his *Critique of Pure Reason*, which he had published two years earlier, but which did not receive much acclaim because it was considered to be too long and winding (I admit that is my rather rude take on it). Just like Kant's prolegomena, mine were also written a year of two later than the text to which it is supposed to be an introduction.

[4] See, for example, our short paper on elementary ring currents. This shows the calculations of the (inertial) mass, radii, and magnetic moments of the proton (and the electron) from first principles. The same paper also briefly explains our view of the photon and the neutrino as carriers of the electromagnetic and nuclear force, respectively. We think no further concepts or hypotheses are needed to explain *measurable reality*.

[5] See our paper on the key concepts related to ring currents and field oscillations, in which we explore the limits of Maxwell's equations and the de Broglie relations. It also explains our interpretation of the (in)famous uncertainty principle, which we think of as a rather simple complementarity principle (to be precise, it is just the physical

exercises, which is this: *what keeps the charge in an elementary ring current in place in the absence of a physical conductor or one or more other charges?*

The immediate answer is this: what keeps the charge in place, is a force acting on a charge. Yes, of course – and we may label such force as electromagnetic or nuclear or as whatever. However, such answer does not answer the deeper question: can we relate such force to some *physicality*?[6]

The honest answer is that we cannot. We can write a lot but, when everything is said and done, we remain stuck in (mathematical) language that describes a strangely fine-tuned *perpetuum mobile*. In ordinary language, this perpetuum mobile is summarized in this statement: **the (orbital or spherical) motion of the charge creates a field, and the field is such that it keeps the charge in *exactly* in the motion that is required to generate that field.** Not approximately, but exactly, thereby satisfying the Planck-Einstein and/or de Broglie relations as well as – apparently – Maxwell's equations.

Two remarks must be made here:

- We only have Maxwell's equations for the time being, and we only have an *electric* charge: there is no such thing as a nuclear charge. Hence, the hypothesis of a nuclear equivalent of the equivalent of the electromagnetic force is rather counterintuitive. In plain language, that amounts to saying that we feel any new force must be considered to be essentially electromagnetic in its nature too.
- However, we must immediately also qualify the statement above: Maxwell's equations apply to charge *densities* rather than pointlike charges.[7] As such, the possibility that some rather special nuclear force fields, at very small scales, and with limited range (otherwise we would have been able to measure these nuclear force fields outside of those small scales) might exist, must be kept open.

[…]

We are tempted to add more remarks, but we cannot say any more without entering the field of ontology. However, that is considered to be a non-scientific field and, hence, we would be stating philosophical rather than physical principles. So, that is what we will, briefly, do in what follows.

Our best guess – but, again, that is a *philosophical* guess rather than a scientific one – is that spacetime itself must have some elasticity which permits two harmonic oscillations only (the electron and proton oscillation, respectively). We have no issue with such conceptualization: if spacetime or the vacuum is associated with properties such as permittivity or permeability, then we see no reason why we would not associate it with a property such as elasticity. We quickly add it is probably useful to distinguish a

equivalent of conjugate relations and variables).

[6] I am very grateful to Dr. Alexander Burinskii, who put this question in very clear terms in early communications when I first started researching the *Zitterbewegung* model of the electron. He effectively wrote the following to us when I first contacted him on the viability on the model: "I know many people who considered the electron as a toroidal photon and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: '*Microgeons with spin*'. Editor E. Lifschitz prohibited me then to write there about *Zitterbewegung* [because of ideological reasons], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?" (email dated 22 December 2018)

[7] See our previously referenced paper on sub-Planck field oscillations.

mathematical and physical spacetime concept: Immanuel Kant told us space and time are categories of the mind (Cartesian or non-Cartesian), but our models – combined with Einstein's general relativity theory – strongly suggest objects come with their own physical spacetime, be it fields or – in the case of elementary particles – the spacetime in which these oscillations *happen*.

The above thoughts are sufficient. Let us talk about the matter at hand – quite literally: we will be talking neutrons. In other words, the reader who – rightly so – skipped all of the verbosity above, should start reading here. 😊

Introduction

We will consistently talk about the neutron and try to develop a neutron model so as to focus our thoughts. The neutron appears as a stable particle in a nucleus only. It does, therefore, not fit with our definition of an elementary matter-particles: these are stable. To be precise, we only consider electrons and protons (and their anti-matter counterparts, of course) to be elementary matter-particles, and we think of their *essence* as charge in motion. Such motion is orbital, and we think of orbital motion as a 2D or 3D oscillation.[8] Let us briefly present our standard model of elementary particles once again, so as to make clear why we cannot apply it to the neutron.

The electron and proton model[9]

This oscillator model of an electron is, essentially, a mass-without-mass model, because we think of it as a *real* oscillation in space. More specifically, we think all of the mass of an elementary particle is the sum of (1) the relativistic mass of the pointlike charge (whose *rest* mass we assume to be zero, and it, therefore, moves about at lightspeed) and (2) the equivalent of the energy in the oscillation. Assuming a circular orbital, this gives us the effective radius of an electron:

$$\left.\begin{aligned} \mathrm{E} &= \hbar\omega \\ \omega &= \frac{a}{c} \end{aligned}\right\} \Rightarrow \mathrm{E} = \frac{\hbar c}{a} \Leftrightarrow a = \frac{\hbar c}{\mathrm{E}} = \frac{\hbar}{\mathrm{m}c} \approx 0.386 \text{ pm}$$

The model itself is summarized by the *wavefunction* of an electron[10]:

$$\psi_{\mathrm{e}} = \frac{\hbar c}{\mathrm{E_e}} e^{\pm i \frac{\mathrm{E_e}}{\hbar} t}$$

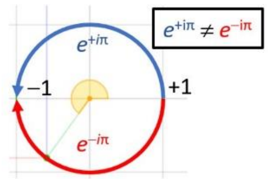
This notation introduces the imaginary unit, which serves as a *rotation operator* and, therefore, denotes

[8] We think de Broglie's frequencies are orbital rather than linear frequencies. Our paper on that (de Broglie's matter-wave: concept and issues) is the most-read of all our writings: we think it shows many of our readers share the view such hypothesis is not too outrageous.

[9] This section was added with the September 2022 revision and overlaps with a development later in the text. However, it is appropriate to already introduce these concepts here.

[10] The reader should make no mistake here: we interpret this wavefunction to represent the electron, which is nothing but Schrodinger's *Zitterbewegung* electron. To be precise, we think of it as modeling the position of the pointlike charge inside of it. As it *zitters* around at lightspeed, and we cannot know its initial condition, we know it must be *somewhere* at *any point in time*, but we cannot predict or measure its position. That is the quantum-mechanical *uncertainty* about it. Nothing more, nothing less. We think H.A. Lorentz was right in his appreciation of the uncertainty principle just before his untimely death: it has nothing to do with a fundamental or *ontological* indetermination (see our short history of quantum-mechanical ideas for a longer discussion).

the plane of oscillation. The sign of the imaginary unit (±) indicates the direction of spin and, interpreting **1** and **–1** as complex numbers (cf. the **boldface** notation), we do *not* treat ± π as a common phase factor: $e^{+i\pi}$ equals $e^{-i\pi}$ in algebra (because both equal −1) but not in geometry! To put it simply, when you go from here to there (+1 to −1, in this particular case), it matters how you get there![11]



I am digressing here, so let me get back to the matter at hand. The above is, basically, our quantum-mechanical electron model.[12] So what about the proton? As mentioned, several times already, we think of the proton oscillation as an orbital oscillation in *three* rather than just two dimensions. We, therefore, have *two* (perpendicular) orbital oscillations, with the frequency of *each* of the oscillators given by $\omega = E/2\hbar = mc^2/2\hbar$ (energy equipartition theorem), and with each of the two perpendicular oscillations packing one *half*-unit of $\hbar$ only.[13] Such spherical view of a proton fits with packing models for nucleons and yields the experimentally measured radius of a proton[14]:

$$\frac{\mathrm{E}}{\mathrm{m_p}} = c^2 = a^2\omega^2 = a^2\left(\frac{\mathrm{m_n}c^2}{2\hbar}\right)^2 \Leftrightarrow a = \frac{4\hbar}{\mathrm{m_p}c} \approx 0.84\ \mathrm{fm}$$

The 4 factor here is the one distinguishing the formula for the surface of a *sphere* ($A = 4\pi r^2$) from the surface of a *disc* ($A = \pi r^2$).[15] We may now write the proton wavefunction as a *combination* of two elementary wavefunctions:

$$\psi_{\mathrm{p}} = \frac{4\hbar}{\mathrm{m_p}c}\cdot\left(e^{\pm i\frac{\mathrm{E_p}}{2\hbar}t} + e^{\pm j\frac{\mathrm{E_p}}{2\hbar}t}\right)$$

A neutron model?

We cannot use the electron or proton model for the neutron – at least *not directly* – because we

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[11] See our paper on *Euler's wavefunction and the double life of –1*, October 2018. This paper is one of our very early papers – a time during which we developed early intuitions – and we were not publishing on RG then. We basically take Feynman's argument on base transformations apart. The logic is valid, but we should probably review and rewrite the paper in light of the more precise intuitions and arguments we developed since then, even if – as mentioned – I have no doubt as to the validity of the argument. In any case, we develop much of the same ideas in a more recent RG paper on the geometry of the wavefunction.

[12] Of course, it is not *ours*. Several authors have further built on Schrodinger's *Zitterbewegung* model. We must, in particular, mention David Hestenes. The problem with reading David Hestenes is that he also invented a new algebra around these models: spacetime algebra (STA). In communications, he urged us to adopt it. However, we see no reason for inventing a different algebra for these models.

[13] Such half-units of $\hbar$ for linearly polarized waves also explains the results of Mach-Zehnder one-photon interference experiments. There is no mystery here.

[14] See our paper on the proton model for more precision in the calculations: the results are well within the confidence interval of the proton radius as established by JLab's PRad experiment.

[15] We also have the same $1/4\pi$ factor in the formula for the electric constant, and for exactly the same reason (Gauss' law).

consider the neutron to combine positive and negative *charge*. We consider the non-zero magnetic moment of the neutron to justify this hypothesis.[16] Because the formalism of the wavefunction, which we introduced above, applies to the motion of *one* charge only, we cannot relate it to our neutron hypothesis.

Now, when we imagine the neutron as a dynamic system of a positive and negative charge, then we can, perhaps, think of a *steady* but *electrically neutral* current.[17] Maxwell's equations[18] then reduce to the

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[16] Again, the neutron is not stable outside of the nucleus. We, therefore, think of it as a composite particle. Of course, the reader who is well versed in the history of nuclear research will probably be aware of Schrödinger's *Platzwechsel* model for nuclei, and so we must say a few words about it (we will also do that in the text itself later on). The *Platzwechsel* theory is an early hypothesis of Schrödinger suggesting that a neutron and a proton swap places or, to be more precise, assemble and disassemble into each other. This may or may not happen, but we see no research supporting this. The Wikipedia article on the Yukawa potential says a few words about it in its history section, but we will let the reader *google* more about it. Indeed, we do not dig into it because modeling the neutron as a combination of a proton and a (nuclear) electron – its decay products – is not in line with our world view here. Indeed, we repeat – once more – that charge is the more fundamental concept when discussing forces and fields. Indeed, a force acts on a *charge*, and we think of *mass* as the *inertia* to a change in the state of motion of the *charge* (or, plural if applicable, charges) inside of a (neutral or charged) particle.

[17] The reader should have no difficulty appreciating the difference between a neutral current and a charged current. A conducting electron in a conductor leaves a positive ion and creates a negative one as it moves through the lattice: the current is, therefore, neutral (the reader may want to review Feynman's analysis of the relativity of electric and magnetic fields here). Hence, we may think of the positive and negative charge in a neutron as playing the same role as the positive and negative ion in a conductor. However, such analogy is very rough. One key difference between a neutral conductor and the supposedly neutral current in a neutron is this: the particle that carries the charge in ordinary neutral currents in a conductor has a *drift* velocity which is of the order of 10^{-5} or even 10^{-6} m/s (*micro*meter) in a copper wire. In contrast, in a mass-without-mass model of elementary particles, we will assume the velocity of the charge to be equal to lightspeed (c).

There is, of course, also the issue of scale. The current in an electron can be calculated as:

$$\mathrm{I} = \mathrm{q_e} f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}$$

That is a humongous value: can we really imagine a household-level current at the sub-atomic scale here? The answer is this: it is consistent with the observed magnetic moment of an electron, for which we quickly add the formula:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

We must also add that electrons are – in normal circumstances – indestructible. Hence, the force that holds them together, so to speak, must be very strong, indeed!

However, the more fundamental point is and remains this: this issue of *scale* may make a simple application of Maxwell's equations *scientifically illegal*. We do not *think* that is the case – on the contrary – but we have to be intellectually honest and admit that their application beyond the limit set by Planck's quantum of action is and remains a speculative conjecture in regard to the range of their validity at atomic or subatomic scales. This also sort of invalidates the assumption that we are actually looking at an electrically neutral current inside of the neutron and, hence, that we can apply simple magnetostatics. However, we soldier on for the benefit of the reader – if only to show where Di Sia goes wrong. This remark is sort of the opposite of saying one cannot just electrostatic potential only to examine what might or might not be going on inside of a nucleus.

[18] We should write Maxwell's equations in integral rather than differential form, but we want to keep the notation rather light here.

equation(s) of *magnetostatics*:

$\nabla \cdot \mathbf{E} = \rho/\varepsilon_0 = 0$	no (net) charge inside the volume and, hence, zero flux of **E** through the (closed) surface
$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t = 0$	the (electrically neutral) ring current generates a *static* magnetic field
$\nabla \cdot \mathbf{B} = 0$	no magnetic charges (no flux of **B**)
$c^2 \nabla \times \mathbf{B} = \boldsymbol{j}/\varepsilon_0 + \partial \mathbf{E}/\partial t = \boldsymbol{j}/\varepsilon_0$	no electric field ($\partial \mathbf{E}/\partial t = \partial \mathbf{0}/\partial t = 0$), steady (neutral) current (***j***), steady magnetic field

The only relevant equation is, therefore, the $c^2 \nabla \times \mathbf{B} = \boldsymbol{j}/\varepsilon_0$ equation. We may add that we can write the magnetic field as the curl of a vector potential(**B** = $\nabla \times \boldsymbol{A}$) and that the magnetic field can be calculated from the current by using the Biot-Savart law. We must quickly note that we think of the charges inside of the neutron as pointlike but *not* infinitesimally small particles which, for the time being, we consider to be spinless.[19] Of course, the neutron itself is *not* pointlike, and does have spin: its charge radius is of the order of a femtometer (10^{-15} m), and we think of it as being determined by the orbital loop(s) of the positive and negative charges.

We should say a few words about deuteron models too, perhaps. Erwin Schrödinger originally considered a *Platzwechsel* model for the deuteron (D = $^2H^+$) nucleus. In classical theory, the deuteron nucleus – which is an ^2H atom without the orbital electron – consists of a proton and a neutron. The *Platzwechsel* model models the deuteron nucleus model as consisting of two protons sharing an electron. However, as explained in more detail in footnote 16 also, we think it is far more productive to think in terms of charge(s) rather in terms of particles. The negative charge would then act as a sort of glue holding the two protons together. A complementary – and, possibly, *alternative* – point of view may be offered by considering the electrically neutral combination of the positive and negative charge to act as an electric dipole generating a $1/r^2$ potential which then traps the third (positive) charge.

An even more radical approach based on the dipole idea, is Di Sia's conception of nucleons as (electrically neutral) *magnetic* dipoles[20], but protons (and, we believe, neutrons) do carry charge, and the repulsive electrostatic force between them can, therefore, not be wished away. The model does stir some thinking, however, because energy difference between the deuteron nucleus (about 1875.613 MeV) and its two constituents (neutron and proton) in their unbound state (939.565 MeV + 938.272 MeV = 1,877.837 MeV) is *negative* and equal to about 2.22 MeV[21], and Di Sia does get energy values of

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[19] Our explanation for the anomalous magnetic moment strongly suggests the charge inside of an electron has a small non-zero *size*, indeed! See our rather elementary paper (pp. 10 ff.) on quantum behavior. This size is the classical electron radius: about 2.818 fm. However, we have a scale issue here when projecting this onto the neutron or proton radius: 2.818 fm is about 3.35 times larger than the radius of a neutron. Hence, the pointlike charge which explains the anomaly in the magnetic moment of a free electron does not *fit* into a neutron.

[20] Paolo Di Sia, *A solution to the 80-year-old problem of the nuclear force*, 2018. We refer to Di Sia's nucleons as zero-charge nucleons because he considers neutral currents only: the electrostatic potential (and the electric field) does not come into play.

[21] Conversely, the energy of a neutron (939.565 MeV) is *larger* than the sum of energies of a free proton (938.272 MeV) and a free electron (0.511 MeV). Such *positive* binding energy (about 0.782 MeV) explains why the (free) neutron (outside of the nucleus) is not stable: it goes into a lower energy state by decaying, which is why the 0.782 MeV is usually referred to as the decay energy. To be precise, the energy difference between a proton and a neutron, which is of the order of about 1.3 MeV, which is about 2.5 times the energy of a free electron. Hence, the energy of the electron would explain only about 40% of the mass difference: the rest (about 60%) is to be explained by some kind of binding energy as well.

the same order of magnitude, which we will discuss below.

Binding energies

The numerical example which Di Sia (2018) provides is for nucleons with an approximate size of 0.5 fm – a rather reasonable ballpark number for the radius of the current loop – which are separated by the typical interproton distance (about 2 fm: this corresponds to the usual value for the range parameter in Yukawa's formula for the nuclear potential[22]).

Interestingly, Di Sia also considers the phase of the currents, which may effectively be in or out of phase, and then calculates energy levels for the magnetic binding using the Biot-Savart law[23], which we can immediately compare with nuclear binding energies. For the mentioned values (0.5 and 2 fm) he gets an energy range between 3.97 KeV and a more respectable 0.127 MeV (the latter value assumes in-phase currents). While this is, without any doubt, significant, it is only 5% of the 2.2 MeV energy difference between the deuteron nucleus (about 1875.613 MeV) and its two constituents (neutron and proton) in their unbound state (939.565 MeV + 938.272 MeV = 1,877.837 MeV). The values get (much) better when changing the parameters (nucleon size and internucleon distance) significantly (2-3 MeV) and, better still, considering paired nucleons creating dipoles acting on other paired nucleons (values up to 5 MeV). The latter point is interesting because, as mentioned above, we may effectively think of a neutron as a paired positive and negative charge: in fact, the neutron is the only nucleon which matches Di Sia's concept of electrically neutral nucleons.

As we are presenting some energy values here, we may note that the energy of a neutron (939.565 MeV) is *larger* than the sum of energies of a free proton (938.272 MeV) and a free electron (0.511 MeV). Such *positive* binding energy (about 0.782 MeV) explains why the (free) neutron (outside of the nucleus) is not stable: it goes into a lower energy state by decaying, which is why the 0.782 MeV value is usually referred to as the decay energy.[24] This 0.782 MeV binding (or decay) energy and the 0.511 MeV energy of the (free) electron add up to the 1.293 MeV energy difference, and so that should be it, right? So did Di Sia solve all problems? Do we have a full-blown model of the nucleus based on electromagnetic theory only here?

Maybe. Maybe not. We request the reader to bear with us and just kindly note the nuclear binding energies are of the same order of magnitude of the electron energy and effectively correspond to dipole field energies at nuclear distance scales, which is why, as Di Sia pointed out in his rather provocative paper, a dipole model makes a lot of intuitive sense. We also ask our reader to note that an added advantage of the dipole model is that it reduces the three-body problem that is inherent to modeling the deuteron nucleus as a combination of *three* charges.

Unlike Di Sia, however, we would rather think in terms of a combined electromagnetic dipole model, rather than reducing all to electric *or* magnetic dipoles, but that might sound like a minor correction to the reader at this point.

The more important question to consider for the reader is this: if the neutron consists of a positive and a

[22] See, for example, Aitchison and Hey, *Gauge Theories in Particle Physics*, 2013 (4th edition), Vol. 1, p. 16.

[23] For a discussion of Di Sia's model, see our paper on an electromagnetic deuteron model.

[24] See, for example, the Wikipedia article on free neutron decay.

negative charge, why does the negative charge (electron) not go sit right on top of the positive charge (proton)? Consider this: perhaps it does, but binding energy is binding energy, and the Planck-Einstein relation (E = *h·f*) tells us the charges might well go and sit right on top of each other – so we would have some kind of *n* = 0 orbital, which is not an orbital at all[25] – but that *we should still have local motion*: a truly *local Zitterbewegung*[26], and the order of magnitude of the frequency is that of the electron ($f_e = \omega_e/2\pi = E/h \approx 0.123\times10^{-21}$ Hz)—but on a femto- rather than a picometer scale.

The femtometer scale needs to be explained, however, and so we do *not* think Di Sia solved all problems. We will, in this paper, effectively want to think of *nuclear* electron orbitals—if only because the assumption of a nuclear force explains the proton (and neutron) radius and, therefore, their rather humongous masses (which suggest humongous forces too!), exceedingly well, as we will show in one of the next sections of this paper.[27] Hence, we are reluctant to drop the idea of a nuclear oscillation altogether, but we readily admit that we should probably revisit Schrödinger's *Platzwechsel* model and think of some linear oscillation—something like the maser[28], perhaps, but with a different scale parameter (a nuclear range parameter, that is).

The nuclear force and potential (1)

The assumption of a combined nuclear and electromagnetic should provide an answer to two basic questions:

1. What keeps the positive and negative charge inside of a neutron?
2. What keeps the positive charges together inside of nuclei?

The nuclear force hypothesis is a *logical* answer because the nuclear force (*i*) keeps positive charges together (the corollary, of course, is that it keeps opposite charges apart[29]), and (*ii*) is much stronger than the electromagnetic force *at short range only* (the latter conveniently explains why we only see the

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[25] This is a reference to the principal quantum number, which gives us a *gross* energy structure. The reader should note we have not brought *spin* angular momentum into the analysis yet: we, therefore, are still analyzing all angular momentum as *orbital* angular momentum, which comes in two states: *up* or *down* (fine structure within the gross structure). Also, if the two charges would be pointlike, spin *coupling* might give rise to a hyperfine structure.

[26] The term was coined by Erwin Schrödinger and, apparently, describes the most trivial solution to Dirac's wave equation. *Zitter* is German for trembling or shaking. Think of the English word '*jittery*'. Dirac made a prominent reference to it in his Nobel Prize Lecture (1933), but we think he did not quite understand its significance because he stuck to the idea of linear rather than orbital motion, just like Louis de Broglie. See our paper on de Broglie's matter-wave idea.

[27] The model(s) we use combines Wheeler's mass-without-mass idea with what is usually referred to as a ring current, *Zitterbewegung*, magneton or toroidal ring model (Parson, 1905; Breit, 1928, Schrödinger, 1930; Dirac, 1933; Hestenes, 2008, 2019).

[28] See our presentation of the maser in our discussion of Feynman's view of two-state systems, in which a nitrogen nucleus constant swaps places in the NH_3 (ammonia) molecule, and whose motion is also based on dipole moments.

[29] While this might solve problems in the deuteron model (two positive and one negative charge), it may raise questions in regard to the neutron model (one positive and one negative charge). However, we think these questions can be answered.

electromagnetic force at work at larger distances).

However, while the introduction of a nuclear field with an a/r^2 potential seems to be easy enough, it raises another rather serious problem: the associated force follows an inverse-*cube* law in space—as opposed to the usual inverse-square law. This violates the energy conservation principle. Of course, one might immediately counter that the force in a dipole field follows an inverse-*cube* law as well but, in this case (dipoles), energy is conserved because the field is *not* spherically symmetric, and the asymmetry is such that energy *is* conserved. This can easily be understood from the equation for the electric field *E*, which can be written in terms of a component along the dipole axis (E_z) and a transverse component ($E_\perp$)[30]:

$$E = \sqrt{E_z^2 + E_\perp^2}$$

Of course, one might suggest fixing the problem of our non-conservative nuclear potential by adding a unit vector ***n*** and assuming the nuclear range parameter is a vector too, whose direction is *fixed* in space. We could then write something like this[31]:

$$\mathrm{U_N}(\boldsymbol{r}) = \boldsymbol{n} \cdot \vec{\boldsymbol{a}} \frac{\mathrm{k_e q_e^2}}{\boldsymbol{r}^2}$$

The vector dot product $\boldsymbol{n}\cdot\boldsymbol{a} = |\boldsymbol{n}|\cdot|\boldsymbol{a}|\cdot\cos\theta = a\cdot\cos\theta$ (the cosθ factor should be positive so *n* must be suitable defined so as to ensure $\pi/2 < \theta < -\pi/2$[32]) introduces a spatial *a*symmetry (think of an oblate spheroid instead of a sphere here), which should then ensure energy is conserved in the absence of an inverse-square law. Alternatively, we could use a vector *cross*-product $\boldsymbol{n}\times\boldsymbol{a} = \boldsymbol{n}\cdot|\boldsymbol{n}|\cdot|\boldsymbol{a}|\cdot\sin\theta = n\cdot a\cdot\boldsymbol{n}\cdot\sin\theta$, but this trick amounts to the same. At first, such *ad hoc* solution might not appeal: how can one possibly *fix* the direction of the ***a*** vector? The answer here is rather straightforward: ***a*** would be directed along the axis connecting the two charges. Are there any other solutions? Of course. We see at least two.

1. One idea might be to try to think of some new curvature of space—something along the lines of how general relativity models gravitation: *not* as a force, but as a *geometric* feature of space. However, this suggestion is unappealing because not only would this require the definition of an entirely new spacetime metric[33], but it also triggers a very obvious question: why would this

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[30] See, for example, Feynman's *Lectures*, II-6-2.

[31] We add a vector arrow to the usual notation for vectors (**boldface**) in the formula to emphasize that its direction, unlike that of **F**, ***n***, and ***r***, is *fixed* in space.

[32] Defining ***a*** such that it broadly points in the same direction of the line along which we want to measure the force **F** should take care of this. Of course, a simple sine or cosine factor does not necessarily ensure energy conservation. Perhaps we should introduce a |cosθ| or $\cos^2\theta$ factor. The point is this: we need to integrate over a volume and ensure that the nuclear potential respects the energy conservation law. We will come back to this at the end of our paper, and argue the dipole model may, effectively, provide the correct equations.

[33] We may refer to electron models using Dirac-Kerr-Newman geometries or, more generally, integrating gravity as a force (e.g., Burinskii, 2021). However, we think gravity is not relevant to the picture here. We can compare force magnitudes by defining a standard parameter. In practice, this means using the same mass – and charge! – in the equations (we take the electron in the equation below) and, when considering the *nuclear* force, equating *r* to *a*:

$$\frac{\frac{\mu_C}{r}}{\frac{\mu_G}{r}} = \frac{\mu_C}{\mu_G} = \frac{\frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r}}{\frac{\mathrm{GMm}}{\mathrm{m}r}} = \frac{\mathrm{k_e q_e^2}}{\mathrm{GMm}} \approx \frac{(9\times10^9\ \mathrm{Nm^2C^{-2}})(1.6\times10^{-19}\ \mathrm{C})^2}{(6.674\times10^{-11}\ \mathrm{Nm^2C^{-2}})(9.1\times10^{-31}\ \mathrm{C})^2} \approx 4\times10^{42}$$

curved space *not* apply to the electromagnetic force?

2. We might think of the nuclear potential as a dynamically changing potential and, therefore, associate a vector potential with the scalar potential, which might ensure (field) energy conservation.

We will come back to the latter possibility. Let us first present the proposed nuclear force and potential without worrying too much about field energy conservation for the time being.

The nuclear force and potential (2)

In previous papers, we introduced the concepts of orbital energies, both electromagnetic and nuclear. These were associated with the Coulomb and nuclear potential, respectively. To be precise, we imagined the neutron as a positive and a negative charge in an oscillation which combines both. Assuming *spherical* potentials (and, therefore, substituting ***r*** for *r* in the formulas), we can write these potentials and the associated forces (the *negative* derivative of the potential) as follows:

$$\begin{cases} \mathrm{U_C}(r) = -\dfrac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\dfrac{1}{r} \\ \mathrm{F_C}(r) = -\dfrac{\partial \mathrm{U_C}(r)}{\partial r} = \dfrac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\dfrac{1}{r^2} \end{cases}$$

$$\begin{cases} \mathrm{U_N}(r) = \dfrac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\dfrac{a}{r^2} \\ \mathrm{F_N}(r) = -\dfrac{\partial \mathrm{U_C}(r)}{\partial r} = -\dfrac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\dfrac{a}{r^3} \end{cases}$$

To help the reader correctly interpret these equations, the following notes may be made:

- The q_e^2 product combines the positive and negative charge and we should, therefore, talk of potential *energy* rather than potential. The concept of a potential effectively assumes a single charge *causing* the potential, while potential *energy* is the energy *between* charges and, therefore, assumes the presence of (at least) two charges.[34]

Hence, the force of gravity – *if* considered a force – is about 10^{42} *weaker* than the (electrostatic) Coulomb force. To demonstrate gravitation is not very relevant, one can also calculate the radius of a black hole with the proton energy using the Schwarzschild formula:

$$r_s = \frac{2\mathrm{Gm}}{c^2} \approx \frac{2 \cdot (6.67 \ldots \times 10^{-11}) \cdot (1.67 \ldots \times 10^{-27})}{(299792458)^2} \approx 2.5 \times 10^{-54}\ \mathrm{m}$$

This clearly shows that, despite the huge energies and forces on these small scales (pico- or femtometer), we should not be worried that we are modeling black holes here. Now that we are here, we should note that the nuclear potential that we will be introducing produces a ratio of standard parameters equal to one. Indeed, equating *r* to *a* and – importantly – using the same mass factor, we get this:

$$\frac{\frac{\mu_\mathrm{N}}{r}}{\frac{\mu_\mathrm{C}}{r}} = \frac{\mu_\mathrm{N}}{\mu_\mathrm{C}} = \frac{\frac{a\mathrm{k_e q_e^2}}{\mathrm{m}a^2}}{\frac{\mathrm{k_e q_e^2}}{\mathrm{m}a}} = \frac{\frac{a}{a^2}}{\frac{1}{a}} = 1$$

Hence, we do get a rather humongous value for nuclear force and field strengths, but these come from the introduction of the nuclear range parameter only: there is no separate nuclear constant ($k_e = k_N$).

[34] The reader will appreciate the distinction when we present the derivation of the dipole potential in the last

- A potential depends on position and may not be static. We should, therefore, write the electromagnetic *scalar* potential as $\phi(\mathbf{r}, t)$, while the vector potential will be denoted as $\mathbf{A}(\mathbf{r}, t) = (A_x(\mathbf{r}, t), A_y(\mathbf{r}, t), A_z(\mathbf{r}, t))$. However, for the time being, we will only consider a static potential ($\partial\phi/\partial t = 0$).

- The physical proportionality constant $k_e = q_e^2/4\pi\varepsilon_0$ is the same for both the electromagnetic and nuclear potential, but the nuclear potential has an added range parameter (a), which defines two ranges: the range where the (electromagnetic) attraction between the positive and negative charge is *larger* than the (nuclear) repulsive force, and the range where electromagnetism loses out, as shown below (Figure 1).

The reasoning here is not as straightforward as it may seem at first. Yukawa's nuclear potential assumed two *like* charges will attract each other, while our neutron model (n = p + e) assumes the opposite. However, we kindly request the reader to forget Yukawa's potential function—if only because the $1/r$ potential in Yukawa's function results in an inconsistency: the physical dimensions do not work out, which is why we replaced it by an a/r^2 dependence.

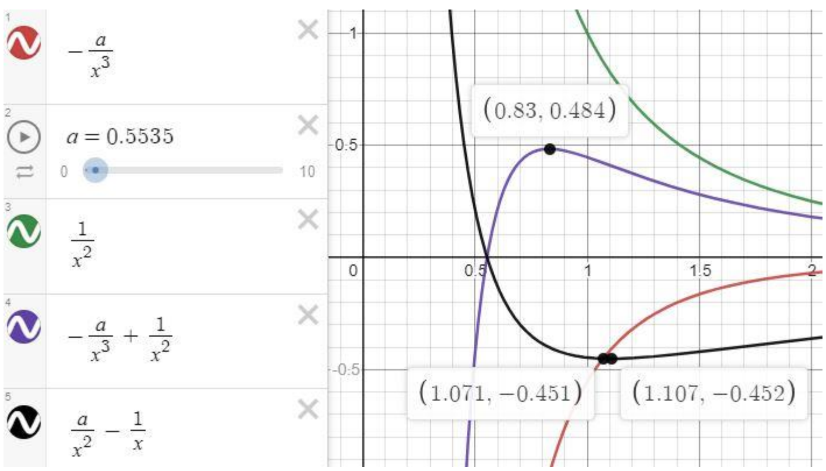

Figure 1: Coulomb and nuclear forces for $a = 2r_e$ and k = 1 (fm units)

Note that, for the convenience of the reader, we show not only the sum of forces, but also the sum of potentials in Figure 1. The model, then, gives us a **potential energy well**, which is nice because it suggests equilibrium: we want the neutron to be stable, of course! As for the value of a, we used a lower range based on a calculation in a previous paper. We found this lower range to be equal to *twice* the classical electron radius (about 2.818 fm), but we now think this calculation was erroneous (we think we made a sign mistake with the potentials).

In any case, the reader should note that we equated the electric constant $k_e = q_e^2/4\pi\varepsilon_0$ to one, so we should not add much importance to the graph and values above: the idea is just to illustrate that the model effectively generates a potential well, and that does not depend on the value of the range parameter a.

section of this paper.

The nuclear range parameter

We can now do a few calculations. We may, for example, apply the usual rule for finding the maximum of the combined $F_N + F_C$ function:

$$\frac{\partial(F_N + F_N)}{\partial r} = \frac{\partial\left(-\frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r^2} + \frac{q_e^2}{4\pi\varepsilon_0}\frac{a}{r^3}\right)}{\partial r} = 0 \Leftrightarrow \frac{q_e^2}{4\pi\varepsilon_0}\left(\frac{2}{r^3} - \frac{3a}{r^4}\right) = 0$$

$$\Leftrightarrow \frac{2}{r^3} - \frac{3a}{r^4} = 0 \Leftrightarrow r = \frac{3a}{2} = 0.83025$$

This value reminds us of the approximate neutron radius (0.82 to 0.84 fm) but that is, of course, a mere coincidence: we can, effectively, not give much *meaning* to the sum of forces here because we equated the Coulomb constant to one in the graph above. To get energies and meaningful values, we must *integrate* forces over a distance and use the correct numerical value for the Coulomb constant. These energies are, of course, the potential energies as well as the kinetic energy of the two pointlike charges. For the nuclear force, we write:

$$\frac{E_N}{m_N} = \frac{v^2}{2} - \frac{a k_e q_e^2}{m_N r^2}$$

The mass factor m_N is the equivalent mass of the energy in the (nuclear) oscillation, which is the sum of the kinetic energy and the (nuclear) potential energy between the two charges. The velocity v is the velocity of the two charges (q_e^+ and q_e^-) as measured in the center-of-mass (*barycenter*) reference frame and may be written as a vector $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{r}) = \boldsymbol{v}(x, y, z) = \boldsymbol{v}(r, \theta, \varphi)$, using either Cartesian or spherical coordinates. Note that we take the potential energy to be *negative* here. This is rather tricky: the sign of the potential energy depends on the U = 0 reference point, which we can choose at either $r = 0$ or $r = \infty$, and we will choose it at $r = 0$ here, so the nuclear potential energy is $-\infty$ at $r = 0$ and 0 at $r = +\infty$. This is not the usual reference point for electromagnetic energy, and so we must swap the sign for U_C as well. We write:

$$\frac{E_C}{m_C} = \frac{v^2}{2} + \frac{k_e q_e^2}{m_C r}$$

Of course, the charges q_e^+ and q_e^- are the same in each equation, and the Coulomb and nuclear energies have to add up, and then the whole equation has to respect the mass-energy equivalence relation: $E = E_N + E_C = (m_N + m_C)\cdot c^2 = m\cdot c^2$. To be precise, this mass-energy equivalence relation *defines* electromagnetic and nuclear mass, respectively, and we may assume that the energy equipartition theorem applies: half of the *total* energy is electromagnetic, and half is nuclear and, therefore, $m_N = m_C = E/2$. We must, therefore, add the U_N and U_C terms. The kinetic energy is the kinetic energy, of course—but now we must wonder: should we add the $v^2/2$ terms too? We have *two* charges: should we, therefore, have twice the kinetic energy?

We suggest the relevant addition is, effectively, equal to $(m_N + m_C)\cdot v^2/2$. Furthermore, the ring current model that we have been using assumes the pointlike charge has no *rest* mass and, therefore, whizzes around at lightspeed: we, therefore, equate v to c to obtain the following sum of equations:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{\mathrm{E}_C}{\mathrm{m}_C} + \frac{\mathrm{E}_N}{\mathrm{m}_N} = \frac{c^2}{2} + \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r} + \frac{c^2}{2} - \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2} = c^2 + \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r} - \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2}$$

$$\Leftrightarrow 0 = \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r} - \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2} \Leftrightarrow \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r} = \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2} \Leftrightarrow a = r$$

What can we say about this statement? What does it *mean*, really? Nothing much: the laws of physics give us a *radius* for the oscillation which must be equal to the nuclear range parameter *a*. So, we have a *truism* here: a self-obvious summary of the laws (plain electromagnetic theory) that we think should apply to elementary particles as well. We must obtain *a empirically*: in this case, it must be the effective charge radius of the neutron which, as mentioned above, is in the same range as the proton radius, for which the CODATA value is about 0.84×10^{-15} m.[35]

Let us redraw the illustration above and equate both a and k_e to 1 to show the result of two other evident calculations: (1) the sum of forces reaches a minimum at r = 3*a*/2 = 1.5·*a* (we calculated that above already), and (2) the sum of potentials reaches a minimum at *r* = 2*a*, so that is the distance at which *the potential well bottoms out*.

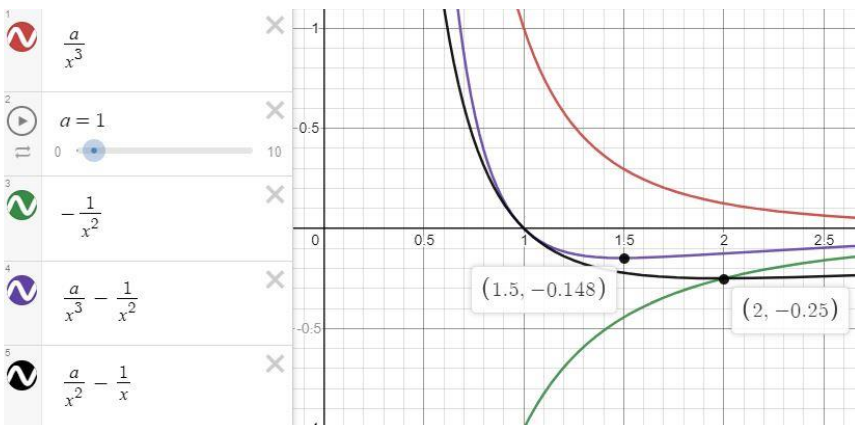

Figure 2: Coulomb and nuclear forces for *a* = k = 1

So far, so good. Let us take a step back and explore some other, related, line of reasoning.

A theoretical value for the proton and neutron radius

We mentioned the *empirical* value for the proton rms charge radius. Can we present a theoretical value? Sure. We think of the proton as a *nuclear* oscillation of a pointlike positive charge in a modified ring current model. However, instead of assuming a 2D oscillation, we imagine the nuclear oscillation to be

[35] The point estimate is 0.8414 fm, with an uncertainty of 0.0019 fm. Hence, the 2σ rule gives a lower value of 0.8376 fm and an upper value of 0.8452 fm. The CODATA value for the proton radius (0.8414 ± 0.0019 fm) takes all past measurements into account but gives very high weightage to the measurements of Pohl (2010) and Antognini (2013), which are both based on muonic-hydrogen spectroscopy. In contrast, the PRad experiment which is based on a proton-electron scattering – quite a different technique – established the following new value for the proton radius: r_p = 0.831 ± 0.007stat ± 0.012syst fm. Prof. Dr. Randolf Pohl is of the opinion that the PRad measurement and the muonic-hydrogen spectroscopy measurements are basically in agreement. He replied this to an email on this: "*There is no difference between the values. You have to take uncertainties seriously (sometimes we spend much more time on determining the uncertainty.*" (email from Prof. Dr. Pohl to the author dated 6 Feb 2020)

driven by two (perpendicular) forces rather than just one, with the frequency of *each* of the oscillators being equal to ω = E/2ħ = mc^2/2ħ. Each of the two perpendicular oscillations would, therefore, pack one *half*-unit of ħ only.

This ω = E/2ħ formula also incorporates the energy equipartition theorem, according to which each of the two oscillations should pack *half* of the total energy of the nuclear particle (so that is the *proton*, in this case). This spherical view of a proton fits nicely with packing models for nucleons and yields the experimentally measured radius of a proton:

$$\frac{\mathrm{E}}{\mathrm{m_p}} = c^2 = a^2\omega^2 = a^2\left(\frac{\mathrm{m_n}c^2}{2\hbar}\right)^2 \Leftrightarrow a = \frac{4\hbar}{\mathrm{m_p}c} \approx 0.84\ \mathrm{fm}$$

You can see that the 4 factor is the same factor 4 as the one appearing in the formula for the surface area of a *sphere* ($A = 4\pi r^2$), as opposed to that for the surface of a *disc* ($A = \pi r^2$). This leads us to represent a proton by a *combination* of two wavefunctions—something like this:

$$\psi_\mathrm{p} = \frac{4\hbar}{\mathrm{m_p}c}\cdot\left(e^{\pm i\frac{\mathrm{E_p}}{2\hbar}t} + e^{\pm j\frac{\mathrm{E_p}}{2\hbar}t}\right)$$

But what about the neutron? The proton and neutron mass do not differ much: 939.565 MeV − 938.272 MeV ≈ 1.3 MeV, so their radii should be more or less the same, and they are.[36] The reader should note that the mass of an electron is 0.511 MeV/c^2, so that is only about 40% of the energy difference, but the kinetic and *binding* energy could make up for the remainder. If anything, the effective charge radius of a neutron might be slightly *smaller* than that of a proton because these ring current models yield an *inverse* proportionality between the energy and the radius of (elementary) particles. To be precise, a 2D ring current model yields the following:

$$\left.\begin{array}{r}\left.\begin{array}{r}\mathrm{E} = \mathrm{m}c^2\\ \mathrm{E} = \hbar\omega\end{array}\right\} \Rightarrow \mathrm{m}c^2 = \hbar\omega\\ c = a\omega \Leftrightarrow a = \dfrac{c}{\omega} \Leftrightarrow \omega = \dfrac{c}{a}\end{array}\right\} \Rightarrow \mathrm{m}a^2\omega^2 = \hbar\omega \Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2 = \hbar\frac{c}{a} \Leftrightarrow a = \frac{\hbar}{\mathrm{m}c}$$

Hence, what is also referred to as the *Zitterbewegung* radius (Schrödinger, 1930; Dirac, 1933; Hestenes, 2008, 2019[37]) is nothing but the *Compton radius* of a particle.[38] Finally, we should note the calculations

[36] CODATA does not list a rms charge radius. We think this is because standard theory considers the neutron to not carry any charge, while our model considers it to be a composite particle. It should be noted that the neutron is *not* stable outside of a nucleus, which we take to confirm Schrödinger's *Platzwechsel* model: it is the *nuclear* electron (or, to be precise, the negative charge) which acts as the *gluon*, so to speak, between protons in the nucleus. The reader should also note that the mass of a proton and an electron add up to *less* than the mass of a neutron, which is why it is only logical that a neutron should decay into a proton and an electron. Binding energies – think of Feynman's calculations of the radius of the hydrogen atom, for example, are effectively measured as *negative* energy.

[37] The *Zitterbewegung* or ring current model actually goes back much further in time. It was established as soon as it became clear that an electron had a magnetic moment (Parson, 1905; Breit, 1928). It is also a direct application of Wheeler's suggested mass-without-mass model of elementary particles, which he pushed as an alternative to mainstream theory in the 1960s.

[38] The reader will be more familiar with the Compton *wavelength* but, paraphrasing Prof. Dr Patrick LeClair, we understand the related radius ($a = \lambda/2\pi$) to be the actual "scale *above* which the electron can be localized in a

above are consistent with the experimentally measured values for the magnetic moment of the proton and neutron.[39]

Having developed the rationale for thinking of a neutron as a proton and a nuclear (or *deep*) electron, we must note the n = p + e model tells us that the proton accounts for most of the mass and, therefore, for most of the energy of the neutron, which seems to contradict the intuition that the nuclear and electromagnetic mass of the neutron must each account for *half* (1/2) of the total mass (energy equipartition theorem). This contradiction may be resolved by considering the scales (femtometer versus picometer[40]) and, more generally, a model which thinks of the neutron as a combination of two electric charges rather than as a combination of a proton and an electron. Such approach would also allow to think of *excited* energy states and more exotic versions of the neutron.[41]

However, let us leave this question aside for the time being, and let us return to the key issue we raised in our introduction: how do we reconcile an a/r^2 or $1/r^2$ potential with the energy conservation principle?

The nuclear potential, energy conservation, gauges, and wave equations

We hoped we have been able to demonstrate that the introduction of a nuclear field with an a/r^2 potential might solve our neutron or deuteron modeling problem, but we also were quite clear we are now stuck with an issue which is at least as problematic as the problems we tried to solve: a force that is associated with a $1/r^2$ potential suggests an inverse-*cube* law in space—as opposed to the usual inverse-square law. This violates the energy conservation principle. The best way to solve this issue, is to suggest the nuclear force is *not* spherically symmetric. We may model this by adding a unit vector ***n*** and assuming the nuclear range parameter is a vector too, whose direction is *fixed* in space. We wrote[42]:

$$\mathrm{U_N}(\boldsymbol{r}) = \boldsymbol{n} \cdot \vec{\boldsymbol{a}} \frac{\mathrm{k_e q_e^2}}{\boldsymbol{r}^2}$$

However, one might also imagine something else. The proposed nuclear potential formula only models a

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particle-like sense." Such interpretation clarifies what Dirac, in his Nobel Prize Lecture (1933), referred to as the law of (elastic or inelastic) scattering of light by an electron"—Compton's law, in other words. Finally, we should add – mainly for the informed reader – that the CODATA value for the Compton wavelength incorporates the same 4 factor which we associated with the *spherical* model of protons and neutrons. We are not sure about the CODATA methodology here.

[39] See our paper on the mass-without-mass model of protons and neutrons.

[40] The Compton radius of an electron is about 0.386×10^{-12} m (pm), i.e., 386 fm, or about 460 times the proton radius. We must note another apparent contradiction here. The small anomaly in the magnetic moment of an electron suggests the pointlike charge is not infinitesimally small: the anomaly may be explained by assuming it has a radius itself, and the anomaly suggests this radius equals the *classical* electron radius (Thomson radius), which is equal to $r_e = \alpha r_C \approx 2.818$ fm. This is the femtometer scale alright, but it is still *much* larger than the neutron radius. Electric charge in Nature seems to have a very *variable geometry*.

[41] We might refer to the ongoing research on deep electron orbitals (Meulenberg and Paillet, 2020), more exotic versions of the hydrogen atom (see, for example, Jerry Va'Vra, 2019) or, more in general, to all of the research on low-energy nuclear and/or anomalous heat reactions (aka cold fusion).

[42] See footnote 31: we add a vector arrow to the usual notation for vectors (**boldface**) in the formula to emphasize that its direction, unlike that of **F**, ***n***, and ***r***, is *fixed* in space.

scalar potential, and it is static: it does not vary with time. This can, of course, not be true: the charges move constantly, and at lightspeed. The potential must, therefore, vary with time, and we must, therefore, also have a *vector* potential.

For electromagnetic oscillations, this corresponds to the distinction between the electric and magnetic force, respectively, and to the distinction between the scalar and vector potential. Indeed, assuming the scalar potential varies with time, one can derive the vector potential **A** from the Lorenz gauge condition:

$$\nabla \cdot \boldsymbol{A} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$$

For a time-independent scalar potential, which is what we have been modeling so far, the Lorenz gauge is zero ($\nabla \cdot \boldsymbol{A} = 0$) because the time derivative is zero: $\partial\phi/\partial t = 0 \Leftrightarrow \nabla \cdot \boldsymbol{A} = 0$.[43] The magnetic field, therefore, vanishes. However, as mentioned above, it is pretty obvious that the time derivative cannot be zero. The question is thus this: can we use the very same Lorenz gauge for the nuclear force fields? We should, effectively, be able to define an equivalent scalar and vector potential for the nuclear force too!

Can we use the Lorenz gauge? We are not sure, but we think it should be possible. The Lorenz gauge incorporates the usual theorems from vector differential and integral calculus (Gauss and Stokes) as well as special relativity. It, therefore, also incorporates the principles of energy (and, most probably, momentum conservation).[44] Hence, if the nuclear force is a conservative force – which it should be (no concepts of entropy or friction here!) – then we should use the Lorenz gauge to relate the nuclear scalar potential to the nuclear vector potential, and then we can use the superposition principle again to add the electromagnetic and nuclear scalar and vector potential to get Dirac's "equations of motion" for everything !

What do we need for that? We need something like the equivalent of Maxwell's equations for the nuclear fields and *that*, we do not have. We, therefore, have no ideas on how to calculate the time derivative of the nuclear scalar potential. Will this question ever be solved? We hope so: readers who are well-versed in math and vector calculus should try their hand at that! There is a lot of useful material out there. Just *google*, for example, for papers on non-paraxial fields. This site (Alonso Research Group, University of Rochester), for example, offers a fine point of departure! 😊

A more direct approach might be to substitute the $1/r_{12}$ factor in the scalar and vector potential integrals below[45] by the a/r_{12}^2 factor but – again – we should probably use some vector product instead to ensure field energy conservation:

[43] The Lorenz gauge does not refer to the Dutch physicist H.A. Lorentz but to the Danish physicist Ludvig Valentin Lorenz.

[44] We say so because the derivation involves the consistent use of relativistic four-vectors. See Feynman, I-14 (work and potential energy), II-2 (vector differential calculus), II-3 (vector integral calculus), II-25 (electrodynamics in relativistic notation), III-26 (Lorentz transformation of the fields) and III-27 (field energy and momentum).

[45] We took these from the excellent overview table of electromagnetic theory in Feynman's *Lectures* (table 15-1 in Chapter 15).

$$\left[\begin{array}{l}\phi(1,t)=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(2,t')}{r_{12}}\,dV_2\\ \quad\text{and}\\ \boldsymbol{A}(1,t)=\frac{1}{4\pi\epsilon_0 c^2}\int\frac{\boldsymbol{j}(2,t')}{r_{12}}\,dV_2\\ \quad\text{with}\\ t'=t-\frac{r_{12}}{c}\end{array}\right.$$

So as to motivate the reader to go into this direction, we note that, *if* we could define the *nuclear* scalar and vector potentials, we could probably use the *combined* electromagnetic and nuclear scalar and vector potentials in a more general wave equation, such as the one which Feynman suggests in his last *Lecture on Physics* (III-21)[46]:

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2\mathrm{m}}(i\hbar\boldsymbol{\nabla}+q\boldsymbol{A})^2\psi+\mathrm{q}\phi\psi$$

If this equation would be, effectively, the most general form of a wave equation, then substituting the ϕ and ***A*** scalar and vector potential, respectively, by the *combined* electromagnetic and nuclear scalar and vector potentials should give us the solutions we are all seeking for.[47]

However, something inside of us tells us the use of one rotation operator only (*i*) might not do the trick: as we show in our previous paper in our previous introductory paper on the nuclear force), we suspect quaternion algebra may be necessary to take the 3D geometry of the nuclear force into account.

Do we need the nuclear force hypothesis?

Let us briefly review Feynman's derivation of the (electrostatic) dipole potential. The point of departure is the (electrostatic) Coulomb potential of zero-spin (electric) charges[48] and the superposition principle (which drives our concern on the need to ensure any model of a nuclear force – if it exists – respects the (field) energy conservation principle). With multiple (zero-spin) charges, the potential at some point ***x*** =

[46] We wrote Feynman's *ħ*/*i* factor as –*iħ*, moved minus signs out of the brackets and combined them into one square factor. We hope we did not make any mistake. Note that Feynman refers to this wave equation as the wave equation for an electron in any electromagnetic field. After re-reading Feynman's *Lectures* several times, we sometimes get the feeling that Feynman might have a secret drawer with the answers to all questions: he just did not want to tell us all! 😊

[47] In another paper on the nuclear potential, we suggested a wave equation using nuclear (kinetic and) potential energy directly (instead of scalar and vector potentials) *directly*. However, the formally correct approach is really to think of how the time derivative of the nuclear (static) potential (modified Yukawa potential combined with Schrödinger's *Platzwechsel* idea for modeling a nucleus) could possibly look like.

[48] Apart from not considering the incongruency of the physical dimensions of his equations, Yukawa also introduced a new *nucleon* (or *nuclear*) charge (g_N), which opened the flood gates to ontologizing mathematical niceties such as *strangeness* and, ultimately, led to the even stranger concepts of quarks and gluons. We think of the Higgs particle as the latest addition (we hope it is the last) to what we refer to as smoking gun physics. We made our viewpoint on the ontological status of these mathematical concepts clear in our paper on the *Zitterbewegung* hypothesis and the *S*-matrix.

(x, y, z)[49] will be equal to:

$$\phi = \sum_j \frac{1}{4\pi\varepsilon_0} \frac{q_j}{r_{xj}}$$

With two (opposite) charges only (remember we are trying to develop an n = $q_e^+ + q_e^-$ model here), separated by the distance *d* along the chosen *z*-axis, this gives us the electrostatic dipole potential:

$$\phi(x, y, z, t = 0) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_e}{\sqrt{[z-(d/2)]^2 + x^2 + y^2}} - \frac{q_e}{\sqrt{[z+(d/2)]^2 + x^2 + y^2}}\right]$$

We now will let Feynman speak. We can expand the term(s) in the denominator above in the (small) distance *d*, and simplify by keeping the first-order terms only:

$$\left(z - \frac{d}{2}\right)^2 \approx z^2 - zd.$$

It is convenient to write

$$x^2 + y^2 + z^2 = r^2.$$

Then

$$\left(z - \frac{d}{2}\right)^2 + x^2 + y^2 \approx r^2 - zd = r^2\left(1 - \frac{zd}{r^2}\right),$$

and

$$\frac{1}{\sqrt{[z-(d/2)]^2 + x^2 + y^2}} \approx \frac{1}{\sqrt{r^2[1-(zd/r^2)]}} = \frac{1}{r}\left(1 - \frac{zd}{r^2}\right)^{-1/2}.$$

Using the binomial expansion again for $[1 - (zd/r^2)]^{-1/2}$—and throwing away terms with the square or higher powers of d—we get

$$\frac{1}{r}\left(1 + \frac{1}{2}\frac{zd}{r^2}\right).$$

Similarly,

$$\frac{1}{\sqrt{[z+(d/2)]^2 + x^2 + y^2}} \approx \frac{1}{r}\left(1 - \frac{1}{2}\frac{zd}{r^2}\right).$$

The difference of these two terms gives for the potential

$$\phi(x, y, z) = \frac{1}{4\pi\epsilon_0}\frac{z}{r^3}qd. \tag{6.9}$$

The potential, and hence the field, which is its derivative, is proportional to qd, the product of the charge and the separation. This product is defined as the *dipole moment* of the two charges, for which we will use the symbol p (do *not* confuse with momentum!):

$$p = qd. \tag{6.10}$$

Equation (6.9) can also be written as

$$\phi(x, y, z) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}, \tag{6.11}$$

since $z/r = \cos\theta$, where θ is the angle between the axis of the dipole and the radius vector to the point (x, y, z)—see Fig. 6–1. The *potential* of a dipole decreases as $1/r^2$ for a given direction from the axis (whereas for a point charge it goes as $1/r$). The electric field $\boldsymbol{E}$ of the dipole will then decrease as $1/r^3$.

Putting all in vector form (defining ***p*** as a vector with magnitude *p* and a direction along the axis of the dipole—from $-q_e$ to + $-q_e$), then yields the magnitude of the electric dipole field:

$$E = \sqrt{E_z^2 + E_\perp^2}$$

So, what does this show? We are not sure, but we offer the following reflections:

- The magnitude of the electric (electrostatic) and magnetic (dynamic) fields (the ***E*** and ***B*** in Lorentz's **F** = q(***E*** + ***v***×***B***) = q(***E*** + ***v***×*i*·**E**/*c*) force law) is equal for ***v*** = ***c***, and Lorentz' force law, therefore, can then be written as **F** = q(***E*** + ***c***×*i*·**E**/*c*) = q·(**E** + **1**×*i*·**E**) = q·(**E** + *j*·**E**) = (1+ *j*)·q·**E**. The magnetic force may be viewed as a relativistic correction to the electric force, but it becomes an equal *component* of the electromagnetic force.
- Dipole moments comes with $1/r^2$, $1/r^3$ and higher-order terms, with complicated directional

[49] Switching to spherical coordinates will usually be more convenient.

factors in the numerator, when expanding the basic electromagnetic equation(s) for the dipole field(s).

- At short (i.e., *nuclear*) range, dipole potentials explain the 0.782 and 2.224 MeV binding energy between the charges in a neutron and a deuteron nucleus, respectively.
- None of our calculations so far considered the *spin* angular momentum of charges—i.e., the *spin of the charges themselves*, as opposed to the orbital angular momentum of charge *orbitals*.

We think all of the above, somehow, justifies an intuition that there may be no need to invoke a nuclear force or potential. Correctly modelling the energy in Schrödinger's *Platzwechsel* model – along the lines of any two-state quantum-mechanical system that is based on the concept of (opposite) *spin* angular momentum and/or opposite dipole moments – might give us a reasonable explanation of the stability, size, and other intrinsic properties (most importantly, the residual or resultant magnetic moment) of both the neutron as well as the deuteron nucleus.

We are far from *proving* this, however, but we hope our reader(s) will be able to do so, one day. To encourage the discussion, we provide some mathematical remarks in the last and final section of this paper.

Power expansions of energies and potentials

As mentioned above, we are not convinced that there is a need to invent a new potential. If such need would be there, then it should probably be based on the idea of *spin* applied to the pointlike charges themselves—which is an entirely different concept than the spin of the (elementary or composite) particles themselves, which we think of *orbital* angular momentum. When analyzing the math which would be required for this, the following rather peculiar power expansions probably merit some attention.

Power expansion of (potential and kinetic) energy

Orbital energy is kinetic and potential, but the *total* energy must obey Einstein's mass-energy equivalence relation. Using the binomial theorem, we can rewrite this E = mc^2 relation as follows:[50]:

$$\mathrm{m}c^2 = \mathrm{m}_0c^2 + \frac{1}{2}\mathrm{m}_0v^2 + \frac{3}{8}\mathrm{m}_0\frac{v^4}{c^2} + \cdots = \mathrm{m}_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right)$$

$$= \mathrm{m}_0c^2\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

The relativistically correct formula for kinetic energy defines kinetic energy as the difference between the *total* energy and the potential energy: KE = E − PE. The potential energy must, therefore, be given by the m_0c^2 term. This term is zero for r = 0 but non-zero because of the potential energy in the radial field

[50] The total energy is given by E = mc^2 = $\gamma\mathrm{m}_0c^2$ which can be expanded into a power series using the binomial theorem (Feynman's *Lectures*, I-15-8 and I-15-9 (relativistic dynamics). He does so by first expanding $\gamma\mathrm{m}_0$:

$$\mathrm{m} = \frac{\mathrm{m}_0}{\sqrt{1 + \frac{v^2}{c^2}}} = \mathrm{m}_0(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots)$$

This is multiplied with c^2 again to obtain the series in the text.

at distances $r > 0$. The *total* energy of a charge in a (static) Coulomb field is given by[51]:

$$U(r) = \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

The potential itself is equal to $V(r) = U(r)/q_e$:

$$V(r) = \frac{U(r)}{q_e} = \frac{q_e}{4\pi\varepsilon_0}\frac{1}{r}$$

We could define the kg (mass) in terms of newton (force) and acceleration (m/s^2). Can we do the same for the *coulomb*? Rewriting the energy equation as a function of the *relative* velocity and the *radial* distance *r* does the trick:

$$mc^2 = \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

We may say this *defines* the mass of the pointlike charge as *electromagnetic* mass only, which now consists of a kinetic and potential piece. The energy in the oscillation, therefore, defines the total mass $m = E/c^2$ of the neutron electron ($n = p + e_n$). The kinetic energy is thus given by[52]:

$$KE = mc^2 - U(r) = \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right) - \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r} = \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 - 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$
$$= \frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r}\left(\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

The first term in the series gives us the non-relativistic kinetic energy $\frac{q_e^2}{4\pi\varepsilon_0}\frac{\beta^2}{2}\frac{1}{r}$. In line with the usual convention for measuring potential energy, we will now set the reference point for potential energy at zero at infinity, and the potential energy will, therefore, be defined as *negative*, going from 0 for $r \to \infty$ to $-\infty$ for $r \to 0$. This makes for a *negative* total energy which is in line with the concept of a *negative* ionization energy for an electron in an atomic orbital which, for a one-proton atom (hydrogen), is given by the Rydberg formula.

Is this power series relevant to the discussion at hand? Should we distinguish a nuclear from the usual electromagnetic potential? Again, we do not know: all that we want to do here is to trigger the imagination of the reader and encourage him to think this through for him- or herself.

Power expansion of the dipole potential

Another interesting power series may be obtained, perhaps, from substituting the electromagnetic

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[51] $U(r) = V(r)\cdot q_e = V(r)\cdot q_e = (k_e\cdot q_e/r)\cdot q_e = k_e\cdot q_e^2/r$ with $k_e \approx 9\times10^9$ N·m^2/C^2. Potential *energy* (U) is, therefore, expressed in *joule* (1 J = 1 N·m), while potential (V) is expressed in *joule*/*Coulomb* (J/C). Since the 2019 revision of the SI units, the electric, magnetic, and fine-structure constants have been co-defined as $\varepsilon_0 = 1/\mu_0 c^2 = q_e^2/2\alpha hc$. The CODATA/NIST value for the standard error on the value ε_0, μ_0, and α is currently set at 1.5×10^{10} F/m, 1.5×10^{10} H/m, and 1.5×10^{10} (no *physical* dimension here), respectively.

[52] In line with the usual convention for measuring potential energy, we will set the reference point for potential energy at zero at infinity.

potential which gives rise to the electric dipole field by the proposed nuclear potential, and see how it changes the derivation that follows from it. We write:

$$\phi_N = \sum_j \frac{a}{4\pi\varepsilon_0}\frac{q_j}{r_{xj}^2}$$

With two (opposite) charges only, this becomes:

$$\phi(x,y,z,t=0) = \frac{a}{4\pi\varepsilon_0}\left[\frac{q_e}{[z-(d/2)]^2+x^2+y^2} - \frac{q_e}{[z+(d/2)]^2+x^2+y^2}\right]$$

We have the same $[z \pm (d/2)]^2$ term in the denominator but no square root function anymore, which should – obviously – affect the subsequent derivation. We leave it to the reader to work it all out: if he or she would obtain something interesting, we would sure hope to get feedback! To be honest, we are rather skeptical. 😊

Tentative conclusion(s)

If reality is that what is the case, what do we believe might be the case?[53] We think it might be this:

1. The (planar) ring current model of an electron, and the (spherical) oscillator of a proton, suggest (electric) charge comes in two fundamental oscillations. The intriguing thing here is the energy level of these two elementary particles: we can *imagine* a lower- or higher-energy electron, or a lower- or higher-energy proton (and apply the Planck-Einstein and mass-energy relations to them to obtain their radius and mass), but the energy of an electron is the energy of an electron, and the energy of a proton is that of a proton. This indicates the negative and positive charge are *not* just each other's opposite.[54]

2. We do not need electromagnetic theory to explain the radius (or mass) of these two elementary particles. The *Zitterbewegung* hypothesis, applied to the idea of pointlike charges (with zero rest mass), combined with the Planck-Einstein and energy-mass equivalence relations, will do. The negative charge is associated with a 2D planar oscillation, while the proton is associated with a 3D (spherical association). To refer to the first as an electromagnetic and the second as a nuclear oscillation may be slightly misleading. We do so because we believe these two different matter-waves are associated with two different types of lightlike particles: the photon and the neutrino, and we only observe the neutrino from nuclear reactions, which explains why the term 'nuclear oscillation' is a convenient shorthand.[55] We also do not need electromagnetic theory to explain unstable composite particles and particles

[53] This is an obvious reference to Wittgenstein's first statement in his *Tractatus Logico-Philosophicus*: "*Die Welt ist alles, was der Fall ist.*" We admire Wittgenstein for his search of a formal language which would encompass each and everything and would be entirely logical/unambiguous, but we are surprised Wittgenstein was apparently unaware of the scientific revolution (especially Einstein's relativity theory) that was taking place. The later Wittgenstein also acknowledged non-ambiguity in language may be unachievable or, worse, that it is actually the key to sense-making and understanding.

[54] For example, the electron comes in a more massive but unstable variant: the muon-electron. The proton does not. Unstable particles can be modeled by combining the elementary wavefunction (complex-valued exponential) and a decay factor (real-valued exponential decay function).

[55] Such nuclear reactions may be low- or high-energy (decay versus high-energy particle collisions).

reactions: the combination of the *zbw* hypothesis and standard matrix algebra will do.[56]

3. However, we do need electromagnetic theory to explain the magnetic moment of the electron, proton, neutron, and composite (stable or unstable) particles: we must assume the *Zitterbewegung* is regular (or regular *enough*), and that the *Zitterbewegung* amounts to a ring current which generates a magnetic field which keeps the charge in motion.[57]

4. A neutron is only stable inside of a nucleus. The smallest nucleus is the deuteron nucleus, which combines two positive charges and one negative charge. The nuclear binding energy is of the order of 2.2 MeV, which can be explained by the dipole field from the neutronic combination within the nucleus. The nuclear potential, therefore, appears as an electromagnetic dipole potential, combining an electric dipole and the magnetic fields of the neutral current from the motion of the positive and negative charges, and the charged current from the motion of the two positive charges. Such potential is, typically, spherically *non*-symmetric but conservative, and the order of magnitude of the presumed nuclear range parameter is the same as that of the distance which separates the charges (femtometer scale).

In short, we believe there is no such thing as a 'nuclear' charge (no g_N, only q_e), nor is there a nuclear equivalent of the electric constant.[58] There is, therefore, no real nuclear force, but we do think of two different fundamental oscillations in spacetime, and combinations thereof. Such combinations, however, can all be explained by standard electromagnetic theory, including electromagnetic dipole theory. We, therefore, tentatively agree with Di Sia's conclusion: the 80-year-old problem of the nuclear force has been resolved.

We started by quoting the first statement of Wittgenstein's *Tractatus Logico-Philosophicus*, and find it appropriate to conclude with his last: "*Wovon man nicht sprechen kann, darüber muss man schweigen.*"

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[56] See our paper on the *Zitterbewegung* hypothesis and the scattering matrix.

[57] The magnetic field is quantized too. The analogy with a superconducting ring and perpetual currents comes to mind. There is no heat: no *thermal* motion of electrons, nuclei or atoms or molecules as a whole and, therefore, no (heat) *radiation*. Also, the perpetual currents in a superconductor behave just like electrons in some electron orbital in an atom: they do *not* radiate their energy out. That is why superconductivity is said to be a quantum-mechanical phenomenon which we can effectively observe at the *macroscopic* level. Hence, we have a magnetic field but no radiation and, since 1961 (the experiments by Deaver and Fairbank in the US and, independently, by Doll and Nabauer in Germany), we know this field is, indeed, quantized. To be precise, the product of the charge (q) and the magnetic flux (Φ), which is the product of the magnetic field B and the area of the loop S, – will always be an integer (n) times h: $q \cdot \Phi = q \cdot B \cdot S = n \cdot h$. However, superconducting rings are made of superconductors. The question in regard to the ring current model of elementary particles is this: what keeps the charge in place? We think of this as the *fine-tuning* problem, but we do not see it as a fundamental problem of the theory: elementary particles in free space may not sit still either. Their motion in space may probably be modelled by some combination of the random walk model (in 3D space) and thermal motion. Both the random walk and thermal motion must respect the Planck-Einstein relation too. Any random walk model should, therefore, be combined with the quantum-mechanical least action principle: the action associated with any path should be equal to h or $\hbar$ (linear versus orbital path), or an integer multiple thereof.

[58] We interpret the 2019 redefinition of SI units as confirming this hypothesis: over the past 100 years, no specifically nuclear-related new constant in Nature has appeared, and our current knowledge of fundamental constants incorporate all laws of physics.

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