Integration of Gravity & Electromagnetism in Terms of a Dirac Polarized Vacuum

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Abstract

Conventionally Maxwell's equations describe transverse elements described as 'EM' waves; but by utilizing the Einstein/de Broglie relations one can derive additional degrees of freedom so that Maxwell's equations are not 'cut off' at the vacuum. Therefore one must employ the $\mu\nu$ fields in addition to the standard EM suggesting also that the photon is piloted. The two sets of coordinates for the EM or $\mu\nu$ fields are mutually exclusive and generally considered to be independent of each other. In this work a method is developed for integrating them in terms of a Dirac covariant polarized vacuum and extended theoretical perspectives.

1. Introduction to Fixing the G/EM Framework

The integration of Gravity and electromagnetism (EM) has been one of the holy grails of physics for the last century. In this chapter Gravity and EM are unified in terms of the covariant density distribution of a real average covariant Dirac vacuum built with extended random elements filling flat space-time. Although the Newton and Coulomb potentials have similar forms the two theories have developed separately leaving their unification an unsolved problem throughout the history of Modern Science. In the past most attempts at unification have been within a frame associating electromagnetism with new geometrical properties of spacetime [1-3]. The approach of this integration is different. Following Puthoff and others [4-7], both fields are represented by four-vector field densities, A_μ; where one considers both types of phenomena as different types of motions within the same real physical zero-point field in a flat spacetime, i.e. as two different vacuum types of collective perturbations carried by a single vacuum field moving in such a space. Our hope is that since this approach suggests new types of experimentation and new interpretations of unexplained effects it could, if confirmed, help to disentangle the present theoretical discussion.

The basis of this model is as follows:

A) The first basis is observational. The universe apparently does not change with distance [8-10] (as it would for Big-Bang type theories). This leads to the possibility of a non-Doppler redshift [11] (which suggests a non-zero photon mass, $m_\gamma \neq 0$) with the velocity of light isotropic in an absolute inertial frame, I_0 , in time.

B) The second basis is that our essential instrument for distance observation (i.e. electromagnetic waves) is more complex than initially thought. De Broglie and Einstein demonstrated that $E = h\nu$ = mc^2 , with m = m_0. $\left(1 - v^2/c^2\right)^{-1/2}$ so that individual massive photon's could be considered as piloted by real non zero-mass Maxwellian waves allowing the electromagnetic field to be represented by a vector density, A_μ. As shown by the Aharonov-Bohm effect, this implies that the EM field is not completely represented by the $\mu\nu$ fields [12].

Maxwell's equations [3] conventionally describe Transverse elements denoted as 'EM' waves; by utilizing the Einstein / de Broglie relation one may derive additional degrees of freedom such that Maxwell's equations are not 'cut off' at the vacuum, but lead to Longitudinal wave components and non-zero electric conductivity of the vacuum. Thus our distinct need for the utility of the $\mu\nu$ fields instead of just the standard 'EM'. This also suggests that the photon is 'piloted'. One must 'fix' the coordinates of either the EM field or the $\mu\nu$ field we have chosen the latter. It should be noted that while c is constant in the rest frame and the velocity of massive photons would be frequency dependent; there is no contradiction because as Dirac himself stated according to coordinate law the pilot wave and the photon decouples [13]. The two sets of coordinates, EM or $\mu\nu$ are mutually exclusive and would generally be independent. In this work a method is developed for integrating them.

It is well known that the usual form of Maxwell's equations in vacuum (describing zero mass photons) possess infinite families of boundary free exact solutions with Longitudinal electric or magnetic fields; this is the usual $\mu\nu$ theory where $B^{(3)} = 0$ and photon mass, $m_\gamma = 0$. This is also true for the vector potential in the Lorentz gauge according to the equation, $A_\mu = 0$. But of interest to the task here, for massive photons there is only one family and one set of boundary conditions!

C) The third basis has its theoretical origin in the introduction by Dirac et al. of a real covariant chaotic physical aether which fills space-time, carries real physical observable wave-like and particle like (soliton-like) perturbations or local extended elements, whose four momenta and angular momenta are statistically and evenly distributed on specific hyperbolic surfaces, at each given point, in all given inertial frames. This vacuum distribution thus appears, as invariant isotropic chaotic and undetectable (except in specific physical cases) for all inertial observers. The form taken by an aether within Relativity Theory carrying both particles and waves is now described in terms of collective motions on the top of a real essentially stochastic covariant background. Such an aether theoretically justifies the statistical productions of Quantum Mechanics (in its causal stochastic interpretation) and SED theory, and has a direct experimental justification in the Casimir effect. This implies a background friction (associated with absolute local conservation of total momentum and angular momentum) and collective motions which provide a new interpretation of the observed cosmological red-shift [11,14] and yields new possibilities to interpret (also in terms of local frictions) the anomalous red-shifts observed by Arp, Tifft and other astronomers [15].

From these bases, section 3, describes the gravitational results of General Relativity in Maxwellian terms. Section 4 develops a possible unification model of both theories. Section 5 briefly discusses possible consequences of the preceding attempt. This aether is locally defined by a particular real Poincairé frame, I_O, in which (measured with real physical instruments) the velocity of light is identical in all directions at all observable frequencies. All observers tied to other frames passing through local inertial motions will see (measure) different space-time properties (associated with their velocity and orientations) defined by the corresponding Poincairé transformations.[1] Local variations of physical properties of the aether correspond to local transitions relating differential inertial frames at neighboring points.

2. Flat Spacetime and a Real Physical Aether

This model depends on the existence of a real physical vacuum (or zero point field) built with extended wave-like individual elements [16,17] centered on points in an external flat space-time, where such elements can overlap and interact (i.e. carry) collective motions corresponding to excess (electromagnetic 'bumps') or defects (gravitational 'holes') in the average density of the local aether elements. The model could be described as a gas of extended elements within flat space-time. These elements can interact locally (i.e. carry collective motions) and the gas' local scalar density thus carries waves (and solitons) associated with excess (electromagnetic) or defects (gravitational) in density, with respect to the average local vacuum density. One thus defines field variables associated with these two possible (excess or defect) local density variations. The vector fields, for example, in this paper, represent localized excess or density defects with respect to the local vacuum density. This model thus implies:

- A description of real physical vacuum properties in terms of real extended vacuum elements average behavior.
- A description of the behavior of its collective excess (above average) associated with recently observed electromagnetic effects.

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[1] To quote Kholmetsky "In order to pass from one arbitrary inertial frame I_1 to another one I_2 it is necessary to carry out the transformation from I_1 to the absolute frames I_O and then from I_0 to I_2" [18].

- A description of the behavior of its collective defects (below average) associated with observed gravitational effects.

Introducing these new concepts into Maxwell's equations and the description of gravitational fields along the same lines (in terms of vector fields, A_μ) suggests a new type of unification of both theories. Instead of looking for a common geometrization of gravity and light (i.e. their unification within a unique form of extended space-time geometry) one could assume the following from Newton and Lorentz :

A) The evolution of extended (fields) and of localized (sources) in terms of 1) vacuum (aether) 2) gravitational fields, 3) the electromagnetic field, reflects the time evolution (motions) and interactions of perturbations of a real material substance moving in a 3-dimensional flat space. This means that all three field and particle sub-elements are localized at given points, at each instant, in this 3-space and move continuously (i.e. locally transform) according to causal laws[2]
This assumption (distinction of space and fields) is now supported by the existence of a special particular experimental inertial cosmological frame I_0 in which

- the 2.7°K microwave radiation frame is isotropic and non rotating
- The average distribution of different types of galaxies (spiral, elliptical, QSO's) is isotropic not changing with distance [15].
- The observable anisotropy of the velocity of light propagation in different directions and around massive objects reflects the real motions of real fields described with respect to the I_0 frame in any real inertial Poincaré frame by covariant (local) four-vector scalar chaotic average density $\rho(x_\mu)$ around each absolute space-time point x_μ in I_0 i.e. by average four-vectors $A_\mu^0(x_\alpha)$ where the $_0$ denotes average measures taken in I_0.[3]

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[2] As a consequence of the failure of the geometrical unification program Einstein was still obliged in 1954 to consider the electromagnetic field as filling curved space-time, but never reached a final satisfying model.

[3] This implies 1) the existence of a basic high density of sub-elements in vacuum, 2) the existence of small density variations above (for light) and below (for gravity) the average density with the possibility of propagating density variation on the top of such a vacuum model as initially suggested by Dirac.

B) That all real physical observations rest on:

- The utilization of real physical apparatus based on electromagnetic fields and gravitational material with charged (or uncharged) particles.
- On observers also built with the same material i.e. influenced by the said fields and particles.

In other terms all observers (and their observations, inertial or not) are an integral part of fields and particles since they are part of the same overall real field and particle distribution. This fact determines their relation with all real phenomena. A physical theory should explicitly provide (within its context) a definition of the means whereby the quantities with which the theory is built and can be measured. The properties of light rays and massive particles are thus sufficient to provide the means of making basic measurements. Since real clocks and rods are the real instruments utilized in physics, we shall thus first define, for an individual inertial observer, the behavior of such instruments with respect to each other: since this determines, for every inertial observer possessing them, the behavior, with respect to I_0, of the material fields around him.

As a consequence of the covariant distribution character observed in I_0, the very small resistance to motion and assumed non-zero photon rest mass, real spin of possible extended vacuum sub-elements and their internal possible motions (and associated local interactions) one can describe the four-momenta and angular momenta of all extended sub-elements passing through a small four-volume with a constant average density on a hyperboloid, Σ_0. The four-momenta and angular momenta of extended elements are distributed at each point $P(x_\mu)$ with constant density $\rho(x_\mu)$ on space-like hyperboloids.

C) Following an idea of Noether the local analysis of moving fields and extended particles at each point by real observers tied to this point, is defined by local clocks and rods which move with the corresponding element. It is thus locally performed at each point of coordinates $x_\mu(\tau)$ which follows world-line L. To this point (in I_0) are attached local, internal variables $b^{(\lambda)}$, which describe its neighborhoods physical properties and thus depend on τ. The evolution is given by $x_\mu(\dot{x}_\mu)$, $b^\lambda(\dot{b}^\lambda)$, where $\dot{}$ denotes the proper time derivative with respect to τ when x_μ describes a world-line L. A scalar Lagrangian thus represents the evolution of the real physical medium in I_0, which depends on a local Lagrangian, L and is thus given by Poisson brackets. This description on I_0 is assumed to correspond to local space-time translations and four dimensional rotations which are determined by a Lagrangian L invariant under the local group of Poincairé transformations (i.e. the inhomogeneous Lorentz group). They contain [8,9]:

1) the operators P_μ of infinitesimal translations of X_μ only and can be described by $P_\mu \cdot X_\lambda = g_{\mu\lambda}$.

2) The operators $M_{\mu\nu}$ of infinitesimal four rotations in I_0 which act simultaneously on X_μ and on the internal variables. We have at X_μ:

$$M_{\mu\nu} x_\lambda = x_\mu g_{\nu\lambda} - x_\nu g_{\mu\lambda}. \tag{1}$$

Their action on internal local variables depends on their choice.

3) A choice of L leads to the momenta

$$G_\mu = \frac{\partial L}{\partial \dot{x}_\mu} \quad and \quad \beta^{(\lambda)} = \frac{\partial L}{\partial \dot{b}^{(\lambda)}} \tag{2}$$

yielding a constant impulsion vector

$$G_\lambda P_\mu x_\lambda = G_\lambda g_{\mu\lambda} = G_\mu : \tag{3}$$

and the total angular momentum:

$$M_{\mu\nu} = G_\lambda M_{\mu\nu} x_\lambda + \beta^{(\lambda)} M_{\mu\nu} b^{(\lambda)},$$

so that

$$M_{\mu\nu} = x_\mu G_\nu - x_\nu G_\mu + S_{\mu\nu}, \tag{4}$$

with

$$S_{\mu\nu} = \beta^{(\lambda)} M_{\mu\nu} \beta^{(\lambda)}.$$

These quantities satisfy the Inhomogeneous Lorentz group commutation relations $[P_\mu, P_\lambda] = 0$

$$[M_{\mu\nu}, P_\alpha] = g_{\alpha\beta} P_\nu - g_{\alpha\nu} P_\mu \tag{5}$$

i.e. Poisson Group Relations :

$$[G_\mu, G_\nu] = 0 \ , \ [M_{\mu\nu}, G_\alpha] = g_{\alpha\beta} G_\nu - g_{\alpha\nu} G_\mu \tag{6}$$
$$[M_{\mu\nu}, M_{\alpha\beta}] = g_{\mu\alpha} M_{\nu\beta} + g_{\nu\beta} M_{\mu\alpha} - g_{\mu\beta} M_{\nu\alpha} - g_{\nu\alpha} M_{\mu\beta}.$$

With these quantities one can also define local conservation laws for free elements i.e.

$$\begin{aligned} \dot{G}_\mu &= 0 \\ \dot{M}_{\mu\nu} &= 0 \\ \dot{S}_{\mu\nu} &= G_\mu \dot{x}_\nu - G_\mu \dot{x}_\mu. \end{aligned} \tag{7}$$

and introduce a constant local mass term M_0 with $G_\mu G_\mu = -M_0^2 \cdot c^2$.

4) An associated center of gravity y_μ is defined by the introduction of the four-vector

$$R_\mu = \left(\frac{1}{(M_0^2 c^2)} \right) \cdot S_{\mu\nu} \cdot G_\nu \tag{8}$$

associated with x_μ i.e.

$$y_\mu = x_\mu - R_\mu; \tag{9}$$

which implies that locally extended real media in I_0 are described by pairs of points as first suggested by Yukawa.

5) An inertial mass (usually not constant*)* μ_0 defined by

$$-M_0 c^2 = G_\mu \cdot \dot{x}_\mu \tag{10}$$

can also be attributed to x_μ: *M_0* being located at y_μ since one has:

$$\dot{y}_\mu = \dot{x}_\mu - \dot{R}_\mu = \dot{x}_\mu - \frac{1}{M_0^2 c^2} (G_\mu \cdot \dot{x}_\nu - G_\nu \dot{x}_\mu) G_\nu = \frac{\mu_0}{M_0^2} \cdot G_\mu \tag{11}$$

so that the motion of y_μ is locally rectilinear and y_μ has a proper time Θ, (with $d\lambda / d\Theta = M_0 / \mu'_0$) and we have :

$$y'_\mu = \dot{y}_\mu \cdot \frac{d\tau}{d\Theta} = G_\mu / M_0 = \text{constant.}$$

and

$$\mu_{\mu\nu} = R_\mu G_\nu - R_\nu G_\mu + S_{\mu\nu}, \qquad (12)$$

with respect to the center of gravity. Local instantaneous four rotations are described by :

- A specific *beigrössen* 4-frame b_μ^ξ (ξ=1,2,3,0) with

  $\dot{x}_\mu = b_\mu^4 = \frac{ic}{6}\varepsilon_{\mu\nu\alpha\beta} \cdot \varepsilon^{rst} b_\nu^r b_\alpha^s b_\beta^t$, $b_\mu^\xi = (i/2)\varepsilon_{\mu\nu\alpha\beta}\dot{x}_\nu S_{\alpha\beta}$ and

  $S_{\alpha\beta} = I \cdot \dot{b}_\alpha^\xi \cdot b_\beta^\xi$.
- A specific four-frame a_μ^ξ centered on y_μ with $M_{\alpha\beta} = K \cdot \dot{a}_\alpha^\xi \cdot a_\beta^\xi$ for a_μ^4 along y'_μ and $a_\mu^3 = (i/2M_0 c) \cdot \varepsilon_{\mu\nu\alpha\beta} G_\nu \mu_{\alpha\beta}$.

This set of relations must be completed by relations which will define the interactions between the extended elements i.e. the propagation in the aether of collective motions corresponding to observed gravitational and electromagnetic phenomena. Before the introduction of such interactions one must recall that such proposals have already been made in the past.

We only mention here:

- Weyssenhof's proposal [16] $S_{\alpha\beta}\dot{x}_\beta = 0$ extensively discussed in the literature.
- Nakano's proposal [19] $S_{\alpha\beta}\dot{x}_\beta = I \cdot \ddot{x}_\alpha$.
- Roscoe's proposal with photon mass [20].

3. General Relativity Represented as a Polarizable Vacuum

Since all observed effects of gravity in distant space rest on light observation (including γ and radio EM waves coming through space from distant sources) a simple model endows the polarizable vacuum with properties that might account for all the phenomena in terms of distortions. This initial proposal of Wilson and Dicke has been recently revived with astonishing success by Puthoff [4] and Krogh [21]. We first summarize their model and will complete it with a supplementary mass term in electro-magnetism.

One starts from the idea that in flat space the electric field moves in a real vacuum medium with a point varying dielectric constant K: so that this D field satisfies the vacuum equation:

$$D = K \cdot \varepsilon_0 \cdot E. \tag{13}$$

This corresponds to a variable fine structure constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot \left(\frac{\mu(K)/\mu_0}{K} \right)^{1/2} : \tag{14}$$

so that the vacuum has permittivity and permeability constants given by

$$\varepsilon_0 \to \varepsilon = K \cdot \varepsilon_0 \quad and \quad \mu_0 \to \mu = K \cdot \mu_0, \tag{15}$$

and an impedance $(\mu/\varepsilon)^{1/2} = (\mu_0/\varepsilon_0)^{1/2}$ to satisfy Eötvos-type experiments. The local velocity of light for a given frequency ν varies like $V_\nu = c/K$ i.e like $1/(\mu\varepsilon)^{1/2}$. The corresponding principle of equivalence implies that the self energy of a system changes when K changes; so that a flat-space energy E_0 in flat space changes into

$$E = E_0 \cdot (K)^{-1/2}; \tag{16}$$

and one has

$$m = m_0 \cdot K^{3/2}. \tag{17}$$

As a consequence the condition $\mathrm{E} = \hbar \cdot \omega$ becomes

$$\omega = \omega_0 (K)^{-1/2} \tag{18}$$

along with the time and length variations Δt *and* Δr given by the relations:

$$\Delta t = \Delta t_0 (K)^{1/2} \quad and \quad \Delta r = \Delta r_0 (K)^{-1/2}. \tag{19}$$

These relations are evidently equivalent to a local curvature of space. Indeed a dx_0 length rod shrinks to $d_x = d_{x_0} \cdot (K)^{-1/2}$ and would measure dx_0, where the rod remains rigid, is now expressed in terms of dx-length rod as $dx_0 = (K)^{1/2} dx$.

Using the same argument for dt and dt_0 we find that one can write:

$$dS^2 = c^2 dt_0^2 - (dx_0^2 + dy_0^2 + dz_0^2) \tag{20}$$

which transforms into

$$dS^2 = \frac{1}{K} c^2 dt^2 - K(dx^2 + dy^2 + dz^2): \qquad \text{(a)}$$

i.e. (21)

$$dS^2 = g_{ij} ... dx^i dx^j, \qquad \text{(b)}$$

with $g_{00} = 1/K, \quad g_{11} = g_{22} = g_{33} = -K \quad and \quad g_{ij} = 0 \quad for \quad i \neq j.$

In the case of a spherically symmetric mass distribution one writes

$$\begin{cases} K = e^{2G \cdot M / rc^2} \\ K = 1 + 2\dfrac{G \cdot M}{rc^2} + \dfrac{1}{2}\left(\dfrac{2GM}{rc^2}\right)^2 + \ldots . \end{cases} \tag{22}$$

where *G* is the gravitational constant, *M* the mass and *r* the distance from its origin located at the center of mass. Puthoff [4] has recently shown that this model accounts (sometimes with better precision) for all known experimental tests of General Relativity in a simple way i.e. one can describe

- The gravitational redshift given by $\omega = \omega_0 /(K)^{1/2}$ (so that $\Delta\omega / \omega \cong (GM / R^2 c^2) h$ has a 1/100 precision).
- The bending of light rays by the sun and stars.
- The advance of the Perihelion of Mercury.

He has also shown that one can derive the form of (22) from a general Lagrangian with a variable *K* i leaving aside vacuum interaction in I_0 :

$$L = -\left[\frac{m_0 c^2}{K^{1/2}}\left(1 - \left(\frac{v}{(c/K)}\right)^2\right)^{1/2} + q \cdot \phi - q \cdot \vec{A}\vec{V}\right]\delta^3(r - \bar{r})$$
$$-\frac{1}{2}\left(B^2 / (K \cdot \mu_0) - K(\varepsilon_0 E^2)\right) - \frac{\lambda}{K^2}\left[(\nabla K)^2 - \frac{1}{(c/K^2)}\left(\frac{\partial K}{\partial t}\right)^2\right] \tag{23}$$

This association of gravitational theory with electromagnetic theory based on the introduction of a variable dielectric vacuum constant *K* has recently been made more explicit by Krogh [21]. Noting that:

a) Electromagnetic theory implies the effects of electromagnetic vector four-potential vectors A_μ on the phases S of quantum mechanical waves so that one has

$$\Delta S = \frac{q}{h}\int \phi dt - \frac{q}{hc}\int \vec{A}\cdot d\vec{S} \qquad (24)$$

for charged particles moving under the influence of the four vector, A_μ.

b) If $m_\gamma \neq 0$ (m_γ is the mass term introduced into Maxwell's equation) the force on charged particles takes the form

$$F = q\left(E + \frac{V\times B}{c}\right) + q\cdot V \qquad (25)$$

where the first term is the usual transverse Poynting force on currents and the second a longitudinal force along currents (resulting from non zero photon mass) recently observed by Graneau [22] and Saumont [23].
c) One can describe gravity with a four-vector density A_μ^g so that the gravitational (Newton) and electromagnetic (Coulomb) potentials have the same form, but different coupling constants. This suggests that both wave fields and singularities are just different aspects of the same fundamental field.

4. Maxwell's Equations Extended

This discussion opens the possibility to test new types of extensions of Maxwell's equations in the laboratory. Since this has already been attempted some results (derived within the frame of the model) are given here:

a) From a non-zero vacuum conductivity coefficient $\sigma \neq 0$ [24,25] we have in vacuum div E =0 with curl H = σE+ $\varepsilon_0 \chi_0 \partial E / \partial t$ and div H = 0 with curl E = $-\mu_0 \chi_m \partial H / \partial t$.

b) From an associated non-zero photon mass term ($m_\gamma \neq 0$) (with $A_\mu A_\mu \rightarrow 0$ where A_μ denotes the total four-potential density in Dirac's aether model. This introduces a non-zero fourth component of the current

$J_\mu = \sigma E, j_0$ (where $j_0 \neq 0$) into the vacuum corresponding to a real detectable space. With present technology this implies that the present vacuum really carries space-charge currents [25] (so that the divergence of the electric field is different from zero in Vacuo) and the corresponding existence of a displacement current (i.e. a curl of the magnetic field) and its associated current density[4].

4.1 *The Infinitesimal Mass of Photons*

Unifying massive spin 1 photons piloted by electromagnetic waves built with massive extended sub-elements has been developed in a series of books by Evans, Vigier et al. [24] The model implies the introduction of spin and mass with an associated energyless magnetic field component $B^{(3)}$ in the direction of propagation and a small electrical conductivity in the Dirac vacuum also implying a new 'tired light' mechanism [11,14,24]. Corresponding equations will be given below.
In the absolute inertial frame I_0 all massive particles are governed by a gravitational potential four-vector $\phi_g, \vec{A}_g / c$, associated with a small mass m_g which can be decomposed into transverse, longitudinal and gradient potentials.

We can thus associate the relations

$$\Box \varphi = -\frac{\rho}{\varepsilon_0} + \mu\varphi \Box \text{and} \Box \vec{A} = -\vec{d}_0 / \varepsilon_0 c + \mu\vec{A} \qquad (26)$$

which represent the electromagnetic field in vacuum in any inertial frame, Σ_0 the relations:

$$\Box \phi_g = 4\pi G m \rho_\mu + \mu_g \cdot \phi_g \quad and \quad \vec{A}_g = 4\pi \cdot G \cdot \vec{j}_m + \mu_g \vec{A}_g \,, \qquad (27)$$

which represent the gravitational field in the same vacuum; where ρ_μ refers to mass density, j_m to mass current and μ and μ_g to EM and gravitational mass (both very small $\cong 10^{-65}$ grams) and $\rho \cdot c_0$ in the $\Box$

[4] Such attempts have been recently published in a book by Lehnert & Roy [25] so we shall only present a summary of some results and assumptions.

terms ($\Box\Box = \nabla^2 - (1/c_0^2)\partial/\partial t^2$) represents the corresponding wave velocities (which except in I_0 depend on the directions in flat space-time) so that one has:

$$c_0 = c \cdot e^{2\phi_g/c^2}; \qquad (28)$$

where c is the value in the absence of a gravitational potential A_μ^g. In this model, one assumes, with Sakharov [5-7], that the gravitational field corresponds to local depressions in the immensely positive energy of the zero-point field; and gravitational fields represent regions of diminished energy (i.e. that their momentum gravity corresponds to holes in vacuum energy or local defects of vacuum elements). Their effective momentum is thus opposite and corresponding gravitational forces are attractive.

Such an association also suggests that although measuring devices (observations) in local inertial Poincairé frames are altered by gravitational potentials (they are part of the same real physical background in this model). There is no effect on the geometry of flat space and time. For any given real inertial local Poincairé frame, Σ_0 real space is Euclidean and one uses Poincairé transformations between Σ_0 and I_0 to describe real motions which include consequences of gravitational potentials. For example a reduction of the velocity of quantum mechanical waves, including light, is taken as a fundamental effect of gravitational potentials. Clocks are slowed and measuring rods shrink in such potentials by a factor e^{ϕ_g/c^2}.

4.2 *Divergence of the Electromagnetic Field*

A non-vanishing divergence of the electric field given below, can be added to Maxwell's equations which results in space-charge distribution. A current density arises in vacuo and longitudinal electric non-transverse electromagnetic terms (i.e. magnetic field components) appears (like $B^{(3)}$) in the direction of propagation.

Both sets of assumptions were anticipated by de Broglie and Dirac. They imply that the real zero-point (vacuum) electromagnetic distribution

- is not completely defined by $F_{\mu\nu}$ but by a four-vector field distribution given by a four-vector density, A_μ associated with a de Broglie-Proca equation i.e.

$$\Box A_\mu(x_\alpha) = -\frac{m_\gamma^2 c^2}{\hbar^2} A_\mu(x_\alpha) \qquad (29)$$

and its complex conjugated equation.

- the A_μ field potential equation also contains a gradient term so one has in vacuum:

$$A_\mu = A_\mu^T + A_\mu^L + \lambda \partial_\mu S \qquad (30)$$

with $A_\mu A^* \to 0$ and a small electrical conductivity in vacuo.

5. Possible New Consequences of the Model

Since such models evidently imply new testable properties of electromagnetic and gravitational phenomena we shall conclude this work with a brief discussion of the points where it differs from the usual interpretations and implies new possible experimental tests.

If one considers gravitational and electromagnetic phenomena as reflecting different behaviors of the same real physical field i.e. as different collective behavior, propagating within a real medium (the aether) one must start with a description of some of its properties.

We thus assume that this aether is built (i.e. describable) by a chaotic distribution $\rho(x_\mu)$ of small extended structures represented by four-vectors $A_\mu(x_\alpha)$ round each absolute point in I_0. This implies

- the existence of a basic local high density of extended sub-elements in vacuum
- the existence of small density variations $\delta\rho(x_\mu)A_\alpha(x\mu)$ above $\delta\rho > 0$ for light and below $(\delta\rho < 0)$ for gravity density at x_μ.
- the possibility to propagate such field variations within the vacuum as first suggested by Dirac [13].

One can have internal variations: i.e. motions within these sub-elements characterized by internal motions associated with the internal behavior of average points (i.e. internal center of mass, centers of charge, internal rotations : and external motions associated with the stochastic behavior, within the aether, of individual sub-elements. As well known the latter can be analyzed at each point in terms of average drift and osmotic motions and A_μ distribution. It implies the introduction of non-linear terms.

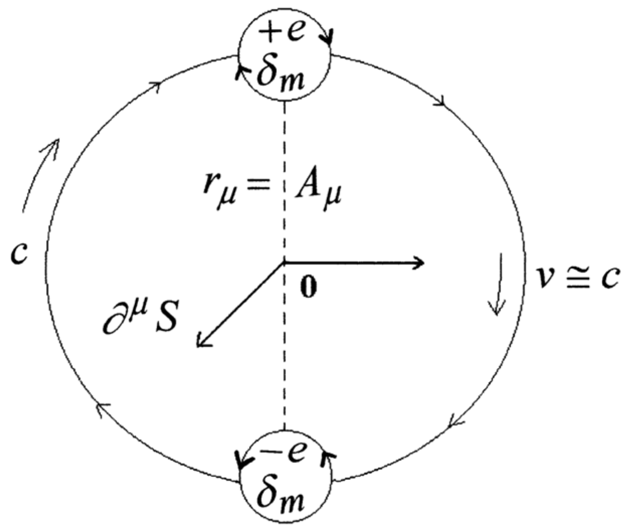

Figure 1. Diagram conceptualizing two oppositely charged sub-elements rotating at $v \cong c$ around a central point 0 behaving like a dipole bump and hole on the topological surface of the covariant polarized Dirac vacuum.

To describe individual non-dispersive sub-elements within I_0, where the scalar density is locally constant and the average A_μ equal to zero, one introduces at its central point $Y_\mu(\theta)$ a space-like radial four-vector $A_\mu = r_\mu \exp(iS/\hbar)$ (with $r_\mu r^\mu$ = a^2 = constant) which rotates around Y_μ with a frequency $\nu = m_\gamma c^2 / h$. At both extremities of a diameter we shall locate two opposite electric charges e^+ and e^- (so that the sub-element behaves like a dipole). The opposite charges attract and rotate around Y_μ with a velocity $\cong c$. The +e and –e electromagnetic pointlike charges correspond to opposite rotations (i.e $\pm\ \hbar/2$) and A_μ rotates around an axis perpendicular to A_μ located at Y_μ, and parallel to the individual sub-element's four momentum $\partial_\mu S$.

Assuming electric charge distributions correspond to $\delta m > 0$ and gravitation to $\delta m < 0$ one can describe such sub-elements as holes ($\delta m <$ 0) around a point 0 around which rotate two point-like charges rotating in opposite directions as shown in Figure 6.1 below.

These charges themselves rotate with a velocity c at a distance $r_\mu = A_\mu$ (with $r_\mu r_\mu$ = Const.). From 0 one can describe this by the equation

$$\Box A_\mu - \frac{m_\gamma^2 c^2}{\hbar^2} \cdot A_\mu = \frac{\left[\Box (A_\alpha^* A_\alpha)\right]^{1/2}}{(A_\alpha^* A_\alpha)^{1/2}} \cdot A_\mu \qquad (31)$$

with $A_\mu = r_\mu \cdot \exp[iS(x_\alpha)/\hbar]$ along with the orbit equations for e^+ and e^- we get the force equation

$$m \cdot \omega^2 \cdot r = e^2 / 4\pi r^2 \qquad (32)$$

and the angular momentum equation:

$$m_\gamma \cdot r^2 \cdot \omega = \hbar / 2 \qquad (33)$$

Eliminating the mass term between (31) and (33) this yields

$$\hbar\omega = e^2 / 2r \qquad (34)$$

where $e^2/2r$ is the electrostatic energy of the rotating pair. We then introduce a soliton-type solution

$$A_\mu^0 = \frac{\sin \cdot K \cdot r}{K \cdot r} \cdot \exp[i(\text{cot} - K_0 x)] \qquad (35)$$

where

$$K = mc/\hbar, \quad \omega = mc^2/\hbar \quad and \quad K_0 = mv/\hbar \qquad (36)$$

satisfies the relation (31) with $r = ((x - vt)^2 \cdot (1 - v^2/c^2)^{-1} + y^2 + z^2)^{1/2}$ i.e.

$$\square A_\mu^0 = 0 : \qquad \square \tag{37}$$

so that one can add to A_μ^0 a linear wave, A_μ (satisfying $\square A_\mu = (m_\gamma^2 c^2 / \hbar^2) A_\mu$) which describes the new average paths of the extended wave elements and piloted solitons. Within this model the question of the interactions of a moving body (considered as excess or defect of field density, above or below the aether's neighboring average density) with a real aether appears immediately[5].

As well known, as time went by, observations established the existence of unexplained behavior of light and some new astronomical phenomena which led to discovery of the Theory of Relativity.

In this work we shall follow a different line of interpretation and assume that if one considers particles, and fields, as perturbations within a real medium filling flat space time, then the observed deviations of Newton's law reflect the interactions of the associated perturbations (i.e. observed particles and fields) with the perturbed average background medium in flat space-time. In other terms we shall present the argument (already presented by Ghosh et al. [26]) that the small deviations of Newton's laws reflect all known consequences of General Relativity

The result from real causal interactions between the perturbed local background aether and its apparently independent moving collective perturbations imply absolute total local momentum and angular momentum conservation resulting from the preceding description of vacuum elements as extended rigid structures.

6 Extending Newton's Model with Inertia and Vacuum Drag

Starting from an aether built with moving small extended structures with an average real distribution isotropic in an inertial frame I_0 (i.e. examining the effects in a given inertial frame I centered on a point Y_μ of the real vacuum distribution on a test particle moving with absolute

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[5] According to Newton massive bodies move in the vacuum, with constant directional velocities, i.e. no directional acceleration, without any apparent relative friction » or drag » term. This is not true for accelerated forces (the equality of inertial and gravitational masses are a mystery) and apparent absolute motions proposed by Newton were later contested by Mach.

velocity V^0 and angular momentum $\omega^0_{\alpha\beta}$) one can evaluate more precisely, the collective interactions carried by this aether between two extended neighboring regions centered on points *A* and *B* with two centers of mass situated at X_A and X_B.

Starting with $\delta\rho < 0$, i.e. for gravitational effects, it appears immediately

a) if one assumes the gravitational potential is spherical in the rest frame I_B of its source *B,*

b) that the motion of *A* undergoes a velocity dependent inertial induction with respect to *A* i.e. a friction depending on the velocity *v* of *A* with respect to *B*

c) that this motion is also submitted to an acceleration dependent inertial with respect to I_B i.e. also an acceleration depending on its acceleration a measured in I_B.

d) possible terms depending on higher order time derivations which we will neglect in the present analysis we can write (6.19) the force on *A* due to *B* in I_B in the form $F = F_S+F_v+F_a$ where

$$F = -G \cdot \frac{m_A \cdot m_B}{r^2} - G' \frac{m_A \cdot m_B}{c^2 r^2} \cdot v^2 \cdot f(\theta)\hat{U}_r - G'' \cdot \frac{m_A \cdot m_B}{c^2 r} \cdot af(\phi)\hat{U}_r \quad (38)$$

The terms *G, G', G''* are scalars possibly dependent on *v*. The terms m_A and m_B are the gravitational masses in I_B, $\hat{U}$, is the unit vector along *r*. $f(\theta)$ and $f(\phi)$ must have the same form i.e. 1/2 cos ϕ or cos $\phi|\cos\phi|$. If we also accept the preceding velocity dependent analysis for contracting rods and retarded clocks then we should write *G* = *G'* in (38) and take *f* (θ) = cos θ $|\cos\phi|$ as done by Ghosh [26]. Moreover, if we compare the form given by Weber to the repulsion of two electric charges of the same sign:

$$F^e_{AB} = \frac{e_A \cdot e_B}{4\pi\varepsilon r^2} \cdot \left[1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2} \cdot \frac{d^2 r}{dt^2}\right] \quad (39)$$

corresponding to electromagnetism, with the recent form given by Assis [27,28] to attracting interacting masses m_A and m_B i.e.

$$F^g_{AB} = -G \cdot \frac{m_A . m_B}{r^2}\left[1 + \frac{6}{c^2}\left[r \cdot \frac{d^2 r}{dt^2} - \frac{1}{2}\left(\frac{dr}{dt}\right)^2\right]\right] \quad (40)$$

we see they have exactly the same form; the difference of their coefficients being compatible (within our interpretation) since they correspond to opposite variations of the average vacuum density. Their interpretation in terms of $\delta\rho > 0$ (for electromagnetism) and $\delta\rho < 0$ (for gravitation) also explains (at last qualitatively) why extended depressions repel or attract when they rotate through parallel or antiparallel directions and only attract when $\delta\rho < 0$. This also explains why a reduction of attraction between two masses has been observed when one puts another mass between them (the LAGEOS satellite). In this model this similarity is indeed comparable to similar behaviors of vortices for gravitation and Tsunamis for electromagnetism on an ocean surface.

If one assumes the absolute local conservation of four-momentum and angular momentum in regions containing the preceding aether carrying its associated collective electromagnetic and gravitational motions one can evaluate the effects of their interactions. With a real physical aether there is no such thing as free electromagnetic or gravitational phenomena. Drag theories (described as inertial induction) are always present and responsible for Casimir type effects in the microscopic domain. Real consequence of the aether appear, at various levels, in the macroscopic and cosmological domains… as has already been suggested in the literature and tested in laboratory or astronomical phenomena. We only mention here:

1) Possible consequences of modifying and testing the Newton and Coulomb forces.
2) The redshift and variable velocity of electromagnetic waves results from the rotational inertial drag of extended photons moving in vacuum: an effect already observed in light traversing around the earth [28].
3) The possible measurable existence of the redshift of transverse gravitational waves… possible in the near future.
4) Observational redshift variations of light emitted by Pioneer close to the solar limb, i.e. also of photons grazing a massive object [28].
5) The observed anisotropy of the Hubble constant in various directions in the sky [28] associated with various galactic densities.
6) Observed torques on rotating spheres in the vicinity of large massive bodies. This also appears in some experiments, i.e.:
    a) Secular retardation of the earth's rotation.

b) Earth-moon rotation in the solar system etc.

7) Apparent evolution with time of angular momentum in the solar-planetary system.
8) Different variation of redshift of light traveling up and down in the Earth’s gravitational field…Which also supports existence of photon mass.

7 Relativistic Maxwell's Equations in Complex Form

We will now outline the relativistic formalism which gives a more comprehensive explanation of the complexification scheme. Such issues as the Higgs (soliton) monopole depend on considering Lorentz invariance and relativistic causality constraints. We will also relate the complexification of Maxwell's equations to models of nonlocality. We examine, for example, the manner in which advanced potentials may explain the remote connectedness which is indicated by the Clauser test of Bell's theorem. Similar arguments apply to Young's double slit experiment. The collective coherent phenomena of superconductivity is also explainable by considering the relativistic field theoretic approach in which wave equations are solved in the complex Minkowski space (such as the Dirac equation).

7.1 *Relativistic conditions on Maxwell's equations in complex geometries and the invariance of the line element*

This section introduces the relativistic form of Maxwell's equations. The fields $\underline{E}$ and $\underline{B}$ are defined in terms of ($\underline{A},\phi$), the four vector potential; and the relativistic form of $\underline{E}$ and $\underline{B}$ is presented in terms of the tensor field, $F_{\mu\nu}$ (where indices μ and ν run 1 to 4). We then complexity $F_{\mu\nu}$ and determine the expression for the four vector potential $A_\mu = \left(\underline{A}_j, \phi\right)$ in terms of $F_{\mu\nu}$. (index j runs 1 to 3). Discussion of line element invariance is given in terms of $F_{\mu\nu}$.

In section 6.8 we describe the complex form of A_μ fields and through the formalism in this section we can relate this to the complex forms of

$\underline{E}$ and $\underline{B}$. We utilize Weyl's action principle to demonstrate the validity of the use of the complex form of $F_{\mu\nu}$. Weyl relates the gravitational potential, $G_{\mu\nu}$, to the EM 'geometerizing' potential A_μ, or geometrical vector, using the principle of stationary action for all variations $\delta G_{\mu\nu}$ and δA_μ. The quantity A_μ, or vector potential, which we identify with A^μ, is related to $F_{\mu\nu}$, the EM force field, by a set of gauge invariant relations. The EM force $F_{\mu\nu}$ is independent of the gauge system. The curl of A_μ has the important property

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \tag{41}$$

where $F_{\mu\nu}$ is antisymmetric or $F_{\mu\nu} = -F_{\nu\mu}$, and changing $\sim A_\mu$ to $A_\mu^{'} = A_\mu + \partial\phi / \partial x_\mu$ is a typical gauge transformation where the intrinsic state of the world remains unchanged.

We define the four vector potential as A_μ, which can be written in terms of the three vector $\underline{A}_j$ and ϕ, where ϕ is the fourth or temporal component of the field. The indices μ, ν run 1 to 4 and *j* runs 1 to 3.

Then we can write Maxwell's equations in compact notation in their usual tensor form in terms of $F_{\mu\nu}$, (for *c* = 1);

$$F_{\mu\nu} = \begin{pmatrix} 0 & -B_z & B_y & E_x \\ B_z & 0 & -B_x & B_y \\ -B_y & B_x & 0 & E_x \\ -E_x & -E_y & -E_z & 0 \end{pmatrix} \tag{42}$$

then the equations $\nabla \times E = -(1/c)(\partial \underline{B} / \partial t)$ and $\nabla \cdot \underline{B} = 0$ can be written

$$\text{as} \frac{\partial F_{\mu\nu}}{\partial x} + \frac{\partial F_{\mu\nu}}{\partial y} + \frac{\partial F_{\mu\nu}}{\partial z} = 0 \tag{43}$$

or $\nabla \times F_{\mu\nu} = 0$ for $x^1 = x,\ \ x^2 = y,\ \ x^3 = z,$ and $x^4 = t$.

To complexity the elements of $F_{\mu\nu}$ we can take conditions, For $\left(F_{41}, F_{42}, F_{43}\right) = i\underline{E}$ and $\left(F_{23}, F_{32}, F_{12}\right) = B$,

or $\left(E_x, E_y, E_z\right) = iE$ and $\left(B_x, B_y, B_z\right) = \underline{B}$. (44)

We can write the complex conjugate of the electric and magnetic fields in terms of the complex conjugate of $\underline{F}$ or $F^*_{\mu\nu} = -F^{\mu\nu}$. There is a useful theorem stating [29] $\nabla_{123} \times \underline{F} = \nabla^4 \cdot F^*$ or $(\nabla_{xyz} \times F = \nabla^t \cdot F^*)$. Then for $\left(F^{23*}, F^{31*}, F^{12*}\right) = i\underline{E}$ and $\left(F^{41*}, F^{42*}, F^{43*}\right) = -B$ we obtain $\partial F^{\mu\nu*} / \partial x^\nu = 0$ or $\nabla \cdot \underline{F}^* = 0$ which gives the same symmetry between real and imaginary components as ours and in Inomata's formalism. [30].

The expressions for the other two Maxwell equations $\nabla \cdot \underline{E} = 4\pi\rho$ and $\nabla \times \underline{B} = \frac{1}{c}\frac{\partial E}{\partial t} + J_e$ can be obtained by introducing the concept of the vector potential in the Lorentz theory as first noticed by Minkowski [31]; we have the four vector forms $\left(\phi_1, \phi_2, \phi_3\right) = \underline{A}$ and $\phi_4 = i\phi$, then $\underline{B} = \nabla \times \underline{A}$ and $\underline{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \underline{A}}{\partial t}$. Then we have $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$ or $\underline{F} = \nabla \times \underline{A}$ for the vector and scalar potentials $A = \left(A_1, A_2, A_3, \phi\right)$. If A is a solution to $F = \nabla \times \underline{A}$ then $\phi'_\mu + \frac{\partial \phi}{\partial x^\mu}$ also is (by gauge invariance) and $\nabla \cdot \underline{A} + \frac{1}{c}\frac{\partial \phi_4}{\partial t} = 0$. We term the fourth component ϕ or ϕ_4 interchangeably. Then from Lorentz theory we have the 4D form as $\frac{\partial A^\mu}{\partial x^\mu} = 0$ or $\nabla \cdot \underline{A} = 0$. We can now write the equations for $\nabla \cdot \underline{F} = 4\pi\rho$ and $\nabla \cdot \underline{B} = \frac{1}{c}\frac{\partial E}{\partial t} + J_e$ as

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = s^\mu \quad \text{or} \quad \nabla \cdot \underline{F} = S. \tag{45}$$

The most general covariant group of transformations of the EM field equations (more general than the Lorentz group) is formed by affine transformations which transform the equation of the light cone, $s^2 = 0$

into itself. (The properties of the spacetime manifold are defined in terms of the constraints of the line element, which relate to the gravitational potential, $g_{\mu\nu}$. We also form an analogy of the metric space invariant to the EM source vector, s_μ) [32].

This group contains the Lorentz transformations as well as inversion with respect to a 4D sphere, or hyperboloid in real coordinates. Frank [33] discusses the Weyl theory and gives a proof that the Lorentz group together with the group of ordinary affine transformations is the only group in which Maxwell's equations are covariant [33]. Recall that an affine transformation acts as $x^\mu = \alpha_\nu^\mu x^\nu$ with an inverse $x^\nu = \alpha_\mu^{-\iota} x'^\mu$. The affine group contains all linear transformations and the group of affine transformations transforms $s^2 = 0$ on the light cone into itself.

In the Weyl geometry, if we have from before, $\underline{F} = \nabla \times \phi$ and

$$\nabla \cdot \underline{F} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} F^\mu}{\partial x^\mu} \qquad (46a)$$

and

$$\nabla^\mu \cdot F = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} F^\nu}{\partial x^\nu} \qquad (46b)$$

with the signature (+,+,+,-). Then using the theorem in W. Pauli [34],

$$\nabla_\mu \cdot \nabla \times \underline{F} = \nabla_\mu \nabla \cdot \underline{F} - \Box F_\mu \qquad (47)$$

and from before, $\nabla \cdot F = S$ and since $\nabla \cdot \phi = 0$ and then $\partial \phi^\mu / \partial x^\mu = 0$ and we have from

$$\nabla_\mu \cdot \nabla \times \underline{A} = \nabla_\mu \nabla \cdot \underline{A} - \Box \phi_\mu = S_\mu \qquad (48)$$

or

$$\Box A_\mu = -S_\mu \qquad (49)$$

for our potential equation, where $\Box$ is the D'Alembertian operator, and

$$\Box = \partial^\mu \partial_\mu = \eta^{\mu\nu} \partial_\nu \partial_\mu = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} = \nabla^2_{R^3} - \frac{\partial^2}{\partial t^2} \qquad (50)$$

The important aspect of this consideration [35] is our ability to relate the EM potential to a corresponding spacetime metric interval s or s^2. Hence we can construct the invariant relations for our fields in terms of our Lorentz invariance four space conditions. We can also relate the introduction of a complex spacetime to the complex expansion of the electric and magnetic fields in this section and demonstrate their self-consistency. We will look at this in more detail at the end of this section where we consider a generalized affine connection. We can relate the EM potential, A_μ and ϕ_4 to $g_{\mu\nu}$ as $\sqrt{g}$ and also to the square root of the invariant, or s.

The key to the relationship of complex $F_{\mu\nu}$ and complex spacetime is the analogy between ϕ and $g_{\mu\nu}$ .We can relate the EM scalar potential into the interval of time as in Eq. (49), $\Box A_\mu = \phi = -S_\mu$ and we make the analogy of A_μ to $g_{\mu\nu}$ which is tied to the invariance conditions on s^2. Both potentials are then related to spacetime or spacetime interval separation. Note that in the $\Box A_\mu = -s$ equation we have a $\sqrt{g}$ factor in order to form the invariant. In the equation for s^2, the invariant is found directly as $s^2 = g_{\mu\nu} x^\mu x^\nu$. We will write a set of invariant relations for the case of complex $\underline{E}$ and $\underline{B}$ fields at the end of this section. We can relate this then to the deSitter algebras and the complex Minkowski metric.

Note that we associate the E_x component of $F_{\mu\nu}$ or $F_{41} = E_x$ with ϕ_4' as follows:

$$F_{41} = E_x = \phi_4' \frac{e}{r^2} \tag{51}$$

in which $4\pi e$ is associated with electric charge on the electron. This approximation is made in the absence of a gravitational field. Maxwell's equations are intended to apply to the case in which no field of force is acting on the system or in the special system of Galilean coordinates $A^\mu = \left(A_x, A_y, A_z, \phi\right)$, where $A^j = \left(A_x, A_y, A_z\right)$ is the vector potential and ϕ is the scalar potential and A^μ is the covariant form. Also, for the contravariant form, we have $A_\mu = \left(-A_x - A_y - A_z, \phi\right)$. And in empty space we have

$$\Box A_\mu = 0 \tag{52}$$

In non-empty space then

$$\Box A^\mu = J^\mu \tag{53}$$

or we can write this as

$$\nabla^2 A^\mu - \mu\varepsilon \frac{\partial^2 A^\mu}{\partial t^2} = -J^\mu \tag{54}$$

which is true only approximately in the assumption of flat space for Galilean coordinates. This is the condition which demands that we consider the weak Weyl limit of the gravitational field.

The invariant integral, $\mathscr{I}$ for $F^{\mu\nu}$ is given by

$$\mathscr{I} = \frac{1}{4}\int F^{\mu\nu} F_{\mu\nu} \sqrt{g}\, d\tau \tag{55}$$

The quantity, $\mathscr{L}$ is called the action integral of the EM field. Weyl [36] demonstrates that the action integral is a Lagrangian function, or

$$\mathscr{L} = \int dt \iiint \frac{1}{2}\left(B_x^2 + B_y^2 + B_z^2 - E_x^2 - E_y^2 - E_z^2\right) dxdydz \tag{56}$$

which is of the form $\mathscr{L} = (T - v)dt$. By describing an electron in a field by Weyl's formalism one has a more general but more complicated formalism than the usual Einstein-Galilean formalism [37]. We can write a generalized Lagrangian in terms of complex quantities. For example, we form a modulus of the complex vector B as $\left|B\right|^2 = BB^* = B_{\mathrm{Re}}^2 + B_{\mathrm{Im}}^2$. This is the Lagrangian form for the real components of E and $\mathbf{B}$ in four space. We can again consider $E = E_{\mathrm{Re}} + iE_{\mathrm{Im}}$ and $B = B_{\mathrm{Re}} + iB_{\mathrm{Im}}$ for the complex forms of $\underline{E}$ and $\underline{B}$. The complex Lagrangian in complex eight-space becomes

$$\mathscr{L} = \iint dt_{\mathrm{Re}} dt_{\mathrm{Im}} \iiint\limits_{\mathrm{Re}} \iiint\limits_{\mathrm{Im}} \frac{1}{2}\left(B_{\mathrm{Re}}^2 - E_{\mathrm{Re}}^2 + B_{\mathrm{Im}}^2 - E_{\mathrm{Im}}^2\right) \tag{57}$$

$dx_{\mathrm{Re}} dy_{\mathrm{Re}} dz_{\mathrm{Re}} dx_{\mathrm{Im}} dy_{\mathrm{Im}} dz_{\mathrm{Im}}$

Note that this is an 8D integral, six over space. Also all quantities of the integrand are real because they are squared quantities. We can also write a generalized Poynting vector and energy relationship. We also have two equations which define a vector quantity A_μ in EM theory which corresponds to the gravitational potential $g_{\mu\nu}$. We have

$$\frac{\partial}{\partial g_{\mu\nu}}\left(\frac{1}{4} F^{\mu\nu} F_{\mu\nu}\right) = \frac{1}{2} E^{\mu\nu} \tag{58}$$

and

$$\frac{\partial}{\partial A_\mu}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right) = -J^\mu \qquad (59)$$

where $E^{\mu\nu}$ is the energy tensor and J^μ is the charge and current vector. Two specific cases are for a region free from electrons, or $T^{\mu\nu} - E^{\mu\nu} = 0$, or a region free of the gravitational potential or in the weak Weyl limit of the gravitational field, $\Box F_{\mu\nu} = J_{\mu\nu} - J_{\nu\mu}$ where $\Box$ is the four space D'Alembertian operator. The solution for this latter case is for the tensor potential $A_{\mu\nu}$,

$$F_{\mu\nu} = \frac{1}{4\pi\gamma}\left(A_{\mu\nu} - A_{\nu\mu}\right)\int\frac{de}{r} \qquad (60)$$

if all parts of the electron are the same or uniform in charge. For the proper charge ρ_0, we have $J^\mu = \rho_0 A^\mu$.

In the limit of $A_\mu^\mu = 0$, then ρ_0, the proper density, is given as

$\rho_0 = -\frac{\gamma^2}{12\pi}J_\mu J^\mu$ for $\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}}$. In Weyl's 4D world then, matter cannot be constituted without electric charge and current. But since the density of matter is always positive the electric charge and current inside an electron must be a space-like vector, the square of its length being negative. To quote from Eddington:

> It would seem to follow that the electron cannot be built up of elementary electrostatic charges but resolves into something more akin to magnetic charges [38].

Perhaps we can use the structure of Maxwell's equations in complex form to demonstrate that this magnetic structure is indeed the complex part of the field.

In considering $F_{\mu\nu}$ and $A_{\mu\nu}$ as complex entities rather than four space real forms, we may need complex forms of the current density. Also the relationship between $F_{\mu\nu}$ and $A_{\mu\nu}$ has a spatial integral over charge. If we consider $F_{\mu\nu}$ and $A_{\mu\nu}$ as complex quantities, we see possible implications for the charge *e* or differential charge *de* being a complex quantity.

Perhaps the expression $e = e_{\mathrm{Re}} + ie_{\mathrm{Im}}$ is not appropriate, but a form for the charge integral is, such as: $\int \frac{de_{\mathrm{Re}} de_{\mathrm{Im}}}{\underline{r}}$ where $r = r_{\mathrm{Re}} + ir_{\mathrm{Im}}$ is more appropriate. Fractional charges such as for quarks, the issue of the source of charge (in an elementary particle) and its fundamental relationship to magnetic phenomena (magnetic domains) are essential considerations and may be illuminated by this or a similar formalism. Neither the source of electrics or magnetics is known, although a great deal is known about their properties.

Faraday's conclusion of the identical nature of the magnetic field of a loadstone and a moving current may need reexamination as well as the issue of Hertzian and non-Hertzian waves. Again, a possible description of such phenomena may come from a complex geometric model [39]. As discussed, one can generalize Maxwell's equations and look at real and imaginary components which comprise a symmetry in the form of the equations. We can examine in detail what the implications of the complex electric and magnetic components have in deriving a Coulomb equation and examine the possible way, given a rotational coordinate, this formalism ties in with the 5D geometries of Kaluza and Klein.

Starting $F^{\mu\nu}$, A_μ and J^μ, Maxwell's equations can be compactly written as $\frac{\partial F^{\mu\nu}}{\partial x_\nu} = J^\mu$ and again, $F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}$ and $F_\nu^{\mu\nu} = J^\mu$. Now suppose that an electron moves in such a way that its own field on the average just neutralizes an applied external field $F'_{\mu\nu}$ in the region occupied by the electron. The value of $F_{\mu\nu}$ averaged for all the elements of change constituting the electron is given by

$$eF_{\mu\nu} = \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\iint \frac{de_1 de_2}{r_{12}}$$

and (61)

$$eF_{\mu\nu} = \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\frac{e^2}{a}$$

where $1/a$ is the average value of $1/r_{12}$ for every pair of points in the electron and a will then be a length comparable to the radius of the sphere throughout which the charge is spread. The mass of the electron is $m = e^2 / 4\pi a.$ We thus have a form of Coulomb's law, as we have shown the complex form of $F^{\mu\nu}$ to be consistent with this and Maxwell's equations and that we will have a real and an imaginary Coulomb's law.

Self-consistency can be obtained in the model by assuming that all physical variables are complex. Thus, as before, we assumed that space, time, matter, energy, charge, etc. were on an equal footing as coordinates of a Cartesian space quantized variable model. It is reasonable then to complexity space and time as well as the electric and magnetic fields and to determine the relationship of the equations governing standard physical phenomena. Also to be examined in detail is any unifying properties of the model in terms of complexifying physical quantities as well as examining any new predictions that can be made.

Faraday discusses some possible implications of considering $A_{\mu\nu}$, rather than $F^{\mu\nu}$ as fundamental in such a way that $A_{\mu\nu}$ may act in a domain where $F^{\mu\nu}$ is not observed [39]. In a later section we present a complexification of $A_{\mu\nu}$ rather than $\underline{E}$ and $\underline{B}$ (in $F^{\mu\nu}$).

Continuing with the relationship of $F^{\mu\nu}$, the vector A^{μ} , and scalar potential ϕ, and the metric space, s^{μ} let us relate our complex EM field, $F^{\mu\nu}$, to complex spacetime. We have the volume element $d\tau = \sqrt{g}\, dxdydz$ for

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{62}$$

and for a particular vector component of $F_{\mu} = \sqrt{g_{\mu\nu}}\, f^{\mu}$.
Then we have

$$\nabla \cdot \underline{F} = \frac{1}{\sqrt{g}} \frac{\partial f^{\mu} \sqrt{g}}{\partial x^{\mu}} \tag{63}$$

For $F = \nabla\phi$ the function f^{μ} is related to the EM potential and gravitational potential as $f^{\mu} = g^{\mu\nu} \frac{\partial \phi}{\partial x^{\nu}}$. As before, $\frac{\partial F_{\mu\nu}}{\partial x_{\nu}} = J_{\mu}$ and $\gamma_{\mu\nu} \frac{\partial F_{\mu\nu}}{\partial x_{\nu}} = J_{\mu}$. As before we also had $\left(F_{41}, F_{42}, F_{43}\right) = i\underline{E}$ and $\left(F_{23}, F_{31}, F_{12}\right) = \underline{B}$ then the generalized complex form of $F^{\mu\nu}$, is

$$F^{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & -\frac{i}{c}E_x \\ -B_z & 0 & B_x & -\frac{i}{c}E_y \\ B_y & -B_x & 0 & -\frac{i}{c}E_z \\ \frac{iE_x}{c} & \frac{iE_y}{c} & \frac{iE_z}{c} & 0 \end{pmatrix} \tag{64}$$

which we can denote as

$$\underline{F} = \left(B, -\frac{i}{c}\underline{E}\right) \quad \text{or} \quad \underline{F^*} = \left(-\frac{iE}{c}, B\right). \tag{65}$$

We can now relate the complex *E* and *B* fields of the complex spacetime coordinates.

Returning to the compact notation for the two homogeneous equations, $\nabla \times \underline{E} + \underline{1}\frac{\partial \underline{B}}{\partial t} = 0$ and $\nabla \cdot \underline{B} = 0$ as

$$\frac{\partial F_{\mu\nu}}{\partial x_{\mathrm{K}}} + \frac{\partial F_{\mathrm{K}\mu}}{\partial x_{\nu}} + \frac{\partial F_{\nu\mathrm{K}}}{\partial x_{\mu}} = 0 \tag{66}$$

It is very clear that introducing the imaginary components into these equations as $\partial / \partial\left(ix_{\mu}\right)$ and $\partial / \partial\left(it\right)$ leaves them unchanged.

Now let us look at the inhomogeneous equations $\nabla \cdot \underline{E} = 4\pi\rho$ and $\nabla \times \underline{B} = \frac{1}{c}\frac{\partial E}{\partial t} + J_e$. Consider then

$$F_{\mu\nu} = \frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}} \tag{67}$$

or

$$\underline{F} = \underline{\nabla} \times \underline{A} \quad \text{for} \sim \underline{A_\mu} = \left(\underline{A_j, \phi}\right)$$

for j runs 1 to 3 and all Greek indices run 1 to 4, as before. Then the inhomogeneous equations become in general form, $\partial F^{\mu\nu} \big/ \partial x^\nu = s^\mu$ which sets the criterion on s for using $\partial \big/ \partial \left(i x_{\mathrm{Im}} \right)$; that is, $s' \rightarrow is.$ To be consistent [40], we can use $A_\mu = \left(A_i, \frac{1}{c}\phi \right)$.

We can consider the group of affine connections for a linear transformation from one system S to another S' where S and S' are two frames of reference and

$$x_\mu^{'} = a_{\mu\nu} x_\nu \tag{68}$$

where $a_{\mu\nu} a_\mu^{\mathrm{K}} = \delta_{\nu\mathrm{K}}$ and $\det \left| a_{\mu\nu} \right| = 1$. In general we can form a 4 x 4 coefficient matrix for the usual diagonal condition where, $a_{11} = 1$, $a_{22} = 1, a_{33} = 1$ and $a_{44} = -1$, all the other elements are zero, i.e. the signature (+++-). We can choose arrays of $a_{\mu\nu}$'s both real and imaginary for the general case so that we obtain forms for space and time components as being complex; for example,

$$x_s^{'} = \gamma \left(x_3 + i\beta x_4 \right) \tag{69}$$

for $x_4 = t,\ \ \gamma = \left(1 - \beta^2\right)^{-1/2}$ *and* $\beta = v/c$. Other examples involve other combinations of complex space and time which must also be consistent with unitarity.

Let us briefly examine the effect of a gravitational field on an electron. Then we will discuss some multidimensional models in which attempts are made to relate the gravitational and EM forces. Some of these multidimensional models are real and some are complex. The structure of the metric may well be determined by the geometric constraints set up by the coupling of the gravitational and EM forces. These geometric constraints govern allowable conditions on such phenomena as types of allowable wave transmission and the manner in which remote space-times are connected. Nonlocality or remote space-time connections have implications for EM phenomena such as Young's double slit experiment and Bell's theorem.

In fact, these experiments are more general than just the properties of the photon, that is, both experiments can be and have been conducted with photons and other particles; and therefore what are exhibited are general quantum mechanical properties. Remote connection and/or transmission and nonlocality are more general than just EM phenomena but certainly have their application in electrodynamics and the nonlocal properties of the space-time metric can be tested by experiments involving classical and quantum electrodynamic properties.

7.2 *Complex E & B in real 4-space & the complex Lorentz condition*

Another attempt to relate the relativistic and electro- magnetic theories is the approach of Wyler in his controversial work at Princeton. The model of Kaluza and Klein use a fifth rotational dimension to develop a model to relate EM and gravitational phenomena. This geometry is one-to-one mappable to our complex Minkowski space. Wyler introduces a complex Lorentz group with similar motives to those of Kaluza and Klein [41,42]. Wyler's formalism appears to relate to our complex Maxwell formalism and to that of Kaluza and Klein. The actual fundamental formalism for the calculation of the fine-structure constant, α , is most interesting but perhaps not definitive.

$$\alpha = \frac{e^2}{\hbar c 4\pi\varepsilon_0} = \frac{e^2 c\mu_0}{2h} \tag{70}$$

where e is elementary charge, ε_0 vacuum permittivity and μ_0 the magnetic constant or vacuum permeability. An anthropic explanation has been given as the basis for the value of the fine-structure constant by Barrow and Tipler. They suggest that stable matter and intelligent living systems would not exist if α were much different because carbon would not be produced in stellar fusion [43].

Wyler [44] introduces a complex description of spacetime by introducing complex generators of the Lorentz group. He shows the Minkowski M^n group is conformally isomorphic to the S0(n,2) group and then introduces a Lie algebra of M^4 which is isomorphic to S0(5,2). From his five and four spaces he generates a set of coefficients that generate the value of the fine structure constant, α . It is through introducing the complex form of the Lorentz group, $L(T^n)$ that he forms an isomorphism to S0(n,2).

Wyler calculates the EM coupling constant in terms of geometric group representations. He expands the generators of the set of linear transformations, T^n, of the group $L(T^n)$. By definition, $L(T^n)$ is isomorphic to the Poincairé group $P(M^n)$, where M^n is the Minkowski space with signature (+++-) or, more generally, (1, n-l). The conformal group $C(M^n)$ is then isomorphic to the S0(n,2) group, which is of quadratic form and signature (n,2).

Wyler then chooses the complex form

$$T^n = R^n + iV^n \qquad (71)$$

(where R^n represents T_{Re} , and V^n represents T_{Im}) for $y\,\varepsilon\, R^n$, or y is an element of R^n and all y's are y > 0. The Poincairé group, $P(M^n)$ is the semidirect product of the Lorentz group S0(l, n-l) and the group of transformations R_n then is $g\,\varepsilon$ S0(n,2).

Then $C(M^4) \cong$ S0(4,2) is the invariance group of Maxwell's equations. The hyperboloids of the 4-mass shell momentum operators are $p_1^2, \ldots, p_4^2 = m^2$ from the representation of the Lie group geometry of M^4 isomorphic to S0(5,2). The intersection of the D5 (five-dimensional) hyperspace with D4 gives a structure reduced on D4 which is colinear to the reduction of a Casimir operator function, f(z) harmonic in D4.

The coefficients of the Poisson group D^n as D^4 and D^5 give the value of $\alpha \sim 1/137.036$. Actually, it is the coefficients of the Poisson nucleus $P^n(z, \xi)$ harmonic in D^n which gives the value of α in terms of z where z is, in general, a complex function and ξ is a spinor. The value is obtained from the isomorphic groups S0(5) x SO(2) and S0(4) x SO(2) which gives $(9/8\pi 4)\,(V(D^5)) = 1/137.037$ where $V(D^5)$ is a Euclidean value of the D5 domain [45].

The expression for the Poisson nucleus is given by Hau [45]. Note that the Wyler calculation is another example of the relationship between a fifth dimension and a complex "space" of Lorentz transformation. The Wyler theory appears to strongly support the fundamental nature of geometric models. If one can calculate the fine structure constant or any other force field coupling constants from first principles, this gives great impetus to the concept that geometric constraints are extremely significant and may potentially be able to explain the origin of scientific law. In particular, we may be able to at least describe the major force fields (nuclear, EM, weak, and gravitational in terms of a geometric structure and, perhaps, by this formalism demonstrate the unifying aspects of major forces of nature [46].

Wyler also associates the conformal group $C(M^n) \cong S0(4,2)$ with the invariant group of Maxwell equations. The mass shell conditions on the hyperboloids of mass form the representation of the Lie algebras of M^4. Isomorphism to S0(5,2) and S(4,2) intersection lead to a model of the intersection of Maxwell's field and the elementary particle field, i.e. a possible unification of EM and weak interactions [47].

In the presence of an external gravitational field, the cosmological term is small and finite and depends on the state of vacuum state polarization. In fact, the cosmological term is given by the sum of all vacuum diagrams. In the supersymmetry then, the cosmological term vanishes and therefore the total zero-point energy density of the free fields vanishes [48].

Let us return to our complex $\underline{E}$ and $\underline{B}$ fields and suggest the relation of our formalism to the Wyler formulation. Using the invariance of line elements $s = X^2 - c^2t^2$ for $r = ct = \sqrt{X^2}$ for $X^2 = x^2 + y^2 + z^2$, to measure the distance from a test charge to an electron charge, we can write for the imaginary part of the complex Maxwell equation $\nabla \times \left(iE_{\mathrm{Im}}\right) = \dfrac{1}{c}\dfrac{\partial\left(iB_{\mathrm{Im}}\right)}{\partial t} + iJ_{\mathrm{Im}}$ then for $E_{\mathrm{Im}} = 0$.

$$\nabla \times \left(iE_{\mathrm{Im}}\right) = 0 \quad or \quad \frac{1}{c}\frac{\partial\left(iB_{\mathrm{Im}}\right)}{\partial t} = iJ_{\mathrm{Im}} \tag{72}$$

or

$$\frac{\partial\left(i\underline{B}_{\mathrm{Im}}\right)}{\partial r} = ic\underline{J}_{\mathrm{Im}} \quad or \quad \frac{\partial \underline{B}_{\mathrm{Im}}}{\partial r} = c\underline{J}_{\mathrm{Im}} \tag{73}$$

for the assumed *i*, B_{Im} commutator relation.

Now let us examine the energy associated with the imaginary part of the magnetic field, $\underline{B}_{\mathrm{Im}}$. We can calculate an energy invariant by squaring and integrating the above equation as [30,49]

$$\mathcal{E} = -\int_\tau J_m^2 R\,d\tau = -\int_\tau \left(\frac{\partial B_z}{\partial r}\right)^2 R\,d\tau \le 0 \tag{74}$$

The distance function $R(r)$ over the volume element $d\tau$ is assumed to be point-symmetrical and vanishes for positive real energy states. The volume $d\tau$ is constructed to include a small real domain where a point charge is located, avoiding possible divergences. The negative value of the energy integral leads us to hypothesize about what the source of this energy may be. Perhaps it can be related to vacuum state polarization in a Fermi sea model, as we have presented before [10]. Another possible association is with advanced potential models such as those of de Beauregard [50,51]. A third and perhaps the most interesting association would be with the complex coordinate space [52,53].

In Weyl's non-Riemannian geometry, [36] he presents a model that does not apply to actual spacetime but to a graphic representation of that relational structure, which is the basis in which both EM and metric variables are interrelated [38]. This is the deep significance of the geometry and relates to work of Hanson and Newman [54] on the complex Minkowski space as well as Wyler's work [44] on complex group theories, such as complex Lorentz invariance, where he attempts to reconcile Maxwell's equations and relativity theory. The examination of the hyperspheres of the de Sitter space is presented by Ellis, where he attempts to unify EM and gravitational theory [55]. Eddington has suggested that the Weyl formalism, developed around 1923, is one of the major advances in the work of Einstein.

There is a significant difference between Einstein's generalization of Galilean geometry and Weyl's generalization of Riemannian geometry. The gravitational force field renders Galilean geometry useless and therefore the move to Riemannian geometry was made. In terms of Weyl's geometry, we find that the EM force, $F_{\mu\nu}$, is comparable to the surface of an electron of 4 x 10^{18} volts/cm, [38] and the size of the charge was compatible with the radius of curvature of space.

For the EM mass, $m_e = e^2 / 4\pi a,$ we have

$$m_g ds = \frac{1}{8\pi} G\sqrt{g}\, d\tau \tag{75}$$

where we denote the curvature R by G for the general case of both gravitational and EM field. The ratio of the masses m_g / m_e relates to the ratio of field strengths.

7.3 *Complex EM Forces in a Gravitational Field*

We have considered the weak Weyl limit of the gravitational force in previous calculations of this chapter. We will briefly outline how the complexification of $F_{\mu\nu}$ can be formulated geometrically. We show that we obtain the same results for the relationship of mass and charge.

Let v^{μ} denote the velocity vector as $v^{\mu} = dX_{\mu} / dS$ of the electron in the field, and ρ_0 denote the proper density of charge, called *e*. Then the current is given by $J^{\mu} = \rho_0 v^{\mu}$. Let $F_{\mu\nu}$ refer to the applied external force of the electron. Returning to Eddington's calculation [38], we then have

$$mA^{\nu} A_{\mu\nu} = -F_{\mu\nu}\rho_0 A^{\nu}. \tag{76}$$

We can also write ρ_0 as *e* in the above equation.

In the limit of our gravitational field we can neglect the gravitational field as an external field or also the gravitational energy of the electron. To discuss the presence of an electron in a gravitational field we start from the field equations with R_{μ}^{ν} the Ricci curvature tensor and g_{μ}^{ν} the metric tensor for the case where no matter is present we have:

$$G_{\mu}^{\nu} = R_{\mu}^{\nu} - \frac{1}{2} g_{\mu}^{\nu} R = -\frac{8\pi}{c^4} GE_{\mu}^{\nu} \tag{77}$$

using the scalar curvature, $R = \dfrac{8\pi GE}{c^4} = 0$. Where Then this equation simplifies to

$$R_{\mu\nu} = -8\pi E_{\mu\nu}. \tag{78}$$

This equation applies to regions that contain EM fields but no matter and no electron charges in the region.

For the only surviving component in the energy considerations, we have

$$F_{41} = -F_{14} = \frac{\partial \phi}{\partial r} \qquad (79)$$

where r is the radial separation. Then $F^{41} = g^{44} F_{41}$ and $\frac{\partial \phi}{\partial r} \propto \frac{\varepsilon}{r^2}$ and $E_1^1 = +E_2^2 = E_3^3 = -E_4^4 = \frac{1}{2}\frac{\partial \phi}{\partial r} = \frac{1}{2}\frac{\varepsilon^2}{r^4}$.

Jeffrey associates m, the mass of the electron, with $4\pi\varepsilon$, giving $\alpha = \frac{2\pi\varepsilon^2}{m} \sim 1.5 \times 10^{-13}$ cm and justifies identifying $4\pi\varepsilon$ with the electrical charge e or

$$F_{41} = \frac{\partial \phi}{\partial r} = \frac{1}{4\pi}\frac{e}{r^2} \qquad (80)$$

We can then use $\square F_{\mu\nu} = J_{\mu\nu} - J_{\nu\mu}$ for $A^\mu = \frac{de}{4\pi r}$ and then

$$\begin{aligned} F_{\mu\nu} &= \int \frac{de\left(A_{\mu\nu} - A_{\nu\mu}\right)}{\left(4\pi\gamma\right)r} \\ &= \frac{1}{4\pi\gamma}\left(A_{\mu\nu} - A_{\nu\mu}\right)\int \frac{de}{r} \end{aligned} \qquad (81)$$

because all parts of the electron have the same relativity where

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu = J^\mu$$

and

$$A^\mu = \frac{1}{4\pi}\frac{ds}{dt} v^\mu \int_\tau \frac{\rho d\tau}{r}. \qquad (82)$$

for velocity, v^μ, we will drop the γ since all measurements will be assumed to be proper time measurements. Now integrating over the electron between pairs of points on the electron surface,

$$\begin{aligned} eF_{\mu\nu} &= \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\iint \frac{de_1 de_2}{r_{12}} \\ &= \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\frac{e^2}{a} \end{aligned} \tag{83}$$

where 1/a is the average value of $1/r_{12}$. We can write Eq. (83) as

$$-eA^{\nu}F_{\mu\nu} = \frac{1}{4\pi}A^{\nu}\left(A_{\mu\nu} - A_{\nu\mu}\right)\frac{e^2}{a} \tag{84}$$

and using the equation from before, relating v^{ν}, $A_{\mu\nu}$, $F_{\mu\nu}$ and A^{ν}, $mv^{\nu}A_{\mu\nu} = -F_{\mu\nu}eA^{\nu}$, so that $m = e^2/4\pi a$ as before.

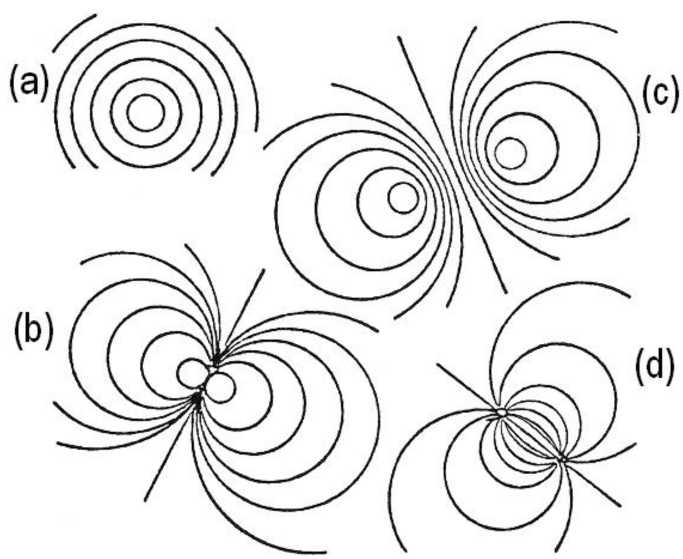

Figure 2 Plotted are the geodesies of the deSitter space which represent the field lines of the EM field. Various conditions for signal propagation are given.

How does this relate to the de Sitter spaces? In the de Sitter algebras the proper time in all inertial frames of intervals is the same (or equivalent). This is the powerful absolute of the de Sitter space. The proper time interval $d\tau$ on its geodesic world-line in the de Sitter picture is given as

$$d\tau^2 = dt^2 - e^{2t}\left(dX^2\right) \tag{85}$$

for $dX^2 = dx^2 + dy^2 + dz^2$ in Euclidean coordinates and t is the cosmic time. The metric form of the de Sitter universe represents the metric form consistent with the observed (approximately flat, low density) universe that we observe. It is constant with Einstein dynamic equations and is therefore consistent with the Hubble's expansion [56].

Ellis [55] suggests that geometry and EM can be unified by a rigorous analysis of time. The hyperspheres of de Sitter space can be represented as a five-dimensional metric manifold which tie the geometric models of gravity and electromagnetism to the structure of matter, and time is not primary but a property of the matter (elementary particles). If $\tau = t$ is allowed in the de Sitter space, then the typical geodesies represent what appears to be EM field lines. This is the manner in which Ellis attempts to describe the EM phenomena as geometric!

The conformal invariant is given as

$$ds^2 = \frac{1}{R^2}\left(dx^2 + dy^2 + dz^2 - dR^2\right) \tag{86}$$

which depends only on the ratios of distances and is thus independent of scale. Let $t = -\ell nR$, then $R = e^{-t}$ and $ds^2 = e^{2t}\left(dx^2 + dy^2 + dz^2\right) - dt^2$ which is the de Sitter metric element. Ellis' geodesies of his angle metric correspond to geodesies of the de Sitter space (Figure 6.3a). In Figure 6.3b, they are time-like subluminal geodesies, and in 3c they are luminal, and in 6.3d they are space-like superluminal. The figures also contain Euclidean space planes as spheres of infinite radii.

Feinberg [57] suggests that the first step in the test of multi-dimensional geometric models is to predict some simple phenomena such as the Coulomb attraction-repulsion; note that Figure 6.3 may point a way to do this, because if we can relate this five-dimensional geometry to the complex geometry, then we can relate this complex geometry to Coulomb interactions.

The curvature of space may then be related to a rotation or angular momentum component as a Kaluza-Klein 5th dimension. We can form an isomorphism of this geometry to an 8D real-complex coordinate geometry which appears to not only unify EM theory and gravitational theory but may also resolve some other apparent paradoxes [58,59].

We have seen that the introduction of the complex E and B fields or complexifying the field, $F^{\mu\nu}$, can be handled in such a way as to not distort the electric charge on the electron. We also find consistency with the five-dimensional geometry of Kaluza and Klein, the 8D Minkowski space, and the deSitter space where the geodesic represents the EM field lines. We can also maintain Lorentz invariance conditions for both real and complex transforms on the line element.

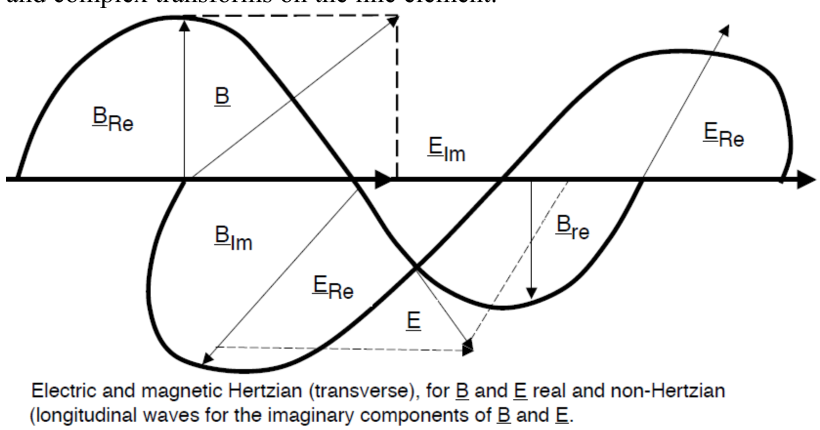

Figure 3 Hertzian and non-Hertzian waves.

8 Summation and Conclusions

This model exploits:

a) the analogy (underlined by Puthoff) between the four vector density representation of gravity and electromagnetism in flat space-time [4].
b) the possibility of describing the causality of quantum mechanical phenomena in terms of extended solitons piloted i.e. by quantum mechanical potentials, by real guiding collective waves on a chaotic, polarizable Dirac-type aether - both moving in a flat space-time [28].

c) the representation of this real vacuum (Dirac aether) in terms of the chaotic distribution of real extended elements moving in the flat space-time.

d) the introduction of internal motions within extended sub-elements and their relation with local collective motions i.e. the $E = mc^2 = h\nu$ relation.

e) the representation of the electron (and its associated pilot-wave) in terms of extended elements with a point-like charge rotating around a center of mass [28].

These assumptions yield realistic physical characteristics to known empirical properties and predict new testable relations besides known properties of elementary particles. The present model must thus be extended, by associating new internal motions to these known properties and interpret them in terms of new strong spin-spin and spin-orbit interactions.

Our attempt is justified by the existence of EM phenomena not explained by Maxwell's equations. Barrett [28] has stated that Maxwell's theory does not explain the Aharonov-Bohm (AB) effect and Altahuler-Aharonov-Spivak (AAS) effects. It does not cover the topological phase question i.e. the Berry-Aharonov-Anandan, Pancharatnam and Chio-Wu phase-rotation effects. An inclusion of Stoke's theorem is necessary and results of Ehrenberg and Siday must be analyzed. The quantum results of Josephson, Hall, de Haas and van Alphen Sagnac-type experiments also need clarification.

The integration of gravity and electromagnetism however, is not finished, because unification is so far only accomplished in terms of bumps and holes rotating on the stochastic surface of the polarized Dirac Vacuum. Unification must also occur in terms of the richer Higher Dimensional (HD) structure of vacuum topology where one would show the geometric origin of charge and how bumps and holes transform into each other through quasi-particle like transitions piloted by advanced and retarded potentials of the fundamental unitary field itself.

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