User's Manual for LINEAR, a FORTRAN Program to Derive Linear Aircraft Models

Eugene L. Duke,
Brian P. Patterson,
and Robert F. Antoniewicz

NASA

User's Manual for LINEAR, a FORTRAN Program to Derive Linear Aircraft Models

Eugene L. Duke,
Brian P. Patterson,
and Robert F. Antoniewicz

Ames Research Center
Dryden Flight Research Facility
Edwards, California

CONTENTS

SUMMARY

This report documents a FORTRAN program that provides a powerful and flexible tool for the linearization of aircraft models. The program LINEAR numerically determines a linear system model using nonlinear equations of motion and a user-supplied nonlinear aerodynamic model. The system model determined by LINEAR consists of matrices for both state and observation equations. The program has been designed to allow easy selection and definition of the state, control, and observation variables to be used in a particular model.

INTRODUCTION

The program LINEAR was developed at the Dryden Flight Research Facility of NASA's Ames Research Center to provide a standard, documented, and verified tool to be used in deriving linear models for aircraft stability analysis and control law design. This development was undertaken to eliminate the need for aircraft-specific linearization programs common in the aerospace industry. Also, the lack of available documented linearization programs provided a strong motivation for the development of LINEAR; in fact, the only available documented linearization program that was found in an extensive literature search of the field is that of Kalviste (1980).

Linear system models of aircraft dynamics and sensors are an essential part of both vehicle stability analysis and control law design. These models define the aircraft system in the neighborhood of an analysis point and are determined by the linearization of the nonlinear equations defining vehicle dynamics and sensors. This report describes LINEAR, a FORTRAN program that provides the user with a powerful and flexible tool for the linearization of aircraft models. LINEAR is a program with well-defined and generalized interfaces to aerodynamic and engine models and is designed to address a wide range of problems without requiring program modification.

The system model determined by LINEAR consists of matrices for both state and observation equations. The program has been designed to provide easy selection and definition of the state, control, and observation variables to be used in a particular model. Thus, the order of the system model is completely under user control. Further, the program provides the flexibility of allowing alternative formulations of both state and observation equations.

LINEAR has several features that make it unique among the linearization programs common in the aerospace industry. The most significant of these features is flexibility. By generalizing the surface definitions and making no assumptions of symmetric mass distributions, the program can be applied to any aircraft in any phase of flight except hover. The unique trimming capabilities, provided by means of a user-supplied subroutine, allow unlimited possibilities for trimming strategies and surface scheduling, which are particularly important for oblique-wing vehicles and aircraft having multiple surfaces affecting a single axis. The formulation of the equations of motion permit the inclusion of thrust-vectoring effects. The ability to select, without program modification, the state, control, and observation variables for the linear models, combined with the large number of observation quantities availiable, allows any analysis problem to be solved with ease.

This report documents the use of the program LINEAR, defining the equations used and the methods employed to implement the program. The trimming capabilities of LINEAR are discussed from both a theoretical and an implementation perspective. The input and output files are described in detail. The user-supplied subroutines required for LINEAR are discussed, and sample subroutines are presented.

NOMENCLATURE

The units associated with the listed variables are expressed in a generalized system (given in parentheses). LINEAR will work equally well with any consistent set of units with two notable exceptions: the printed output and the atmospheric model. Both the printed output and the atmospheric model assume English units. Where applicable, quantities are defined with respect to the body axis system.

Variables

A	state matrix of the state equation, $\dot{x} = Ax + Bu$; or, axial force (force)
A'	state matrix of the state equation, $C\dot{x} = A'x + B'u$
a	speed of sound in air (length/sec)
a_n	normal acceleration (g)
$a_{n,i}$	normal acceleration, accelerometer not at center of gravity (g)
a_{nx}	x body axis accelerometer output, accelerometer at center of gravity (g)
$a_{nx,i}$	x body axis accelerometer output, accelerometer not at center of gravity (g)
a_{ny}	y body axis accelerometer output, accelerometer at center of gravity (g)
$a_{ny,i}$	y body axis accelerometer output, accelerometer not at center of gravity (g)
a_{nz}	z body axis accelerometer output, accelerometer at center of gravity (g)
$a_{nz,i}$	z body axis accelerometer output, accelerometer not at center of gravity (g)
a_x	acceleration along the x body axis (g)
a_y	acceleration along the y body axis (g)
a_z	acceleration along the z body axis (g)

2

B   control matrix of the state equation, $\dot{x} = Ax + Bu$

B'   control matrix of the state equation, $C\dot{x} = A'x + B'u$

b   wingspan (length)

C   C matrix of the state equation, $C\dot{x} = A'x + B'u$; or, force or moment coefficient

C_D coefficient of drag

C_L coefficient of lift

C_ℓ coefficient of rolling moment

C_m coefficient of pitching moment

C_n coefficient of yawing moment

C_Y coefficient of sideforce

CM_i center of mass of ith engine

$\bar{c}$ mean aerodynamic chord (length)

D   dynamic interaction matrix for state equation, $\dot{x} = Ax + Bu + Dv$; or, drag force (force)

D'   dynamic interaction matrix for the state equation, $C\dot{x} = A'x + B'u + D'v$

E   dynamic interaction matrix for the observation equation, $y = Hx + Fu + Ev$

E'   dynamic interaction matrix for the observation equation,
$$y = H'x + G\dot{x} + F'u + E'v$$

E_s specific energy (length)

F   feedforward matrix of the observation equation, $y = Hx + Fu$

F   total aerodynamic force acting at the aerodynamic center

F_p engine thrust vector

F'   feedforward matrix of the observation equation, $\dot{y} = H'x + G\dot{x} + F'u$

fpa	flightpath acceleration (g)
G	G matrix of the observation equation, $y = H'x + G\dot{x} + F'u$
g	acceleration due to gravity (length/sec^2)
H	observation matrix of the observation equation, $y = Hx + Fu$
H'	observation matrix of the observation equation, $y = H'x + G\dot{x} + F'u$
h	altitude (length)
h_e	angular momentum of engine rotor (mass-length^2/sec)
I	aircraft inertia tensor (mass-length^2)
I_e	rotational inertia of the engine (mass-length^2)
I_x	x body axis moment of inertia (mass-length^2)
I_{xy}	x-y body axis product of inertia (mass-length^2)
I_{xz}	x-z body axis product of inertia (mass-length^2)
I_y	y body axis moment of inertia (mass-length^2)
I_{yz}	y-z body axis product of inertia (mass-length^2)
I_z	z body axis moment of inertia (mass-length^2)
L	total body axis aerodynamic rolling moment (length-force); or, total aerodynamic lift (force)
ℓ	generalized length (length)
M	Mach number; or, total body axis aerodynamic pitching moment (length-force)
m	aircraft total mass (mass)
m_e	mass of engine
N	normal force (force); or, total body axis aerodynamic yawing moment (length-force)
n	load factor
P_s	specific power (length/sec)

4

p	roll rate (rad/sec); or, pressure (force/length^2)
p_a	ambient pressure (force/length^2)
p_t	total pressure (force/length^2)
q	pitch rate (rad/sec)
$\bar{q}$	dynamic pressure (force/length^2)
q_c	impact pressure (force/length^2)
R_ε, R_ξ	axis transformation matrices
Re	Reynolds number
Re'	Reynolds number per unit length (length^{-1})
r	yaw rate (rad/sec)
S	wing planform area (length^2)
T	ambient temperature (K); or, total angular momentum (mass-length^2/sec^2); or thrust (force)
T_t	total temperature (K)
u	velocity in x-axis direction (length/sec)
$\mathbf{u}$	control vector
V	total velocity (length/sec)
V_c	calibrated airspeed (knots)
V_e	equivalent airspeed (knots)
v	velocity in y-axis direction (length/sec)
$\mathbf{v}$	dynamic interaction vector
W	vehicle weight (force)
w	velocity in z-axis direction (length/sec)
X	total force along the x body axis (force)
X_T	thrust along the x body axis (force)
$\mathbf{x}$	state vector

Y	sideforce (force)
Y_T	thrust along the y body axis (force)
y	observation vector
Z	total force along the z body axis (force)
Z_T	thrust along the z body axis (force)
α	angle of attack (rad)
β	angle of sideslip (rad)
γ	flightpath angle (rad)
Δr	displacement of aerodynamic reference point from center of gravity
Δx	displacement from center of gravity along x body axis (length)
Δy	displacement from center of gravity along y body axis (length)
Δz	displacement from center of gravity along z body axis (length)
δ_a	lateral trim parameter
δ_{dt}	differential stabilator trim parameter
δ_e	longitudinal trim parameter
δ_{jk}	Kronecker delta
δ_r	directional trim parameter
δ_{sb}	speed brake trim parameter
δ_{th}	thrust trim parameter
δL	incremental rolling moment (length-force)
δM	incremental pitching moment (length-force); or, incremental Mach
δN	incremental yawing moment (length-force)
δX	incremental x body axis force (force)
δY	incremental y body axis force (force)
δZ	incremental z body axis force (force)
ε	angle from the thrust axis of engine to the x-y body axis plane (rad)

6

ζ angle from the projection of F_p onto the engine x-y plane to the local x axis (rad)

η angle from F_p to the engine x-y plane (rad)

θ pitch angle (rad)

μ coefficient of viscosity

ξ angle from the projection of F_p onto the x-y body axis plane to the x body axis (rad)

ρ density of air (mass/length^3)

ΣL total body axis rolling moment (length-force)

ΣM total body axis pitching moment (length-force)

ΣN total body axis yawing moment (length-force)

τ torque from engines (length-force)

τ_g gyroscopic torque from engine (length-force)

ϕ roll angle (rad)

ϕ_L tilt angle of acceleration normal to the flightpath from the vertical plane (rad)

ψ heading angle (rad)

ω total rotational velocity of the vehicle

ω_e engine angular velocity (rad/sec)

Superscripts

$\char"5E$ nondimensional version of variable

$\cdot$ derivative with respect to time

T   transpose of a vector or matrix

Subscripts

ar   aerodynamic reference point

D   total drag

E   engine

7

h	altitude
L	total lift
ℓ	rolling moment
M	Mach number
m	pitching moment
max	maximum
min	minimum
n	yawing moment
o	offset from center of gravity
p	roll rate
q	pitch rate
r	yaw rate
s	stability axis
TP	thrust point
x	along the x body axis
Y	sideforce
y	along the y body axis
z	along the z body axis
0	standard day, sea level conditions; or, along the reference trajectory

FORTRAN Variables

AIX	inertia about the x body axis
AIXE	engine inertia
AIXY	inertial coupling between the x and y body axes
AIXZ	inertial coupling between the x and z body axes
AIY	inertia about the y body axis
AIYZ	inertial coupling between the y and z body axes
AIZ	inertia about the z body axis

ALP	angle of attack
ALPDOT	time rate of change of angle of attack
AMCH	Mach number
AMSENG	total rotor mass of the engine
AMSS	aircraft mass
B	wingspan
BTA	angle of sideslip
BTADOT	time rate of change of angle of sideslip
CBAR	mean aerodynamic chord
CD	total coefficient of drag
CDA	coefficient of drag due to angle of attack
CDDE	coefficient of drag due to symmetric elevator deflection
CDO	drag coefficient at zero angle of attack
CDSB	coefficient of drag due to speed brake deflection
CL	total coefficient of rolling moment
CLB	coefficient of rolling moment due to angle of sideslip
CLDA	coefficient of rolling moment due to aileron deflection
CLDR	coefficient of rolling moment due to rudder deflection
CLDT	coefficient of rolling moment due to differential elevator deflection
CLFT	total coefficient of lift
CLFTA	coefficient of lift due to angle of attack
CLFTAD	coefficient of lift due to angle-of-attack rate
CLFTDE	coefficient of lift due to symmetric elevator deflection
CLFTO	lift coefficient at zero angle of attack
CLFTQ	coefficient of lift due to pitch rate
CLFTSB	coefficient of lift due to speed brake deflection
CLP	coefficient of rolling moment due to roll rate

CLR	coefficient of rolling moment due to yaw rate
CM	total coefficient of pitching moment
CMA	coefficient of pitching moment due to angle of attack
CMAD	coefficient of pitching moment due to angle-of-attack rate
CMDE	coefficient of pitching moment due to symmetric elevator deflection
CMO	pitching moment coefficient at zero angle of attack
CMQ	coefficient of pitching moment due to pitch rate
CMSB	coefficient of pitching moment due to speed brake deflection
CN	total coefficient of yawing moment
CNB	coefficient of yawing moment due to sideslip
CNDA	coefficient of yawing moment due to aileron deflection
CNDR	coefficient of yawing moment due to rudder deflection
CNDT	coefficient of yawing moment due to differential elevator deflection
CNP	coefficient of yawing moment due to roll rate
CNR	coefficient of yawing moment due to yaw rate
CY	total coefficient of sideforce
CYB	coefficient of sideforce due to sideslip
CYDA	coefficient of sideforce due to aileron deflection
CYDR	coefficient of sideforce due to rudder deflection
CYDT	coefficient of sideforce due to differential elevator deflection
DAS	longitudinal trim parameter
DC	surface deflection and thrust control array
DELX	displacement of the aerodynamic reference along the x body axis from the center of gravity
DELY	displacement of the aerodynamic reference along the y body axis from the center of gravity
DELZ	displacement of the aerodynamic reference along the z body axis from the center of gravity
DES	lateral trim parameter

DRS	directional trim parameter
DXTHRS	distance between the center of gravity of the engine and the thrust point
EIX	rotational inertia of the engine
ENGOMG	rotational velocity of the engine
GMA	flightpath angle
H	altitude
HDOT	time rate of change of altitude
P	roll rate
PDOT	time rate of change of roll rate
PHI	roll angle
PHIDOT	time rate of change of roll angle
PSI	heading angle
PSIDOT	time rate of change of heading angle
Q	pitch rate
QBAR	dynamic pressure
QDOT	time rate of change of pitch rate
R	yaw rate
RDOT	time rate of change of yaw rate
S	wing area
T	time
TDOT	time rate of change of time
THA	pitch angle
THADOT	time rate of change of pitch angle
THRSTX	thrust trim parameter
THRUST	thrust generated by each engine
TLOCAT	location of the engine in the x, y, and z axes from the center of gravity

TVANXY	orientation of the thrust vector in the x-y engine axis plane
TVANXZ	orientation of the thrust vector in the x-z engine axis plane
UB	velocity along the x body axis
V	velocity
VB	velocity along the y body axis
VCAS	calibrated airspeed
VDOT	time rate of change of total vehicle velocity
VEAS	equivalent airspeed
WB	velocity along the z body axis
X	position north from an arbitrary reference point
XDOT	time rate of change of north-south position
XYANGL	orientation of engine axis in x-y body axis plane
XZANGL	orientation of engine axis in x-z body axis plane
Y	position east from an arbitrary reference point
YDOT	time rate of change of east-west position

PROGRAM OVERVIEW

The program LINEAR numerically determines a linear system model using nonlinear equations of motion and a user-supplied nonlinear aerodynamic model. LINEAR is also capable of extracting linearized gross engine effects (such as net thrust, torque, and gyroscopic effects) and including these effects in the linear system model. The point at which this linear system model is defined is determined either by specifying the state and control variables or by selecting an analysis point on a trajectory, selecting a trim option, and allowing the program to determine the control variable and remaining state variables to satisify the trim option selected.

Because the program is designed to satisfy the needs of a broad class of users, a wide variety of options has been provided. Perhaps the most important of these options are those that allow user specification of the state, control, and observation variables to be included in the linear model derived by LINEAR.

Within the program, the nonlinear equations of motion include 12 states representing a rigid aircraft flying in a stationary atmosphere over a flat nonrotating earth. Thus, the state vector x is computed internally as

$$x = [p \quad q \quad r \quad V \quad \alpha \quad \beta \quad \phi \quad \theta \quad \psi \quad h \quad x \quad y]^T$$

The nonlinear equations used to determine the derivatives of the quantities are presented in the following section (Equations of Motion). The internal control vector u can contain up to 30 controls. The internal observation vector y contains 120 variables, including the state variables, the time derivatives of the state variables, the control variables, and a variety of other parameters of interest. Thus, within the program,

$$y = \begin{bmatrix} x^T & \dot{x}^T & u^T & y_1^T & y_2^T & y_3^T & y_4^T & y_5^T & y_6^T & y_7^T & y_8^T \end{bmatrix}^T$$

where

$$y_1 = \begin{bmatrix} a_x & a_y & a_z & a_{nx} & a_{ny} & a_{nz} & a_n & a_{nx,i} & a_{ny,i} & a_{nz,i} & a_{n,i} & n \end{bmatrix}^T$$

$$y_2 = \begin{bmatrix} a & Re & Re' & M & \bar{q} & q_c & p_a & q_c/p_a & p_t & T & T_t & V_e & V_c \end{bmatrix}^T$$

$$y_3 = \begin{bmatrix} \gamma & fpa & \dot{\gamma} & \ddot{h} & \dot{h}/57.3 \end{bmatrix}^T$$

$$y_4 = \begin{bmatrix} E_s & P_s \end{bmatrix}^T$$

$$y_5 = \begin{bmatrix} L & D & N & A \end{bmatrix}^T$$

$$y_6 = \begin{bmatrix} u & v & w & \dot{u} & \dot{v} & \dot{w} \end{bmatrix}^T$$

$$y_7 = \begin{bmatrix} \alpha_{,i} & \beta_{,i} & h_{,i} & \dot{h}_{,i} \end{bmatrix}^T$$

$$y_8 = \begin{bmatrix} T & p_s & q_s & r_s \end{bmatrix}^T$$

The equations defining these quantities are presented in the Observation Equations section.

From the internal formulation of the state, control, and observation variables, the user must select the specific variables desired in the output linear model (described in the Selection of State, Control, and Observation Variables section). Figure 1 illustrates the selection of variables in the state vector for a requested linear model. From the internal formulation on the right of the figure, the requested model is constructed, and the linear system matrices are selected in accordance with the user specification of the state, control, and observation variables.
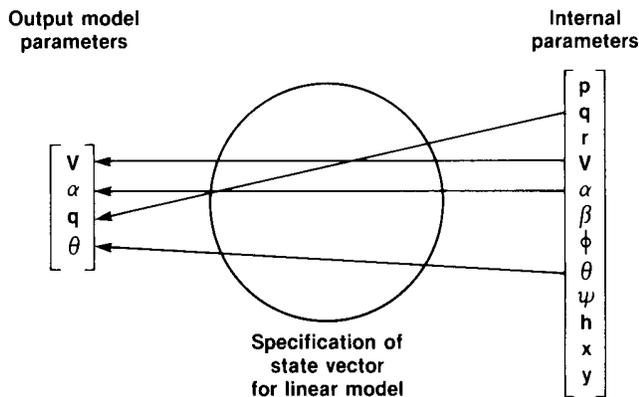
Figure 1. Selection of state variables for linear model.

13

The linear model derived by LINEAR is determined at a specific analysis point. LINEAR allows this analysis point to be defined as a true steady-state condition on a specified trajectory (a point at which the rotational and translational accelerations are zero) or as a totally arbitrary state on a trajectory. LINEAR provides the user with several options, described in detail in the Analysis Point Definition section. These analysis point definition options allow the user to trim the aircraft in wings-level flight, pushovers, pullups, level turns, or zero-sideslip maneuvers. Also included is a nontrimming option in which the user defines a totally arbitrary condition about which the linear model is to be derived.

The linear system matrices are determined by numerical perturbation and are the first-order terms of a Taylor series expansion about the analysis point, as described in the Linear Models section. The formulation of the output system model is under user control. The user can select state equation matrices corresponding to either the standard formulation of the state equation,

$$\dot{x} = Ax + Bu$$

or the generalized equation,

$$C\dot{x} = A'x + B'u$$

The observation matrices can be selected from either of two formulations corresponding to the standard equation,

$$y = Hx + Fu$$

or the generalized equation,

$$y = H'x + G\dot{x} + F'u$$

In addition to the linear system matrices, LINEAR also computes the nondimensional stability and control derivatives at the analysis point. These derivatives are discussed in the Nondimensional Stability and Control Derivatives section.

The input file for LINEAR is an ASCII file that defines the geometry and mass properties of the aircraft and selects various program options. Within this input file, the state, control, and observation vectors desired in the output linear model are defined, and the analysis point options are selected. The details of the input file are discussed in the Input Files section.

The output of LINEAR is three files, one containing the linear system matrices and two documenting the options and analysis points selected by the user. The first is intended to be used with follow-on design and analysis programs. The other two contain all the information contained in the first file and also include the details of the analysis point and the nondimensional stability and control derivatives. These files are described in the Output Files section.

To execute LINEAR, five user-supplied subroutines are required. These routines, discussed in the User-Supplied Subroutines section, define the nonlinear aerodynamic

14

model, the gross engine model, the gearing between the LINEAR trim inputs and the
surfaces modeled in the aerodynamic model, and a model of the mass and geometry
properties of the aircraft. The gearing model (fig. 2) defines how the LINEAR trim
inputs will be connected to the surface models and allows schedules and nonstandard
trimming schemes to be employed. This last feature is particularly important for
oblique-wing aircraft.
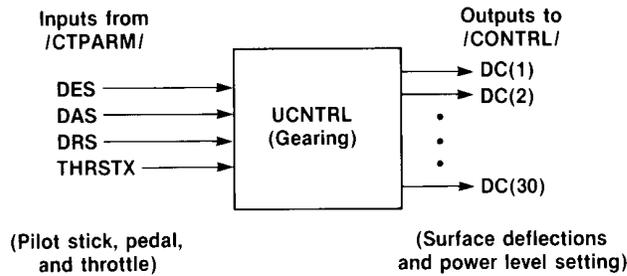
Figure 2. *Inputs to and outputs of the*
user-supplied subroutine UCNTRL.

EQUATIONS OF MOTION

The nonlinear equations of motion used in the linearization program are general
six-degree-of-freedom equations representing the flight dynamics of a rigid aircraft
flying in a stationary atmosphere over a flat nonrotating earth. The assumption of
nonzero forward motion also is included in these equations; because of this assump-
tion, these equations are invalid for vertical takeoff and landing or hover. These
equations contain no assumptions of either symmetric mass distribution or aerody-
namic properties and are applicable to asymmetric aircraft (such as oblique-wing
aircraft) as well as to conventional symmetric aircraft. These equations of motion
were derived by Etkin (1972), and the derivation will be detailed in a proposed NASA
Reference Publication, "Derivation and Definition of a Linear Aircraft Model," by
Eugene L. Duke, Robert F. Antoniewicz, and Keith D. Krambeer (in preparation).

The following equations for rotational acceleration are used for analysis point
definition:

$$\dot{p} = [(\Sigma L)I_1 + (\Sigma M)I_2 + (\Sigma N)I_3 - p^2(I_{xz}I_2 - I_{xy}I_3)$$

$$+ \ pq(I_{xz}I_1 - I_{yz}I_2 - D_zI_3) - pr(I_{xy}I_1 + D_yI_2 - I_{yz}I_3)$$

$$+ \ q^2(I_{yz}I_1 - I_{xy}I_3) - qr(D_xI_1 - I_{xy}I_2 + I_{xz}I_3)$$

$$- \ r^2(I_{yz}I_1 - I_{xz}I_2)]/\det I$$

15

$$\dot{q} = [(\Sigma L)I_2 + (\Sigma M)I_4 + (\Sigma N)I_5 - p^2(I_{xz}I_4 - I_{xy}I_5)$$

$$+ pq(I_{xz}I_2 - I_{yz}I_4 - D_zI_5) - pr(I_{xy}I_2 + D_yI_4 - I_{yz}I_5)$$

$$+ q^2(I_{yz}I_2 - I_{xy}I_5) - qr(D_xI_2 - I_{xy}I_4 + I_{xz}I_5)$$

$$- r^2(I_{yz}I_2 - I_{xz}I_4)]/\det I$$

$$\dot{r} = [(\Sigma L)I_3 + (\Sigma M)I_5 + (\Sigma N)I_6 - p^2(I_{xz}I_5 - I_{xy}I_6)$$

$$+ pq(I_{xz}I_3 - I_{yz}I_5 - D_zI_6) - pr(I_{xy}I_3 + D_yI_5 - I_{yz}I_6)$$

$$+ q^2(I_{yz}I_3 - I_{xy}I_6) - qr(D_xI_3 - I_{xy}I_5 + I_{xz}I_6)$$

$$- r^2(I_{yz}I_3 - I_{xz}I_5)]/\det I$$

where

$$\det I = I_xI_yI_z - 2I_{xy}I_{xz}I_{yz} - I_xI_{yz}^2 - I_yI_{xz}^2 - I_zI_{xy}^2$$

$$I_1 = I_yI_z - I_{yz}^2$$

$$I_2 = I_{xy}I_z + I_{yz}I_{xz}$$

$$I_3 = I_{xy}I_{yz} + I_yI_{xz}$$

$$I_4 = I_xI_z - I_{xz}^2$$

$$I_5 = I_xI_{yz} + I_{xy}I_{xz}$$

$$I_6 = I_xI_y - I_{xy}^2$$

$$D_x = I_z - I_y$$

$$D_y = I_x - I_z$$

$$D_z = I_y - I_x$$

Here, the body axis rates are designated p, q, and r, corresponding to roll rate, pitch rate, and yaw rate. The total moments about the x, y, and z body axes (rolling, pitching, and yawing moments) are designated ΣL, ΣM, and ΣN, respectively. These total moments are the sums of all aerodynamic moments and powerplant-induced moments due to thrust asymmetries and gyroscopic torques. (The equations used to determine the change in moment coefficients due to the noncoincidence of the vehicle center of gravity and the reference point of the nonlinear aerodynamic model

are derived in appendix A. The equations defining the engine torque and gyroscopic contributions to the total moments are derived in appendix B.) The body axis moments and products of inertia are designated I_x, I_y, I_z, I_{xy}, I_{xz}, and I_{yz}. These moments and products of inertia are elements of the inertia tensor I, defined as

$$I = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix}$$

To derive the state equation matrices for the generalized formulation,

$$C\dot{x} = Ax + Bu$$

(where A and B are the state and control matrices of the state equation), the rotational accelerations are cast in a decoupled-axes formulation. The equations used to derive the linearized matrices are

$$\begin{bmatrix} 1 & \dfrac{-I_{xy}}{I_x} & \dfrac{-I_{xz}}{I_x} \\ \dfrac{-I_{xy}}{I_y} & 1 & \dfrac{-I_{yz}}{I_y} \\ \dfrac{-I_{xz}}{I_z} & \dfrac{-I_{yz}}{I_z} & 1 \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\Sigma L}{I_x} - rp\,\dfrac{I_{xy}}{I_x} + pq\,\dfrac{I_{xz}}{I_x} + rq\,\dfrac{I_y}{I_x} + (q^2 - r^2)\,\dfrac{I_{yz}}{I_x} - qr\,\dfrac{I_z}{I_x} \\[2em] \dfrac{\Sigma M}{I_y} - rp\,\dfrac{I_x}{I_y} + rq\,\dfrac{I_{xy}}{I_y} - pq\,\dfrac{I_{yz}}{I_y} + (r^2 - p^2)\,\dfrac{I_{xz}}{I_y} + pr\,\dfrac{I_z}{I_y} \\[2em] \dfrac{\Sigma N}{I_z} + qp\,\dfrac{I_x}{I_z} - qr\,\dfrac{I_{xz}}{I_z} + pr\,\dfrac{I_{yz}}{I_z} + (p^2 - q^2)\,\dfrac{I_{xy}}{I_z} - pq\,\dfrac{I_y}{I_z} \end{bmatrix}$$

The translational acceleration equations used in the program LINEAR for both analysis point definition and perturbation are

$$\dot{V} = [-D \cos \beta + Y \sin \beta + X_T \cos \alpha \cos \beta + Y_T \sin \beta + Z_T \sin \alpha \cos \beta$$

$$- mg(\sin \theta \cos \alpha \cos \beta - \cos \theta \sin \phi \sin \beta - \cos \theta \cos \phi \sin \alpha \cos \beta)]/m$$

$$\dot{\alpha} = [-L + Z_T \cos \alpha - X_T \sin \alpha + mg(\cos \theta \cos \phi \cos \alpha + \sin \theta \sin \alpha)]/Vm \cos \beta$$

$$+ q - \tan \beta (p \cos \alpha + r \sin \alpha)$$

$$\dot{\beta} = [D \sin \beta + Y \cos \beta - X_T \cos \alpha \sin \beta + Y_T \cos \beta - Z_T \sin \alpha \sin \beta$$

$$+ mg(\sin \theta \cos \alpha \sin \beta + \cos \theta \sin \phi \cos \beta - \cos \theta \cos \phi \sin \alpha \sin \beta)]/Vm$$

$$+ p \sin \alpha - r \cos \alpha$$

where α, β, θ, and ϕ are angles of attack, sideslip, pitch, and roll, respectively; X_T, Y_T, and Z_T are thrust along the x, y, and z body axes; and D is drag force, g gravitational acceleration, L total aerodynamic lift, m total aircraft mass, V total velocity, and Y sideforce.

The equations defining the vehicle attitude rates are

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = q \sin \phi \sec \theta + r \cos \phi \sec \theta$$

where ψ is heading angle.

The equations defining the earth-relative velocities are

$$\dot{h} = V(\cos \beta \cos \alpha \sin \theta - \sin \beta \sin \phi \cos \theta - \cos \beta \sin \alpha \cos \phi \cos \theta)$$

$$\dot{x} = V[\cos \beta \cos \alpha \cos \theta \cos \psi + \sin \beta (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi)$$

$$+ \cos \beta \sin \alpha (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)]$$

$$\dot{y} = V[\cos \beta \cos \alpha \cos \theta \sin \psi + \sin \beta (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi)$$

$$+ \cos \beta \sin \alpha (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)]$$

where h is altitude.

OBSERVATION EQUATIONS

The user-selectable observation variables computed in LINEAR represent a broad class of parameters useful for vehicle analysis and control design problems. These

18

variables include the state, time derivatives of state, and control variables. Also included are air data parameters, accelerations, flightpath terms, and other miscellaneous parameters. The equations used to calculate those parameters are derived from a number of sources (Clancy, 1975; Dommasch et al., 1967; Etkin, 1972; Gainer and Hoffman, 1972; Gracy, 1980). Implicit in many of these observation equations is an atmospheric model. The model included in LINEAR is derived from the U.S. Standard Atmosphere (1962).

The vehicle body axis accelerations constitute the set of observation variables that, except for state variables themselves, are most commonly used in the aircraft control analysis and design problem. These accelerations are measured in g units and are derived directly from the body axis forces defined in the previous section for translational acceleration. The equations used in LINEAR for the body axis accelerations a_x, a_y, and a_z are

$$a_x = (X_T - D \cos \alpha + L \sin \alpha - gm \sin \theta)/g_0 m$$

$$a_y = (Y_T + Y + gm \cos \theta \sin \phi)/g_0 m$$

$$a_z = (Z_T - D \sin \alpha - L \cos \alpha + gm \cos \theta \cos \phi)/g_0 m$$

where subscript 0 denotes standard day, sea level conditions. The equations for the outputs of the body axis accelerometers (denoted by subscript n) that are at vehicle center of gravity are

$$a_{nx} = (X_T - D \cos \alpha + L \sin \alpha)/g_0 m$$

$$a_{ny} = (Y_T + Y)/g_0 m$$

$$a_{nz} = (Z_T - D \sin \alpha - L \cos \alpha)/g_0 m$$

$$a_n = (-Z_T + D \sin \alpha + L \cos \alpha)/g_0 m$$

For orthogonal accelerometers that are aligned with the vehicle body axes but are not at vehicle center of gravity (denoted by subscript ,i), the following equations apply:

$$a_{nx,i} = a_{nx} - [(q^2 + r^2)x_x - (pq - \dot{r})y_x - (pr + \dot{q})z_x]/g_0$$

$$a_{ny,i} = a_{ny} + [(pq + \dot{r})x_y - (p^2 + r^2)y_y + (qr - \dot{p})z_y]/g_0$$

$$a_{nz,i} = a_{nz} + [(pr - \dot{q})x_z + (qr + \dot{p})y_z - (q^2 + p^2)z_z]/g_0$$

$$a_{n,i} = a_n - [(pr - \dot{q})x_z + (qr + \dot{p})y_z - (q^2 + p^2)z_z]/g_0$$

where the subscripts x, y, and z refer to the x, y, and z body axes, respectively, and the symbols x, y, and z refer to the x, y, and z body axis locations of the

sensors relative to the vehicle center of gravity. Also included in the set of acceleration equations is load factor, $n = L/W$, where L is the total aerodynamic lift and W is the vehicle weight.

The air data parameters having the greatest application to aircraft dynamics and control problems are the sensed parameters and the reference and scaling parameters. The sensed parameters are impact pressure q_c, ambient or free-stream pressure p_a, total pressure p_t, ambient or free-stream temperature T, and total temperature T_t. The selected reference and scaling parameters are Mach number M, dynamic pressure $\bar{q}$, speed of sound a, Reynolds number Re, Reynolds number per unit length, Re', and the Mach meter calibration ratio q_c/p_a. These quantities are defined as

$$a = \left(1.4 \frac{p_0}{\rho_0 T_0} T\right)^{1/2}$$

$$M = \frac{V}{a}$$

$$Re = \frac{\rho V \ell}{\mu}$$

$$Re' = \frac{\rho V}{\mu}$$

$$\bar{q} = \frac{\rho V^2}{2}$$

$$q_c = \begin{cases} \left[(1.0 + 0.2M^2)^{3.5} - 1.0\right]p_a & (M < 1.0) \\ \left[1.2M^2 \left(\frac{5.76M^2}{5.6M^2 - 0.8}\right)^{2.5} - 1.0\right] p_a & (M > 1.0) \end{cases}$$

$$\frac{q_c}{p_a} = \begin{cases} (1.0 + 0.2M^2)^{3.5} - 1.0 & (M < 1.0) \\ \left[1.2M^2 \left(\frac{5.76M^2}{5.6M^2 - 0.8}\right)^{2.5}\right] - 1.0 & (M > 1.0) \end{cases}$$

$$p_t = p_a + q_c$$

$$T_t = T(1.0 + 0.2M^2)$$

where ℓ is length, p pressure, ρ the density of the air, and μ the coefficient of viscosity. Free-stream pressure, free-stream temperature, and the coefficient of viscosity are derived from the U.S. Standard Atmosphere (1962).

Also included in the air data calculations are two velocities: equivalent airspeed V_e and calibrated airspeed V_c, both computed in knots. The calculations assume that internal units are in the English system. The equation used for equivalent airspeed is

$$V_e = 17.17 \sqrt{\overline{q}} \ (lb/ft^2)$$

which is derived from the definition of equivalent airspeed,

$$V_e = \sqrt{\frac{2\overline{q}}{\rho_0}}$$

where $\rho_0 = 0.002378$ slug/ft^3 and V_e is converted from feet per second to knots. Calibrated airspeed is derived from the following definition of impact pressure:

$$q_c = \begin{cases} p_0 \left[\left(1.0 + \frac{\rho_0}{7.0 p_0} V_c^2\right)^{3.5} - 1\right] & (V_c \leqslant a_0) \\ 1.2 \left(\frac{V_c}{a_0}\right)^2 p_0 \left[\dfrac{5.76}{5.6 - 0.8 \ (a_0/V_c)^2}\right]^{2.5} - p_0 & (V_c > a_0) \end{cases}$$

For the case where $V_c \leqslant a_0$, the equation for V_c is

$$V_c = 1479.116 \sqrt{\left(\frac{q_c}{p_0} + 1.0\right)^{2/7} - 1.0} \qquad (V_c \leqslant a_0)$$

Calibrated airspeed is found using an iterative process for the case where $V_c > a_0$:

$$V_c = 582.95174 \sqrt{\left(\frac{q_c}{p_0} + 1.0\right) \left[1.0 - \frac{1.0}{7.0(V_c/a_0)^2}\right]^{2.5}} \qquad (V_c > a_0)$$

is executed until the change in V_c from one iteration to the next is less than 0.001 knots.

Also included in the observation variables are the flightpath-related parameters (described in app. D), including flightpath angle γ, flightpath acceleration fpa, vertical acceleration $\ddot{h}$, flightpath angle rate $\dot{\gamma}$, and (for lack of a better category in which to place it) scaled altitude rate $\dot{h}/57.3$. The equations used to determine these quantities are

$$\gamma = \sin^{-1}\left(\frac{\dot{h}}{V}\right)$$

$$fpa = \frac{\dot{V}}{g_0}$$

$$\ddot{h} = a_x \sin \theta - a_y \sin \phi \cos \theta - a_z \cos \phi \cos \theta$$

$$\dot{\gamma} = \frac{V\ddot{h} - \dot{h}\dot{V}}{V\sqrt{V^2 - \dot{h}^2}}$$

Two energy-related terms are included in the observation variables: specific energy E_s and specific power P_s, defined as

$$E_s = h + \frac{V^2}{2g}$$

$$P_s = \dot{h} + \frac{V\dot{V}}{g}$$

The set of observation variables available in LINEAR also includes four force parameters: total aerodynamic lift L, total aerodynamic drag D, total aerodynamic normal force N, and total aerodynamic axial force A. These quantities are defined as

$$L = \bar{q}SC_L$$

$$D = \bar{q}SC_D$$

$$N = L\cos\alpha + D\sin\alpha$$

$$A = -L\sin\alpha + D\cos\alpha$$

where C_D and C_L are coefficients of drag and lift, respectively.

Six body axis rates and accelerations are available as observation variables. These include the x body axis rate u, the y body axis rate v, and the z body axis rate w. Also included are the time derivatives of these quantities, $\dot{u}$, $\dot{v}$, and $\dot{w}$. The equations defining these quantities are

$$u = V\cos\alpha\cos\beta$$

$$v = V\sin\beta$$

$$w = V\sin\alpha\cos\beta$$

$$\dot{u} = \left(\frac{X_T - gm\sin\theta - D\cos\alpha + L\sin\alpha}{m}\right) + rv - qw$$

$$\dot{v} = \left(\frac{Y_T + gm\cos\theta\sin\phi + Y}{m}\right) + pw - ru$$

$$\dot{w} = \left(\frac{Z_T + gm\cos\theta\cos\phi - D\sin\alpha - L\cos\alpha}{m}\right) + qu - pv$$

The final set of observation variables available in LINEAR is a miscellaneous collection of other parameters of interest in analysis and design problems. The

first group consists of measurements from sensors not located at the vehicle center of gravity. These represent angle of attack $\alpha_{,i}$, angle of sideslip $\beta_{,i}$, altitude $h_{,i}$, and altitude rate $\dot{h}_{,i}$ measurements displaced from the center of gravity by some x, y, and z body axis distances. The equations used to compute these quantities are

$$\alpha_{,i} = \alpha - \left(\frac{qx - py}{V}\right)$$

$$\beta_{,i} = \beta + \left(\frac{rx - pz}{V}\right)$$

$$h_{,i} = h + x \sin\theta - y \sin\phi \cos\theta - z \cos\phi \cos\theta$$

$$\dot{h}_{,i} = \dot{h} + \dot{\theta}(x \cos\theta + y \sin\phi \sin\theta + z \cos\phi \sin\theta)$$

$$- \dot{\phi}(y \cos\phi \cos\theta - z \sin\phi \cos\theta)$$

The remaining miscellaneous parameters are total angular momentum T, stability axis roll rate p_s, stability axis pitch rate q_s, and stability axis yaw rate r_s, defined as

$$T = \frac{1}{2}(I_x p^2 - 2I_{xy}pq - 2I_{xz}pr + I_y q^2 - 2I_{yz}qr + I_z r^2)$$

$$p_s = p \cos\alpha + r \sin\alpha$$

$$q_s = q$$

$$r_s = -p \sin\alpha + r \cos\alpha$$

SELECTION OF STATE, CONTROL, AND OBSERVATION VARIABLES

The equations in the two preceding sections define the state and observation variables available in LINEAR. The control variables are defined by the user in the input file. Internally, the program uses a 12-state model, a 120-variable observation vector, and a 30-parameter control vector. These variables can be selected to specify the formulation most suited for the specific application. The order and number of parameters in the output model is completely under user control. Figure 1 illustrates the selection of variables for the state vector of the output model. However, it should be noted that no model order reduction is attempted. Elements of the matrices in the internal formulation are simply selected and reordered in the formulation specified by the user.

The selection of specific state, control, and observation variables for the formulation of the output matrices is accomplished by alphanumeric descriptors in the input file. The use of these alphanumeric descriptors is described in the Input Files section. Appendix C lists the state variables and their alphanumeric descriptors. Appendix D lists the observation variables and their alphanumeric descriptors. The alphanumeric descriptors for the selection of control parameters to be included

in the observation vector are the control variable names defined by the user in the input file, as described in the Input Files section.

LINEAR MODELS

The linearized system matrices computed by LINEAR are the first-order terms of a Taylor series expansion about the analysis point (Dieudonne, 1978; Kwakernaak and Sivan, 1972; proposed NASA RP by Duke, Antoniewicz, and Krambeer, in preparation) and are assumed to result in a time-invariant linear system. The validity of this assumption is discussed in the Analysis Point Definition section. The technique employed to obtain these matrices numerically is a simple approximation to the partial derivative, that is,

$$\frac{\partial f}{\partial x} \approx \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2 \Delta x}$$

where f is a general function of x, an arbitrary independent variable. The Δx may be set by the user but it defaults to 0.001 for all state and control parameters with the single exception of velocity V, where Δx is multiplied by a, the speed of sound, to obtain a reasonable perturbation size.

From the generalized nonlinear state,

$$T\dot{x} = f(x, \dot{x}, u)$$

and observation equations,

$$y = g(x, \dot{x}, u)$$

the program determines the linearized matrices for the generalized formulation of the system:

$$C \delta\dot{x} = A' \delta x + B' \delta u$$

$$\delta y = H' \delta x + G \delta\dot{x} + F' \delta u$$

where

$$C = T - \frac{\partial f}{\partial \dot{x}}$$

$$A' = \frac{\partial f}{\partial x}$$

$$B' = \frac{\partial f}{\partial u}$$

$$H' = \frac{\partial g}{\partial x}$$

$$G = \frac{\partial g}{\partial \dot{x}}$$

$$F' = \frac{\partial g}{\partial u}$$

with all derivatives evaluated along the nominal trajectory defined by the analysis point $(x_0, \dot{x}_0, u_0)$; the state, time derivative of state, and control vectors can be expressed as small perturbations $(\delta x, \delta \dot{x}, \delta u)$ about the nominal trajectory, so that

$$x = x_0 + \delta x$$

$$\dot{x} = \dot{x}_0 + \delta \dot{x}$$

$$u = u_0 + \delta u$$

In addition to the matrices for this generalized system, the user has the option of requesting linearized matrices for a standard formulation of the system:

$$\delta \dot{x} = A \, \delta x + B \, \delta u$$

$$\delta y = H \, \delta x + F \, \delta u$$

where

$$A = \left[T - \frac{\partial f}{\partial \dot{x}} \right]^{-1} \frac{\partial f}{\partial x}$$

$$B = \left[T - \frac{\partial f}{\partial \dot{x}} \right]^{-1} \frac{\partial f}{\partial u}$$

$$H = \frac{\partial g}{\partial x} + \frac{\partial g}{\partial \dot{x}} \left[T - \frac{\partial f}{\partial \dot{x}} \right]^{-1} \frac{\partial f}{\partial x}$$

$$F = \frac{\partial g}{\partial u} + \frac{\partial g}{\partial \dot{x}} \left[T - \frac{\partial f}{\partial \dot{x}} \right]^{-1} \frac{\partial f}{\partial u}$$

with all derivatives evaluated along the nominal trajectory defined by the analysis point $(x_0, \dot{x}_0, u_0)$.

LINEAR also provides two nonstandard matrices, D and E, in the equations

$$\dot{x} = Ax + Bu + Dv$$

$$y = Hx + Fu + Ev$$

or D' and E' in the equations

$$\dot{C\mathbf{x}} = A'\mathbf{x} + B'\mathbf{u} + D'\mathbf{v}$$

$$\mathbf{y} = H'\mathbf{x} + G\dot{\mathbf{x}} + F'\mathbf{u} + E'\mathbf{v}$$

These dynamic interaction matrices include the effect of external forces and moments acting on the vehicle. The components of the dynamic interaction vector $\mathbf{v}$ are incremental body axis forces (δX, δY, δZ) and moments (δL, δM, δN):

$$\mathbf{v} = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \\ \delta L \\ \delta M \\ \delta N \end{bmatrix}$$

Thus,

$$D' = \frac{\partial f}{\partial \mathbf{v}}$$

$$E' = \frac{\partial g}{\partial \mathbf{v}}$$

and

$$D = \left[T - \frac{\partial f}{\partial \dot{\mathbf{x}}} \right]^{-1} \frac{\partial f}{\partial \mathbf{v}}$$

$$E = \frac{\partial g}{\partial \mathbf{v}} + \frac{\partial g}{\partial \dot{\mathbf{x}}} \left[T - \frac{\partial f}{\partial \dot{\mathbf{x}}} \right]^{-1} \frac{\partial f}{\partial \mathbf{v}}$$

These matrices allow the effects of unusual subsystems or control effectors to be easily included in the vehicle dynamics.

The default output matrices for LINEAR are those for the standard system formulation. However, the user can select matrices for either generalized or standard state and observation equations in any combination. Internally, the matrices are computed for the generalized system formulation and then combined appropriately to accommodate the system formulation requested by the user.

ANALYSIS POINT DEFINITION

The point at which the nonlinear system equations are linearized is referred to as the analysis point. This can represent a true steady-state condition on the specified trajectory (a point at which the rotational and translational accelerations

are zero; Perkins and Hage, 1949; Thelander, 1965) or a totally arbitrary state on a
trajectory. LINEAR allows the user to select from a variety of analysis points.
Within the program, these analysis points are referred to as trim conditions, and
several options are available to the user. The arbitrary state and control option
is designated NOTRIM, and in selecting this option the user must specify all nonzero
state and control variables. For the equilibrium conditions, the user specifies a
minimum number of parameters, and the program numerically determines required state
and control variables to force the rotational and translational accelerations to
zero. The analysis point options are described in detail in the following sections.

For all the analysis point definition options, any state or control parameter
may be input by the user. Those state variables not required to define the analysis
point are used as initial estimates for the calculation of the state and control
conditions that result in zero rotational and translational accelerations. As each
state variable is read into LINEAR, the name is compared to the list of alternative
state variables names listed in appendix C. All state variables except velocity
must be specified according to this list. Velocity can also be defined by specify-
ing Mach number (see alternative observation variable names in app. D). Appendix E
lists analysis point definition identifiers that are recognized by LINEAR.

It should be noted that the option of allowing the user to linearize the system
equations about a nonequilibrium condition raises theoretical issues (beyond the
scope of this report) of which the potential user should be aware. Although all the
analysis point definition options provided in LINEAR have been found to be useful in
the analysis of vehicle dynamics, not all the linear models derived about these anal-
ysis points result in the time-invariant systems assumed in this report. However,
the results of the linearization provided by LINEAR do give the appearance of being
time invariant.

The linearization process as defined in this report is always valid for some
time interval beyond the point in the trajectory about which the linearization is
done. However, for the resultant system to be truly time invariant, the vehicle
must be in a sustainable, steady-state flight condition. This requirement is
something more than merely a trim requirement, which is typically represented as
$\dot{x}(t) = 0$, indicating that for trim, all the time derivatives of the state vari-
ables must be zero. (This is not the case, however: Trim is achieved when the
acceleration-like terms are identically zero; no constraints need to be placed on
the velocity-like terms in $\dot{x}$. Thus, for the model used in LINEAR, only $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{V}$,
$\dot{\alpha}$, and $\dot{\beta}$ must be zero to satisfy the trim condition.) The trim condition is
achieved for the straight-and-level, pushover-pullup, level turn, thrust-stabilized
turn, and beta trim options described in the following sections. In general, the
no-trim and specific power analysis point definition options do not result in a trim
condition.

Of these analysis point options resulting in a trim condition, only the straight-
and-level and level turn options force the model to represent sustainable flight
conditions. In fact, only in the special case where the flightpath angle is zero
does this occur for these options.

As previously stated, the linearization of a nonlinear model and its represen-
tation as a time-invariant system are always valid for some time interval beyond the
analysis point on the trajectory. This time interval is determined by several fac-

tors (such as trim and sustainable flight conditions) and ultimately by accuracy requirements placed on the representation. Thus, in using the linear models provided by this program, the user should exercise some caution.

Untrimmed

For the untrimmed option, the user specifies all state and control variables that are to be set at some value other than zero. The number of state variables specified is entirely at the user's discretion. If any of the control variables are to be nonzero, the user must specify the control parameter and its value. The untrimmed option allows the user to analyze the vehicle dynamics at any flight condition, including transitory conditions.

Straight-and-Level Trim

The straight-and-level trims available in LINEAR are in fact wings-level, constant-flightpath-angle trims. Both options available for straight-and-level trim allow the user to specify either a flightpath angle or an altitude rate. However, since the default value for these terms is zero, the default for both types of straight-and-level trim is wings-level, horizontal flight.

The two options available for straight-and-level trim require the user to specify altitude and either an angle of attack or a Mach number. If a specific angle of attack and altitude combination is desired, the user selects the "Mach-trim" option, which determines the velocity required for the requested trajectory. Likewise, the "alpha-trim" option allows the user to specify Mach number and altitude, and the trim routines in LINEAR determine the angle of attack needed for the requested trajectory.

The trim condition for both straight-and-level options are determined within the following constraints:

$$p = q = r = 0$$

$$\phi = 0$$

The trim surface positions, thrust, angle of sideslip, and either velocity or angle of attack are determined by numerically solving the nonlinear equations for the translational and rotational acceleration. Pitch attitude θ is determined by iterative solution of the altitude rate equation.

Pushover-Pullup

The pushover-pullup analysis point definition options result in wings-level flight at $n \neq 1$. For $n > 1$, the analysis point is the minimum altitude point of a pullup when $\dot{h} = 0$. For $n < 1$, this trim results in a pushover at the maximum altitude with $\dot{h} = 0$. There are two options available for the pushover-pullup analysis point definition: (1) the "alpha-trim" option in which angle of attack is determined from the specified altitude, Mach number, and load factor and (2) the "load-

factor-trim" option in which angle of attack, altitude, and Mach number are specified and load factor is determined according to the constraint equations.

The analysis point is determined at the specified conditions, subject to the following contraints:

$$p = r = 0$$

$$q = \frac{1}{Vm \cos \beta} [mg(n - \cos(\theta - \alpha)) - Z_T \cos \alpha + X_T \sin \alpha]$$

$$\phi = 0$$

The expression for q is derived from the $\dot{\alpha}$ equation by setting $\dot{\alpha} = 0$ and $\dot{\phi} = 0$; θ is derived from the $\dot{h}$ equation. The trim surface positions, thrust, angle of sideslip, and either angle of attack or load factor are determined by numerically solving the nonlinear equations for the translational and rotational accelerations.

Level Turn

The level turn analysis point definition options result in non-wings-level, constant-turn-rate flight at n > 1. The vehicle model is assumed to have sufficient excess thrust to trim at the condition specified. If thrust is not sufficient, trim will not result, and the analysis point thus defined will have a nonzero (in fact, negative) velocity rate.

The level trim options available in LINEAR require the specification of an altitude and a Mach number. The user can then use either angle of attack or load factor to define the desired flight condition. These two options are referred to as "alpha-trim" and "load-factor-trim," respectively. For either option, the user may also request a specific flightpath angle or altitude rate. Thus, these analysis point definitions may result in ascending or descending spirals, although the default is for the constant-altitude turn.

The constraint equations for the coordinated level turn analysis point definitions are derived by Chen (1981) and Chen and Jeske (1981). Using the requested load factor, the tilt angle of the acceleration normal to the flightpath from the vertical plane, ϕ_L, is calculated using the equation

$$\phi_L = \pm\tan^{-1} \left[\frac{(n^2 - \cos^2 \gamma)^{1/2}}{\cos \gamma} \right]$$

where the positive sign is used for a right turn and the negative sign is used for a left turn. From ϕ_L, turn rate can be calculated as

$$\dot{\psi} = \frac{g}{V} \tan \phi_L$$

Using these two definitions, the state variables can be determined:

$$q = \dot{\psi} \sin^2 \phi_L \left[-\sin \gamma \sin \beta \pm \left(\sin^2 \gamma \sin^2 \beta - \frac{\sin^2 \gamma - \cos^2 \beta}{\sin^2 \phi_L} \right)^{1/2} \right]$$

$$r_s = \frac{q}{\tan \phi_L \cos \beta}$$

$$p_s = - \frac{\dot{\psi} \sin \gamma}{\cos \beta} - q \tan \beta$$

$$p = p_s \cos \alpha - r_s \sin \alpha$$

$$r = p_s \sin \alpha + r_s \cos \alpha$$

$$\theta = \sin^{-1}(-p/\dot{\psi})$$

$$\phi = \tan^{-1}(q/r)$$

The trim surface positions, thrust, angle of sideslip, and either angle of attack or load factor are determined by numerically solving the nonlinear equations for the translational and rotational accelerations.

Thrust-Stabilized Turn

The thrust-stabilized turn analysis point definition results in a constant-throttle, non-wings-level turn with a nonzero altitude rate. The two options available in LINEAR are "alpha-trim" and "load-factor-trim." These options allow the user to specify either the angle of attack or the load factor for the analysis point. The altitude and Mach number at the analysis point must be specified for both options. The user also must specify the value of the thrust trim parameter by assigning a value to the variable THRSTX in the input file after the trim has been selected.

The constraint equations for the thrust-stabilized turn are the same as those for the level turn. However, for this analysis point definition, flightpath angle is determined by LINEAR.

Beta Trim

The beta trim analysis point definition results in non-wings-level, horizontal flight with heading rate $\dot{\psi}$ = 0 at a user-specified Mach number, altitude, and angle of sideslip. This trim option is nominally at 1 g, but as β varies from zero, normal acceleration decreases and lateral acceleration increases. For an aerodynamically symmetric aircraft, a trim to β = 0 using the beta trim option results in the same trimmed condition as the straight-and-level trim. However, for an aerodynamically asymmetric aircraft, such as an oblique-wing vehicle, the two trim options are not equivalent.

The constraint equations used with the beta trim option are

$$p = q = r = 0$$

The trim surface positions, thrust, angle of attack, and bank angle are determined by numerically solving the nonlinear equations for translational and rotational acceleration. Pitch attitude θ is derived from the equation for flightpath angle γ with $\gamma = 0$:

$$\theta = \tan^{-1} \left(\frac{\sin \beta \sin \phi + \cos \beta \sin \alpha}{\cos \beta \cos \alpha} \right)$$

Specific Power

The specific power analysis point definition results in a level turn at a user-specified Mach number, altitude, thrust trim parameter, and specific power. Unlike the other trim options provided in LINEAR, the specific power option does not, in general, attempt to achieve velocity rate $\dot{V} = 0$. In fact, because the altitude rate $\dot{h} = 0$ and specific power is defined by

$$P_S = \dot{h} + \frac{V\dot{V}}{g}$$

the resultant velocity rate will be

$$\dot{V} = \frac{P_S g}{V}$$

However, the other acceleration-like terms ($\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{\alpha}$, and $\dot{\beta}$) will be zero if the requested analysis point is achieved.

The constraint equations used with the specific power analysis point definition option can be derived from the load factor tilt angle equation used in the level turn analysis point definition with $\gamma = 0$:

$$\phi_L = \pm\tan^{-1}(n^2 - 1)^{1/2}$$

(where the positive sign is used for a right turn and the negative sign is used for a left turn),

$$\dot{\psi} = \frac{g}{V} \tan \phi_L$$

$$q = \dot{\psi} \sin \phi_L \cos \beta$$

$$r_s = \frac{q}{\tan \phi_L \cos\beta}$$

$$p_s = -q \tan \beta$$

$$p = p_s \cos \alpha - r_s \sin \alpha$$

$$r = p_s \sin \alpha + r_s \cos \alpha$$

$$\theta = \sin^{-1}\left(-\frac{p}{\dot{\psi}}\right)$$

$$\phi = \tan^{-1}\frac{q}{r}$$

The analysis point surface positions, load factor, angle of attack, and angle of sideslip are determined by numerically solving the nonlinear equations for translational and rotational accelerations.

NONDIMENSIONAL STABILITY AND CONTROL DERIVATIVES

The nondimensional stability and control derivatives computed by LINEAR from the nonlinear aerodynamic model assume broadly formulated linear aerodynamic equations. These equations include effects of what are normally considered exclusively lateral-directional parameters in the longitudinal force and moment coefficient equations and, conversely, effects of longitudinal parameters in the lateral-directional equations. The reason for this is two-fold: application to a larger class of vehicle types, such as oblique-wing aircraft, and computational ease.

The nondimensional stability and control derivatives assume the following equations for the aerodynamic force and moment coefficients:

$$C_\ell = C_{\ell_0} + C_{\ell_\alpha}\alpha + C_{\ell_\beta}\beta + C_{\ell_h}\delta h + C_{\ell_M}\delta M + \sum_{i=1}^{n} C_{\ell_{\delta_i}}\delta_i$$

$$+ C_{\ell_p}\hat{p} + C_{\ell_q}\hat{q} + C_{\ell_r}\hat{r} + C_{\ell_{\dot{\alpha}}}\hat{\dot{\alpha}} + C_{\ell_{\dot{\beta}}}\hat{\dot{\beta}}$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_\beta}\beta + C_{m_h}\delta h + C_{m_M}\delta M + \sum_{i=1}^{n} C_{m_{\delta_i}}\delta_i$$

$$+ C_{m_p}\hat{p} + C_{m_q}\hat{q} + C_{m_r}\hat{r} + C_{m_{\dot{\alpha}}}\hat{\dot{\alpha}} + C_{m_{\dot{\beta}}}\hat{\dot{\beta}}$$

$$C_n = C_{n_0} + C_{n_\alpha}\alpha + C_{n_\beta}\beta + C_{n_h}\delta h + C_{n_M}\delta M + \sum_{i=1}^{n} C_{n_{\delta_i}}\delta_i$$

$$+ C_{n_p}\hat{p} + C_{n_q}\hat{q} + C_{n_r}\hat{r} + C_{n_{\dot{\alpha}}}\hat{\dot{\alpha}} + C_{n_{\dot{\beta}}}\hat{\dot{\beta}}$$

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_\beta}\beta + C_{L_h}\delta h + C_{L_M}\delta M + \sum_{i=1}^{n} C_{L_{\delta_i}}\delta_i$$

$$+ C_{L_p}\hat{p} + C_{L_q}\hat{q} + C_{L_r}\hat{r} + C_{L_{\dot{\alpha}}}\hat{\dot{\alpha}} + C_{L_{\dot{\beta}}}\hat{\dot{\beta}}$$

$$C_D = C_{D_0} + C_{D_\alpha}\alpha + C_{D_\beta}\beta + C_{D_h}\delta h + C_{D_M}\delta M + \sum_{i=1}^{n} C_{D_{\delta_i}}\delta_i$$

$$+ C_{D_p}\hat{p} + C_{D_q}\hat{q} + C_{D_r}\hat{r} + C_{D_{\dot{\alpha}}}\hat{\dot{\alpha}} + C_{D_{\dot{\beta}}}\hat{\dot{\beta}}$$

$$C_Y = C_{Y_0} + C_{Y_\alpha}\alpha + C_{Y_\beta}\beta + C_{Y_h}\delta h + C_{Y_M}\delta M + \sum_{i=1}^{n} C_{Y_{\delta_i}}\delta_i$$

$$+ C_{Y_p}\hat{p} + C_{Y_q}\hat{q} + C_{Y_r}\hat{r} + C_{Y_{\dot{\alpha}}}\hat{\dot{\alpha}} + C_{Y_{\dot{\beta}}}\hat{\dot{\beta}}$$

where the stability and control derivatives have the usual meaning,

$$C_{\epsilon_x} \equiv \frac{\partial C_\epsilon}{\partial x}$$

with C_ϵ being an arbitrary force or moment coefficient (subscript ℓ denoting rolling moment, m pitching moment, L total lift, D total drag, n yawing moment, and Y force along the y body axis) and x an arbitrary nondimensional variable (α denoting angle of attack, β angle of sideslip, h altitude, M Mach number, p roll rate, q pitch rate, and r yaw rate). The rotational terms in the equations are nondimensional versions of the corresponding state variable with

$$\hat{p} = \frac{bp}{2V}$$

$$\hat{q} = \frac{\bar{c}q}{2V}$$

$$\hat{r} = \frac{br}{2V}$$

$$\hat{\dot{\alpha}} = \frac{\bar{c}\dot{\alpha}}{2V}$$

$$\hat{\dot{\beta}} = \frac{b\dot{\beta}}{2V}$$

where b is wingspan and $\bar{c}$ is mean aerodynamic chord. The δ_i in the summations are the n control variables defined by the user. The effects of altitude and velocity (Mach) are included in the derivatives with respect to those parameters and in the incremental multipliers

$$\delta h = h - h_0$$

$$\delta M = M - M_0$$

where the subscript zero represents the current analysis point $(x_0, \dot{x}_0, u_0)$, described in the Analysis Point Definition section.

All stability derivatives are computed as nondimensional terms except the altitude and velocity parameters. The control derivatives are in whatever units are used in the nonlinear aerodynamic model. The derivatives with respect to velocity are multiplied by the speed of sound (at the analysis point altitude) to convert them to derivatives with respect to Mach number. Derivatives with respect to angle of attack and angle of sideslip can be obtained in units of reciprocal degrees. These derivatives are simply the corresponding nondimensional derivatives multiplied by $180.0/\pi$.

INPUT FILES

The LINEAR input file (defined in table 1) is separated into seven major sections: case title and file selection information; project title; geometry and mass data for the aircraft; state, control, and observation variable definitions for the state-space model; trim parameter specification; additional control surfaces that may be specified for each case; and various test case specifications. All the input data can be written on one file or various files according to the divisions specified above and according to the input format defined in table 1. An example input file is listed in appendix F. All the input records to LINEAR are written in ASCII form.

TABLE 1. — INPUT FORMAT FOR LINEAR

Input record	Format
Title and file selection information	
Case title	(20A4)
Input file names	(6A10)
Project title	
Project title	(20A4)
Geometry and mass data	
S b $\bar{c}$ Weight	(4F13.0)
I_x I_y I_z I_{xz} I_{xy} I_{yz}	(6F13.0)
DELX DELY DELZ LOGCG	(3F10.0,12A4)
α_{min} α_{max}	(2F13.0)

TABLE 1. — Continued.

Input record	Format
State, control, and observation variable definitions	
NUMSAT EQUAT STAB	(I10,A4,11X,A4)
STATE 1 DRVINC 1	(5A4,F10.0)
STATE 2 DRVINC 2	(5A4,F10.0)
STATE 3 DRVINC 3	(5A4,F10.0)
$\cdot$ $\cdot$	$\cdot$
$\cdot$ $\cdot$	$\cdot$
$\cdot$ $\cdot$	$\cdot$
NUMSUR	(I10)
CONTROL 1 LOCCNT 1 CONVR 1 CNTINC 1	(5A4,I10,A4,6X,F10.0)
CONTROL 2 LOCCNT 2 CONVR 2 CNTINC 2	(5A4,I10,A4,6X,F10.0)
CONTROL 3 LOCCNT 3 CONVR 3 CNTINC 3	(5A4,I10,A4,6X,F10.0)
$\cdot$ $\cdot$ $\cdot$ $\cdot$	$\cdot$
$\cdot$ $\cdot$ $\cdot$ $\cdot$	$\cdot$
$\cdot$ $\cdot$ $\cdot$ $\cdot$	$\cdot$
NUMYVC EQUAT	(I10,A4)
MEASUREMENT 1 PARAM 1 (1 to 3)	(5A4,3F10.0)
MEASUREMENT 2 PARAM 2 (1 to 3)	(5A4,3F10.0)
MEASUREMENT 3 PARAM 3 (1 to 3)	(5A4,3F10.0)
$\cdot$ $\cdot$	$\cdot$
$\cdot$ $\cdot$	$\cdot$
$\cdot$ $\cdot$	$\cdot$
Trim parameter specification	
δe_{min} δe_{max} δa_{min} δa_{max} δr_{min} δr_{max} δth_{min} δth_{max}	(8F10.0)
Additional surface specification	
NUMXSR	(1X,I2)
ADDITIONAL SURFACE 1 LOCCNT 1 CONVR1	(5A4,I10,A4)
ADDITIONAL SURFACE 2 LOCCNT 2 CONVR2	(5A4,I10,A4)

TABLE 1. — Concluded.

Input records	Format
Additional surface specification (continued)	
ADDITIONAL SURFACE 3 LOCCNT 3 CONVR3	(5A4,I10,A4)
• • •	•
• • •	•
• • •	•
Test case specification	
ANALYSIS POINT DEFINITION OPTION	(20A4)
ANALYSIS POINT DEFINITION SUBOPTION	(A4)
VARIABLE 1 VALUE 1	(5A4,F15.5)
VARIABLE 2 VALUE 2	(5A4,F15.5)
VARIABLE 3 VALUE 3	(5A4,F15.5)
• •	•
• •	•
• •	•
NEXT	(A4)
ANALYSIS POINT DEFINITION OPTION	(20A4)
ANALYSIS POINT DEFINITION SUBOPTION	(A4)
VARIABLE 1 VALUE 1	(5A4,F15.5)
VARIABLE 2 VALUE 2	(5A4,F15.5)
VARIABLE 3 VALUE 3	(5A4,F15.5)
• •	•
• •	•
• •	•
END	(A4)

There are seven input files for the batch linearizer. The first is a file containing the vehicle title and the names of the six data input files. The six data input files are

1. project title,

2. mass and geometry properties,

3. state, control, and observation vectors,

4. trim parameter limits,

5. trim definition cases, and

6. additional surfaces to be set.

This is the order in which they must be defined. However, the sixth file will be read before the fifth file.

Case Title, File Selection Information, and Project Title

There are two title records that need to be specified for LINEAR: a title for the individual test cases (case title) being analyzed and the name of the specific vehicle (project title). These records are read with a 20A4 format and are separated by a file selection record. Both titles appear on each page of the line printer output file. The file selection record contains the names of the files from which the data are read. The data contained on the files specified by the six fields of the file selection record are shown in table 2. The input file names are written in character strings 10 columns long, and if not specified, the data are assumed to be on the same file as the first title record and the file selection record. The local name of the file containing these first two records must be attached to FORTRAN device unit 1.

TABLE 2. — DEFINITION OF FILES SPECIFIED
IN FILE SELECTION RECORD

Field number	Data contained on selected file
1	Project title
2	Geometry and mass data
3	State, control, and observation variable definitions
4	Trim parameter specifications
5	Test case specification
6	Additional surface definitions

Geometry and Mass Data

The geometry and mass data file consists of four records that can either follow the title and file selection records on unit 1 or be stored on a separate file

defined by the first of the input file name definitions on the file selection record. The geometry and mass data records define the geometry, mass properties, location of the aerodynamic reference with respect to the center of gravity, and angle-of-attack range for the vehicle model being analyzed. The first record of this set of five records is a vehicle title record in which the vehicle name and specific configuration can be defined to describe the information presented in the remaining records.

The second and third records define the vehicle geometry, mass, and mass distribution. The second record defines the wing planform area S, in units of length squared, the wingspan b, in units of length, the mean aerodynamic chord of the wing, $\bar{c}$, in units of length, and the sea level weight of the vehicle, Weight, in units of mass-length per second squared. The third record defines the vehicle moments and products of inertia in units of mass-length squared. The fourth record defines the location of the vehicle center of gravity with respect to the aerodynamic reference point. This fourth record defines the offset of the aerodynamic reference point with respect to the center of gravity of the vehicle in the normal right-handed body axis reference system with the positive x axis forward and DELX, DELY, and DELZ in table 1 representing the x, y, and z body axis displacements of the aerodynamic reference in units of length (see app. A). The fourth variable of the fourth record is an alphanumeric variable, read using a 12A4 format, to specify if corrections due to an offset of the aerodynamic reference with respect to the center of gravity are to be computed in LINEAR or in the user-supplied aerodynamic subroutine CCALC, described in the User-Supplied Subroutines section. The variable LOGCG defaults to a state that causes the aerodynamic reference offset calculations to be performed by subroutines within LINEAR. Any of the following statements in the LOGCG field will cause LINEAR not to make these corrections:

NO CG CORRECTIONS BY LINEAR

CCALC WILL CALCULATE CG CORRECTIONS

FORCE AND MOMENT CORRECTIONS CALCULATED IN CCALC

The final record of this geometry and mass data set defines the angle-of-attack range for which the user-supplied nonlinear aerodynamic model (CCALC) is valid. These parameters, α_{min} and α_{max}, specify the minimum and maximum values of angle of attack to be used for trimming the aircraft model. These parameters are in units of degrees.

State, Control, and Observation Variable Definitions

The state, control, and observation variables to be used in the output formulation of the linearized system are defined in records that either follow the last of the previously described sets of records on FORTRAN unit 1 or are stored on a separate file defined by the second of the input file selection definitions on the file selection record. The number of records in the state, control, and observation variable definition set is determined by the number of such variables defined by the user.

The states to be used in the output formulation of the linearized system are defined in the first set of records in the state, control, and observation variable definitions. The first record of this set defines the number of states to be used

(NUMSAT), the formulation of the output equation (EQUAT), and whether the nondimensional stability derivatives with respect to angle of attack and angle of sideslip are to be output in units of reciprocal radians or degrees. The variable EQUAT is read using an A4 format and is tested against the following list:

NONSTANDARD
NON-STANDARD
GENERALIZED
EXTENDED

If EQUAT matches the first four characters of any of the listed words, the output formulation of the state equation is

$$C\dot{x} = A'x + B'u$$

If EQUAT is read in as STANDARD or does not match the preceding list, then the default standard bilinear formulation of the state equation is assumed, and the output matrices are consistent with the equation

$$\dot{x} = Ax + Bu$$

The variable STAB is also read using an A4 format and is compared with the following list:

DEGREES
DGR

If STAB matches the first four characters of either of these words, the nondimensional stability derivatives with respect to angle of attack and angle of sideslip are printed in units of reciprocal degress on the printer file. Otherwise, these derivatives are printed in units of reciprocal radians.

The remaining records of the state variable definition set are used to specify the state variables to be used in the output formulation of the linearized system and the increments to be used for the numerical perturbation described in the Linear Models section. The state variable names are checked for validity against the state variable alphanumeric descriptors listed in appendix C. If a name is not recognized, the variable is ignored and a warning message is written to the printer file. The increment to be used with any state variable (in calculating the A' and H' matrices) and the time derivative of that state variable (in calculating the C and G matrices) can be specified using the DRVINC variable. The units are the units of the state variable or its time derivative except for derivatives with respect to velocity. When DRVINC is specified for velocity, DRVINC specifies a Mach number increment. If DRVINC is not specified by the user, the default value of 0.001 is used.

The next set of records in the state, control, and observation variable definitions are those defining the variables to be used in the control vector of the output model. The first record of this set defines the number of control parameters to be used (NUMSUR). The remaining records define the names of these variables (CONTROL), their location (LOCCNT) in the common block /CONTRL/ (see the User-Supplied Subroutines section), the units associated with these control variables (CONVR), and the increments (CNTINC) to be used with these variables in determining the B' and F' matrices.

Because LINEAR has no default control variable names, the control variable names input by the user are used for subsequent identification of the control variables. Therefore, consistency in the use of control variable names is extremely important, particularly when the user attempts to establish control variable initial conditions when using the untrimmed analysis point definition option.

The CONVR field in the control variable records is used to specify if the control variables are given in degrees or radians. CONVR is read using an A4 format and is compared to the following list:

> DEGREES
> DGR
> RADIANS
> RAD

If CONVR agrees with the first four characters of either of the first two names, it is assumed that the control variable is specified in units of degrees. If CONVR agrees with the first four characters of either of the last two listed names, it is assumed that the control variable is specified in units of radians. No units are assumed if CONVR does not agree with any of the listed names. When it is assumed that the control variable is specified in units of radians, the initial value of the control variable is converted to degrees before being written to the printer file.

The variable CNTINC can be used to specify the increments used for a particular control surface when the B' and F' matrices are being calculated. It is assumed that the units of CNTINC agree with those used for the surface, and no unit conversion is attempted on these increments. If CNTINC is not specified for a particular surface, a default value of 0.001 is used.

The final set of records in the state, control, and observation variable definitions pertain to the specification of parameters associated with the observation vector, observation equation, and observation parameters. The first record of this set defines the number of observation variables (NUMYVC) to be used in the output linear model and the formulation of the output equation (EQUAT). The remaining records in this set specify the variables to be included in the observation vector (MEASUREMENT) and any position information (PARAM) that may be required to compute the output model for a sensor not located at the vehicle center of gravity.

The variable used to specify the formulation of the observation equation (EQUAT) is compared with the same list of names used to determine the formulation of the state equation. If it is determined that the generalized formulation is desired, the observation equation

$$y = H'x + G\dot{x} + F'u$$

is used. Otherwise, the standard formulation is assumed, and the form of the observation equation used is

$$y = Hx + Fu$$

The records defining the observation variables to be used in the output formulation of the linear model contain a variable that includes the parameter name (MEASUREMENT) and three fields (PARAM) defining, when appropriate, the location of

40

the sensor relative to the vehicle center of gravity. The parameter name is com-
pared with the list of observation variables given in appendix D. If the parameter
name is recognized as a valid observation variable name, that observation variable
is included in the formulation of the output observation vector. If the param-
eter name is not recognized, an error message is printed and the parameter named
is ignored. The three variables represented by PARAM(1), PARAM(2), and PARAM(3)
provide the x-axis, y-axis, and z-axis locations, respectively, of the measurement
with respect to the vehicle center of gravity if the selected observation is one of
the following:

$$a_{x,i}$$

$$a_{y,i}$$

$$a_{z,i}$$

$$a_{n,i}$$

$$h_{,i}$$

$$\dot{h}_{,i}$$

$$\alpha_{,i}$$

$$\beta_{,i}$$

The unit associated with these variables is length. If the selected observation
variable is not in the preceding list, the PARAM variables are not used. The sole
exception to this occurs when Reynolds number is requested as an observation vari-
able. In that case, PARAM(1) is used to specify the characteristic length. When

no value is input for PARAM(1), the mean aerodynamic chord $\bar{c}$ is used as the char-
acteristic length.

Trim Parameter Specification

There is one record in the trim parameter specification set that is associated
with the subroutine UCNTRL (described in the User-Supplied Subroutines section).
This record specifies the limits to be used for the trim parameters δ_e, δ_a, δ_r, and
δ_{th}, representing the longitudinal, lateral, directional, and thrust trim parame-
ters, respectively. The units associated with these parameters are defined by the
implementation of UCNTRL.

Additional Surface Specification

The first record of this set of additional surface specifications defines the
number of additional controls to be included in the list of recognized control names
(NUMXSR). The purpose of defining these additional controls is to allow the user to
set such variables as landing gear position, wing sweep, or flap position. Only the
controls are defined in the additional surface specification records; actual control

positions are defined in the analysis point definition records. Because there may be no additional controls, these secondary trim parameter specification records may not be present. If such controls are defined, the records defining them will have the format specified in table 1. The control variable name (ADDITIONAL SURFACE), location (LOCCNT) in the common block /CONTRL/, and the units associated with this control variable (CONVR) are specified for each additional control.

Test Case Specification

The test case specification records allow the user to define the flight condition, or analysis point, at which a linear model is to be derived. Multiple cases can be included in the test case specification records. The final record in each set directs the program to proceed (NEXT) or to stop (END) execution.

The first record of a test case specification set determines the analysis point, or trim, option to be used for the current case. The ANALYSIS POINT DEFINITION OPTION parameter is read in and checked against the list of analysis point definition identifiers described in appendix E. The second record of a test case specification set, defining an analysis point definition suboption (ANALYSIS POINT DEFINITION SUBOPTION), will be read only if the requested analysis point definition option has a suboption associated with it. These suboptions are defined in the Analysis Point Definition section. The valid alphanumeric descriptors for these suboptions are described in appendix E.

The remaining records in a test case specification set define test conditions or initial conditions for the trimming subroutines. These records consist of a field defining a parameter name (VARIABLE) and its initial condition (VALUE). These records may be in any order; however, if initial Mach number is to be defined, the initial altitude must be specified before Mach number if the correct initial velocity is to be determined. The parameter names are checked against all name lists used within LINEAR. Any recognized state, time derivative of state, control, or observation variable will be accepted. Trim parameters also can be set in these records.

In general, setting observation variables and time derivatives of the state variables has little effect. However, for some of the trim options defined in the Analysis Point Definition section, Mach number and load factor are used. The thrust trim parameter only affects the specific power trim. For the untrimmed option, the initial values of the state and control variables determine the analysis point completely. For all other trim options, only certain states are not varied; all controls connected to the aerodynamic and engine model are varied.

OUTPUT FILES

There are three output files from LINEAR: a general-purpose analysis file, a printer file containing the calculated case conditions and the state-space matrices for each case, and a printer file containing the calculated case conditions only.

The general-purpose analysis file contains the title for the cases being analyzed; the state, control, and observation variables used to define the state-space model; and the state and observation matrices calculated in LINEAR. The C and G

matrices are printed only if the user has selected an appropriate formulation of
the state and observation equations. The output for this file is assigned to
FORTRAN device unit 15. An example of a general-purpose analysis file is pre-
sented in appendix G, corresponding to the format of table 3.

TABLE 3. — ANALYSIS FILE FORMAT

Variable	Format
Title of the case	(4A20)
Title of the aircraft	(4A20)
Case number	(//,64X,I3)
Number of states, controls, and outputs	(17X,I2,22X,I2,22X,I3)
State equation formulation	(36X,2A4)
Observation equation formulation	(36X,2A4) (////)
State variable names, values, and units	(1X,5A4,3X,E12.6,2X,A20) (////)
Control variable names, values, and units	(1X,5A4,3X,E12.6,2X,A20) (////)
Dynamic interaction variable names and units	(1X,5A4,17X,A20) (////)
Output variable names, values, and units	(1X,5A4,3X,E12.6,2X,A20)
Matrix name	(//,A8,/)
A matrix	(5(E13.6))
Matrix name	(//,A8,/)
B matrix	(5(E13.6))
Matrix name	(//,A8,/)
D matrix	(5(E13.6))
Matrix name	(//,A8,/)
C matrix (if general form chosen)	(5(E13.6))
Matrix name	(//,A8,/)
H matrix	(5(E13.6))

TABLE 3. -- Concluded.

Variable	Format
Matrix name	(//,A8,/)
F matrix	(5(E13.6))
Matrix name	(//,A8,/)
E matrix	(5(E13.6))
Matrix name	(//,A8,/)
G matrix (if general form chosen)	(5(E13.6))

The titles are written on the first two records of the file in 80-character strings and are specified in LINEAR as the title of the vehicle and the title for the cases. The next record contains the number of the case as defined in LINEAR (999 cases maximum).

The number of states, controls, and outputs used to define each case are written on the following record. The formulation of the state and observation equations are listed next, followed by the names and values of the states, controls, dynamic interaction variables, and outputs. These values are followed by the matrices that describe each case.

The titles records appear only at the beginning of the file; all other records are duplicated for each subsequent case calculated in LINEAR. The matrices are written row-wise, five columns at a time, as illustrated in the following tabulation, which shows a system containing 7 states, 3 controls, and 11 outputs using the general state equation and standard observation equation.

Size of matrix	Output formulation
A = [7 x 7]	A = [7 x 5]
	[7 x 2]
B = [7 x 3]	B = [7 x 3]
D = [7 x 6]	D = [7 x 5]
	[7 x 1]
C = [7 x 7]	C = [7 x 5]
	[7 x 2]
H = [11 x 7]	H = [11 x 5]
	[11 x 2]
F = [11 x 3]	F = [11 x 3]
E = [11 x 6]	E = [11 x 5]
	[11 x 1]

The second output file, which is assigned to FORTRAN device unit 3, contains the values calculated in LINEAR describing each case. The first section of this file contains a listing of the input data defining the aircraft's geometry and mass properties, variable names defining the state-space model, and various control surface limits characteristic of the given aircraft. Appendix H presents an example of this output file.

The second section of this file contains the trim conditions of the aircraft at the point of interest. These conditions include the type of trim being attempted, whether trim was achieved, and parameters defining the trim condition.

The values for the variables of the state-space model at the trim condition are also printed. If trim was not achieved, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{V}$, $\dot{\beta}$, and $\dot{\alpha}$ (calculated from the equations of motion) and the force and moment coefficients are printed. Changes in the geometry and mass properties are also printed.

The third section of this output file contains the nondimensional stability and control derivatives for the trim condition calculated. The static margin of the aircraft at the given flight condition is also printed.

The final section of this output file contains the state and observation matrices for the given flight condition. The formulation of the state equations and only the terms of the matrices chosen by the user to define the model are printed. A subset of this output file containing only the trim conditions is assigned to FORTRAN device unit 2.

The third output file, which is assigned to FORTRAN device unit 2, contains the trim conditions of the aircraft at the point of interest. These conditions include the type of trim being attempted, whether trim was achieved, parameters defining the trim condition, and the static margin of the aircraft at the given flight condition. Appendix H presents an example of this file.

USER-SUPPLIED SUBROUTINES

There are five subroutines that must be supplied by the user to interface LINEAR with a specific aircraft's subsystem models: ADATIN, CCALC, IFENGN, UCNTRL, and MASGEO. The first two subroutines constitute the aerodynamic model. The subroutine IFENGN is used to provide an interface between LINEAR and any engine modeling routines the user may wish to incorporate. UCNTRL converts the trim parameters used by LINEAR into control surface deflections for the aerodynamic modeling routines. The subroutine MASGEO allows the user to define the mass and geometry properties of the vehicle as a function of flight condition or control setting. The use of these subroutines is illustrated in figure 3, which shows the program flow and the interaction of LINEAR with the user-supplied subroutines. These subroutines are described in detail in the following sections. Examples of these subroutines are given in appendix I.

Main program

User-supplied
subroutines

```
┌─────────────────┐                    ┌─────────────────┐
│  Initialization │ ─────────────────► │   Aeromodel     │
└─────────────────┘                    │   (ADATIN)      │
                                       └─────────────────┘
┌─────────────────┐
│ Read aircraft-  │
│ specific        │
│ input data files│
└─────────────────┘
┌─────────────────┐
│ Read in data    │
│ for next        │
│ analysis point  │
└─────────────────┘
```

Interface common
block names
/CONTRL/,/CTPARM/

```
        ◇ Was trim                    ┌─────────────────┐
  No    option requested              │ Control surface │
◄────── ?                             │   gearing       │
                                      │   (UCNTRL)      │
         Yes                          └─────────────────┘
```

/CONTRL/,/ENGSTF/

```
┌─────────────────┐                   ┌─────────────────┐
│ Determine       │                   │ Thrust effector │
│ analysis        │                   │   model         │
│ trim point      │  /CONTRL/,/CGSHFT/ │   (IFENGN)      │
│ conditions      │                   └─────────────────┘
└─────────────────┘
```

/CGSHFT/,/CLCOUT/,
/CONTRL/,
/DATAIN/,/DRVOUT/

```
┌─────────────────┐                   ┌─────────────────┐
│ Determine       │                   │ Aerodynamic     │
│ linear model    │                   │   model         │
└─────────────────┘                   │   (CCALC)       │
                                      └─────────────────┘
┌─────────────────┐
│ Select matrix   │
│ elements        │
│ for output      │
└─────────────────┘
```

/OBSERV/,/CGSHFT/,
/DATAIN/,/CONTRL/,
/ENGSTF/

```
┌─────────────────┐                   ┌─────────────────┐
│  Output data    │                   │ Mass and        │
└─────────────────┘                   │ geometry        │
                                      │   model         │
                                      │   (MASGEO)      │
        ◇ Run                         └─────────────────┘
  Yes   another case
◄────── ?

         No

      ┌────────┐
      │  Stop  │
      └────────┘
```

Figure 3. Program flow diagram showing communication
with user-supplied subroutines.

Aerodynamic Model

It is assumed that the aerodynamic models consist of two main subroutines,
ADATIN and CCALC. ADATIN is used to input the basic aerodynamic data from remote
storage and can also be used to define aerodynamic data. CCALC is the subroutine
that uses these aerodynamic data, the state variables, and the surface positions
to determine the aerodynamic coefficients. Either routine may call other subrou-
tines to perform related or required functions; however, from the point of view
of the interface to LINEAR, only these two subroutines are required for an aero-
dynamic model.

Externally, ADATIN has no interface to the program LINEAR. The subroutine is called only once when the aerodynamic data are input or defined. The calling program must provide ADATIN with the input devices it requires, but no other accommodation is necessary. The aerodynamic data are communicated from ADATIN to CCALC through named common blocks that occur in only these two routines.

The interface between CCALC and the calling program is somewhat more complicated. However, the interface is standard, and this feature provides a framework about which a general-purpose tool can be built. This interface consists of several named common blocks that are used to pass state variables, air data parameters, surface positions, and force and moment coefficients between CCALC and the calling program. CCALC is executed whenever new aerodynamic coefficients are required (for example, once per frame for a real-time simulation).

The main transfer of data into the subroutine CCALC is through six named common blocks. These common blocks contain the state variables, air data parameters, and surface positions. The transfer of data from CCALC is through a named common block containing the aerodynamic force and moment coefficients. The details of these common blocks follow.

The common block /DRVOUT/ contains the state variables and their derivatives with respect to time. The structure of this common block is as follows:

```
COMMON /DRVOUT/ T      ,

                P      , Q     , R      ,

                V      , ALP   , BTA    ,

                THA    , PSI   , PHI    ,

                H      , X     , Y      ,

                TDOT   ,

                PDOT   , QDOT  , RDOT   ,

                VDOT   , ALPDOT, BTADOT,

                THADOT, PSIDOT, PHIDOT,

                HDOT   , XDOT  , YDOT
```

The body axis rates p, q, and r appear as P, Q, and R, respectively. Total velocity is represented by the variable V, altitude by H; angle of attack (ALP), angle of sideslip (BTA), and their derivatives with respect to time (ALPDOT and BTADOT, respectively) are also contained within this common block.

The common block /SIMOUT/ contains the main air data parameters required for the function generation subroutine. The variables in this common block are

COMMON /SIMOUT/ AMCH, QBAR, GMA, DEL,

UB , VB , WB ,

VEAS, VCAS

Mach number and dynamic pressure are the first entries in the common block, symbolized by AMCH and QBAR, respectively. The body axis velocities u, v, and w are included as UB, VB, and WB, respectively.

The /CONTRL/ common block contains the surface position information. The exact definition of each of the elements used for a particular aerodynamic model is determined by the implementer of that model. The structure of the common block /CONTRL/ is as follows:

COMMON /CONTRL/ DC (30)

The common block /DATAIN/ contains geometry and mass information.

COMMON /DATAIN/ S , B , CBAR, AMSS,

AIX , AIY , AIZ , AIXZ,

AIXY, AIYZ, AIXE

The first three variables in the common block, S, B, and CBAR, represent wing area, wingspan, and mean aerodynamic chord, respectively. The vehicle mass is represented by AMSS.

The information on the displacement of the aerodynamic reference point of the aerodynamic data with respect to the aircraft center of gravity is contained in the /CGSHFT/ common block:

COMMON /CGSHFT/ DELX, DELY, DELZ

The displacements are defined along the vehicle body axis with DELX, DELY, and DELZ representing the displacements in the x, y, and z axes, respectively.

The common block /SIMACC/ contains the accelerations, accelerometer outputs, and normal accelerometer output at the center of gravity of the aircraft.

COMMON /SIMACC/ AX , AY , AZ ,

ANX, ANY, ANZ,

AN

The output common block /CLCOUT/ contains the variables representing the aerodynamic moment and force coefficients:

COMMON /CLCOUT/ CL, CM, CN, CD, CLFT, CY

48

The variables CL, CM, and CN are the symbols for the coefficients of rolling moment C_l, pitching moment C_m, and yawing moment C_n, respectively; these terms are body axis coefficients. The stability axis forces are represented by CD (coefficient of drag C_D), CLFT (coefficient of lift C_L), and CY (sideforce coefficient C_Y).

Control Model

The program LINEAR attempts to trim the given condition using control inputs similar to those of a pilot: longitudinal stick, lateral stick, rudder, and throttle. The UCNTRL subroutine utilizes these trim output control values to calculate actual surface deflections and power level angles for the given aircraft (fig. 2). The location of each surface and power level angle in the /CONTRL/ common block is specified by the user in the input file (maximum of 30 surfaces). The limits for the control parameters in pitch, roll, yaw, and thrust are user specified (see Trim Parameter Specification in the Input Files section) and must agree in units with the calculations in CCALC.

The common block /CTPARM/ contains the four trim parameters that must be related to surface deflections in the subroutine UCNTRL:

COMMON /CTPARM/ DES, DAS, DRS, THRSTX

The output from UCNTRL is through the common block /CONTRL/ described previously in the Aerodynamic Model section. The variables DES, DAS, DRS, and THRSTX are used to trim the longitudinal, lateral, directional, and thrust axes, respectively.

For an aircraft using feedback (such as a statically unstable vehicle), state variables and their derivatives are available in the common block /DRVOUT/. If parameters other than state variables and their time derivatives are required for feedback, the user may access them using the common block /OBSERV/ described in the Mass and Geometry Model section of this report.

Engine Model

The subroutine IFENGN computes individual engine parameters used by LINEAR to calculate force, torque, and gyroscopic effects due to engine offset from the centerline. The parameters for each engine (maximum of four engines) are passed through common /ENGSTF/ as follows:

COMMON /ENGSTF/ THRUST(4), TLOCAT(4,3), XYANGL(4), XZANGL(4),
                TVANXY(4), TVANXZ (4), DXTHRS(4),
                EIX   (4), AMSENG (4), ENGOMG(4)

The variables in this common block correspond to thrust (THRUST); the x, y, and z body axis coordinates of the point at which thrust acts (TLOCAT); the orientation of the thrust vector in the x-y body axis plane (XYANGL), in degrees; the orientation of the thrust vector in the x-z body axis plane (XZANGL), in degrees; the orientation of the thrust vector in the x-y engine axis plane (TVANXY), in degrees; the orientation of the thrust vector in the x-z engine axis plane (TVANXZ), in degrees; the distance between the center-of-gravity of the engine and the thrust point (DXTHRS), measured positive in the negative x engine axis; the rotational inertia of

the engine (EIX); mass (AMSENG); and the rotational velocity of the engine (ENGOMG). The variables are all dimensioned to accommodate up to four engines.

While the common block structure within LINEAR is designed for engines that do not interact with the vehicle aerodynamics, propeller-driven aircraft can be easily modeled by communicating the appropriate parameters from the engine model (IFENGN) to the aerodynamic model (CCALC).

Mass and Geometry Model

The subroutine MASGEO allows the user to vary the center-of-gravity position and vehicle inertias automatically. Nominally, this subroutine must exist as one of the user subroutines, but it may be nothing more than RETURN and END statements. MASGEO is primarily for variable-geometry aircraft (such as an oblique-wing or variable-sweep configurations) or for modeling aircraft that undergo significant mass or inertia changes over their operating range. The center-of-gravity position and inertias may be functions of flight condition or any surface defined in the /CONTRL/ common block. Changes in center-of-gravity position are passed in the /CGSHFT/ common block, and inertia changes are passed in the /DATAIN/ common block.

Care must be taken when using the subroutine MASGEO in combination with selecting an observation vector that contains measurements away from the center of gravity. If measurements are desired at a fixed location on the vehicle, such as a sensor platform or nose boom, the moment arm to the new center-of-gravity location must be recomputed as a result of the center-of-gravity shift for accurate results. This can be accomplished by implementing the moment arm calculations in one of the user subroutines and passing the new moment arm values through the /OBSERV/ common block:

COMMON /OBSERV/ OBVEC(120), PARAM(120,6)

The common block /OBSERV/ allows the user to access all the observation variables during trim as well as to pass parameters associated with the observations back to LINEAR. The common block /OBSERV/ contains two single-precision vectors: OBVEC(120), and PARAM(120,6). A list of the available observations and parameters is given in table 4. Access to the observation variables allows the user to implement trim strategies that are functions of observations, such as gain schedules and surface management schemes. The parameters associated with the observations are used primarily to define the moment arm from the center of gravity to the point at which the observation is to be made. If the user subroutine MASGEO is used to change the center-of-gravity location and observations are desired at fixed locations on the vehicle, then the moment arm from the new center-of-gravity location to the fixed sensor location ((x, y, z), in feet) must be computed in one of the user subroutines and passed back in the first three elements of the PARAM vector associated with the desired observation (PARAM(n = 1 to 3), where n is the number of the desired observation). Additional information on observations and parameters can be found in the State, Control, and Observation Variable Definitions section.

TABLE 4. — OBSERVATION VARIABLES AVAILABLE IN THE USER-SUPPLIED SUBROUTINES
USING THE OBVEC ARRAY

Location (index) in OBVEC	Variable
State Variables	
1	Roll rate
2	Pitch rate
3	Yaw rate
4	Velocity
5	Angle of attack
6	Angle of sideslip
7	Pitch attitude
8	Heading
9	Roll attitude
10	Altitude
11	Displacement north
12	Displacement east
Derivatives of state variables	
13	Roll acceleration
14	Pitch acceleration
15	Yaw acceleration
16	Velocity rate
17	Angle-of-attack rate
18	Angle-of-sideslip rate
19	Pitch attitude rate
20	Heading rate
21	Roll attitude rate
22	Altitude rate
23	Velocity north
24	Velocity east
Accelerations	
25	x body axis acceleration
26	y body axis acceleration
27	z body axis acceleration
28	x body axis accelerometer at vehicle center of gravity
29	y body axis accelerometer at vehicle center of gravity
30	z body axis accelerometer at vehicle center of gravity
31	Normal acceleration
32	x body axis accelerometer not at vehicle center of gravity
33	y body axis accelerometer not at vehicle center of gravity

TABLE 4. — Continued.

Location (index) in OBVEC	Variable
Accelerations (continued)	
34	z body axis accelerometer not at vehicle center of gravity
99	Normal accelerometer not at vehicle center of gravity
98	Load factor
Air data parameters	
91	Speed of sound
37	Reynolds number
103	Reynolds number per unit length
35	Mach number
36	Dynamic pressure
56	Impact pressure
55	Ambient pressure
57	Impact/ambient pressure ratio
58	Total pressure
59	Temperature
60	Total temperature
92	Equivalent airspeed
93	Calibrated airspeed
Flightpath-related parameters	
39	Flightpath angle
38	Flightpath acceleration
40	Flightpath angle rate
43	Scaled altitude rate
Energy-related terms	
46	Specific power
47	Specific energy
Force parameters	
94	Lift force
95	Drag force
96	Normal force
97	Axial force
Body axis parameters	
52	x body axis velocity
53	y body axis velocity
54	z body axis velocity

TABLE 4. — Concluded.

Location (index) in OBVEC	Variable
Body axis parameters (continued)	
100	Rate of change of velocity in x body axis
101	Rate of change of velocity in y body axis
102	Rate of change of velocity in z body axis
Miscellaneous measurements not at vehicle center of gravity	
44	Angle of attack not at vehicle center of gravity
45	Angle of sideslip not at vehicle center of gravity
41	Altitude instrument not at vehicle center of gravity
42	Altitude rate instrument not at vehicle center of gravity
Other miscellaneous parameters	
48	Vehicle total angular momentum
49	Stability axis roll rate
50	Stability axis pitch rate
51	Stability axis yaw rate
Control surface parameters	
61 to 90	Control surfaces DC(1) to DC(30)
Trim parameters	
104	Longitudinal trim parameter
105	Lateral trim parameter
106	Directional trim parameter
107	Thrust trim parameter

CONCLUDING REMARKS

The FORTRAN program LINEAR was developed to provide a flexible, powerful, and documented tool to derive linear models for aircraft stability analysis and control law design. This report discusses LINEAR from the perspective of a potential user, defining the nonlinear equations from which the linear model is derived and describ-

ing the interfaces to user-supplied subroutines and input files. The output from LINEAR is also described. Examples of the user-supplied subroutines are presented in the appendixes. A microfiche listing of the program for a VAX 11/750 with the VMS operating system is included with this report. (Revisions to the microfiche supplement are given in app. J.)

The program LINEAR numerically determines a linear system model using nonlinear equations of motion and a user-supplied nonlinear aerodynamic model. LINEAR is also capable of extracting linearized engine effects (such as net thrust, torque, and gyroscopic effects) and including these effects in the linear system model. The point as which this linear system model is defined is determined either by completely specifying the state and control variables or by specifying an analysis point on a trajectory, selecting a trim option, and directing the program to determine the control variables and remaining state variables.

The system model determined by LINEAR consists of matrices for both state and observation equations. The program has been designed to provide an easy selection and definition of the state, control, and observation variables to be used in a particular model. Thus, the order of the system model is completely under user control. Further, the program provides the flexibility of allowing alternative formulations of both state and observation equations.

LINEAR has several features that make it unique among the linearization programs common in the aerospace industry. The most significant of these features is flexibility. By generalizing the surface definitions and making no assumptions of symmetric mass distributions, the program can be applied to any aircraft in any phase of flight except hover. The unique trimming capability, implemented through a user-supplied subroutine, allows unlimited possibilities of trimming strategies and surface scheduling, which are particularly important for oblique-wing vehicles and aircraft having multiple surfaces affecting a single axis. The formulation of the equations of motion permit the inclusion of thrust-vectoring effects. The ability to select, without program modification, the state, control, and observation variables for the linear models, combined with the large number of observation quantities available, allows any analysis problem to be solved with ease.

This report documents the use of the program LINEAR, defining the equations used and the methods employed to implement the program. The trimming capabilities of LINEAR are discussed from both theoretical and implementation perspectives. The input and output files are described in detail. The user-supplied subroutines required for LINEAR are discussed, and sample subroutines are presented.

APPENDIX A: CORRECTION TO AERODYNAMIC COEFFICIENTS FOR A CENTER OF GRAVITY
             NOT AT THE AERODYNAMIC REFERENCE POINT

The point on the vehicle at which the nonlinear force and moment coefficients
are defined is referred to as the aerodynamic reference point. All aerodynamic
effects are modeled at this aerodynamic reference point. Thus, when this point
and the vehicle center of gravity are not coincident, the forces acting at the
aerodynamic reference point effectively induce moments that act incrementally on
the moments defined at the aerodynamic reference point by the nonlinear aerody-
namic model.

The total aerodynamic moment **M** acting at the vehicle center of gravity is
defined as

$$M = M_{ar} + \Delta r \times F$$

where

$$M_{ar} = [L_{ar} \quad M_{ar} \quad N_{ar}]^T$$

is the total aerodynamic moment acting at the aerodynamic reference point (denoted
by subscript ar) of the vehicle,

$$\Delta r = [\Delta x \quad \Delta y \quad \Delta z]^T$$

is the displacement of the aerodynamic reference point from the vehicle center of
gravity, and

$$F = [X \quad Y \quad Z]^T$$

is the total aerodynamic force acting at the aerodynamic center, where X, Y, and Z
are total forces along the x, y (sideforce), and z body axes. Thus,

$$M = \begin{bmatrix} L_{ar} + \Delta y\, Z - \Delta z\, Y \\ M_{ar} + \Delta z\, X - \Delta x\, Z \\ N_{ar} + \Delta x\, Y - \Delta y\, X \end{bmatrix}$$

The total aerodynamic moment acting at the vehicle center can be expressed in
terms of the force and moment coefficients derived from the user-supplied nonlinear
aerodynamic modeling subroutine CCALC by defining the body axis forces in terms of
stability axis force coefficients:

$$X = \bar{q}S(-C_D \cos \alpha + C_L \sin \alpha)$$

$$Y = \bar{q}SC_y$$

$$Z = \bar{q}S(-C_D \sin \alpha - C_L \cos \alpha)$$

Substituting these equations into the definition of the total aerodynamic moment
equation and applying the definitions of the total aerodynamic moments,

$$L_{ar} = \bar{q}SbC_{\ell_{ar}}$$

$$M_{ar} = \bar{q}S\bar{c}C_{m_{ar}}$$

$$N_{ar} = \bar{q}SbC_{n_{ar}}$$

expressions for total aerodynamic moment coefficients corrected to the vehicle center of gravity can be derived as follows:

$$C_{\ell} = C_{\ell_{ar}} + \frac{\Delta y}{b}(-C_D \sin \alpha - C_L \cos \alpha) - \frac{\Delta z}{b}C_Y$$

$$C_m = C_{m_{ar}} + \frac{\Delta z}{\bar{c}}(-C_D \cos \alpha + C_L \sin \alpha) - \frac{\Delta x}{\bar{c}}(-C_D \sin \alpha - C_L \cos \alpha)$$

$$C_n = C_{n_{ar}} + \frac{\Delta x}{b}C_Y - \frac{\Delta y}{b}(-C_D \cos \alpha + C_L \sin \alpha)$$

These calculations are normally performed within LINEAR in the subroutine CGCALC. However, if the user selects, the calculation can be performed within the user-supplied aerodynamic model, CCALC.

APPENDIX B: ENGINE TORQUE AND GYROSCOPIC EFFECTS MODEL

Torque and gyroscopic effects represent (after thrust) the main contributions of the engines to the aircraft dynamics. The torque effects arise due to thrust vectors not acting at the vehicle center of gravity. The gyroscopic effects are a consequence of the interaction of the rotating mass of the engine and the vehicle dynamics. These effects can be either major or virtually negligible, depending on the vehicle.

The torque effects can be modeled by considering the thrust of an engine, F_p, where the thrust vector is aligned with the local x axis of the engine acting at some point Δr from the center of gravity of the vehicle, as shown in figure 4.
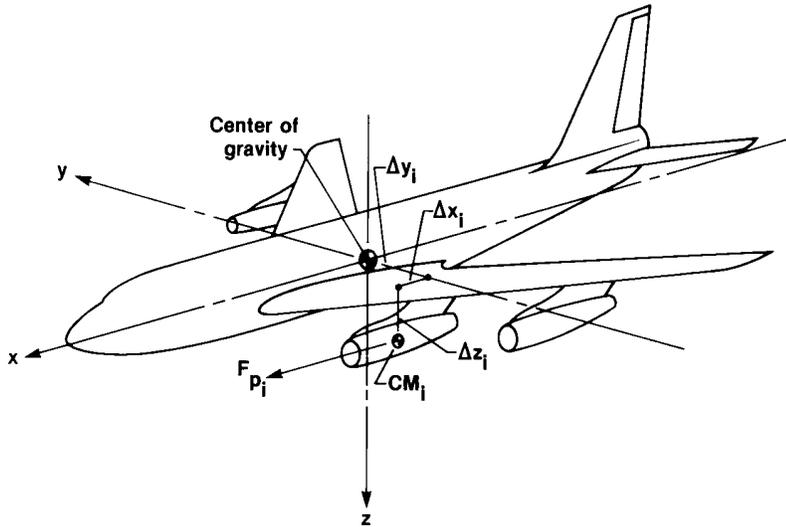
Figure 4. Definition of location of engine center of mass (CM$_i$) relative to vehicle center of gravity.

The thrust vector for the ith engine, F_{p_i}, can be defined as

$$F_{p_i} = \begin{bmatrix} F_{p_{x_i}} & F_{p_{y_i}} & F_{p_{z_i}} \end{bmatrix}^T$$

where $F_{p_{x_i}}$, $F_{p_{y_i}}$, and $F_{p_{z_i}}$ are the components of thrust in the x, y, and z body axes, respectively. From figures 5 and 6 it can be seen that the following relationships hold:

$$F_{p_{x_i}} = \left| F_{p_i} \right| \cos \varepsilon_i \cos \xi_i$$

$$F_{p_{y_i}} = \left| F_{p_i} \right| \cos \varepsilon_i \sin \xi_i$$

$$F_{p_{z_i}} = - \left| F_{p_i} \right| \sin \varepsilon_i$$

57

where $\left| F_{p_i} \right|$ represents the magnitude of the thrust due to the ith engine, ε_i the angle from the thrust axis of the engine to the x-y body axis plane, and ξ_i the angle from the projection of F_{p_i} onto the x-y body axis plane to the x body axis.
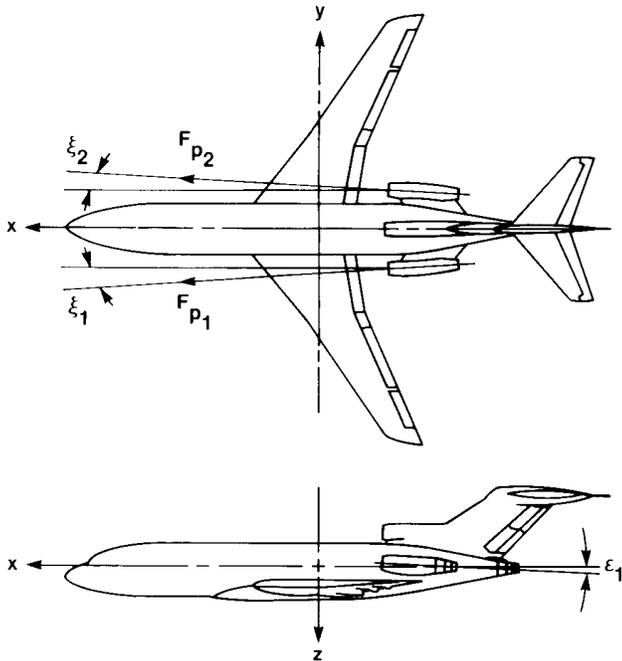
Figure 5. Orientation of the engines in the x-y and x-z body axis planes.
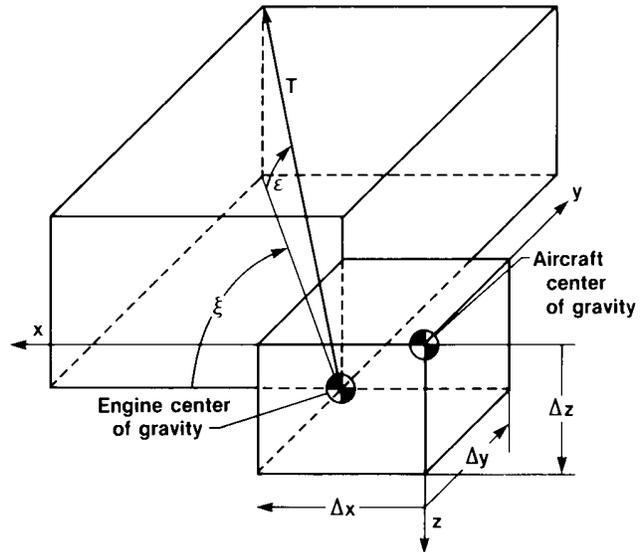
Figure 6. Detailed definition of engine location and orientation parameters.

Denoting the point at which the thrust from the ith engine acts as Δr_i, this offset vector can be defined as

$$\Delta r_i = [\Delta x_i \quad \Delta y_i \quad \Delta z_i]^T$$

where Δx_i, Δy_i, and Δz_i are the x, y, and z body axis coordinates, respectively, of the origin of the ith thrust vector.

The torque due to offset from the center of gravity of the ith engine, $\Delta \tau_{o_i}$, is then given by

$$\Delta \tau_{o_i} = \Delta r_i \times F_{p_i}$$

Thus,

$$\Delta \tau_{o_i} = \begin{bmatrix} \Delta y \, F_{p_{z_i}} - \Delta z \, F_{p_{y_i}} \\ \Delta z \, F_{p_{x_i}} - \Delta x \, F_{p_{z_i}} \\ \Delta x \, F_{p_{y_i}} - \Delta y \, F_{p_{x_i}} \end{bmatrix}$$

The total torque due to engines offset from the center of gravity of the vehicle, τ_o, is given by

58

$$\tau_0 = \sum_{i=1}^{n} \Delta\tau_{o_i} = \sum_{i=1}^{n} (\Delta r_i \times \Delta F_{p_i})$$

where n is the number of engines.

For the case of vectored thrust, the equations for torque produced at the vehicle center of gravity from the ith engine, $\Delta\tau_{o_i}$, are somewhat more complicated. Figure 7 schematically represents an engine with thrust vectoring whose center of gravity is located at Δr_i relative to the vehicle's center of gravity.
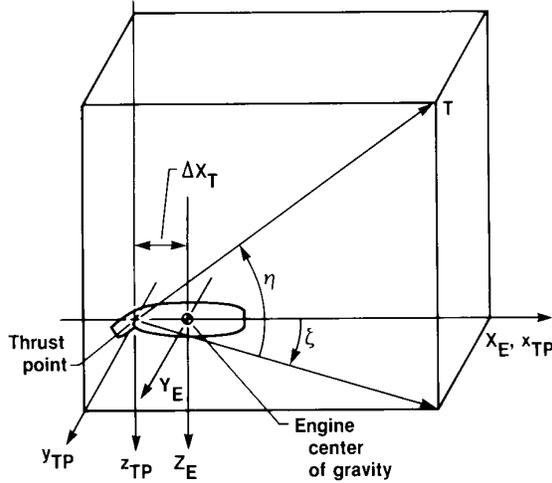
Figure 7. Detailed definition of thrust-vectoring parameters.

The thrust is assummed to act at $-\Delta x_T$ in the local (engine) x axis, with the engine center of gravity being the origin of this local coordinate system. The thrust is also assumed to be vectored at angles η_i and ζ_i relative to the local coordinate axes, with η_i being the angle from the thrust vector to the engine x-y plane and ζ_i the angle from the projection of the thrust vector onto the engine x-y plane to the local x axis. Thus, letting $F'_{p_{x_i}}$, $F'_{p_{y_i}}$, and $F'_{p_{z_i}}$ represent the x, y, and z thrust components in the local engine coordinate system, respectively, these terms can be defined in terms of the total thrust for the ith engine, F_{p_i}, and the angles η_i and ζ_i as

$$F'_{p_{x_i}} = |F_{p_i}| \cos \eta_i \cos \zeta_i$$

$$F'_{p_{y_i}} = |F_{p_i}| \cos \eta_i \sin \zeta_i$$

$$F'_{p_{z_i}} = -|F_{p_i}| \sin \eta_i$$

where

$$F_{p_i} = \begin{bmatrix} F'_{p_{x_i}} & F'_{p_{y_i}} & F'_{p_{z_i}} \end{bmatrix}^T$$

To transform this equation from the ith engine axis system to the body axis system, the transformation matrix

$$\begin{bmatrix} \cos \epsilon_i \cos \xi_i & -\sin \xi_i & \sin \epsilon_i \cos \xi_i \\ \cos \epsilon_i \cos \xi_i & \cos \xi_i & \sin \epsilon_i \cos \xi_i \\ -\sin \epsilon_i & 0 & \cos \epsilon_i \end{bmatrix}$$

is used.

The resultant force in body axis coordinates is

$$F_{p_i} = \begin{bmatrix} F_{p_{x_i}} \\ F_{p_{y_i}} \\ F_{p_{z_i}} \end{bmatrix} = \begin{bmatrix} \cos \epsilon_i \cos \xi_i & -\sin \xi_i & \sin \epsilon_i \cos \xi_i \\ \cos \epsilon_i \sin \xi_i & \cos \xi_i & \sin \epsilon_i \cos \xi_i \\ -\sin \epsilon_i & 0 & \cos \epsilon_i \end{bmatrix} \begin{bmatrix} F'_{p_{x_i}} \\ F'_{p_{y_i}} \\ F'_{p_{z_i}} \end{bmatrix}$$

so that

$$F_{p_{x_i}} = |F_{p_i}| (\cos \eta_i \cos \zeta_i \cos \epsilon_i \cos \xi_i - \cos \eta_i \sin \zeta_i \sin \xi_i$$
$$- \sin \eta_i \sin \epsilon_i \cos \xi_i)$$

$$F_{p_{y_i}} = |F_{p_i}| (\cos \eta_i \cos \zeta_i \cos \epsilon_i \sin \xi_i + \cos \eta_i \sin \zeta_i \cos \xi_i$$
$$- \sin \eta_i \sin \epsilon_i \cos \xi_i)$$

$$F_{p_{z_i}} = |F_{p_i}| (-\cos \eta_i \cos \zeta_i \sin \epsilon_i - \sin \eta_i \cos \epsilon_i)$$

The moment arm through which the vectored thrust acts is

$$\Delta r'_i = \begin{bmatrix} \Delta x_i - \Delta x_{T_i} \cos \epsilon_i \cos \xi_i \\ \Delta y_i - \Delta x_{T_i} \cos \epsilon_i \sin \xi_i \\ \Delta z_i + \Delta x_{T_i} \sin \epsilon_i \end{bmatrix}$$

and the total torque due to thrust vectoring is

$$\tau_0 = \sum_{i=1}^{n} \Delta \tau_{0_i} = \sum_{i=1}^{n} (\Delta r'_i \times \Delta F_{p_i})$$

60

The engine inertia tensor must be defined in an axis system oriented consistent with the vehicle body axis system. This is done in two steps. These steps involve rotating the engine inertia tensor into a coordinate system orthogonal to the aircraft body axis system. First, the ith engine inertia tensor is rotated through an angle ϵ_i about the local y axis so that the new inertia tensor is oriented with its local x-y body axis plane parallel to the x-y body axis plane of the vehicle. The second step requires a rotation through an angle ξ_i about the local z axis so that the local x, y, and z axes are orthogonal to the x, y, and z body axes of the vehicle. As determined by Gainer and Sherwood (1972) and Thelander (1965), this rotation is a similarity transformation that yields a new inertia tensor I_{e_i}, such that

$$I_{e_i} = R_{\xi_i} R_{\epsilon_i} I'_{x_{e_i}} R_{\epsilon_i}^{-1} R_{\xi_i}^{-1}$$

where R_ξ and R_ϵ are axis transformation matrices that perform the previously described rotations through ξ and ϵ, respectively. These matrices are given as

$$R_{\epsilon_i} = \begin{bmatrix} \cos \epsilon_i & 0 & \sin \epsilon_i \\ 0 & 1 & 0 \\ -\sin \epsilon_i & 0 & \cos \epsilon_i \end{bmatrix}$$

$$R_{\xi_i} = \begin{bmatrix} \cos \xi_i & -\sin \xi_i & 0 \\ \sin \xi_i & \cos \xi_i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

so that

$$R_{\xi_i} R_{\epsilon_i} = \begin{bmatrix} \cos \xi_i \cos \epsilon_i & -\sin \xi_i & \cos \xi_i \sin \epsilon_i \\ \sin \xi_i \cos \epsilon_i & \cos \xi_i & \sin \xi_i \sin \epsilon_i \\ -\sin \epsilon_i & 0 & \cos \epsilon_i \end{bmatrix}$$

Because

$$R_{\epsilon_i}^{-1} = R_{\epsilon_i}^T$$

$$R_{\xi_i}^{-1} = R_\xi^T$$

and the matrices are unitary,

$$R_{\epsilon_i}^{-1} R_{\xi_i}^{-1} = R_{\epsilon_i}^T R_{\xi_i}^T = \left(R_{\xi_i} R_{\epsilon_i} \right)^T$$

$$\left(R_{\varepsilon_i}R_{\xi_i}\right)^T = \begin{bmatrix} \cos\xi_i\cos\varepsilon_i & \sin\xi_i\cos\varepsilon_i & -\sin\varepsilon_i \\ -\sin\xi_i & \cos\xi_i & 0 \\ \cos\xi_i\sin\varepsilon_i & \sin\xi_i\sin\varepsilon_i & \cos\varepsilon_i \end{bmatrix}$$

Therefore,

$$I_{e_i} = I_{x_{e_i}}\begin{bmatrix} \cos^2\varepsilon_i\cos^2\xi_i & \cos^2\varepsilon_i\cos\xi_i\sin\xi_i & -\cos\varepsilon_i\sin\varepsilon_i\cos\xi_i \\ \cos^2\varepsilon_i\cos\xi_i\sin\xi_i & \sin^2\xi_i\cos^2\varepsilon_i & -\sin\varepsilon_i\cos\varepsilon_i\sin\xi_i \\ -\cos\varepsilon_i\sin\varepsilon_i\cos\xi_i & -\sin\varepsilon_i\cos\varepsilon_i\sin\xi_i & \sin^2\varepsilon_i \end{bmatrix}$$

The angular momentum of the ith engine, h_{e_i}, can now be expressed as

$$h_{e_i} = I_{e_i}\omega_{e_i} = \begin{bmatrix} h_{e_{i_1}} & h_{e_{i_2}} & h_{e_{i_3}} \end{bmatrix}^T$$

with

$$h_{e_{i_1}} = p_{e_i}\left[I_{x_{e_i}}\cos^2\varepsilon_i\cos^2\xi_i\right]$$
$$+ q_{e_i}\left[I_{x_{e_i}}\cos^2\varepsilon_i\cos\xi_i\sin\xi_i\right]$$
$$+ r_{e_i}\left[-I_{x_{e_i}}\cos\varepsilon_i\sin\varepsilon_i\cos^2\xi_i\right]$$

$$h_{e_{i_2}} = p_{e_i}\left[I_{x_{e_i}}\cos\varepsilon_i\cos\xi_i\sin\xi_i\right]$$
$$+ q_{e_i}\left[I_{x_{e_i}}\sin^2\xi_i\cos^2\varepsilon_i\right]$$
$$+ r_{e_i}\left[-I_{x_{e_i}}\sin\varepsilon_i\cos\xi_i\sin\xi_i\right]$$

$$h_{e_{i_3}} = p_{e_i}\left[-I_{x_{e_i}}\cos\varepsilon_i\sin\varepsilon_i\cos^2\xi_i\right]$$
$$+ q_{e_i}\left[-I_{x_{e_i}}\sin\varepsilon_i\cos\xi_i\sin\xi_i\right]$$
$$+ r_{e_i}\left[I_{x_{e_i}}\sin^2\varepsilon_i\right]$$

Thus, the gyroscopic moment induced by the ith engine, τ_{g_i}, can be expanded to

$$\tau_{g_i} = \begin{bmatrix} qh_{e_{i_3}} - rh_{e_{i_2}} \\ rh_{e_{i_1}} - ph_{e_{i_3}} \\ ph_{e_{i_1}} - qh_{e_{i_2}} \end{bmatrix}$$

and the total moment induced by gyroscopic interaction of the vehicle dynamics and the rotating engine components is

$$\tau_g = \sum_{i=1}^{n} \tau_{g_i}$$

Engine torque and gyroscopic effects are modeled within the subroutine ENGINE using information provided by the user from the engine modeling subroutine IFENGN. These effects are calculated as incremental moments and are included directly in the equations of motion for both analysis point definition and derivation of the linearized system matrices.

APPENDIX C: STATE VARIABLE NAMES RECOGNIZED BY LINEAR

This appendix lists the alphanumeric descriptors specifying state variables that
are recognized by LINEAR. In the input file, the field containing these descriptors
uses a 54A format, and all characters are left-justified. The input alphanumeric
descriptor specified by the user serves both to identify the state variable selected
by the user within the program itself and to identify state variables on the printed
output of LINEAR, as described in the Output Files section.

State variable	Units	Symbol	Alphanumeric descriptor
Roll rate	rad/sec	p	P
			ROLL RATE
Pitch rate	rad/sec	q	Q
			PITCH RATE
Yaw rate	rad/sec	r	R
			YAW RATE
Velocity	ft/sec	V	V
			VELOCITY
			VEL
			VTOT
Angle of attack	rad	α	ALP
			ALPHA
			ANGLE OF ATTACK
Sideslip angle	rad	β	BTA
			BETA
			SIDESLIP ANGLE
			ANGLE OF SIDESLIP
Pitch angle	rad	θ	THA
			THETA
			PITCH ATTITUDE
Heading	rad	ψ	PSI
			HEADING
			HEADING ANGLE
Roll angle	rad	ϕ	PHI
			ROLL ATTITUDE
			BANK ANGLE
Altitude	length	h	H
			ALTITUDE
Displacement north	length	x	X
Displacement east	length	y	Y

This appendix lists all observation variable names recognized by LINEAR except for state and control variable names. If state variables are specified as elements in the observation vector, the alphanumeric descriptor must correspond to the names defined in appendix C. When control variables are to be included in the observation vector these variables must be identified exactly as they were specified by the user.

The input file is formatted 5A4 with the alphanumeric data left-justified. The floating-point fields are used to define sensor locations not at the center of gravity. The input name specified by the user for an observation variable serves both to identify the observation variable selected within the program itself and to identify observation variables on the printed output of LINEAR.

An asterisk preceding the variable name indicates measurements at some point other than the vehicle center of gravity. The program LINEAR uses the quantities defined in the first three floating-point fields as definitions of the location of the sensor with respect to the vehicle center of gravity. The three parameters define the x body, y body, and z body location, in that order, of the sensor. These offsets from the vehicle center of gravity are defined in units of length.

Observation variable	Units	Symbol	Alphanumeric descriptor
		Derivatives of state variables	
Roll acceleration	rad/sec^2	$\dot{p}$	PDOT ROLL ACCELERATION
Pitch acceleration	rad/sec^2	$\dot{q}$	QDOT PITCH ACCELERATION
Yaw acceleration	rad/sec^2	$\dot{r}$	RDOT YAW ACCELERATION
Velocity rate	$length/sec^2$	$\dot{V}$	VDOT VELOCITY RATE
Angle-of-attack rate	rad/sec	$\dot{\alpha}$	ALPDOT ALPHA DOT ALPHADOT
Angle-of-sideslip rate	rad/sec	$\dot{\beta}$	BTADOT BETA DOT BETADOT
Pitch attitude rate	rad/sec	$\dot{\theta}$	THADOT THETA DOT
Heading rate	rad/sec	$\dot{\psi}$	PSIDOT PSI DOT

Observation variable	Units	Symbol	Alphanumeric descriptor
Derivatives of state variables (continued)			
Roll attitude rate	rad/sec	$\dot{\phi}$	PHIDOT PHI DOT
Altitude rate	length/sec	$\dot{h}$	HDOT ALTITUDE RATE
Velocity north	length/sec	$\dot{x}$	XDOT
Velocity east	length/sec	$\dot{y}$	YDOT
Accelerations			
x body axis acceleration	g	a_x	AX LONGITUDINAL ACCEL X-AXIS ACCELERATION X AXIS ACCELERATION X-BODY AXIS ACCEL X BODY AXIS ACCEL
y body axis acceleration	g	a_y	AY Y-AXIS ACCELERATION Y AXIS ACCELERATION Y-BODY AXIS ACCEL Y BODY AXIS ACCEL LATERAL ACCELERATION LAT ACCEL LATERAL ACCEL
z body axis acceleration	g	a_z	AZ Z-BODY AXIS ACCEL Z BODY AXIS ACCEL
x body axis accelerometer at vehicle center of gravity	g	a_{nx}	ANX X-AXIS ACCELEROMETER X AXIS ACCELEROMETER
y body axis accelerometer at vehicle center of gravity	g	a_{ny}	ANY Y-AXIS ACCELEROMETER Y AXIS ACCELEROMETER

Observation variable	Units	Symbol	Alphanumeric descriptor
Accelerations (continued)			
z body axis accelerometer at vehicle center of gravity	g	a_{nz}	ANZ Z-AXIS ACCELEROMETER Z AXIS ACCELEROMETER
Normal acceleration	g	a_n	AN NORMAL ACCELERATION NORMAL ACCEL GS G'S
*x body axis accelerometer not at vehicle center of gravity	g	$a_{nx,i}$	AX,I ANX,I
*y body axis accelerometer not at vehicle center of gravity	g	$a_{ny,i}$	AY,I ANY,I
*z body axis accelerometer not at vehicle center of gravity	g	$a_{nz,i}$	AZ,I ANZ,I
*Normal accelerometer not at vehicle center of gravity	g	$a_{n,i}$	AN,I
Load factor	(Dimensionless)	n	N LOAD FACTOR
Air data parameters			
Speed of sound	length/sec	a	A SPEED OF SOUND
[a]Reynolds number	(Dimensionless	Re	RE REYNOLDS NUMBER
Reynolds number per unit length	length^{-1}	Re'	RE PRIME R/LENGTH R/FEET R/UNIT LENGTH

[a]Reynolds number is defined in terms of an arbitrary unit of length that is input by the user. This length is input using the first floating-point field; however, if no value is input, $\bar{c}$ is used as the default value.

Observation variable	Units	Symbol	Alphanumeric descriptor
Flightpath-related parameters (continued)			
Flightpath angle rate	rad/sec	$\dot{\gamma}$	GAMMA DOT GAMMADOT
Vertical acceleration	length/sec^2	$\ddot{h}$	VERTICAL ACCELERATION HDOTDOT H-DOT-DOT HDOT-DOT
Scaled altitude rate	length/sec	$\dot{h}/57.3$	H-DOT/57.3 HDOT / 57.3
Energy-related terms			
Specific energy	length	E_S	ES E-SUB-S SPECIFIC ENERGY
Specific power	length/sec	P_S	PS P-SUB-S SPECIFIC POWER SPECIFIC THRUST
Force parameters			
Lift force	force	L	LIFT
Drag force	force	D	DRAG
Normal force	force	N	NORMAL FORCE
Axial force	force	A	AXIAL FORCE
Body axis parameters			
x body axis velocity	length/sec	u	UB X-BODY AXIS VELOCITY X BODY AXIS VELOCITY X-BODY AXIS VEL X BODY AXIS VEL U-BODY U BODY

Observation variable	Units	Symbol	Alphanumeric descriptor

Body axis parameters (continued)

Observation variable	Units	Symbol	Alphanumeric descriptor
y body axis velocity	length/sec	v	VB Y-BODY AXIS VELOCITY Y BODY AXIS VELOCITY Y-BODY AXIS VEL Y BODY AXIS VEL V-BODY V BODY
z body axis velocity	length/sec	w	WB Z-BODY AXIS VELOCITY Z BODY AXIS VELOCITY Z-BODY AXIS VEL Z BODY AXIS VEL W-BODY W-BODY
Rate of change of velocity in x body axis	length/sec^2	$\dot{u}$	UBDOT UB DOT
Rate of change of velocity in y body axis	length/sec^2	$\dot{v}$	VBDOT VB DOT
Rate of change of velocity in z body axis	length/sec^2	$\dot{w}$	WBDOT WB DOT

Miscellaneous measurements not at vehicle center of gravity

Observation variable	Units	Symbol	Alphanumeric descriptor
*Angle of attack not at vehicle center of gravity	rad	$\alpha_{,i}$	ALPHA,I ALPHA INSTRUMENT AOA INSTRUMENT
*Angle of sideslip not at vehicle center of gravity	rad	$\beta_{,i}$	BETA,I BETA INSTRUMENT SIDESLIP INSTRUMENT
*Altitude instrument not at vehicle center of gravity	length	$h_{,i}$	H,I ALTITUDE INSTRUMENT
*Altitude rate instrument not at vehicle center of gravity	length/sec	$\dot{h}_{,i}$	HDOT,I

Observation variable	Units	Symbol	Alphanumeric descriptor
	Other miscellaneous parameters		
Vehicle total angular momentum	mass-length^2/sec^2	T	ANGULAR MOMENTUM ANG MOMENTUM
Stability axis roll rate	rad/sec	p_s	STAB AXIS ROLL RATE
Stability axis pitch rate	rad/sec	q_s	STAB AXIS PITCH RATE
Stability axis yaw rate	rad/sec	r_s	STAB AXIS YAW RATE

APPENDIX E: ANALYSIS POINT DEFINITION IDENTIFIERS

Analysis point definition options are selected using alphanumeric descriptors. The first record read for each analysis case contains these descriptors. All these descriptors are read using a 5A4 format. The following table associates the analysis point definition options with their alphanumeric descriptors.

Analysis point definition option	Alphanumeric descriptor
Untrimmed	UNTRIMMED NO TRIM NONE NOTRIM
Straight-and-level	STRAIGHT AND LEVEL WINGS LEVEL LEVEL FLIGHT
Pushover-pullup	PUSHOVER/PULLUP PULLUP PUSHOVER PUSHOVER AND PULLUP PUSHOVER / PULLUP PUSHOVER / PULL-UP PUSHOVER PULLUP PUSH OVER PULL UP
Level turn	LEVEL TURN WINDUP TURN
Thrust-stabilized turn	THRUST STABILIZED TURN THRUST LIMITED TURN FIXED THROTTLE TURN FIXED THRUST TURN
Beta	BETA SIDESLIP
Specific power	SPECIFIC POWER PS P-SUB-S

Each of these analysis point definitions except the untrimmed, beta, and specific power options has two suboptions associated with it. The suboptions are requested using alphanumeric descriptors read using an A4 format. These suboptions

73

are defined in the Analysis Point Definition section. The following table defines
these suboptions and the alphanumeric descriptors associated with each.

Analysis point definition suboption	Alphanumeric descriptor
Straight-and-level	
Alpha-trim	ALP ALPH ALPHA
Mach-trim	MACH AMCH
Pushover-pullup	
Alpha-trim	ALP ALPH ALPHA
Load-factor-trim	LOAD GS G'S AN
Level turn	
Alpha-trim	ALP ALPH ALPHA
Load-factor-trim	LOAD GS G'S AN
Thrust-stabilized turn	
Alpha-trim	ALP ALPH ALPHA
Load-factor-trim	LOAD GS G'S AN

The following listing is an example of an input file to LINEAR. This file was used with the example subroutines listed in appendix I to generate the analysis and printer output files listed in appendixes G and H, respectively.

```
LINEARIZER TEST AND DEMONSTRATION CASES

USER'S GUIDE
 6.080000E+02 4.280000E+01 1.595000E+-1 4.500000E+04
 2.870000E+04 1.651000E+-5 1.879000E+05-5.200000E+02 0.        0.
 0.         0.         0.         CCALC WILL CALCULATE CG CORRECTIONS
-1.000000E+01 4.000000E+01
          4STAN          RADI
ALPHA
Q
THETA
VEL
          3
ELEVATOR               5
THROTTLE              12
SPEED BRAKE           10
          2STAN
AN
AY
-2.900E+00 5.430E+00-4.000E+00 4.000E+00-3.250E+00 3.250E+00-1.000E+00 1.000E+00
  0 ADDITIONAL SURFACES
WINDUP TURN
ALPHA
H                     20000.0
MACH                  0.90
AN                    3.00
BETA                  0.0
NEXT
LEVEL FLIGHT
ALPHA
H                     20000.0
MACH                  0.9
GAMMA                 10.0
END
```

This input file is for a case called (record 1) LINEARIZER TEST AND DEMONSTRA-TION CASES, and all input data are on logical device unit 1, signified by the second record being blank. The project title is USER'S GUIDE (record 3). Record 4 specifies the mass and geometric properties of the vehicle as

$$S = 608 \text{ ft}^2$$

$$b = 42.8 \text{ ft}$$

$$\bar{c} = 15.95 \text{ ft}$$

$$w = 45,000 \text{ lb}$$

Record 5 defines the moments and products of intertia of the vehicle (all in units of slug-ft^2) as

$$I_x = 28,700$$

$$I_y = 165,100$$

$$I_z = 187,900$$

$$I_{xz} = 520$$

$$I_{xy} = 0$$

$$I_{yz} = 0$$

Record 6 defines the location of the vehicle center of gravity to be coincident with the aerodynamic reference point of the nonlinear aerodynamic model by setting

$$\Delta x = 0$$

$$\Delta y = 0$$

$$\Delta z = 0$$

Record 6 also specifies that LINEAR should not use its internal model to make corrections for the offset in the vehicle center of gravity from the aerodynamic reference point, either because the aerodynamic model includes such corrections or because none are to be made.

Record 7 defines the angle-of-attack range of the aerodynamic model.

Record 8 specifies that there will be four state variables in the output, that the output formulation of the state equation will be in the standard form $(\dot{x} = Ax + Bu)$, and that the output for the nondimensional stability and control derivatives with respect to angle of attack and angle of sideslip should be scaled as reciprocal radians.

The next four records (9 to 12) define the output formulation of the state vector to be

$$x = \begin{bmatrix} \alpha \\ q \\ \theta \\ V \end{bmatrix}$$

Record 13 specifies that the output model will have three parameters in the control vector. The following three records (14 to 16) specify that

$$u = \begin{bmatrix} \text{elevator} \\ \text{throttle} \\ \text{speed brake} \end{bmatrix}$$

and that elevator, throttle, and speed brake are located in DC(5), DC(12), and DC(10) of the /CONTROL/ common block.

76

Record 17 specifies that two observation variables will be output and that the observation equation will be in the standard form,

$$y = Hx + Fu$$

The next two records (18 and 19) define the elements of the output vector to be

$$y = \begin{bmatrix} a_n \\ a_y \end{bmatrix}$$

Record 20 specifies the ranges for the trim parameters, DES, DAS, DRS, and THRSTX, used to trim the longitudinal, lateral, and directional axes and thrust, respectively. The ranges for these parameters are defined by record 20 to be

$$-2.9 \leqslant DES \leqslant 5.43$$
$$-4.0 \leqslant DAS \leqslant 4.0$$
$$-3.25 \leqslant DRS \leqslant 3.25$$
$$-1.0 \leqslant THRSTX \leqslant 1.0$$

The first three parameters essentially represent stick and rudder limits and are so specified because of the implementation of the subroutine UCNTRL (discussed in app. I). The thrust trim parameter is used, again because of the implementation of UCNTRL, to schedule speed brake when THRSTX < 0 and to command thrust when THRSTX > 0.

Record 21 specifies that no additional control surfaces are to be set.

The next seven records (22 to 28) define an analysis point option. These records request a level turn trim option at

$$h = 20,000 \text{ ft}$$

$$M = 0.9$$

$$a_n = 3.0 \text{ g}$$

$$\beta = 0$$

The second record of this set (record 23) indicates which level turn suboption is requested. The alphanumeric descriptor ALPHA indicates that angle of attack is to be varied until the specified 3.0-g turn is achieved. The final record of this analysis point option definition set contains the key word NEXT to indicate both an end to the current analysis point option definition and that another analysis point option definition follows.

The final six records (records 29 to 34) define a straight-and-level analysis point option at

$$h = 20,000 \text{ ft}$$

$$M = 0.9$$

$$\gamma = 10.0°$$

The second record of this set (record 30) identifies the "Alpha-trim" suboption in which angle of attack is varied until trim is achieved at the specified condition. The final record of this set contains the key word END to indicate the termination of the current analysis point definition as well as the termination of input cases.

The following listing is an example analysis file output on unit 15.   This file was produced using the example input file listed in appendix F and the example user-supplied subroutines listed in appendix I.

```
LINEARIZER TEST AND DEMONSTRATION CASES
USER'S GUIDE

TEST CASE #   ***************************************************   1
X - DIMENSION = 4      U - DIMENSION = 3      Y - DIMENSION = 2
STATE EQUATION FORMULATION:        STANDARD
OBSERVATION EQUATION FORMULATION:  STANDARD


STATE VARIABLES

ALPHA            = 0.465695D-01  RADIANS
Q                = 0.921683D-01  RADIANS/SECOND
THETA            = 0.159885D-01  RADIANS
VEL              = 0.933232D+03  FEET/SECOND


CONTROL VARIABLES

ELEVATOR         = 0.538044D-01
THROTTLE         = 0.214105D+00
SPEED BRAKE      = 0.000000D+00


DYNAMIC INTERACTION VARIABLES

X-BODY AXIS FORCE = 0.000000D+00  POUNDS
Y-BODY AXIS FORCE = 0.000000D+00  POUNDS
Z-BODY AXIS FORCE = 0.000000D+00  POUNDS
PITCHING MOMENT   = 0.000000D+00  FOOT-POUNDS
ROLLING MOMENT    = 0.000000D+00  FOOT-POUNDS
YAWING MOMENT     = 0.000000D+00  FOOT-POUNDS


OBSERVATION VARIABLES

AN               = 0.300163D+01   GS
AY               = 0.941435D+00   GS


A-MATRIX FOR:  DX / DT  = A * X  + B * U + D * V

-0.121436D+01 0.100000D+01 0.136756D-02-0.121605D-03
-0.147423D+01-0.221451D+01-0.450462D-02 0.294019D-03
 0.000000D+00 0.331812D+00 0.000000D+00 0.000000D+00
-0.790853D+02 0.000000D+00-0.320822D+02-0.157297D-01


B-MATRIX FOR:  DX / DT  = A * X  + B * U + D * V

-0.141961D+00-0.164948D-02-0.928933D-02
-0.220778D+02 0.543324D-02-0.135074D+02
 0.000000D+00 0.000000D+00 0.000000D+00
-0.105186D+02 0.342817D+02-0.155832D+02


D-MATRIX FOR:  DX / DT = A * X + B * U + D * V

-0.343642D-07 0.000000D+00 0.737378D-06 0.000000D+00 0.000000D+00
 0.113192D-06 0.000000D+00-0.242885D-05 0.605694D-05 0.000000D+00
 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00 0.000000D+00
 0.714203D-03 0.398492D-06 0.332842D-04 0.000000D+00 0.000000D+00
```

```
    0.000000D+00
    0.000000D+00
    0.000000D+00
    0.000000D+00


H-MATRIX FOR:   Y  =  H  *  X  +  F  *  U  +  E  *  V

0.351752D+02 -0.191245D-05 0.150046D-02 0.640771D-02
0.000000D+00 0.000000D+00 -0.150534D-01 0.274248D-09


F-MATRIX FOR:   Y  =  H  *  X  +  F  *  U  +  E  *  V

0.412845D+01 -0.180978D-02 0.291699D+00
0.000000D+00 0.000000D+00 0.000000D+00


E-MATRIX FOR:   Y  =  H  *  X  +  F  *  U  +  E  *  V

-0.377037D-07 0.000000D+00 -0.214132D-04 0.000000D+00 0.000000D+00
 0.000000D+00 0.222222D-04 0.000000D+00 0.000000D+00 0.000000D+00
 0.000000D+00
 0.000000D+00
```

TEST CASE   ** 2
X - DIMENSION = 4   U - DIMENSION = 3   Y - DIMENSION = 2
STATE EQUATION FORMULATION: STANDARD
OBSERVATION EQUATION FORMULATION: STANDARD

STATE VARIABLES

ALPHA	= -.126650D-01	RADIANS
Q	= 0.000000D+00	RADIANS/SECOND
THETA	= 0.161868D+00	RADIANS
VEL	= 0.933232D+03	FEET/SECOND

CONTROL VARIABLES

ELEVATOR	= 0.637734D-01
THROTTLE	= 0.225092D+00
SPEED BRAKE	= 0.000000D+00

DYNAMIC INTERACTION VARIABLES

X-BODY AXIS FORCE	= 0.000000D+00	POUNDS
Y-BODY AXIS FORCE	= 0.000000D+00	POUNDS
Z-BODY AXIS FORCE	= 0.000000D+00	POUNDS
PITCHING MOMENT	= 0.000000D+00	FOOT-POUNDS
ROLLING MOMENT	= 0.000000D+00	FOOT-POUNDS
YAWING MOMENT	= 0.000000D+00	FOOT-POUNDS

OBSERVATION VARIABLES

AN	= 0.985228D+00	GS
AY	= 0.000000D+00	GS

A-MATRIX FOR: DX / DT = A * X + B * U + D * V

```
-0.120900D+01  0.100000D+01 -0.575730D-02 -0.701975D-04
-0.149189D+01 -0.221451D+01  0.189640D-01  0.231368D-03
 0.000000D+00  0.100000D+01  0.000000D+00  0.000000D+00
-0.576868D+02  0.000000D+00 -0.316251D+02 -0.460435D-02
```

B -MATRIX FOR: DX / DT = A * X + B * U + D * V

```
-0.141961D+00  0.448742D-03-0.928932D-02
-0.220778D+02-0.147812D-02-0.135074D+02
 0.000000D+00  0.000000D+00  0.000000D+00
-0.105186D+02  0.343162D+02-0.155832D+02
```


D -MATRIX FOR: DX / DT + A * X * B * U + D * V

```
 0.934880D-08  0.000000D+00  0.738119D-06  0.000000D+00  0.000000D+00
-0.307941D-07  0.000000D+00-0.243129D-05  0.605694D-05  0.000000D+00
 0.000000D+00  0.000000D+00  0.000000D+00  0.000000D+00  0.000000D+00
 0.714920D-03  0.495829D-13-0.905497D-05  0.000000D+00  0.000000D+00
 0.000000D+00
 0.000000D+00
 0.000000D+00
 0.000000D+00
```


H -MATRIX FOR: Y = H * X + F * U

```
 0.350424D+02-0.128333D-06-0.632314D-02  0.203434D-02
 0.000000D+00  0.000000D+00  0.000000D+00  0.000000D+00
```

F -MATRIX FOR: Y = H * X + F * U

```
 0.411323D+01  0.492845D-03  0.263288D+00
 0.000000D+00  0.000000D+00  0.000000D+00
```


E -MATRIX FOR: Y + H * X + F * U + E * V

```
 0.102676D-07  0.000000D+00-0.214116D-04  0.000000D+00  0.000000D+00
 0.000000D+00  0.222222D-04  0.000000D+00  0.000000D+00  0.000000D+00
 0.000000D+00
 0.000000D+00
```

APPENDIX H: EXAMPLE PRINTER OUTPUT FILES

The following listings are the printer output files generated by LINEAR using the example input file in appendix F and the example user supplied subroutines listed in appendix I.

Example printer output file 1 (unit 3)

```
GEOMETRY AND MASS DATA FOR:
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

WING AREA                608.000  (FT**2)
WING SPAN                 42.800  (FT)
MEAN CHORD                15.950  (FT)
VEHICLE WEIGHT         45000.000  (LB)
IX                     28700.000  (SLUG-FT**2)
IY                    165100.000  (SLUG-FT**2)
IZ                    187900.000  (SLUG-FT**2)
IXZ                     -520.000  (SLUG-FT**2)
IXY                        0.000  (SLUG-FT**2)
IYZ                        0.000  (SLUG-FT**2)

VECTOR DEFINING REFERENCE POINT OF AERODYNAMIC MODEL
WITH RESPECT TO VEHICLE CENTER OF GRAVITY:

DELTA X                    0.000  (FT)
DELTA Y                    0.000  (FT)
DELTA Z                    0.000  (FT)

FORCE AND MOMENT COEFFICIENT CORRECTIONS DUE
TO THE OFFSET OF THE REFERENCE POINT TO THE
AERODYNAMIC MODEL FROM THE VEHICLE CG ARE
CALCULATED IN CCALC.

MINIMUM ANGLE OF ATTACK       -10.000 (DEG)
MAXIMUM ANGLE OF ATTACK        40.000 (DEG)


PARAMETERS USED IN THE STATE VECTOR FOR:
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

ALPHA
Q
THETA
VEL


THE STANDARD FORMULATION OF THE STATE EQUATION
HAS BEEN SELECTED.  THE FORM OF THE EQUATION IS:

        DX/DT = A * X + B * U + D * V

SURFACES TO BE USED FOR THE CONTROL VECTOR FOR:
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE
                              LOCATION IN /CONTROL/

ELEVATOR                           5
THROTTLE                          12
SPEED BRAKE                       10
```

```
PARAMETERS USED IN THE OBSERVATION VECTOR FOR:
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

AN
AY


THE STANDARD FORMULATION OF THE OBSERVATION EQUATION
HAS BEEN SELECTED.  THE FORM OF THE EQUATION IS:

          Y = H * X + F * U + E * V

LIMITS FOR TRIM OUTPUT PARAMETERS:
                                    MINIMUM      MAXIMUM
       PITCH AXIS PARAMETER          -2.900        5.430
       ROLL AXIS PARAMETER           -4.000        4.000
       YAW AXIS PARAMETER            -3.250        3.250
       THRUST PARAMETER              -1.000        1.000
    NO ADDITIONAL SURFACES TO BE SET WERE DEFINED



TRIM CONDITIONS FOR CASE #  1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

LEVEL TURN WHILE VARYING ALPHA

TRIM ACHIEVED

COEFFICIENT OF LIFT                 =        0.40144
COEFFICIENT OF DRAG                 =        0.03058
LIFT                   (LBS)        =    134741.68807
DRAG                   (LBS)        =     10265.70661
ALTITUDE               (FT)         =       20000.
MACH                                =        0.90000
VELOCITY               (FT/SEC)     =      933.23196
EQUIVALENT AIRSPEED    (KTS)        =      403.42303
SPEED OF SOUND         (FT/SEC)     =     1036.92440
GRAVITATIONAL ACCEL    (FT/SEC**2)  =       32.11294
NORMAL ACCELERATION    (G-S)        =        3.00163
LOAD FACTOR                         =        2.99995
DYNAMIC PRESSURE       (LBS/FT**2)  =      552.05302
DENSITY                (SLUG/FT**3) =        0.00126774
WEIGHT (@ALTITUDE)     (LBS)        =    44914.60434
BETA                   (DEG)        =        0.03193
ALPHA                  (DEG)        =        2.66824
PHI                    (DEG)        =       70.62122
THETA                  (DEG)        =        0.91607
ALTITUDE RATE          (FT/SEC)     =        0.00000
GAMMA                  (DEG)        =        0.00000
ROLL RATE              (DEG/SEC)    =       -0.08951
PITCH RATE             (DEG/SEC)    =        5.28086
YAW RATE               (DEG/SEC)    =        1.85749
THRUST                 (LBS)        =    10277.03515
SUM OF THE SQUARES                  =        0.00000

TRIM PARAMETERS

TRIM PITCH AXIS PARAMETER           =       -0.66958
TRIM ROLL AXIS PARAMETER            =       -0.01526
TRIM YAW AXIS PARAMETER             =       -0.02125
TRIM THRUST PARAMETER               =        0.21410
```

CONTROL VARIABLES

ELEVATOR	=	0.05380
THROTTLE	=	0.21410
SPEED BRAKE	=	0.00000

OBSERVATION VARIABLES

AN	=	3.00163339	GS
AY	=	0.94136286	GS

NON-DIMENSIONAL STABILITY AND CONTROL DERIVATIVES FOR CASE #   1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ROLLING MOMENT	PITCHING MOMENT	YAWING MOMENT	DRAG	LIFT	SIDE FORCE
ZERO COEFFICIENTS	-4.02966D-05	4.22040D-02	2.25747D-04	1.42882D-04	1.57360D-01	5.42882D-04
ROLL RATE (RAD/SEC)	-2.00000D-01	0.00000D+00	-3.37210D-02	0.00000D+00	0.00000D+00	0.00000D+00
PITCH RATE (RAD/SEC)	0.00000D+00	3.89530D+00	0.00000D+00	0.00000D+00	-1.72320D+01	0.00000D+00
YAW RATE (RAD/SEC)	1.50990D-01	0.00000D+00	-4.04710D-01	0.00000D+00	0.00000D+00	0.00000D+00
VELOCITY (FT/SEC)	-1.27955D-07	-3.28739D-06	3.21096D-07	0.00000D+00	1.45433D-05	0.00000D+00
MACH NUMBER	-1.32680D-04	-3.40878D-03	3.32952D-04	0.00000D+00	1.50803D-02	0.00000D+00
ALPHA (RAD)	0.00000D+00	-1.68820D-01	0.00000D+00	3.72570D-01	4.87060D+00	0.00000D+00
BETA (RAD)	-1.33450D-01	0.00000D+00	1.29960D-01	0.00000D+00	0.00000D+00	-9.74030D-01
ALTITUDE (FT)	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00
ALPHA DOT (RAD/SEC)	0.00000D+00	-1.18870D+01	0.00000D+00	0.00000D+00	1.72320D+01	0.00000D+00
BETA DOT (RAD/SEC)	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00
ELEVATOR	0.00000D+00	-6.95280D-01	0.00000D+00	4.38310D-02	5.72960D-01	0.00000D+00
THROTTLE	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00
SPEED BRAKE	0.00000D+00	-4.17500D-01	0.00000D+00	6.49350D-02	3.74920D-02	0.00000D+00

VEHICLE STATIC MARGIN IS   3.5% MEAN AERODYNAMIC CHORD STABLE
AT THIS FLIGHT CONDITION.

MATRIX A USING THE FORMULATION OF THE STATE EQUATION:

$$DX / DT = A * X + B * U + D * V$$

FOR CASE #   1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

DERIVATIVES WRT TIME OF:	ALPHA	Q	THETA	VEL
ALPHA	-0.121436D+01	0.100000D+01	0.136756D-02	-0.121605D-03
Q	-0.147423D+01	-0.221451D+01	-0.450462D-02	0.294019D-03
THETA	0.000000D+00	0.331812D+00	0.000000D+00	0.000000D+00
VEL	-0.790853D+02	0.000000D+00	-0.320822D+02	-0.157297D-01

MATRIX B USING THE FORMULATION OF THE STATE EQUATION:

$$DX / DT = A * X + B * U + D * V$$

FOR CASE #   1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

84

	ELEVATOR	THROTTLE	SPEED BRAKE
DERIVATIVES WRT TIME OF:			
ALPHA	-0.141961D+00	-0.164948D-02	-0.928933D-02
Q	-0.220778D+02	0.543324D-02	-0.135074D+02
THETA	0.000000D+00	0.000000D+00	0.000000D+00
VEL	-0.105186D+02	0.342817D+02	-0.155832D+02

MATRIX D USING THE FORMULATION OF THE STATE EQUATION

$$DX \; / \; DT \; = A * X + B * U + D * V$$

FOR CASE # 1
LINEARIZER DEMONSTRATION AND TEST CASES

FOR THE PROJECT:
USER'S GUIDE

	X-BODY AXIS FORCE	Y-BODY AXIS FORCE	Z-BODY AXIS FORCE	PITCHING MOMENT	ROLLING MOMENT
DERIVATIVES WRT TIME OF:					
ALPHA	-0.343642D-07	0.000000D+00	0.737378D-06	0.000000D+00	0.000000D+00
Q	0.113192D-06	0.000000D+00	-0.242885D-05	0.605694D-05	0.000000D+00
THETA	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00
VEL	0.714203D-03	0.398492D-06	0.332842D-04	0.000000D+00	0.000000D+00

	YAWING MOMENT
DERIVATIVES WRT TIME OF:	
ALPHA	0.000000D+00
Q	0.000000D+00
THETA	0.000000D+00
VEL	0.000000D+00

MATRIX H USING THE FORMULATION OF THE OBSERVATION EQUATION:

$$Y \; = \; H * X \; + \; F * U + E * V$$

FOR CASE # 1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ALPHA	Q	THETA	VEL
AN	0.351752D+02	-0.191245D-05	0.150046D-02	0.640771D-02
AY	0.000000D+00	0.000000D+00	-0.150534D-01	0.274248D-09

MATRIX F USING THE FORMULATION OF THE OBSERVATION EQUATION:

$$Y \; = \; H * X \; + \; F * U + E * V$$

FOR CASE # 1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ELEVATOR	THROTTLE	SPEED BRAKE
AN	0.412845D+01	-0.180978D-02	0.291699D+00
AY	0.000000D+00	0.000000D+00	0.000000D+00

MATRIX E USING THE FORMULATION OF THE OBSERVATION EQUATION:

$$Y = H * X + F * U + E * V$$

FOR CASE # 1
LINEARIZER DEMONSTRATION AND TEST CASES

FOR THE PROJECT:
USER'S GUIDE

	X-BODY AXIS FORCE	Y-BODY AXIS FORCE	Z-BODY AXIS FORCE	PITCHING MOMENT	ROLLING MOMENT
AN	-0.377037D-07	0.000000D+00	-0.214132D-04	0.000000D+00	0.000000D+00
AY	0.000000D+00	0.222222D-04	0.000000D+00	0.000000D+00	0.000000D+00

YAWING MOMENT

AN	0.000000D+00
AY	0.000000D+00

TRIM CONDITIONS FOR CASE # 2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

STRAIGHT AND LEVEL TRIM WHILE VARYING ALPHA

TRIM ACHIEVED

COEFFICIENT OF LIFT		=	0.13221
COEFFICIENT OF DRAG		=	0.00895
LIFT	(LBS)	=	44376.86258
DRAG	(LBS)	=	3004.93778
ALTITUDE	(FT)	=	20000.
MACH		=	0.90000
VELOCITY	(FT/SEC)	=	933.23196
EQUIVALENT AIRSPEED	(KTS)	=	403.42303
SPEED OF SOUND	(FT/SEC)	=	1036.92440
GRAVITATIONAL ACCEL	(FT/SEC**2)	=	32.11294
NORMAL ACCELERATION	(G-S)	=	0.98523
LOAD FACTOR		=	0.98803
DYNAMIC PRESSURE	(LB/FT**2)	=	552.05302
DENSITY	(SLUG/FT**3)	=	0.00126774
WEIGHT (@ALTITUDE)	(LBS)	=	44914.60434
BETA	(DEG)	=	0.00000
ALPHA	(DEG)	=	-0.72565
PHI	(DEG)	=	0.00000
THETA	(DEG)	=	9.27435
ALTITUDE RATE	(FT/SEC)	=	162.05403
GAMMA	(DEG)	=	10.00000
ROLL RATE	(DEG/SEC)	=	0.00000
PITCH RATE	(DEG/SEC)	=	0.00000
YAW RATE	(DEG/SEC)	=	0.00000
THRUST	(LBS)	=	10804.39673
SUM OF THE SQUARES		=	0.00000

TRIM PARAMETERS

TRIM PITCH AXIS PARAMETER	=	-0.79364
TRIM ROLL AXIS PARAMETER	=	0.00000
TRIM YAW AXIS PARAMETER	=	0.00000
TRIM THRUST PARAMETER	=	0.22509

CONTROL VARIABLES

ELEVATOR	=	0.06377	
THROTTLE	=	0.22509	
SPEED BRAKE	=	0.00000	

OBSERVATION VARIABLES

AN	=	0.98522771	GS
AY	=	0.00000000	GS

NON-DIMENSIONAL STABILITY AND CONTROL DERIVATIVES FOR CASE #2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ROLLING MOMENT	PITCHING MOMENT	YAWING MOMENT	DRAG	LIFT	SIDE FORCE
ZERO COEFFICIENTS	-9.37149D-20	4.22040D-02	-6.11595D-19	1.08760D-02	1.57360D-01	4.31432D-19
ROLL RATE (RAD/SEC)	-2.00000D-01	0.00000D+00	-3.37210D-02	0.00000D+00	0.00000D+00	0.00000D+00
PITCH RATE (RAD/SEC)	0.00000D+00	3.89530D+00	0.00000D+00	0.00000D+00	-1.72320D+01	0.00000D+00
YAW RATE (RAD/SEC)	1.50990D-01	0.00000D+00	-4.04710D-01	0.00000D+00	0.00000D+00	0.00000D+00
VELOCITY (FT/SEC)	8.28111D-17	-1.01777D-09	7.42372D-16	0.00000D+00	7.18527D-09	0.00000D+00
MACH NUMBER	8.58688D-14	-1.05535D-06	7.69784D-13	0.00000D+00	7.45058D-06	0.00000D+00
ALPHA (RAD)	0.00000D+00	-1.68820D-01	0.00000D+00	3.72570D-01	4.87060D+00	0.00000D+00
BETA (RAD)	-1.33450D-01	0.00000D+00	1.29960D-01	0.00000D+00	0.00000D+00	-9.74030D-01
ALTITUDE (FT)	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00
ALPHA DOT (RAD/SEC)	0.00000D+00	-1.18870D+01	0.00000D+00	0.00000D+00	1.72320D+01	0.00000D+00
BETADOT (RAD/SEC)	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00
ELEVATOR	0.00000D+00	-6.95280D-01	0.00000D+00	4.38310D-02	5.72960D-01	0.00000D+00
THROTTLE	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00	0.00000D+00
SPEED BRAKE	0.00000D+00	-4.17500D-01	0.00000D+00	6.49350D-02	3.74920D-02	0.00000D+00

VEHICLE STATIC MARGIN IS   3.5% MEAN AERODYNAMIC CHORD STABLE
AT THIS FLIGHT CONDITION.

MATRIX A USING THE FORMULATION OF THE STATE EQUATION:

$$DX / DT = A * X + B * U + D * V$$

FOR CASE # 2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ALPHA	Q	THETA	VEL
TIME DERIVATIVES WRT:				
ALPHA	-0.120900D+01	0.100000D+01	-0.575730D-02	-0.701975D-04
Q	-0.149189D+01	-0.221451D+01	0.189640D-01	0.231368D-03
THETA	0.000000D+00	0.100000D+01	0.000000D+00	0.000000D+00
VEL	-0.576868D+02	0.000000D+00	-0.316251D+02	-0.460435D-02

MATRIX B USING THE FORMULATION OF THE STATE EQUATION:

$$DX / DT = A * X + B * U + D * V$$

FOR CASE # 2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

87

TIME DERIVATIVES WRT:	ELEVATOR	THROTTLE	SPEED BRAKE
ALPHA	-0.141961D+00	0.448742D-03	-0.928932D-02
Q	-0.220778D+02	-0.147812D-02	-0.135074D+02
THETA	0.000000D+00	0.000000D+00	0.000000D+00
VEL	-0.105186D+02	0.343162D+02	-0.155832D+02

MATRIX D USING THE FORMULATION OF THE STATE EQUATION:

$$DX / DT = A * X + B * U + D * V$$

FOR CASE #2
LINEARIZER DEMONSTRATION AND TEST CASES

FOR THE PROJECT:
USER'S GUIDE

DERIVATIVES WRT TIME OF:	X-BODY AXIS FORCE	Y-BODY AXIS FORCE	Z-BODY AXIS FORCE	PITCHING MOMENT	ROLLING MOMENT
ALPHA	0.934880D-08	0.000000D+00	0.738119D-06	0.000000D+00	0.000000D+00
Q	-0.307941D-07	0.000000D+00	-0.243129D-05	0.605694D-05	0.000000D+00
THETA	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00
VEL	0.714920D-03	0.495829D-13	-0.905497D-05	0.000000D+00	0.000000D+00

DERIVATIVES WRT TIME OF:	YAWING MOMENT
ALPHA	0.000000D+00
Q	0.000000D+00
THETA	0.000000D+00
VEL	0.000000D+00

MATRIX H USING THE FORMULATION OF THE OBSERVATION EQUATION:

$$Y = H * X + F * U + E * V$$

FOR CASE # 2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ALPHA	Q	THETA	VEL
AN	0.350424D+02	-0.128333D-06	-0.632324D-02	0.203434D-02
AY	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00

MATRIX F USING THE FORMULATION OF THE OBSERVATION EQUATION:

$$Y = H * X + F * U + E * V$$

FOR CASE # 2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

	ELEVATOR	THROTTLE	SPEED BRAKE
AN	0.411323D+01	0.492845D-03	0.263288D+00
AY	0.000000D+00	0.000000D+00	0.000000D+00

MATRIX E USING THE FORMULATION OF THE OBSERVATION EQUATION:

$$Y = H * X + F * U + E * V$$

FOR CASE # 2
LINEARIZER DEMONSTRATION AND TEST CASES

FOR THE PROJECT:
USER'S GUIDE

	X-BODY AXIS FORCE	Y-BODY AXIS FORCE	Z-BODY AXIS FORCE	PITCHING MOMENT	ROLLING MOMENT
AN	0.102676D-07	0.000000D+00	-0.214116D-04	0.000000D+00	0.000000D+00
AY	0.000000D+00	0.222222D-04	0.000000D+00	0.000000D+00	0.000000D+00

YAWING MOMENT

AN	0.000000D+00
AY	0.000000D+00

Example printer output file 2 (unit 2)

TRIM CONDITIONS FOR CASE # 1
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

LEVEL TURN WHILE VARYING ALPHA

TRIM ACHIEVED

COEFFICIENT OF LIFT		=	0.40144
COEFFICIENT OF DRAG		=	0.03058
LIFT	(LBS)	=	134741.68807
DRAG	(LBS)	=	10265.70661
ALTITUDE	(FT)	=	20000.
MACH		=	0.90000
VELOCITY	(FT/SEC)	=	933.23196
EQUIVALENT AIRSPEED	(KTS)	=	403.42303
SPEED OF SOUND	(FT/SEC)	=	1036.92440
GRAVITATIONAL ACCEL	(FT/SEC**2)	=	32.11294
NORMAL ACCELERATION	(G-S)	=	3.00163
LOAD FACTOR		=	2.99995
DYNAMIC PRESSURE	(LBS/FT**2)	=	552.05302
DENSITY	(SLUG/FT**3)	=	0.00126774
WEIGHT (@ALTITUDE)	(LBS)	=	44914.60434
BETA	(DEG)	=	0.03193
ALPHA	(DEG)	=	2.66824
PHI	(DEG)	=	70.62122
THETA	(DEG)	=	0.91607
ALTITUDE RATE	(FT/SEC)	=	0.00000
GAMMA	(DEG)	=	0.00000
ROLL RATE	(DEG/SEC)	=	-0.08951
PITCH RATE	(DEG/SEC)	=	5.28086
YAW RATE	(DEG/SEC)	=	1.85749
THRUST	(LBS)	=	10277.03515
SUM OF THE SQUARES		=	0.00000

TRIM PARAMETERS

TRIM PITCH AXIS PARAMETER	=	-0.66958
TRIM ROLL AXIS PARAMETER	=	-0.01526
TRIM YAW AXIS PARAMETER	=	-0.02125
TRIM THRUST PARAMETER	=	0.21410

CONTROL VARIABLES

ELEVATOR		=	0.05380
THROTTLE		=	0.21410
SPEED BRAKE	(DEG)	=	0.00000

OBSERVATION VARIABLES

```
AN                                  =        3.00163339  GS
AY                                  =        0.94136286  GS
```

VEHICLE STATIC MARGIN IS 3.5% MEAN AERODYNAMIC CHORD STABLE
AT THIS FLIGHT CONDITION

TRIM CONDITIONS FOR CASE #   2
LINEARIZER TEST AND DEMONSTRATION CASES

FOR THE PROJECT:
USER'S GUIDE

STRAIGHT AND LEVEL TRIM WHILE VARYING ALPHA

TRIM ACHIEVED

```
COEFFICIENT OF LIFT                 =           0.13221
COEFFICIENT OF DRAG                 =           0.00895
LIFT                   (LBS)        =       44376.86258
DRAG                   (LBS)        =        3004.93778
ALTITUDE               (FT)         =       20000.
MACH                                =           0.90000
VELOCITY               (FT/SEC)     =         933.23196
EQUIVALENT AIRSPEED    (KTS)        =         403.42303
SPEED OF SOUND         (FT/SEC)     =        1036.92440
GRAVITATIONAL ACCEL    (FT/SEC**2)  =          32.11294
NORMAL ACCELERATION    (G-S)        =           0.98523
LOAD FACTOR                         =           0.98803
DYNAMIC PRESSURE       (LB/FT**2)   =         552.05302
DENSITY                (SLUG/FT**3) =           0.00126774
WEIGHT (@ALTITUDE)     (LBS)        =       44914.60434
BETA                   (DEG)        =           0.00000
ALPHA                  (DEG)        =          -0.72565
PHI                    (DEG)        =           0.00000
THETA                  (DEG)        =           9.27435
ALTITUDE RATE          (FT/SEC)     =         162.05403
GAMMA                  (DEG)        =          10.00000
ROLL RATE              (DEG/SEC)    =           0.00000
PITCH RATE             (DEG/SEC)    =           0.00000
YAW RATE               (DEG/SEC)    =           0.00000
THRUST                 (LBS)        =       10804.39673
SUM OF THE SQUARES                  =           0.00000
```

TRIM PARAMETERS

```
TRIM PITCH AXIS PARAMETER           =          -0.79364
TRIM ROLL AXIS PARAMETER            =           0.00000
TRIM YAW AXIS PARAMETER             =           0.00000
TRIM THRUST PARAMETER               =           0.22509
```

CONTROL VARIABLES

```
ELEVATOR                            =           0.06377
THROTTLE                            =           0.22509
SPEED BRAKE                         =           0.00000
```


OBSERVATION VARIABLES

```
AN                                  =           0.98522771  GS
AY                                  =           0.00000000  GS
```

VEHICLE STATIC MARGIN IS 3.5% MEAN AERODYNAMIC CHORD STABLE
AT THIS FLIGHT CONDITION

The following subroutines are examples of user-supplied routines that provide the aerodynamic, control, and engine models to LINEAR. These subroutines are based on an F-15 aircraft simulation and are typical of the routines needed to interface LINEAR to a set of nonlinear simulation models. These subroutines are meant to illustrate the use of the named common blocks to communicate between LINEAR and the user's routines. These subroutines are used with all examples in this report. Included with this report are microfiche listings of these subroutines.

Aerodynamic Model Subroutines

The following two subroutines define a linear aerodynamic model. Even though this model is greatly simplified from the typical nonlinear aerodynamic model, the example illustrates the functions of the subroutines ADATIN and CCALC.

```
      SUBROUTINE ADATIN
C
C....EXAMPLE AERODYNAMIC DATA DEFINITION (OR INPUT) SUBROUTINE
C
C....ROUTINE TO DEFINE STABILITY AND CONTROL DERIVATIVES FOR THE
C    AERODYNAMIC MODEL
C
C
C         COMMON BLOCK TO COMMUNICATE AERODYNAMIC DATA BETWEEN THE
C         SUBROUTINES ADATIN AND CCALC
C
      COMMON /ARODAT/ CLB    ,CLP    ,CLR    ,
     .                CLDA   ,CLDR   ,CLDT   ,
     .                CMO    ,CMA    ,CMQ    ,CMAD   ,
     .                CMDE   ,CMSB   ,
     .                CNB    ,CNP    ,CNR    ,
     .                CNDA   ,CNDR   ,CNDT   ,
     .                CDO    ,CDA    ,CDDE   ,CDSB   ,
     .                CLFTO  ,CLFTA  ,CLFTQ  ,CLFTAD,
     .                CLFTDE,CLFTSB,
     .                CYB    ,CYDA   ,CYDR   ,CYDT
C
C....ROLLING MOMENT DERIVATIVES.
C
C         STABILITY DERIVATIVES WITH RESPECT TO
C         SIDESLIP, ROLL RATE, YAW RATE . . .
C
      CLB    = -1.3345 E-01
      CLP    = -2.0000 E-01
      CLR    =  1.5099 E-01
C
C         CONTROL DERIVATIVES WITH RESPECT TO
C         AILERON, RUDDER, DIFFERENTIAL TAIL . . .
C
      CLDA   =  2.6356 E-02
      CLDR   = -2.3859 E-03
      CLDT   =  4.0107 E-02
C
C....PITCHING MOMENT DERIVATIVES.
C
C         STABILITY DERIVATIVES WITH RESPECT TO ANGLE OF
C         ATTACK, PITCH RATE, ANGLE OF ATTACK RATE . . .
C
      CMO    =  4.2204 E-02
C
      CMA    = -1.6882 E-01
      CMQ    =  3.8953
      CMAD   = -1.1887 E+01
C
```

```
C              CONTROL DERIVATIVES WITH RESPECT TO
C              ELEVATOR, SPEED BRAKE . . .
C
      CMDE  = -6.9528 E-01
      CMSB  = -4.1750 E-01
C
C....YAWING MOMENT DERIVATIVES.
C
C              STABILITY DERIVATIVES WITH RESPECT TO
C              SIDESLIP, ROLL RATE, YAW RATE . . .
C
      CNB   =  1.2996 E-01
      CNP   = -3.3721 E-02
      CNR   = -4.0471 E-01
C
C              CONTROL DERIVATIVES WITH RESPECT TO
C              AILERON, RUDDER, DIFFERENTIAL TAIL . . .
C
      CNDA  =  2.1917 E-03
      CNDR  = -6.9763 E-02
      CNDT  =  3.0531 E-02
C
C....COEFFICIENT OF DRAG DERIVATIVES.
C
C              STABILITY DERIVATIVES WITH RESPECT TO
C              ANGLE OF ATTACK . . .
C
      CDO   =  1.0876 E-02
C
      CDA   =  3.7257 E-01
C
C
C              CONTROL DERIVATIVES WITH RESPECT TO
C              ELEVATOR, SPEED BRAKE . . .
C
      CDDE  =  4.3831 E-02
      CDSB  =  6.4935 E-02
C
C....COEFFICIENT OF LIFT DERIVATIVES.
C
C              STABILITY DERIVATIVES WITH RESPECT TO ANGLE OF
C              ATTACK, PITCH RATE, ANGLE OF ATTACK RATE . . .
C
      CLFTO =  1.5736 E-01
C
      CLFTA =  4.8706
      CLFTQ = -1.7232 E+01
      CLFTAD=  1.7232 E+01
C
C              CONTROL DERIVATIVES WITH RESPECT TO
C              ELEVATOR, SPEED BRAKE . . .
C
      CLFTDE=  5.7296 E-01
      CLFTSB=  3.7492 E-02
C
C....SIDEFORCE COEFFICIENT DERIVATIVES.
C
C              STABILITY DERIVATIVES WITH RESPECT TO
C              SIDESLIP . . .
C
      CYB   = -9.7403 E-01
C
C              CONTROL DERIVATIVES WITH RESPECT TO
C              AILERON, RUDDER, DIFFENTIAL TAIL
C
      CYDA  = -1.1516 E-03
      CYDR  = -1.5041 E-01
      CYDT  = -7.9315 E-02
C
      RETURN
      END
```

```
      SUBROUTINE CCALC
C
C....EXAMPLE AERODYNAMIC MODEL.
C
C....ROUTINE TO CALCULATE THE AERODYNAMIC FORCE AND MOMENT COEFFICIENTS.
C
C             COMMON BLOCKS CONTAINING STATE, CONTROL, AND AIR
C             DATA PARAMETERS
C
      COMMON /DRVOUT/ F(13),DF(13)
      COMMON /CONTRL/ DC(30)
C
      COMMON /DATAIN/ S,B,CBAR,AMSS,AIX,AIY,AIZ,AIXZ,AIXY,AIYZ,AIXE
      COMMON /TRIGFN/ SINALP,COSALP,SINBTA,COSBTA,SINPHI,COSPHI,
     .               SINPSI,COSPSI,SINTHA,COSTHA
      COMMON /SIMOUT/ AMCH,QBAR,GMA,DEL,UB,VB,WB,VEAS,VCAS
      COMMON /CGSHFT/ DELX,DELY,DELZ
C
C             COMMON BLOCK TO OUTPUT AERODYNAMIC FORCE AND MOMENT
C             COEFFICIENTS
C
      COMMON /CLCOUT/ CL    ,CM    ,CN    ,CD    ,CLFT  ,CY
C
C             COMMON BLOCK TO COMMUNICATE AERODYNAMIC DATA BETWEEN
C             THE SUBROUTINES ADATIN AND CCALC
C
      COMMON /ARODAT/ CLB   ,CLP   ,CLR   ,
     .                CLDA  ,CLDR  ,CLDT  ,
     .                CMO   ,CMA   ,CMQ   ,CMAD  ,
     .                CMDE  ,CMSB  ,
     .                CNB   ,CNP   ,CNR   ,
     .                CNDA  ,CNDR  ,CNDT  ,
     .                CDO   ,CDA   ,CDDE  ,CDSB  ,
     .                CLFTO ,CLFTA ,CLFTQ ,CLFTAD,
     .                CLFTDE,CLFTSB,
     .                CYB   ,CYDA  ,CYDR  ,CYDT
C
C             EQUIVALENCE VARIABLE NAMES
C
      EQUIVALENCE (T      , F( 1)),
     .            (P      , F( 2)),(Q      , F( 3)),(R      , F( 4)),
     .            (V      , F( 5)),(ALP    , F( 6)),(BTA    , F( 7)),
     .            (THA    , F( 8)),(PSI    , F( 9)),(PHI    , F(10)),
     .            (H      , F(11)),(X      , F(12)),(Y      , F(13)),
     .            (TDOT   ,DF( 1)),
     .            (PDOT   ,DF( 2)),(QDOT   ,DF( 3)),(RDOT   ,DF( 4)),
     .            (VDOT   ,DF( 5)),(ALPDOT ,DF( 6)),(BTADOT ,DF( 7)),
     .            (THADOT ,DF( 8)),(PSIDOT ,DF( 9)),(PHIDOT ,DF(10)),
     .            (HDOT   ,DF(11)),(XDOT   ,DF(12)),(YDOT   ,DF(13))
C
      EQUIVALENCE (DA     ,DC( 1)),(DE     ,DC( 5)),(DT     ,DC( 8)),
     .            (DR     ,DC( 9)),(DSB    ,DC(10))
C
C....COMPUTE TERMS NEEDED WITH ROTATIONAL DERIVATIVES.
C
      V2    = 2.0    *V
      B2V   =B       /V2
      C2V   =CBAR    /V2
C
C....ROLLING MOMENT COEFFICIENT.
C
      CL    =CLB     * BTA    +CLDA   *DA     +CLDR   *DR     +CLDT   *DT
     .      +B2V     *(CLP    *P      +CLR    *R      )
C
C....PITCHING MOMENT COEFFICIENT.
C
      CM    =CMO     + CMA    *ALP    +CMDE   *DE     +CMSB   *DSB
     .      +C2V     *(CMQ    *Q      +CMAD   *ALPDOT)
C
C....YAWING MOMENT COEFFICIENT.
```

93

```
C
      CN    =CNB     *  BTA    +CNDA   *DA    +CNDR   *DR     +CNDT   *DT
      .     +B2V     *(CNP    *P      +CNR    *R      )
C
C....COEFFICIENT OF DRAG.
C
      CD    =CDO     +  CDA   *ALP    +CDDE   *DE     +CDSB   *DSB
C
C....COEFFICIENT OF LIFT.
C
      CLFT  =CLFTO   + CLFTA  *ALP    +CLFTDE*DE       +CLFTSB*DSB
      .     +C2V     *(CLFTQ  *Q      +CLFTAD*ALPDOT )
C
C....SIDEFORCE COEFFICIENT.
C
      CY    =CYB     *BTA     +CYDA   *DA     +CYDR   *DR     +CYDT  *DT
C
      RETURN
      END
```

Engine Model Interface Subroutine

The following subroutine, IFENGN, both defines an engine model and provides the interface to LINEAR. In normal usage this subroutine would call subroutines that provide the detailed thrust, engine rotation, and fuel consumption modeling; the information from these subroutines would be transferred into the /ENGSTF/ common block.

```
      SUBROUTINE IFENGN
C
C....EXAMPLE SUBROUTINE TO PROVIDE PROPULSION SYSTEM MODEL.
C
C....ROUTINE TO COMPUTE PROPULSION SYSTEM INFORMATION FOR LINEAR
C
C    THIS SUBROUTINE IS THE INTERFACE BETWEEN THE DETAILED ENGINE
C    MODELING SUBROUTINES AND LINEAR.
C
C             INPUT COMMON BLOCK CONTAINING INFORMATION ON THRUST
C             REQUEST TO ENGINE
C
      COMMON /CONTRL/ DC(30)
C
C             OUTPUT COMMON BLOCK CONTAINING DETAILED INFORMATION
C             ON EACH OF UP TO FOUR SEPARATE ENGINES
C
C....ROUTINE TO COMPUTE ENGINE PARAMETERS:
C
C               THRUST(I) = THRUST CREATED BY EACH ENGINE
C              TLOCAT(I,J) = LOCATION OF EACH ENGINE IN THE X-Y-Z PLANE
C               XYANGL(I) = ANGLE IN X-Y BODY AXIS PLANE AT WHICH EACH
C                           ENGINE IS MOUNTED
C               XZANGL(I) = ANGLE IN X-Z BODY AXIS PLANE AT WHICH EACH
C                           ENGINE IS MOUNTED
C               TVANXY(I) = ANGLE IN THE X-Y ENGINE AXIS PLANE OF THE
C                           THRUST VECTOR
C               TVANXZ(I) = ANGLE IN THE X-Z ENGINE AXIS PLANE OF THE
C                           THRUST VECTOR
C               DXTHRS(I) = DISTANCE BETWEEN THE ENGINE C.G. AND THE
C                           THRUST POINT
C                  EIX(I) = ROTATIONAL INERTIA OF EACH ENGINE
```

```
C                 AMSENG(I) = MASS OF EACH ENGINE
C                 ENGOMG(I) = ROTATIONAL VELOCITY (RAD/SEC)
C
      COMMON /ENGSTF/ THRUST(4),TLOCAT(4,3),XYANGL(4),XZANGL(4),
     .                TVANXY(4),TVANXZ(4),DXTHRS(4),
     .                EIX(4),AMSENG( 4),ENGOMG(4)
C
C            EQUIVALENCE VARIABLE NAMES
C
      EQUIVALENCE (THR    ,DC(12))
C
C....ASSUME THRUST PER ENGINE IS HALF VEHICLE WEIGHT.
C
      THRUST(1)= 24000.0 *THR
      THRUST(2)= 24000.0 *THR
C
C....LET ALL OTHER PARAMETERS DEFAULT TO ZERO
C
      RETURN
      END
```

Control Model Subroutine

The subroutine UCNTRL provides an interface between the trim parameters and the surface deflections. Figure 8 illustrates the gearing model implemented in the example UCNTRL subroutine. The thrust demand parameter is also set and passed to the subroutine IFENGN.
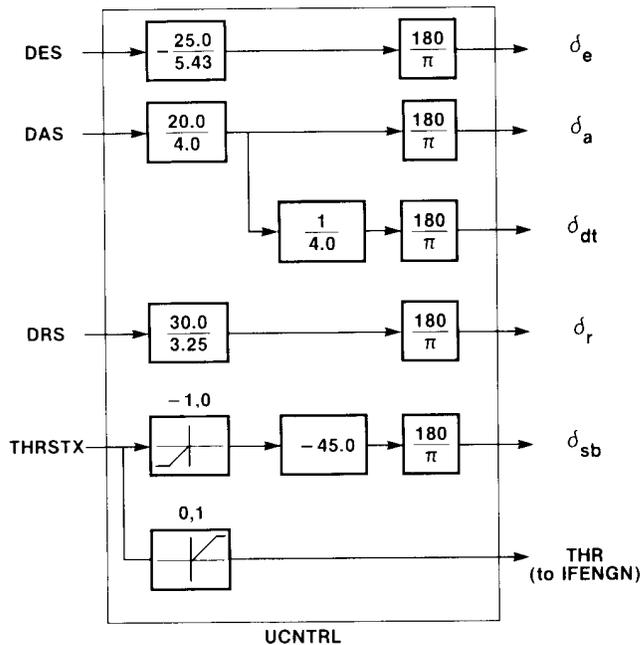
Figure 8. Gearing model in example UCNTRL subroutine.

```
      SUBROUTINE UCNTRL
C
C....EXAMPLE TRIM/CONTROL SURFACE INTERFACE ROUTINE.
C
C
C....ROUTINE TO CONVERT TRIM INPUTS INTO CONTROL SURFACE DEFLECTIONS.
C
C              INPUT COMMON BLOCK CONTAINING TRIM PARAMETERS
C
      COMMON /CTPARM/ DES   ,DAS   ,DRS   ,THRSTX
C
C              OUTPUT COMMON BLOCK CONTAINING CONTROL SURFACE DEFLECTIONS
C
      COMMON /CONTRL/ DC(30)
C
C              EQUIVALENCE VARIABLE NAMES.
C
      EQUIVALENCE (DA    ,DC( 1)),(DE    ,DC( 5)),(DT    ,DC( 8)),
     .            (DR    ,DC( 9)),(DCB   ,DC(10)),(THR   ,DC(12))
C
      DATA DGR   /57.29578/
C
C....CONVERT FROM INCHES OF STICK AND PEDAL TO DEGREES OF SURFACE
C    DEFLECTION
C
      DA    =DAS   *( 20.0 / 4.0 )
      DE    =DES   *(-25.0 / 5.43)
      DR    =DRS   *( 30.0 / 3.25)
C
C....SET DIFFERENTIAL TAIL BASED ON AILERON COMMAND.
C
      DT    =DA    /   4.0
C
C....CONVERT THRUST TRIM PARAMETER TO PERCENT THROTTLE COMMAND.
C
      THR     = 0.0
      IF(THRSTX.GE. 0.0 ) THR   =THRSTX
C
C....USE SPEED BRAKE IF NEEDED.
C
      DSB    = 0.0
      IF(THRSTX.LT. 0.0 ) DSB   =THRSTX*(-45.0 )
C
C....CONVERT SURFACE COMMANDS TO RADIANS.
C
      DA    =DA    /DGR
      DE    =DE    /DGR
      DR    =DR    /DGR
      DT    =DT    /DGR
      DSB   =DSB   /DGR
C
      RETURN
      END
```

Mass and Geometry Model Subroutine

The following subroutine, MASGEO, is an example of the mass and geometry subroutine that the user must provide to LINEAR. If the mass and geometry characteristics of the aircraft do not change or can be easily specified as a function of flight condition and surface setting (vehicle configuration), then the mass and geometry characteristics of the vehicle can be defined using the input file, and MASGEO would be a dummy subroutine, as is the following example. However, for an aircraft in which these mass and geometry properties are complicated or already defined using FORTRAN subroutines, MASGEO provides an interface between LINEAR and those subroutines.

```
      SUBROUTINE MASGEO
C
C.... SUBROUTINE TO COMPUTE THE MASS AND GEOMETRY PROPERTIES OF THE
C     AIRCRAFT.
C
      COMMON /CONPOS/ DAP,FLAT,DATRIM,DEP,FLON,DETRIM,DRP,FPED,DRTRIM,
     .                DSBP,DFP,PLAPL,PLAPR,THSL,THSR
      COMMON /CONTRL/ DC(30)
      COMMON /DATAIN/ S,B,CBAR,AMSS,AIX,AIY,AIZ,AIXZ,AIXY,AIYZ,AIXE
      COMMON /SIMOUT/ AMCH,QBAR,GMA,DEL,UB,VB,WB,VEAS,VCAS
      COMMON /DRVOUT/ F(13),DF(13)
      EQUIVALENCE(T      , F( 1)),
     .          (P      , F( 2)),(Q      , F( 3)),(R      , F( 4)),
     .          (V      , F( 5)),(ALP    , F( 6)),(BTA    , F( 7)),
     .          (THA    , F( 8)),(PSI    , F( 9)),(PHI    , F(10)),
     .          (H      , F(11)),(X      , F(12)),(Y      , F(13)),
     .          (TDOT   ,DF( 1)),
     .          (PDOT   ,DF( 2)),(QDOT   ,DF( 3)),(RDOT   ,DF( 4)),
     .          (VDOT   ,DF( 5)),(ALPDOT ,DF( 6)),(BTADOT ,DF( 7)),
     .          (THADOT ,DF( 8)),(PSIDOT ,DF( 9)),(PHIDOT ,DF(10)),
     .          (HDOT   ,DF(11)),(XDOT   ,DF(12)),(YDOT   ,DF(13)),
C
      RETURN
      END
```

APPENDIX J: REVISIONS TO MICORFICHE SUPPLEMENT

    The following listing of subroutine ENGINE incorporates revisions not contained in the microfiche supplement included with this report (Program LINEAR and Subroutines Utilized by LINEAR). This listing should be used in place of the subroutine ENGINE listing on the microfiche supplement.

```
0001          SUBROUTINE ENGINE
0002    C
0003    C...  ROUTINE TO COMPUTE EFFECTS OF ENGINE(S).
0004    C...
0005    C     WRITTEN FEBRUARY 1,1983
0006    C     LEE DUKE NASA/DFRF
0007    C
0008    C...  MODIFIED FOR DIFFERENT ANGLE DEFINITION JUNE, 1985
0009    C     JOE PAHLE NASA/DFRF
0010    C
0011          IMPLICIT DOUBLE PRECISION (A-H)
0012          IMPLICIT DOUBLE PRECISION (O-Z)
0013    C
0014          CHARACTER * 4 NAMYVC
0015    C
0016          DIMENSION XTHRST(4),YTHRST(4),ZTHRST(4)
0017          DIMENSION HENGIN(4,3),GYRO(3),AINRTA(3,3)
0018          DIMENSION TORQUE(3)
0019          DIMENSION TVECLC(4,3)
0020          DIMENSION CSANXY(4),SNANXY(4),CSANXZ(4),SNANXZ(4)
0021    C
0022    C...  THE COMMON BLOCKS /LLL016/ AND /LLL015/ ARE THE MAIN I/O
0023    C     COMMON BLOCKS FOR THIS ROUTINE. THE VARIABLES IN /LLL016/
0024    C     ARE DEFINED IN THE USER SUPPLIED ROUTINE IFENGN. THIS
0025    C     ROUTINE COMPUTES THE VALUES OF THE VARIABLES IN /LLL015/
0026    C     AND SUPPLIES THEM TO THE REST OF LINEAR.
0027    C
0028    C     THE VARIABLES IN THESE COMMON BLOCKS ARE:
0029    C
0030    C     /LLL016/
0031    C
0032    C           THRUST(I) THE TOTAL THRUST OF THE I-TH
0033    C                     ENGINE (LBS)
0034    C           TLOCAT(I,J)     THE DISPLACEMENT OF THE I-TH ENGINE
0035    C                     FROM THE AIRCRAFT REFERENCE C.G. (FEET),
0036    C                     WHERE
0037    C                          J=1 CORRESPONDS TO THE
0038    C                              THE X-BODY AXIS
0039    C                              COORDINATE OF THE
0040    C                              I-TH ENGINE
0041    C                          J=2 CORRESPONDS TO THE
0042    C                              THE Y-BODY AXIS
0043    C                              COORDINATE OF THE
0044    C                              I-TH ENGINE
0045    C                          J=3 CORRESPONDS TO THE
0046    C                              THE Z-BODY AXIS
0047    C                              COORDINATE OF THE
0048    C                              I-TH ENGINE
0049    C           XYANGL(I) THE ANGLE IN THE X-Y BODY AXIS
0050    C                     PLANE FROM THE X-BODY AXIS TO
0051    C                     THE PROJECTION OF THE I-TH ENGINE
0052    C                     AXIS ONTO THE X-Y PLANE. (DEG)
0053    C           XZANGL(I)       THE ANGLE BETWEEN THE X-Y BODY AXIS
0054    C                     PLANE AND THE I-TH ENGINE AXIS
0055    C                     PROJECTED ON THE X-Z' BODY/ENGINE AXIS
0056    C           EIX(I)          THE MOMENT OF INERTIA ABOUT THE
0057    C                     X-ENGINE AXIS OF THE I-TH ENGINE
```

```
0058      C               (SLUG-FT**2).
0059      C     AMSENG(I) THE MASS OF THE ROTATING MACHINERY
0060      C               IN THE I-TH ENGINE (SLUGS)
0061      C     ENGOMG(I) THE ROTATIONAL VELOCITY OF THE
0062      C               I-TH ENGINE. POSITIVE ROTATION IS
0063      C               MEASURED USING THE R.H.R. ABOUT THE
0064      C               X-ENGINE AXIS. (RAD/SEC)
0065      C     TVANXY(I) THE ANGLE IN THE X-Y ENGINE AXIS
0066      C               PLANE FROM THE X-ENGINE AXIS TO
0067      C               THE PROJECTION OF THE I-TH THRUST VECTOR
0068      C               ONTO THE X-Y ENGINE PLANE. (DEG)
0069      C     TVANXZ(I) THE ANGLE BETWEEN THE X-Y ENGINE AXIS
0070      C               PLANE AND THE I-TH THRUST VECTOR
0071      C               PROJECTED ON THE X-Z' ENGINE/THRUST AXIS
0072      C     DXTHRS(I) THE DISTANCE BETWEEN THE C.G. OF THE
0073      C               ENGINE AND THE THRUST POINT MEASURED
0074      C               POSITIVE IN THE NEGATIVE ENGINE X-AXIS
0075      C               DIRECTION
0076      C
0077      C     /LLL015/
0078      C
0079      C     FPX               TOTAL THRUST IN THE X-BODY AXIS
0080      C     FPY               TOTAL THRUST IN THE Y-BODY AXIS
0081      C     FPZ               TOTAL THRUST IN THE Z-BODY AXIS
0082      C     DCM               TOTAL PITCHING MOMENT INCREMENT
0083      C                       DUE TO ENGINES
0084      C     DCL               TOTAL ROLLING MOMENT INCREMENT
0085      C                       DUE TO ENGINES
0086      C     DCN               TOTAL YAWING MOMENT INCREMENT
0087      C                       DUE TO ENGINES
0088      C
0089            COMMON /LLL016/ THRUST(4),TLOCAT(4,3),XYANGL(4),XZANGL(4),
0090           .                TVANXY(4),TVANXZ(4),DXTHRS(4),
0091           .                EIX(4),AMSENG(4),ENGOMG(4)
0092            COMMON /LLL015/ FPX    ,FPY    ,FPZ    ,DCM    ,DCL    ,DCN
0093      C
0094            COMMON /LLL013/ F(13),DF(13)
0095            LOGICAL STDOBS
0096      C
0097            COMMON /LLL048/ AX,AY,AZ,ANX,ANY,ANZ,AN
0098            COMMON /LLL012/ S,B,CBAR,AMSS,AIX,AIY,AIZ,AIXZ,AIXY,AIYZ,AIXE
0099            COMMON /LLL019/ TLFT,DRAG,SIDFOR,ALM,AMM,ANM
0100            COMMON /LLL028/ GS,ALOADF
0101            COMMON /LLL054/ SINALP,COSALP,SINBTA,COSBTA,SINPHI,COSPHI,
0102           .                SINPSI,COSPSI,SINTHA,COSTHA
0103            COMMON /LLL008/ CL,CM,CN,CD,CLFT,CY
0104            COMMON /LLL049/ AMCH,QBAR,GMA,DEL,UB,VB,WB,VEAS,VCAS
0105            COMMON /LLL007/ DELX,DELY,DELZ
0106            COMMON /LLL001/ A,RHO,G,PA,TMPR
0107      C
0108            COMMON /LLL041/ NUMYVC,LOCYVC(120),STDOBS,
0109           .                VARPOS(120, 6)        ,OBSVEC(120)
0110            COMMON /LLL040/ NAMYVC(120, 5)
0111            COMMON /LLL032/ VMU
0112      C
0113            EQUIVALENCE (T        , F( 1)),
0114           .            (P        , F( 2)),(Q        , F( 3)),(R        , F( 4)),
```

```
0115        .                   (V      , F( 5)),(ALP    , F( 6)),(BTA    , F( 7)),
0116        .                   (THA    , F( 8)),(PSI    , F( 9)),(PHI    , F(10)),
0117        .                   (H      , F(11)),(X      , F(12)),(Y      , F(13)),
0118        .                   (TDOT   ,DF( 1)),
0119        .                   (PDOT   ,DF( 2)),(QDOT   ,DF( 3)),(RDOT   ,DF( 4)),
0120        .                   (VDOT   ,DF( 5)),(ALPDOT,DF( 6)),(BTADOT,DF( 7)),
0121        .                   (THADOT,DF( 8)),(PSIDOT,DF( 9)),(PHIDOT,DF(10)),
0122        .                   (HDOT   ,DF(11)),(XDOT   ,DF(12)),(YDOT   ,DF(13))
0123        C
0124              DATA DGR    / 57.2958/
0125        C
0126        C..... CALL USER ENGINE MODEL INTERFACE ROUTINE
0127        C
0128              CALL INTRF2
0129        C
0130        C..... COMPUTE COMPONENTS OF THRUST.
0131        C
0132              DO 1 I=1,4
0133        C
0134              ANGLXZ = XZANGL(I)/DGR
0135              ANGLXY = XYANGL(I)/DGR
0136        C
0137              CSANXZ(I) = DCOS(ANGLXZ)
0138              SNANXZ(I) = DSIN(ANGLXZ)
0139              CSANXY(I) = DCOS(ANGLXY)
0140              SNANXY(I) = DSIN(ANGLXY)
0141        C
0142              TANGXY = TVANXY(I)/DGR
0143              TANGXZ = TVANXZ(I)/DGR
0144        C
0145              CSTAXY = DCOS(TANGXY)
0146              SNTAXY = DSIN(TANGXY)
0147              CSTAXZ = DCOS(TANGXZ)
0148              SNTAXZ = DSIN(TANGXZ)
0149        C
0150              XTHRSI =  THRUST(I) * CSTAXZ * CSTAXY
0151              YTHRSI =  THRUST(I) * CSTAXZ * SNTAXY
0152              ZTHRSI = -THRUST(I) * SNTAXZ
0153        C
0154              XTHRST(I) =  XTHRSI * CSANXZ(I) * CSANXY(I) -
0155        .                  YTHRSI * SNANXY(I) +
0156        .                  ZTHRSI * SNANXZ(I) * CSANXY(I) +
0157              YTHRST(I) =  XTHRSI * CSANXZ(I) * SNANXY(I) +
0158        .                  YTHRSI * CSANXY(I) +
0159        .                  ZTHRSI * SNANXZ(I) * SNANXY(I) +
0160              ZTHRST(I) = -XTHRSI * SNANXZ(I) +
0161        .                  ZTHRSI * CSANXZ(I)
0162        C
0163            1 CONTINUE
0164        C
0165        C..... COMPUTE TOTAL X-AXIS AND Z-AXIS THRUST.
0166        C
0167              FPX    = 0.0
0168              FPY    = 0.0
0169              FPZ    = 0.0
0170        C
0171              DO 2 I=1,4
```

ENGINE

VAX FORTRAN V4.7-271
SYSDA1:[ANTON]ENGINE.FOR;1

26-Oct-1987 15:10:03
26-Oct-1987 15:09:35

102

```
0172        C
0173                    FPX      = FPX      + XTHRST(I)
0174                    FPY      = FPY      + YTHRST(I)
0175                    FPZ      = FPZ      + ZTHRST(I)
0176        C
0177            2  CONTINUE
0178        C
0179        C.... COMPUTE ROTATIONAL EFFECTS (TORQUE) DUE TO ENGINE OFFSET FROM
0180        C     CENTERLINE, AND THRUST VECTORING .
0181        C
0182                    TORQUE(1) = 0.0
0183                    TORQUE(2) = 0.0
0184                    TORQUE(3) = 0.0
0185        C
0186                    DO 3 I=1,4
0187        C
0188                    TVECLC(I,1) = TLOCAT(I,1) - DXTHRS(I)  *  CSANXZ(I)  *  CSANXY(I)
0189                    TVECLC(I,2) = TLOCAT(I,2) - DXTHRS(I)  *  CSANXZ(I)  *  SNANXY(I)
0190                    TVECLC(I,3) = TLOCAT(I,3) + DXTHRS(I)  *  SNANXZ(I)
0191        C
0192                    TORQUE(1) = TORQUE(1)
0193              .             +ZTHRST(I)*TVECLC(I,2)-YTHRST(I)*TVECLC(I,3)
0194                    TORQUE(2) = TORQUE(2)
0195              .             +XTHRST(I)*TVECLC(I,3)-ZTHRST(I)*TVECLC(I,1)
0196                    TORQUE(3) = TORQUE(3)
0197              .             +YTHRST(I)*TVECLC(I,1)-XTHRST(I)*TVECLC(I,2)
0198        C
0199            3  CONTINUE
0200        C
0201        C.... COMPUTE ANGULAR MOMENTUM OF ENGINES
0202        C
0203                    DO 5 I=1,4
0204        C
0205                    OMEGX  =   ENGOMG(I)  *  CSANXZ(I)  *  CSANXY(I)
0206                    OMEGY  =   ENGOMG(I)  *  CSANXZ(I)  *  SNANXY(I)
0207                    OMEGZ  =  -ENGOMG(I)  *  SNANXZ(I)
0208        C
0209                    EIXE      = EIX(I)
0210                    EX        = TLOCAT(I,1)
0211                    EY        = TLOCAT(I,2)
0212                    EZ        = TLOCAT(I,3)
0213        C
0214        C.... COMPUTE THE INERTIA TENSOR OF THE I-TH ENGINE
0215        C     ABOUT THE AIRCRAFT REFERENCE C.G.
0216        C
0217                    AINRTA(1,1) =    EIXE*CSANXZ(I)*CSANXZ(I)*CSANXY(I)*CSANXY(I)
0218                    AINRTA(1,2) =    EIXE*CSANXZ(I)*CSANXZ(I)*SNANXY(I)*CSANXY(I)
0219                    AINRTA(1,3) =  - EIXE*CSANXZ(I)*SNANXZ(I)*CSANXY(I)
0220                    AINRTA(2,1) =    AINRTA(1,2)
0221                    AINRTA(2,2) =    EIXE*SNANXY(I)*SNANXY(I)*CSANXZ(I)*CSANXZ(I)
0222                    AINRTA(2,3) =  - EIXE*SNANXZ(I)*CSANXZ(I)*SNANXY(I)
0223                    AINRTA(3,1) =    AINRTA(1,3)
0224                    AINRTA(3,2) =    AINRTA(2,3)
0225                    AINRTA(3,3) =    EIXE*SNANXZ(I)*SNANXZ(I)
0226        C
0227                    DO 4 J=1,3
0228        C
```

```
0229        HENGIN(I,J) =   OMEGX*AINRTA(J,1) + OMEGY*AINRTA(J,2)
0230                      + OMEGZ*AINRTA(J,3)
0231        .
0232  C
0233  C   4 CONTINUE
0234  C
0235  C   5 CONTINUE
0236  C
0237  C.... COMPUTE GYROSCOPIC EFFECTS.
0238  C
0239        GYRO  (1)=  0.0
0240        GYRO  (2)=  0.0
0241        GYRO  (3)=  0.0
0242  C
0243        DO 6 I=1,4
0244  C
0245        GYRO  (1)=GYRO  (1) + Q      *HENGIN(I,3)
0246                            - R      *HENGIN(I,2)
0247        GYRO  (2)=GYRO  (2) + R      *HENGIN(I,1)
0248                            - P      *HENGIN(I,3)
0249        GYRO  (3)=GYRO  (3) + P      *HENGIN(I,2)
0250                            - Q      *HENGIN(I,1)
0251  C
0252    6 CONTINUE
0253  C
0254  C.... COMPUTE TOTAL MOMENT INCREMENT DUE TO ENGINES.
0255  C
0256        DCL  = GYRO  (1) + TORQUE(1)
0257        DCM  = GYRO  (2) + TORQUE(2)
0258        DCN  = GYRO  (3) + TORQUE(3)
0259  C
0260        RETURN
              END
```

REFERENCES

Chen, Robert T.N.: Kinematic Properties of Rotary-Wing and Fixed-Wing Aircraft in Steady Coordinated High-g Turns. AIAA-81-1855, Aug. 1981.

Chen, Robert T.N.; and Jeske, James A.: Kinematic Properties of the Helicopter in Coordinated Turns. NASA TP-1773, 1981.

Clancy, L.J.: Aerodynamics. John Wiley & Sons, New York, 1975.

Dieudonne, James E.: Description of a Computer Program and Numerical Techniques for Developing Linear Perturbation Models From Nonlinear Systems Simulations. NASA TM-78710, 1978.

Dommasch, Daniel O.; Sherby, Sydney S.; and Connolly, Thomas F.: Airplane Aerodynamics, Fourth edition. Pitman, New York, 1967.

Etkin, Bernard: Dynamics of Atmospheric Flight. John Wiley & Sons, New York, 1972.

Gainer, Thomas G.; and Hoffman, Sherwood: Summary of Transformation Equations and Equations of Motion Used in Free-Flight and Wind-Tunnel Data Reduction and Analysis. NASA SP-3070, 1972.

Gracey, William: Measurement of Aircraft Speed and Altitude. NASA RP-1046, 1980.

Kalviste, Juri: Fixed Point Analysis Program. NOR 80-165, Northrop Corporation, Aircraft Division, Hawthorne, California, Nov. 1980.

Kwakernaak, Huibert; and Sivan, Raphael: Linear Optimal Control Systems. Wiley-Interscience, New York, 1972.

Perkins, Courtland D.; and Hage, Robert E.: Airplane Performance Stability and Control. John Wiley & Sons, New York, 1949.

Thelander, J.A.: Aircraft Motion Analysis. FDL-TDR-64-70, Air Force Flight Dynamics Laboratory, Mar. 1965

U.S. Standard Atmosphere, 1962. U.S. Government Printing Office, 1962.

1. Report No. NASA TP-2768	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle User's Manual for LINEAR, a FORTRAN Program to Derive Linear Aircraft Models		5. Report Date **December 1987**
		6. Performing Organization Code
7. Author(s) Eugene L. Duke, Brian P. Patterson, and Robert F. Antoniewicz		8. Performing Organization Report No. H-1259
9. Performing Organization Name and Address NASA Ames Research Center Dryden Flight Research Facility P.O. Box 273 Edwards, CA 93523-5000		10. Work Unit No. RTOP 505-66-11
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		13. Type of Report and Period Covered Technical Paper
		14. Sponsoring Agency Code

15. Supplementary Notes
  A listing of the program LINEAR is provided in the microfiche supplement
  included with this report (3 sheets total).

16. Abstract

This report documents a FORTRAN program that provides a
powerful and flexible tool for the linearization of aircraft
models. The program LINEAR numerically determines a linear
system model using nonlinear equations of motion and a user-
supplied nonlinear aerodynamic model. The system model
determined by LINEAR consists of matrices for both state and
observation equations. The program has been designed to
allow easy selection and definition of the state, control,
and observation variables to be used in a particular model.

17. Key Words (Suggested by Author(s)) Aircraft model Computer program Control law design Linearization	18. Distribution Statement Unclassified — Unlimited Subject category 66		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 108	22. Price* A06