Why there is no apparent noon–midnight red shift in the global positioning system: Comparing spin stabilized and gravity-gradient stabilized frames

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Abstract: The recent paper, "Why there is no noon–midnight red shift in the GPS" by Ashby and Weiss [arXiv:1307.6525 (2013)], is critically reviewed herein. While their own criticism of Hoffmann's paper [Phys. Rev. **121**, 337 (1961)] is valid, their alternate solution is contradicted by direct evidence from the global positioning system (GPS) itself. They claim their solution is "….. based on fundamental relativity principles." But fundamental relativity principles need to be based upon fundamental physics. GPS conclusively shows that the physics is not correct in their solution. Looking first at the underlying physics of spin stabilized and gravity-gradient stabilized frames reveals the errors found in the papers of both Hoffmann and of Ashby and Weiss. The physical principles that lead to an apparent absence of a noon–midnight red shift also reveal the mechanism for transforming the solar speed of light into the local speed of light in the earth's frame and gives rise to the apparent relativity of simultaneity. © *2014 Physics Essays Publication*.
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Résumé: Un examen critique de la publication récente "Why there is no noon–midnight red shift in the GPS" par Ashby et Weiss [arXiv:1307.6525 (2013)], est présenté. Quoique leur critique de la publication de 1961 de Hoffman soit valable, les résultats du système GPS permettent de réfuter leur solution alternative. Leur affirmation est basée sur les principes fondamentaux de la relativité. Cependant, ceux-ci doivent être basés sur les principes fondamentaux de la physique. Le système GPS permet de vérifier de façon certaine que certains principes utilisés dans leur solution sont erronés. Une analyse des principes de physique des systèmes de référence stabilisés à l'aide de gradients gravimétriques et de spin permet d'abord de vérifier les erreurs commises par Hoffmann et par Ashby et Weiss. Les principes qui aboutissent à une absence apparente du décalage midi-minuit vers le rouge permettent également d'établir le mécanisme de transformation de la vitesse de la lumière solaire à la vitesse de la lumière locale dans le système de référence terrestre et qui résulte à la relativité de la simultanéité apparente.

Key words: Noon–Midnight Redshift; GPS; Equivalence Principle; Acceleration; Gravitational Potential; Selleri Transformation; Lorentz Transformation; Scale Change; Clock Bias.

I. INTRODUCTION

The question as to why there are no observable effects upon satellite clocks caused by the gravitational potential of the sun has been addressed by several people. Such effects might be expected since at noon clocks have a lower solar gravitational potential than they do at midnight. Thus, one could expect a global positioning system (GPS) clock at noon to exhibit a clock frequency that is slower than the clock at midnight. As far as I am aware, the first to address the issue was Banish Hoffman.[1] Unfortunately, his analysis as applied to the earth was clearly faulty and is recognized as such by almost all who have considered the issue. I also addressed the issue in two prior papers,[2,3] but it is not apparent from the titles of the papers that the issue was discussed therein. The most recent paper[4] to address the topic is that of Ashby and Weiss (A&W) and it was the underlying motivation for this paper.

I was asked to review the A&W paper by another journal for possible publication therein. Their response to my criticism and suggestions was rejected and so I questioned the editor of that journal as to the possibility of publishing a rebuttal within the same issue. The response was, yes, such a rebuttal could be considered but they would have the right to also publish their response to the rebuttal also within the same issue. As a result, I submitted a rebuttal but A&W rejected the whole concept and proceeded to immediately publish their paper on-line.

As a result, I rewrote the rebuttal and submitted it to *Physics Essays* for publication. However, additional criticism from a reviewer stimulated a thorough revision with additional background material. To make the criticism of both the Hoffman and the A&W paper much more apparent, two background segments are added. Specifically, a section on spin stabilized frames in orbit around a gravitational body

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and a section on gravity-gradient stabilized frames around a gravitational body are now included.

Three specific effects in the "child" frame (e.g., the earth's frame) are the result of three specific mechanisms acting within the "parent" frame (e.g., the solar frame). The effects considered in the child frame are: (1) effects upon the clock frequency and time; (2) effects upon the speed of light and mechanical motion; and (3) effects upon the frame units. The mechanisms in the parent frame which drive these effects are: (1) the speed (kinetic energy) relative to the parent frame; (2) the gravitational potential (energy) relative to the parent frame; and (3) the acceleration of the child frame relative to the parent frame.

The frequency effects are perhaps most important and to simplify later equations are considered here briefly. The first two of these mechanisms affects the child frame clock frequency (and time) directly. The third mechanism (acceleration) induces a Doppler frequency path effect and only impacts the apparent frequency as it is received. It arises from a path length change during the transit time of the signal from satellite to the receiver, i.e., the acceleration of the earth during the transit time of the signal from a satellite orbiting the earth to a receiver upon the earth. While it does not affect the clock rate (and thereby accumulated time) of either the satellite or receiver clocks, it does affect the received frequency which is compared with the receiver clock rate. This acceleration Doppler causes an apparent aliasing of the satellite frequency at the receiver.

The fractional frequency change caused by each of the above three mechanisms is converted into a simplified form below. The earth's orbital speed affects the clocks on the earth and in its vicinity due to the velocity scale factor, γ. Actually, it would be more logical to refer to it as the kinetic energy scale factor since the effect is a function of the speed squared. Specifically,

$$f_m = \frac{f}{\gamma} = f\sqrt{1 - \frac{V^2}{c^2}} \approx f\left[1 - \frac{V^2}{2c^2}\right]. \tag{1}$$

The subscript m indicates the modified frequency. The velocity scale factor, γ, is defined within the equation.

This equation can be restructured in terms of the fractional frequency change

$$\frac{\Delta f}{f} \approx \frac{V^2}{2c^2}. \tag{2}$$

In the rest of the paper, this simplified form for velocity effects on frequency will be used.

The gravitational potential (energy) effects of the sun upon clocks on the earth and its vicinity are similar in form to Eq. (1). Specifically,

$$f_m = f\sqrt{1 - \frac{2GM}{Rc^2}} \approx f\left[1 - \frac{GM}{Rc^2}\right]. \tag{3}$$

The mass, M, designates the mass of the sun, and the R designates the orbital distance from the sun to the earth. Putting Eq. (3) into the same fractional frequency form as Eq. (2) gives

$$\frac{\Delta f}{f} \approx \frac{GM}{Rc^2}. \tag{4}$$

In the rest of the paper, this simplified form for the gravitational potential effects on frequency will be used.

Finally, the acceleration induced fractional frequency effects can be directly put into equation form,

$$\frac{\Delta f}{f} = -\frac{a\tau}{c} = -\frac{GM}{R^2}\left(\frac{1}{c}\right)\left(\frac{y}{c}\right) = -\frac{GM}{Rc^2}\left(\frac{y}{R}\right). \tag{5}$$

In Eq. (5), a is the solar acceleration of the earth, τ is the transit time from a satellite signal to the receiver on the earth (the negative sign is assigned for the case when the earth is falling away from the satellite), and y is the component of distance in the direction of the fall over which the signal travels from satellite to receiver.

Equations (2), (4), and (5) will be referred to several places throughout the paper. The first background segment addressing a spin stabilized orbiting frame is addressed next.

II. THE SPIN STABILIZED FRAME—ILLUSTRATED BY THE EARTH AROUND THE SUN

The spin stabilized frame illustrated by the earth in orbit around the sun has some interesting characteristics. If we temporarily remove the spin (without changing the stabilization) every element of the frame, i.e., the earth and its vicinity would orbit the sun at precisely the same orbital speed. This allows us to treat separately the spin velocity and the orbital velocity as is done in the development below.

A. The apparent Lorentz transformation

The material in this section is intended to show that there are natural physical processes in the solar frame, which automatically cause the speed of light in the earth's frame to appear to be isotropic and with a speed commonly accepted as the invariant speed of light. This mathematical and numerical demonstration conflicts with some of the commonly accepted concepts taught as part of the special relativity theory (SRT). Thus, it is critical that this background be understood to appreciate the flaws in the A&W paper. The material in this section largely follows the approach presented in a prior paper.[5]

1. Clocks—their rate and time reading

Idealized clocks are assumed in this paper, i.e., clocks that if stationary in a given gravitational potential will run at a specific frequency without any disturbing noise. Furthermore, since clocks are physical objects, it is assumed that the rate at which they run is independent of the frame in which they are assumed to reside. (Note: this does not exclude different numerical rates due to the units of time assigned to the frame.) But it is also recognized that clocks can be biased by setting their initial reading. In addition, as is illustrated by the GPS satellite clocks, their rate can also be adjusted so that they run at a rate equivalent to a similar unmodified clock not moving at the same speed and/or not at the same gravitational potential.

It is well demonstrated that in a given inertial frame clocks run slower as a function of their speed (kinetic energy) in that given frame. It is also a generally recognized fact that clocks run faster, the higher their gravitational potential energy. These two clock rate mechanisms are assumed in this paper.

2. Longitudinal length contraction and anisotropic velocity

The evidence usually cited for length contraction is the Michelson–Morley experiment. But there is additional simple mechanical evidence which supports the concept. For example, the GPS satellites orbit the earth as the earth orbits the sun. Assume for simplicity that the orbit of the satellite is in the ecliptic plane. When the satellite orbital velocity is added to the earth's orbital velocity, the increased inertial mass with speed means that the orbital speed has to slow a small amount or else the conservation of momentum would be violated. But the earth's orbital velocity is not measurably affected. Thus, the slowing of the speed of the satellite integrated into its position means that its orbit is slightly contracted in the along velocity direction when in front of the earth. When the satellite velocity subtracts from the earth's orbital velocity, these effects are reversed but the net effect is the same, i.e., the orbit of the satellite is slightly contracted when in back of the earth. Working out the mathematics, it is found that this length contraction is precisely identical to that required for the null result of the Michelson–Morley experiment.

In addition to the length contraction in the earth's along-orbit direction, two other effects are noteworthy. First, the satellite velocity (fore and aft) in the direction of the earth's orbital velocity is slightly anisotropic even in the earth's frame. Second, this anisotropic satellite velocity means that it takes more time to traverse the forward direction of the orbit than it does the backward direction. These two effects will be discussed further below.

3. The Selleri transformation (ST)

Assuming clock slowing defines smaller time units in a moving frame and longitudinal length contraction defines shorter longitudinal lengths in a moving frame, one can develop a transformation from an absolute frame (or a frame one assumes is absolute) and its associated units into a frame moving with respect to that absolute frame. The first to specify such a transformation was apparently Tangherlini.[6] However, I prefer to identify it as the ST since Selleri[7] developed it more thoroughly by specifying its inverse and its repeated application. Selleri himself referred to it as the "inertial transformation."

The common velocity or kinetic energy scale factor was defined implicitly in Eq. (1) above. It is used in the following ST equations. The ST from the absolute frame to the moving frame becomes

$$t = T/\gamma \quad \text{and} \quad f = \gamma F, \tag{6}$$

$$x = \gamma(X - VT) = \gamma(X - X_0), \tag{7}$$

$$y = Y, \tag{8}$$

$$z = Z. \tag{9}$$

This mapping defines the transformation of the measured values in the stationary or absolute frame into the measured values of the moving frame. It is important to note that the velocity units also change. The larger units of time together with the shorter units of length in the contracted x direction mean that the velocities in the stationary frame map into larger speeds in the moving frame. This includes the frame velocity itself. Thus,

$$v = \gamma^2 V. \tag{10}$$

A general velocity in the X direction maps from the stationary frame into the moving frame as

$$\dot{x} = \gamma_{\dot{X}}^2(\dot{X} - V). \tag{11}$$

Note that the x velocity in this equation uses a γ which is a function of the X velocity, i.e., not the velocity, V, which causes the x velocity to become slightly anisotropic in the fore and aft directions.

Since only the time units affect the velocity in the Y and Z directions, the mappings of the measured velocities in these orthogonal directions is much simpler,

$$\dot{y} = \gamma\dot{Y}, \tag{12}$$

$$\dot{z} = \gamma\dot{Z}. \tag{13}$$

It is not difficult to invert these equations to get the inverse ST. First, note that the origin of the moving frame, X_0, in the stationary frame units maps into the origin in the moving frame units as

$$x_0 = \gamma X_0. \tag{14}$$

The inverse ST transformation becomes

$$T = \gamma t \quad \text{and} \quad F = f/\gamma, \tag{15}$$

$$X = \frac{1}{\gamma}(x + vt) = \frac{1}{\gamma}(x + x_0), \tag{16}$$

$$Y = y, \tag{17}$$

$$Z = z. \tag{18}$$

The inverse velocity transformations become

$$\dot{X} = \frac{1}{\gamma_{\dot{x}}^2}(\dot{x} + v). \tag{19}$$

See note following Eq. (11):

$$\dot{Y} = \dot{y}/\gamma, \tag{20}$$

$$\dot{Z} = \dot{z}/\gamma. \tag{21}$$

4. The ST plus clock bias as a function of x position results in the apparent Lorentz transformation (ALT)

At this point, it is instructive to compare the ST with the Lorentz transformation (LT). But the LT from the stationary frame to the moving frame is identical with the ST equations (6) through Eq. (9) except for the time transformation given in Eq. (6). The LT time transformation is

$$t' = \gamma\left(T - \frac{VX}{c^2}\right). \tag{22}$$

The prime is used to distinguish the Lorentz time mapping from the Selleri time mapping.

It is instructive to form the difference between these two time mappings. Specifically,

$$\begin{aligned}\Delta t = t' - t &= \left(1 - \frac{1}{\gamma^2}\right)\gamma T - \frac{\gamma VX}{c^2} \\ &= -\frac{V}{c^2}\gamma(X - VT).\end{aligned} \tag{23}$$

Now substituting the result of Eq. (7) into the final form of Eq. (23) gives

$$\Delta t = -\frac{Vx}{c^2}. \tag{24}$$

This shows the very important result that the ST can be converted into the ALT by the simple implementation of a clock bias that is a function of the x position, i.e., by Eq. (24). The motivation for calling it an ALT rather than simply an LT is described below.

5. The source of the clock bias required for the ALT

There are several mechanisms whereby the clock bias given by Eq. (24) may arise.

a. Manual synchronization. Since clocks are not perfect instruments, it is often necessary to perform manual synchronization of remote clocks. There are two simple manual synchronization methods by which this is done. Specifically, Einstein synchronization is accomplished by assigning one-half of the two-way light time to the one-way light time of travel. Since the two-way light speed has been experimentally verified to have the constant numerical value of "c," Einstein synchronization practically involves assuming the one-way light path between clocks is c. Thus, knowing the separation distance, d, allows the remote clock to be set using an assumed travel time of d/c. Alternatively, one can simply use slow clock transport, i.e., synchronize two collocated clocks and then carry one slowly to the remote location.

In a moving frame, it is not difficult to show that both of these synchronization methods result in a clock bias precisely as defined by Eq. (24) above. In a prior paper,[8] rather than showing directly that assuming the speed of light in a moving frame leads to the appropriate clock bias, we simply showed the inverse, i.e., using the clock bias of Eq. (24) causes the speed of light to be converted from the anisotropic value of $(c \pm V)$ to the isotropic value of c numerically. However, while it is numerically invariant, it is not physically invariant. Its physical value in the new frame is measured in smaller units. Those units are smaller by the square of the velocity scale factor, γ.

To show that slow clock transport leads to the same clock bias is even easier. Specifically, the clock frequency is affected by both the slow transport velocity, v, and by the frame velocity V. Thus, using the vector sum of the two velocities in the form of Eq. (2) above gives

$$\frac{\Delta f}{f} \approx -\frac{V^2}{2c^2} - \frac{V \bullet v}{c^2} - \frac{v^2}{2c^2}. \tag{25}$$

The first term of Eq. (25) affects all clocks in the moving frame including those not moving in that frame. The last term is made negligibly small by the slow movement of the clock. This leaves only the dot product middle term, which can be integrated to reveal the resulting bias on the clock resulting from the slow clock movement.

$$\begin{aligned}\Delta t &= \int -\frac{V \bullet v}{c^2}dt = -\frac{V}{c^2}\int vCos(\theta)dt \\ &= -\frac{V}{c^2}\int \dot{x}dt = -\frac{V}{c^2}\int dx = -\frac{Vx}{c^2}.\end{aligned} \tag{26}$$

In this equation, θ is the angle between the frame velocity and the velocity within the frame.

Q.E.D. Slow clock transport leads to the same clock bias as Eq. (20), which converts the Selleri Transformation into an ALT.

b. Automatic synchronization (bias generation) of clocks on the earth. There are two critical processes involved in the automatic generation of clock biases for earth based clocks (or satellite clocks in orbit around the earth). First, we temporarily assume the sun is an absolute, i.e., a stationary, frame. Next, we temporarily ignore any solar gravitational potential or force from the sun and allow the earth to simply move in a straight line path in the solar frame at a velocity, V. Under these assumptions, a clock on the spinning earth or a clock on an orbiting satellite moving at a velocity, v, will give rise to a clock bias as a function of the x component of motion. Equation (25) still applies but the interpretation of the terms needs to be slightly modified to handle the orbiting satellite or the spinning earth.

Like the slow clock transport, the first term due to the orbital speed of the earth is common to all clocks on the earth. In addition, the third term due to earth's spin is also included along with the variation in the gravitational potential of mean sea level with latitude (the earth flattening effect). The result is that all clocks on the earth at mean sea level run at the same rate and include the contribution of the first and last terms. As was true of the slow clock transport, the middle term is integrated as was done in Eq. (26), i.e., the same clock bias form is obtained from the spin of the earth as is obtained from Einstein synchronization and slow clock transport.

In the case of the GPS satellites, by using the small v to indicate the orbital velocity, the same result is obtained as when the small v is used to represent the earth's spin velocity. The GPS satellite clocks are preset before launch at a rate such that the first and third terms in Eq. (25), together with the different nominal earth's gravitational potential term, cause the satellite clocks once they are in orbit to run at the same average rate as clocks on the surface of the earth. (Clock effects from the eccentricity of the satellite orbit are separately compensated in the GPS receiver processing.) The satellite clock middle term integrates into the same form of clock bias as a function of the x position as that shown in Eq. (26).

It was assumed above that the earth traveled in a straight line in the solar frame while the earth spin was unchanged. In one year's time, this would cause the clocks near the equator on the earth's surface to cycle through 366.24 clock bias cycles. In other words, there would be one clock bias cycle per sidereal day. Plugging in the equatorial radius of the earth into Eq. (24) indicates that the cyclic error would vary from 2.1 μs negative as the clock was on the leading edge of the earth's path to 2.1 μs positive as the clock moved to the trailing edge of the earth's path.

It is now time to address the second critical process mentioned above. Specifically, we need to remove the restriction that the earth follows a straight line path. In addition, it is clear that in the circular orbit with the earth spinning, we need to have one less cycle of clock bias variation per year, if the clock bias directions are to remain aligned with the orbital direction. In other words, when in circular orbit around the sun, the clock bias at the equator needs to have a daily 24 h cycle not a daily sidereal cycle. Clearly, the orbital path of the earth is such that the along-velocity direction is constantly changing by about one degree per day. The changing path of the earth is caused by the solar gravitational force given by the radial gradient of the solar gravitational potential. It turns out that the radial gradient of the gravitational potential also causes the clocks on earth (and in orbit around the earth) on the dark side of the earth to run faster than the clocks on the sunlit side of the earth. This gradient of the solar gravitational potential causes the direction of the clock biases to change in concert with the force, which changes the direction of the orbital velocity.

To illustrate the solar gravitational potential effect on the clocks most clearly, assume that we stop the earth's spin and locate a clock at the equator at precisely midnight on the vernal equinox. This clock will run faster than a clock on the surface which is the same distance from the sun as the earth's center, but since the direction of the polar axis of the earth does not change in space as the earth orbits the sun, the distance to the equatorial clock changes and thus the relative clock rates change. Using the approximation given by Eq. (4) above, the net difference in the fractional frequency between a clock on the surface of the earth and one at the center, i.e., the gradient of the fractional frequency effect gives

$$\frac{\Delta f}{f} \approx \left[-\frac{GM}{(R + rCos(\theta))c^2}\right] - \left[-\frac{GM}{Rc^2}\right] \approx \frac{GM}{Rc^2}\left(\frac{r}{R}\right)Cos(\theta). \tag{27}$$

But the gravitational potential term of the sun is mathematically equivalent to earth's orbital velocity squared. So Eq. (27) is mathematically equivalent to

$$\frac{\Delta f}{f} \approx \frac{v}{c^2}\left(rCos(\theta)\frac{V}{R}\right) = \frac{V}{c^2}rCos(\theta)\dot{\theta}. \tag{28}$$

Note that putting the gravitational effect in terms of the orbital velocity does not imply the effect is caused by the velocity. It simply allows the integral to be performed easier and expresses the result in the form desired.

Integrating this over time and converting the time derivative to the orbital angle derivative gives

$$\Delta t = \frac{V}{c^2}\int rCos(\theta)d\theta = \frac{VrSin(\theta)}{c^2} = \frac{Vx}{c^2}. \tag{29}$$

Thus, the clock bias caused by the solar gravitational potential will subtract one cycle (note the sign difference) from 366.24 cycles caused by the earth's spin in one year, i.e., the cyclic clock bias will be once per solar day rather than once per sidereal day.

Obviously from the development above, the same automatic generation of clock biases applies to the GPS clocks in orbit around the earth. Yes, a different number of clock bias cycles occur, but the gradient of the gravitational potential of the sun subtracts one bias cycle per year to keep the apparent speed of light at c relative to the GPS satellites.

This automatically generated clock bias converts the natural ST into an ALT and results in a numerical isotropic speed of light on the earth of "c."

The automatic generation of the clock bias not only converts the anisotropic speed of light in the earth's frame to the isotropic value of "c," but also converts the anisotropic mechanical speed of motion of two equal masses given identical forces acting in opposite directions into equal, i.e., isotropic speeds within the earth's frame. In other words, it counteracts the use of the modified velocity scale factor in Eqs. (11) and (19) which introduced the anisotropic mechanical velocities in the earth's frame.

6. *Why call the transformation an ALT rather than simply a Lorentz Transformation (LT)?*

It is true that adding the appropriate clock bias, which is automatically generated on the earth and in its vicinity, to the ST does result in mathematics which are identical to the LT. But there are several reasons to apply a different name to the transformation. This list of reasons here is largely a repetition of reasons given in a prior paper.[5]

(a) In most instances, the transformation from one frame to another clearly involves a transformation from one frame (call it the "parent" frame) to another frame embedded within it (call it the "child" frame). For example, the sun is the parent frame to the earth. The galactic frame is the parent frame to the sun and the Cosmic Microwave Background (CMB) frame is most likely the parent frame to the galactic frame. The LT does not distinguish the frame hierarchies and therefore cannot properly account for the embedded scale changes which are implied by them. The ALT as developed above shows that there are scale changes in time, length, and velocity that are hidden in the LT. For example, in the case of light although its speed is numerically invariant due to the scale changes in length and time, its physical speed is slower in the child frame, i.e., it is not physically invariant—as is assumed when the LT is applied. When interframe measurements are made this blind use of the LT will lead to inaccurate results. But recognizing the scale changes of the ALT from its component parts of a ST together with a clock bias will give accurate results.

(b) Another significant implication of the above development arises from the parent frame/child frame relationship. Specifically, it becomes apparent, from the parent/child frame hierarchy, that an ultimate absolute frame must exist. That frame is most likely the CMB frame.

(c) When the ALT is from the child frame back to the parent frame, the clock bias must be removed first and then the reverse ST applied. Mathematically, the result is the inverse LT but includes the inverse scaling which is hidden in the LT. Again, the ALT recognizes the measurement mapping and the numerical invariance of the speed of light. The LT assumes the numerical invariance implies a physical invariance as well.

(d) Another important difference is that the ALT differs from the LT in that it clarifies the requirement for a specific mechanism or source for the clock bias generation. In the SRT, it is generally assumed that if a frame undergoes acceleration the speed of light is automatically maintained at the value of "c" as the speed of the frame is changed. In fact, this claim is specifically made in Goldstein's textbook on classical mechanics.[9] This claim is made in many forms including in the typical explanation for Thomas Precession. However, this flies in the face of all the experimental evidence that acceleration never directly produces a change in the rate at which clocks run. Acceleration may (or may not) cause the speed (i.e., kinetic energy) to change and that speed change can result in a clock rate change. However, there is no mechanism whereby an identical linear acceleration of two clocks can cause one clock to run at a different rate than the other. But if the speed of light is to be measured with a value of c as a moving frame is linearly accelerated the two separated clocks would have to run at different rates. *This is commonly ignored by SRT and General Relativity Theory (GRT) in their use of infinitesimal frames*.

This completes the development of the ALT.

B. Implications of the ALT applied to the earth-centered inertial (ECI) frame

From the development above, there are several interesting implications of the ALT applied to the ECI frame. Perhaps most important, it is clear that the clock biases cause the speed of light to be isotropic and with a numerical value of "c" relative to the ECI frame. Thus, no orbital Sagnac effect is required in the ECI frame. But this in turn means that the speed of light is anisotropic with respect to the Earth-Centered Earth-Fixed (ECEF) frame because of the spin of that frame relative to the ECI frame. This means that a spin Sagnac correction is required to compensate for the anisotropic speed of light in the ECEF frame. It is also clear that the ALT clock biases compensate for the increase of mass with speed such that the momentum of particles of the same initial mass moving in opposite directions relative to the earth center appear to have isotropic speeds relative to the ECI frame.

In addition to the isotropy of light and mechanical momentum in the ECI frame, the units of the frame differ from the units in the parent solar frame. The time units are increased by the velocity scale factor. The along-orbit length units are contracted. The along-orbit velocity units (length units divided by time units) are contracted as the square of the velocity scale factor. The orthogonal length units are unchanged and the orthogonal velocity units are contracted by the velocity scale factor.

These unit changes and the scale changes involved in the ALT and inverse ALT are not currently considered in mapping measurements of solar frame phenomena taken within the earth's frame. This may explain some of the small anomalies found in solar system measurements. Some of these anomalies were addressed in a prior paper.[10]

C. Why there is no apparent noon or midnight frequency shift due to the solar gravitational potential effect upon GPS satellite clocks

The absence of apparent solar gravitational potential effects upon GPS clocks is the originating motivation for this and other prior papers. At this point, it is pretty easy to demonstrate the reason for the absence. Equation (27) is valid for both transmitter clocks in the GPS satellites and for receivers on the earth's surface. It can be put into an equivalent form to give

$$\frac{\Delta f}{f} \approx \frac{GM}{Rc^2}\left(\frac{r}{R}\right)Cos(\theta) = \frac{GM}{Rc^2}\left(\frac{y}{R}\right). \tag{30}$$

From this equation, the true difference in clock frequency caused by the solar gravitational potential at the satellite minus the effect at the receiver is given by

$$\frac{\Delta f}{f} = \frac{GM}{Rc^2}\left(\frac{y_s - y_r}{R}\right). \tag{31}$$

Equation (31) gives the true solar gravitational potential difference in fractional frequency between the satellite clock and the receiver clock. However, the Doppler shift which arises due to the acceleration of the earth from the solar force on the earth has not been included. This Doppler shift is caused by the movement of the earth during the transit time of the signal from the satellite to the receiver. But the equation for the Doppler shift was given in the introduction as Eq. (5). Since the y distance (component of signal traveled in the direction of fall) in that equation is precisely the distance $(y_s - y_r)$ Eq. (5) becomes

$$\frac{\Delta f}{f} = -\frac{GM}{Rc^2}\left(\frac{y_s - y_r}{R}\right). \tag{32}$$

This means that the acceleration induced Doppler shift of the frequency obscures the true clock difference between the satellite transmitter and the receiver on the earth.

Note the above development shows why there is no apparent clock frequency shift from the solar gravitational potential effect upon the measured difference in the satellite and receiver clock frequencies. But this Doppler cancelation of the true frequency difference in the frequency measurements does not affect the biases generated by the true clock time difference which results from the integration of the clock frequency differences. This is because the acceleration Doppler is only a transmission path effect. It is not integrated into either the satellite or receiver clock time.

D. Summary of "parent" frame (e.g., solar frame) effects upon an orbiting spin stabilized "child" frame (e.g., ECI frame)

Three fundamental mechanisms in the parent frame were listed in the introduction, which affect the clock frequency and other parameters in the child frame. The speed of matter within the child frame relative to the parent frame (together with conservation of momentum) causes the combined effects of length contraction, of anisotropic velocities of both light and mechanical motion, and of clock slowing. These combined effects are characterized by the ST as developed above.

The interaction of orbital velocity and spin velocity induces a cyclic clock frequency offset which integrates into a cyclic clock bias with a period equal to the spin period. Other than the spin interaction with the orbital velocity there is no other direct velocity effect that needs to be considered since (ignoring the spin) all elements of the frame move at the same orbital velocity in the parent frame. The gradient of the parent frame gravitational potential creates a cyclic clock frequency which integrates into a cyclic clock bias with a period equal to the orbital period. This latter effect causes the removal of one cyclic clock bias per orbital period from the total cyclic clock biases generated by the spin mechanism. The net result of the induced cyclic clock biases is to convert the ST into the ALT. The ALT causes the anisotropic velocities of the ST to be converted into the isotropic velocities of the ALT. In other words, it removes the need for an orbital Sagnac correction for the anisotropic speed of light in the ST. The ALT is distinct from the Lorentz Transformation (LT) in that it clarifies the changes of units between the parent and child frames.

The remaining acceleration Doppler effect upon offset frequencies has only one role to play. It causes the actual offset frequency due to the gravitational gradient in the parent frame (which gave rise to the once per orbit cyclic clock bias) to be cancelled out in any measurements of the frequency at a receiver in the child frame. In other words, it explains the absence of any noon–midnight effect which would otherwise be present in measurements of the GPS clock frequency by receivers located upon the earth. This is actually very convenient since it removes the otherwise awkward evidence of the parent frame from needing to be considered within the child frame.

III. THE GRAVITY-GRADIENT (OR EQUIVALENT) STABILIZED FRAME

The moon's orbit around the earth can be used to illustrate the characteristics of the gravity-gradient stabilized frame. The major characteristic of the lunar frame is that the clock biases which are generated by the spin of the earth in its frame around the sun are missing in the orbit of the moon around the earth, i.e., no spin generated biases are present in clocks located on the moon.

In addition, there is a velocity scale factor (i.e., kinetic energy) gradient of positions on the surface of the moon relative to the earth that is missing in earth's orbit around the sun. It is not difficult to show that the effects upon clocks of this gradient over the lunar surface exactly counteract (except for small libration effects) the effects upon clocks of the gradient of the earth's gravitational potential over the lunar surface. This means that clock biases which are generated once per orbit as the earth orbits the sun are missing, i.e., they are not generated once per orbit as the moon orbits the earth.

This cancellation of kinetic and potential energy gradients is easy to show. First, the gravitational potential gradient is developed starting with the equivalent of Eq. (27) above, except now the gravity-gradient rotation of the frame cancels out the theta rotation of once per orbit relative to the orbit center in the spin stabilized frame. Changing the nomenclature such that R represents the lunar orbital radius, M the mass of the earth, and y. the additional distance of a point on the lunar surface from the earth gives us

$$\frac{\Delta f}{f} \approx \left[-\frac{GM}{(R+y)c^2}\right] - \left[-\frac{GM}{Rc^2}\right] \approx \frac{GM}{Rc^2}\left(\frac{y}{R}\right). \tag{33}$$

Now converting the potential energy into the numerical equivalent of an orbital velocity (kinetic energy) as was done in Eq. (28) gives the fractional frequency rate in terms of the orbital angular rate.

$$\frac{\Delta f}{f} \approx \frac{V}{c^2}\left(y\frac{V}{R}\right) = \frac{Vy\dot{\theta}}{c^2}. \tag{34}$$

Next, the gradient of the orbital kinetic energy effect upon the frequency can be computed by finding the difference in frequency which results from a small radial distance offset (radial gradient) in the velocity scale factor. In this case, the spin velocity is the orbital spin velocity, v, and is replaced by the offset distance y multiplied by the orbital angular rate.

$$\frac{\Delta f}{f} \approx \left[-\frac{V^2}{2c^2} - \frac{Vy\dot{\theta}}{c^2} - \frac{(y\dot{\theta})^2}{2c^2}\right] - \left[\frac{V^2}{2c^2}\right]$$
$$\approx -\frac{Vy\dot{\theta}}{c^2} - \frac{(y\dot{\theta})^2}{2c^2} \approx -\frac{Vy\dot{\theta}}{c^2}. \quad (35)$$

The last step in the equation above was to drop the final term because it is very small due to the small distance y and the slow orbital angular rate. Thus, the residual term of the clock kinetic energy gradient with respect to radial distance cancels out the gravitational gradient term of Eq. (34).

While the gradient of the orbital kinetic energy (velocity scale factor) of a gravity-gradient stabilized frame counteracts the gravitational scale factor gradient of the parent frame, the acceleration Doppler shift in the frequency between a transmitter and receiver in the frame would still occur. In most cases, the received frequency offset may well be small enough to ignore, i.e., it would simply alias into a small offset in the receiver frequency. However, in specific cases it may need to be removed before any receiver measurements are processed, particularly if integrated carrier phase measurements are employed.

Note that if we had not taken the radial gradient of the velocity scale factor, the first term of the first bracket in Eq. (35) would have remained. But it is common to all satellites in the lunar vicinity and all clocks could be set to simply run at a rate which counteracts this constant rate term.

The result is that for a gravity-gradient stabilized frame (or any equivalent frame that rotates once per orbit), there are no clock biases generated due to spin stabilization or to the gravity-gradient process. Interestingly, this opens up two different frame synchronization approaches described below. It appears that the simplest to implement for the moon would be the synchronization described in Section III B below.

A. Synchronization of clocks to the parent frame

The gravity-gradient (lunar) frame can be synchronized to the parent (earth's ECI) frame if the initial clocks are synchronized within the frame by adjusting for the anisotropic speed of light by using an orbital Sagnac correction. This means that within the child frame there are no relative clock biases present or generated. To make this synchronization work, the clock rates must be adjusted such that the orbital velocity and different gravitational potential effects upon the clock rates in the child frame are compensated such that they run at the same rate as the reference clocks of the parent frame. In the case of a lunar child frame, the gravitational potential of the moon as well as the gravitational potential of the earth must be included in the adjustment. Since the child frame is not rotating there is no need for a spin Sagnac correction. However, an orbit Sagnac correction in the child frame appears as if it was a spin Sagnac correction in the parent frame.

B. Synchronization of clocks within the child frame via Einstein synchronization or slow clock transport

If clocks on the child frame (moon) were initially synchronized using Einstein or slow clock transport, the clocks would remain synchronized since there is no mechanism (other than clock frequency noise) to drive them out of synchronization. The speed of light in such a frame would have an isotropic numerical value of c. Furthermore, the generated clock biases would remove the need for an orbital Sagnac correction. However, since the lunar inertial frame so designed does not rotate with the orbit, a small spin Sagnac correction is needed to account for the orbital spin rate. Though no orbital clock bias is generated on the lunar surface to cause the clock biases to rotate into the along track direction of the orbit, the biased clocks themselves rotate in their position such that the initialized clock biases are preserved.

If it were desired to implement a lunar global positioning system (LGPS) by orbiting satellites around the moon, the lunar satellite clocks would, due to their motion relative to the earth, generate clock biases as a result of both the satellites velocity with respect to the earth and clock biases as a result of the gradient of the earth's gravitational potential. Like the earth's GPS satellites, their clock rate needs to be adjusted before launch from the lunar surface to account for the difference in the lunar gravitational potential and their speed relative to the nonrotating lunar frame. The resulting frame would be an apparent nonrotating lunar frame with a valid ALT transformation of measurements from the earth's ECI frame to the moon's lunar centered inertial (LCI) frame. In addition, such a LGPS could be used to automatically set the lunar clocks with the appropriate clock biases to maintain the speed of light as c within the frame. As stated above, a small Sagnac correction would be required to correct for the orbital rotation during the transit time of any signal from either a satellite or other transmitter to the receiver. In addition, the acceleration Doppler effect from an LGPS satellite to a receiver on the moon would not quite cancel out the gravitational clock difference between the satellite and the receiver on the moon. Specifically, the acceleration which occurs during the transit path from satellite to receiver cancels out the gravitational potential between the satellite and the receiver. But at the receiver the orbital rotation velocity has already canceled out the gravitational potential at the receiver. Thus, a very small effect from the acceleration Doppler frequency will remain in the apparent received frequency.

C. Another illustration of a gravity-gradient equivalent frame

The two GRACE satellites in orbit around the earth with one trailing the other by about 200 km in almost identical orbits could be used to define a common frame that appears to be stabilized as an effective gravity-gradient frame. Like the lunar frame, no clock biases are generated between the two clocks due to either velocity gradients or potential gradients. In the case of the GRACE satellites, the simplest clock setting is done using the on-board GPS receivers. This automatically sets the clocks to the ECI frame. In the ECI frame, there is a spin Sagnac effect which is required for signals between the two satellites. In the rotating GRACE frame, that same process looks like an orbital Sagnac correction. To maintain synchronization of the clocks at the two GRACE satellites with clocks on the earth's surface, the clocks need to be adjusted to counteract the gravitational potential at the two satellites and the orbital velocity of the two satellites. To synchronize using either the Einstein method or slow clock transport is not particularly practical and would serve no useful purpose other than maintaining the speed of light between the two satellites at the numerical value of c. For signals between the two satellites, the acceleration Doppler due to their "fall" in the earth's frame is negligible.

D. Summary of "parent" frame (e.g., earth frame) effects upon an orbiting gravity-gradient stabilized "child" frame (e.g., lunar frame)

In the gravity-gradient stabilized frame, length contraction and clock effects should lead to an ST from the parent frame to the child frame just as it does in the spin stabilized frame. However, in the case of the moon in orbit around the earth, the slow orbital velocity relative to the earth is apt to make the contraction of the orbit in the along-orbit direction too small to measure.

Unlike the spin stabilized frames, there is no automatic clock bias generation from either the spin interaction with the orbital velocity or from the gradient of the gravitational potential. In fact the gradient of orbital velocity cancels out the gradient of the gravitational potential. Thus, the two automatic bias generating mechanisms of the spin stabilized frame are absent from the gravity-gradient stabilized frame. Unless some other method of synchronizing the clocks is used, the gravity-gradient frame would require an orbital Sagnac correction. The acceleration Doppler shift of frequency generated during the transit time from a transmitter would remain and potentially cause a small bias in the received frequency. In some instances, this may require a computed compensation of the receiver measurements.

IV. THE HOFFMAN PAPER

The Hoffman paper is generally judged faulty and it is not difficult to see why. There are two faults. The first fault is a major error and is generally recognized by all. His second fault is relatively minor but well worth addressing.

A. Hoffman's major fault—confusing the earth stabilization mechanism

His major error is to alias part of the earth's spin into the equivalent of an orbital rotation of once per year. He does this by assigning a constant position to the noon and midnight locations on the earth's surface. Such an assignment does not recognize the difference between a spin rate of once per sidereal period with the actual once per solar day period. In a nut shell, he treats the spin stabilized frame of the earth as if it is a gravity-gradient stabilized frame. Both my earlier papers and the Ashby and Weiss recent paper recognized this fault.

B. Hoffman's minor fault—misapplication of the equivalence principle

There is a second fault that is generally ignored. Specifically, Hoffman calls upon the equivalence principle for the cancellation of clock effects induced by the gravitational potential energy. There are two problems with this. First, the cancellation is only valid if the frame is a gravity-gradient stabilized frame, which it is not. Second, if it were a gravity-gradient stabilized frame, the actual mechanism for the cancellation is related to the orbital velocity of the clocks (kinetic energy) rather than the equivalence principle. This is a misuse of the equivalence principle and is entirely unnecessary.

As was shown above, even in the case of a gravity-gradient stabilized frame, the cancellation of the two effects is straightforward and does not rely upon the equivalence principle. The equivalence principle is claimed to specifically apply to infinitesimal frames or approximately infinitesimal frames. There is no such limitation inherent in the development above which equated the radial gradient of kinetic energy (velocity) effects with the radial gradient of gravitational potential effects. While the radial gradient of the gravitational potential energy is force and the radial gradient of orbital velocity is acceleration, neither force nor acceleration directly results in a change in frequency. Furthermore, the actual cancellation mechanism is a function of the orbital kinetic energy or velocity squared, not simply velocity. The equivalence principle applies when the integration of the gradient of potential energy is equal to the integration of the kinetic energy. But that integration loses the constant of integration of both the gravitational potential energy and of the kinetic energy. The two constants have different effects upon the clock rates—which is one reason the equivalence principle applies only over infinitesimal regions.

Hoffman's use of the equivalence principle would only be valid in the circular orbit of a gravity-gradient stabilized frame, i.e., when the change in the kinetic energy and the change in the potential energy within the frame are precisely equal. Under these conditions the clock effects do cancel. Furthermore, these effects are not limited to an infinitesimal frame. Interestingly, clocks on the earth exhibit a similar effect as a function of latitude. At mean sea level, as the latitude is increased on the earth, the spin kinetic energy decreases, but the gravitational potential energy decreases by precisely the same amount. Also interestingly, in an elliptic orbit there is a kind of antiequivalence principle at work. As the clock in an elliptic orbit approaches periapse, the kinetic energy increases while the gravitational potential energy decreases. Rather than cancelling the two clock frequency effects, the two effects precisely add together, doubling the individual effects.

V. THE ASHBY AND WEISS PAPER

While the Hoffman paper is generally judged faulty, I believe that the Ashby and Weiss (A&W) paper is likewise faulty. However, I think the A&W paper is of more concern because the errors in their solution are much more subtle. There are actually two major errors in the A&W paper. The first major error leads to a faulty result and the second major error is stimulated by an attempt to rectify the result of that first error.

A. A&W first major error—improper use of the equivalence principle

The Ashby and Weiss alternative solution to the problem is, they say, "…based on fundamental relativity principles." The authors assert the equivalence principle to support their claims but pay little regard to the physical basis of the equivalence principle. They do correctly state the equivalence principle as: "Over a sufficiently small region of space and time the effect of acceleration cannot be distinguished from a real gravitational field." But as we shall see, they ignore the implied limit on a small interval of time.

They accept the above and claim that the solar acceleration of the earth creates an effective gravitational field in the earth's vicinity that cancels the linear component of the actual solar gravitational field. Such a claim depends upon a long history of misunderstanding of the physics involved. In that light, it is appropriate to review a bit of history of the equivalence principle.

1. An equivalence principle detour

The first major issue in the A&W paper is the appropriateness of ascribing acceleration effects as equivalent to gravitational potential effects.

This problem with the equivalence principle can be traced back to an early misinterpretation by Einstein[11] which was repeated more recently by Feynman.[12] Indeed several contemporary relativists have noted the discrepancy but have either dismissed it as insignificant or have given poor reasons to excuse it. The problem is revealed in a review of Einstein's and Feynman's development of the equivalence principle.

The problem is illustrated by considering the rocket example which Feynman used to explain how Einstein applied the equivalence principle to argue that clocks (apparently) run faster at higher gravitational potentials. Assume an accelerating rocket that has two clocks designed to run at the same rate, f_0, with clock A at the front and clock B at the rear. Further, assume that clock A's frequency is transmitted to clock B and there compared with Clock B's frequency. Assume also: (1) that the distance between the two clocks is H; (2) that the speed of light between the two clocks remains at c in the frame in which the rocket was initially stationary; and (3) that the velocity of the rocket remains much less than c during the time interval of interest. Under these conditions, the speed of the rocket will change during the transit time, τ, of the clock signal from the front to the rear and the frequency will appear to be higher by the induced (first order) Doppler shift caused by that change in velocity. Specifically, the frequency received (subscript r) will be larger than the frequency of the rear clock B (subscript zero). Thus,

$$f_r = f_0\left(1+\frac{a\tau}{c}\right) = f_0\left(1+\frac{aH}{c^2}\right) \tag{36}$$

Einstein then argued by his equivalence principle that if the acceleration is g, as in the gravitational field at the earth's surface, that clocks must also (appear) to run faster in a gravitational potential as a function of the height separation, e.g.,

$$f_r = f_0\left(1+\frac{g\tau}{c}\right) = f_0\left(1+\frac{gH}{c^2}\right) = f_0\left(1+\frac{\Delta\Phi}{c^2}\right). \tag{37}$$

But Einstein and Feynman each claimed that both the acceleration effect and the gravitational potential difference effect (falling blue shift) *actually occurred during the signal transit process* so that both clocks A and B continued to run at the same true rate in each instance. They each supported this claim with three illustrations and the claim is critical to the validity of the equivalence principle as used by A&W.

2. Direct clock effects of gravitational potentials cannot be cancelled by acceleration effects

However, evidence is readily available from the GPS system[5] (and from the prior TRANSIT system) that in a gravitational potential clock A at a higher potential actually runs faster than clock B at a lower gravitational potential, i.e., the gravitational potential effect is a direct clock rate effect and is not an artifact (blue shift) of the signal in transit. By contrast, the Doppler effect from acceleration is clearly an effect which occurs during the transit of the clock signal and does not affect the two clock rates themselves. The important difference is that the gravitational potential effect is integrated into the clock readings (generating clock time biases) while the acceleration induced Doppler effect does not affect the receiving clock frequency or time. This significant difference in the clock time reading clearly invalidates the use of the equivalence principle to ascribe acceleration effects as "equivalent" to a gravitational potential. In fact, the limit of the equivalence principle to an "infinitesimal" (or nearly so) time interval clearly invalidates the use of the principle within the GPS system, since the GPS satellite clock frequency is integrated continually to give the satellite time. Thus, even otherwise tiny frequency offsets are integrated into significant clock biases in the GPS satellite time.

This rather heretical claim that the equivalence principle does not allow a cancelation of direct clock effects (gravitational potential) with acceleration induced signal path effects has such strong significance it is worth exploring it in some detail.

Clifford Will[13] realized there was evidence that a gravitational potential difference created a clock rate difference rather than a frequency shift in transit. He says:

> A question that is often asked is; Do the intrinsic rates of the emitter and receiver of the clocks change, or is it the light signal that changes frequency during its flight? The answer is that it doesn't matter. Both descriptions are physically equivalent. Put differently, there is no operational way to distinguish between the two descriptions… the observable phenomenon is unambiguous: to ask for more is to ask questions without observational meaning.

Will actually goes on to admit that one can take one clock to a higher potential, leave it for a while, and then bring it back down and compare it with a second clock that was not moved. Thus, he admits that it was the clock rate which changed, but since it was "after the fact" and cannot be known in real time, Will claims that the ambiguity remains. (This is strange logic! Perhaps Will recognizes that acknowledging the obvious challenges the equivalence principle as well as General Relativity theory.[14]) Using GPS, we can actually show in real-time that a gravitational potential causes a clock rate change and that the signal in transit does not change in frequency as a result of the potential difference between source and receiver.

Ashby and Spilker's[15] logic is also faulty. They state:

> Second, the *strong equivalence principle* implies that light traveling downward in a gravitational field is shifted to a higher frequency: i.e., it is blue shifted and gains energy. As a consequence, atomic clocks at a higher elevation in a gravitational field run faster.

One must ask after this quote: Which is it? If clocks run faster, falling radiation does not gain energy else the effect on the received signal would be doubled. Thus, their statement is self-contradictory. Ashby[16] makes another ambiguous statement elsewhere when he states: "The negative sign in this result means that the standard clock in orbit is beating too fast, primarily because its frequency is gravitationally blue shifted." Again we must ask, Does Ashby believe it is the frequency of the clock that is blue shifted or the signal in transit that is blue shifted?

In the context of Ashby's second statement above, he explains that the satellite clock frequency is adjusted lower before launch to counteract the gravitationally induced increase in frequency. (The orbital velocity effect on frequency is also removed by a lesser adjustment, higher in frequency before launch.) That the satellite clocks keep the correct time (quite closely) orbit after orbit shows that the adjustment was necessary to correct the clock frequency itself (not a blue shift in transit) in order to agree with the time of the earth based clocks.

It is also true that with GPS, we can clearly in real time (contrary to Will's claim) show that the emitted GPS clock frequencies are ***NOT*** blue shifted in transit or gain energy (contrary to the Ashby and Spilker claim and also contrary to Einstein's and Feynman's original assumptions) as the signals move downward in the gravitational potential of the earth. This is quite easy to do. The GPS code or pseudorange measurements are based upon a timed code rate (frequency) which allows the receiver to compute the apparent transit time (multiplied by the speed of light to give the range) between the satellite clock and the receiver clock. If the frequency increased in transit, the measurement would be somewhat larger and appear as if the range were increased fractionally. But carrier phase measurements are also available which allow the satellite clock frequency to be compared with the receiver clock frequency together with the path Doppler effects. By integrating these carrier phase measurements one obtains a measure of the range change over the integration interval. Because the phase measurements are more precise than the code measurements, they can be used to smooth the code measurements to obtain more accurate range measurements. This process is often referred to as a Hatch filter and is used in the majority of GPS receivers. But if the frequency suffered a blue shift from the satellite to the receiver the carrier phase measurements would cumulate that measurement into larger and larger changes in range as the integration time was increased. This would cause a divergence between the code and carrier phase measurements and the Hatch filter would result in decreased accuracy rather than an increased accuracy in GPS receiver positioning.

The earlier TRANSIT navigation system also showed that the frequency did not increase in frequency as the signal "fell." The system depended upon sequential integrals of the received Doppler shift in the frequency due to the motion between the satellite and receiver. Its accuracy depended upon every cycle transmitted being received and only those cycles being received. But if the signal frequency increased as the signal fell extra cycles would have been received. Such extra cycles would have aliased into an extra Doppler shift and thereby corrupted the navigation accuracy.

Finally, if one needs more evidence, millisecond pulsars[17] external to the solar system can be used as a very stable frequency source. Indeed they show that in fact clocks on the earth do run at slightly different rates at noon and midnight due to the linear component of the solar gravitational potential.

Recapping the evidence in the rocket example, accelerations affect the apparent clock rates via the Doppler effect caused by the change in velocity during the transit time. This is distinct from the change in the actual clock rate due to a higher gravitational potential. The former does not integrate into the clock time, the latter does. This means that an acceleration effect cannot be modeled as a gravitational potential effect. Thus, the equivalence principle is falsified.

Ashby and Weis compute the gravitational frequency shift caused by the solar gravitational potential upon GPS clocks and they even properly ascribe it to a changed frequency of the GPS clocks themselves. They then improperly claim that such large effect would be seen if it were actually present. But they are wrong. The acceleration Doppler effect is a signal path effect and does cause the increased frequency at the satellite to be cancelled at the receiver. But the satellite clock frequency is increased and it integrates into a clock bias as describe above in Section II A 4 above. By contrast, acceleration of a receiver does have an effect upon the received clock frequency due to the Doppler effect of the change in velocity during the transit time of the signal, but it does not integrate into the clock time.

The use of the equivalence principle by both Hoffman and by A&W can be contrasted. Hoffman attempted (improperly) to use the gradient of the orbital velocity effect (of a gravity-gradient stabilized frame) to cancel out the effect of the gradient of the gravitational potential effect upon the clocks. Since both of these two effects are true clock frequency effects his use of the equivalence principle was not a major error other than the fact that a spin stabilized frame does not encounter any pure differential orbit velocity. The equivalence principle was simply unnecessary. Hoffman did not address the acceleration Doppler effect at all. A&W by contrast correctly rule out any gradient of the orbital velocity effect upon clocks in a spin stabilized frame. But then they improperly use the equivalence principle and an acceleration Doppler effect (which only affects the receiver clock frequency measurement and has no impact on the real clock frequency at either transmitter or receiver) to cancel out a real satellite clock frequency arising from the gradient of gravitational potential.

By assuming acceleration causes a gravitational potential that cancels out the true gravitational potential, Ashby and Weiss effectively assume that both the satellite clock frequency and the receiver clock frequency are affected equally by the two phenomena (potential and acceleration effects), i.e., both are path induced effects only or both are direct clock effects only. Thus, they falsely cancel out the true frequency offset at the satellite and fail to recognize that the gravitational potential induced frequency offset remains present in the satellite clock and integrates into the satellite clock time even though it appears at the satellite receiver to be cancelled.

When the solar gravitational potential effect upon GPS and receiver clocks is properly handled (i.e., not cancelled out by improper use of the equivalence principle), the induced frequency offset is integrated into a clock bias that results in the apparent relativity of simultaneity and causes the apparent local speed of light to be c relative to the earth as was developed above in Section II.

B. A&W second major error—Improper use of the relativity of simultaneity

By using the equivalence principle to create in effect a counteracting gravitational potential A&W falsely cancel out the gravitationally induced change in the clock frequencies. This means that they require an alternate explanation for the generation of the clock biases needed to make the speed of light appear to be isotropic on the earth. Their solution in Section III of their paper is to reverse cause and effect and claim that the speed of light induces the required clock biases. In other words, assume the speed of light is locally c independent of any physical effect upon the clocks and from that assumption prove that a clock bias must exist. This is an example of reverse reasoning; assume the answer and then derive the implied physical result of that assumption.

So how do they get the result that they want? They improperly map the acceleration into an equivalent gravitational field and use a sequence of instantaneous inertial frames, in which they *imagine* time synchronization signals. It is faulty use of the equivalence principle to claim that the speed of light automatically adjusts to a value of c in a locally accelerated frame.

Here, we wish to quote several sentences of their argument:

> As the earth orbits around the sun, earth's velocity continually changes. We can describe the relativistic effects by introducing a succession of instantaneous inertial frames, each with its origin at earth's center, and having velocity V equal to earth's instantaneous velocity, and maintaining axes in fixed directions relative to the stars. In each one of these instantaneous frames, we can ***imagine*** the process of synchronization of a GPS clock is continually being repeated, or is being repeated as often as necessary.

The italics and bold print were not in the original.

Two different mechanisms above have been proposed to explain the relativity of simultaneity in the earth's frame. Obviously, both cannot be correct—one must be wrong. I will use "IIFM" for instantaneous inertial frame mechanism to designate the mechanism which Ashby and Weiss have proposed. I believe the true mechanism is due to the time biases which result from the integration of the solar gravitational potential induced frequency offsets as described in the first part of this paper. I will use GPM for gravitational potential mechanism as shorthand for this effect. Two arguments against the A&W IIFM argument are given below. The first simply uses an argument by analogy. The second argument digs into the details of what they propose a bit more to identify evidence of faulty logic.

1. *First argument using analogy of the gravity-gradient stabilized frame*

In Section II above, it was shown that the gravity-gradient stabilized frame does not generate any automatic clock biases, which is the same result A&W get by an improper use of the equivalence principle. The moon and the GRACE satellites were used to illustrate such a frame. It was argued in Section II that a manual synchronization could be performed which would allow the use of either synchronization to the earth's frame or synchronization to a new rotating frame.

The problem with the A&W mechanism is that it could always be applied and thereby rules out any choice in the manual synchronization process. By always requiring the speed of light to be c in the moving frame and claiming that the speed of light itself somehow creates the required clock biases, A&W in effect always require that the speed of light be c in any moving frame. This defies any logical mechanism to set the clock biases for that required result.

2. *Second argument tracing the implications of the IIFM process*

The second argument against the IIFM is that it contradicts the fact that acceleration has never been shown to cause a true change in clock frequency. Yet, what is being proposed is that the acceleration does cause two separated clocks to run at different rates in order to create a clock bias required for the relativity of simultaneity and cause the speed of light to appear as c in the earth's frame. Actually, what is being proposed is an imagined synchronization signal that is sent at the speed c in the local instantaneous frame which adjusts the clock time to include the needed biases. Note that the instantaneous frame is an infinitesimal frame because the acceleration is constantly changing the direction of the earth's velocity. How is a signal sent at a speed of c across an infinitesimal distance used to synchronize the two clocks an infinitesimal distance apart?

The details of my second argument against IIFM rests upon Friedman's[18] claims:

> Thus, although physics texts often claim that freely falling frames are "locally" equivalent to inertial frames; this assertion is strictly false if "local" has its usual mathematical meaning… Freely falling frames are only "infinitesimally" equivalent to inertial frames: only at a single point or on a single trajectory. (Of course, freely falling frames *approximate* inertial frames on a neighborhood of σ: the smaller the neighborhood, the better the approximation.)

Note that the italics and parentheses are in the original.

From Friedman's logic Ashby and Weiss' use of instantaneous inertial frames to map the acceleration effects are, in fact, equivalent to using a sequence of infinitesimal Lorentz transformations. This argument is supported by their Eq. (18) followed by the comment about instantaneous inertial frames cited above. This allows us to consider their arguments for the validity of an IIFM as an argument for the validity of Infinitesimal Lorentz Transformations (ILTs).

So are ILTs valid? I am aware of only one single piece of evidence that supports the conclusion that ILTs are valid. Specifically, ILTs are used by Goldstein in his text on Classical Mechanics[5] to explain the Thomas precession of the electron. Goldstein's explanation for the motivation of the ILT is stated as:

> Consider a particle moving in the laboratory system with a velocity v that is not constant. Since the system in which the particle is at rest is accelerated with respect to the laboratory, the two systems should not be connected by a Lorentz transformation. We can circumvent this difficulty by a frequently used stratagem (elevated by some to the status of an additional postulate of relativity). We imagine an infinity of inertial systems moving uniformly relative to the laboratory system, one of which instantaneously matches the velocity of the particle. The Particle is thus instantaneously at rest in an inertial system that can be connected to the laboratory system by a Lorentz transformation. It is assumed that this Lorentz transformation will also describe the properties of the particle and its true rest system as seen from the laboratory system.

However, Muller[19] provides an alternate explanation for the Thomas precession that obviates the requirement for the validity of the ILT. Muller came up with an alternate explanation for the Thomas precession because he was puzzled by the fact that ILTs seemed to cause a rotation without any torque being applied. So he found a mechanism that supplied the torque—but falsely assumed it was equivalent to, i.e., not different from, the ILT explanation. If both explanations were valid, the Thomas precession effect would be doubled. Since ILTs can only provide a Thomas rotation via a torque free process, the preponderance of evidence is that Muller's alternate explanation is valid and that ILTs are invalid.

Rather than ILTs to handle accelerations, a more limited hypothesis from Goy[20] is more consistent with the concept that accelerations can never affect a clock's frequency directly, let alone cause two separated clocks to run at different rates. Goy calls his hypothesis the "clock hypothesis" in the quote below.

> The "clock hypothesis" states that the rate of an ideal clock accelerated relative to an inertial frame is identical to the rate of a similar clock in the instantaneously commoving inertial frame. With other words, the rate of clocks is not influenced by accelerations per se, when seen from inertial frames. It is also supposed that real clocks exist in nature, which approach the conditions of the clock hypothesis. To our knowledge, this assumption was first implicitly used by Einstein in 1905[8] and was superbly confirmed in the CERN muon storage ring experiment,[14] where the muons had a time decay depending only on their velocity (in agreement with the time dilation formula) despite the fact that their acceleration was 10^{18} g.

(Note that Goy's Refs. 8 and 14 appear as Refs. 21 and 22, respectively, in this paper.)

Reiterating, acceleration has never been shown to affect clock rate directly. Yet with their IIFM Ashby and Weiss are asking us to believe that if we go to instantaneous, i.e., infinitesimal, frames and ***IMAGINE*** a synchronization signal, we can cause two separated clocks to achieve the required inter-clock bias to cause the speed of light to be c. They claim this without telling us how the imagined synchronization signal could actually cause the clock time to be changed. I know of no infinitesimal clock adjustment mechanism which the synchronization signal can affect. The normal adjustment mechanism would be the frequency, but we know that acceleration does not affect a clock's frequency.

VI. CONCLUSION

As developed in the first part of this paper, the earth spin rate compounded with the orbital velocity cause a frequency variation which integrates into a cyclic clock bias with a sidereal day period. It was shown that the hidden effect of the solar gravitational potential effect upon clock rate integrates into an annual cyclic clock bias which converts the sidereal period of the clock bias into a daily clock bias. The result of the combined clock biases is to cause the apparent relativity of simultaneity and the apparent speed of light to be c in the earth's frame as the earth orbits the sun. By contrast Ashby and Weiss have improperly used the equivalence principle, independent of its physical basis, to claim the relativity of simultaneity in the earth's moving frame. There is no physical basis for acceleration to affect the clock rate. However, acceleration does induce a Doppler effect that causes the solar gravitational clock rate difference to be obscured and not show up in the direct measurements. Evidence has been given supporting the gravitational potential effect of the sun and against an acceleration equivalence to gravitational potential. While A&W arrive at the correct result, they do so by employing a second error to counteract the first error in their derivation. It is important for progress to get the physics correct. Accelerations do NOT generate a counteracting gravitational field—acceleration effects do not integrate into the clock time while potential effects do. The integration effects violate the infinitesimal or limited time intervals required by the equivalence principle.

Perhaps the most compelling evidence against the A&W mechanism is that it contradicts the results of Section II above which reveals that the Lorentz transformation contains hidden scale factors between parent and child frames. In addition, the scale factor differences imply the existence of an absolute frame. The apparent Lorentz transformation name change is proposed to clarify these differences. The Ashby and Weiss mechanism implies that the numerical invariance of the speed of light is also a physical invariance. This defies the clear evidence of change in time units, independent of the evidence for length contraction.

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