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Phase Considerations in a Rotating System

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ABSTRACT

A neutron interferometer in constant absolute rotation will exhibit a certain phase-shift between its two beams, a phenomenon shared with the classic Sagnac or Michelson-Gale-Pearson experiments or with the modern laser-gyrocompass composed of lasers in a ring. To first order in the rotational frequency it is possible to understand by employing only rudimentary theory the essence of this phenomenon to any degree of relativisticness of the participating particle. This paper is mainly paedagogical, noting the similarity anent rotation between photon-, electron- and neutron-interferometers. Future experimentation, aside from corroborating well believed tenets, may hope with improving precision to bring new approaches to measurement of fundamental effects.

Introduction

The sensitivity achieved in recent years with neutron interferometers (Bonse & Hart 1965; Bonse & Hart 1966; Rauch, Treimer and Bonse 1974) (Overhauser & Colella 1974; Colella, Overhauser & Werner 1975; Werner et al 1975) recalls to mind the classic experiment (Sagnac 1913) in which an optical interferometer encompassing an area $\sim 10^3$ cm^2 was rotated at several revolutions per second resulting in a discernible phase-shift between two beams, the one rotating progressively, the other retrogressively. Likewise Michelson, Gale and Pearson (Michelson 1925) employed an optical system embracing some 10^5 meter^2 fixed to the earth and demonstrated the earth's rotation with respect to the fixed stars by means of the similar phase-shift. In this paper we examine such phase-shift in a paradigmatic interferometer letting the beam particle be alternatively non-relativistic, mediumly relativistic, or completely relativistic (as with the photon). The theory is especially simple if we restrict attention to a response linear in the rotation frequency, Ω_0 rad/sec.

The paradigm

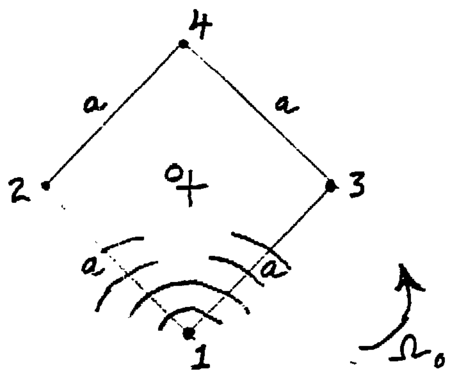

Fig. 1

Arranged in a square for simplicity, we consider a sender of waves at position (1), the ultimate receiver at (4), with identical transceivers at positions (2) and (3) as depicted in Fig. 1. The discussion is kinematic only; the dynamic details of eg how the sender produces two coherent beams, how the emitters (2) and (3) work, are not specified. The four active elements have only small extent with respect to dimension a.

To obviate time dilation, Lorentz contraction and the like we require the speed parameter $\Omega_0 a/c$ to be negligible in second order; and to obviate direct consideration of aberration or transverse Doppler shift we require the relevant phase velocity to greatly exceed the speed $\Omega_0 a$. With relativistic Schroedinger waves or with elĕctromagnetic waves in vacuum the first (mild) restriction implies the second. Nonetheless the seat of the phase-shift is in fact special relativity as will be explicitly shown. Looking forward to forming wave-packets we stipulate that the group velocity greatly exceed $\Omega_0 a$ to minimize centrifugal effects.

We might term the 'Mach Lab' that inertial system in which our interferometer (Fig. 1) is rotating anticlockwise at fixed Ω_0 about its centre 0. We note the effective absence of a Doppler shift; thus in virtue of the constant angular velocity and the symmetry all four active elements send/receive at the same frequency namely that at which (1) emits in its own comoving system.

Photon, viewed in Mach Lab

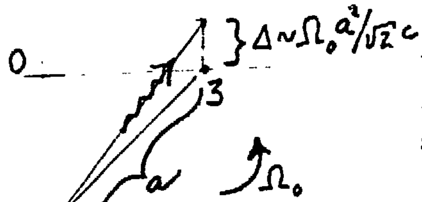

Fig. 2

To first order in $\Omega_0 a/c$ we see that the path from (1) to reflector (3) is increased from a to $a(1 + \frac{a\Omega_0}{2c})$; and similarly the path from (3) to destination (4). Inasmuch as (4) receives non Doppler shifted light via path 3 just as via path 2 we end up with the phase-shift, path 3 minus path 2,

$$\Delta\phi_{32} = + \frac{2}{\hbar}\left(\frac{\hbar\omega}{c^2}\right) a^2 \Omega_0 \quad \text{, a well known result.}$$

Notice that this same result would apply for an extremely fast particle of relativistic mass $M = E/c^2$. In which case we can write

$$\Delta\Phi = \frac{2M}{\hbar} a^2 \Omega_0 \qquad \ldots\ldots\ldots 1$$

Technically E is the particle's energy in a momentarily comoving system for any of the stations (1) through (4) for that portion of trajectory with which the given station deals of course.

Wave of any phase velocity, viewed in certain comoving system(s)

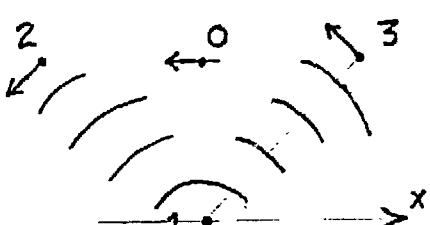

Fig. 3

At the instant in the Mach Lab when sender (1) is moving just along x we make a Lorentz transformation along x at $\tilde{\beta} = \Omega_0 a/\sqrt{2}c$, which makes the speed of (1) to be zero but not its acceleration, which latter is to be ignored to first order in Ω_0. This Lorentz system can be called CM-1 since momentarily at least it comoves with (1).

In the system CM-1 we note that whatever the phase velocity (be it c or larger) transverse Doppler effect and aberration are to be ignored by our earlier stipulation. Since (see Fig. 3) the longitudinal Doppler effect is absent, both (3) and (2) receive a given wavefront at the same time in CM-1. However, and this is essential, such $\Delta t_{CM-1} = 0$ corresponds to a $\Delta t_{Mach} \cong +\tilde{\beta} \frac{\Delta x_{CM1}}{c}$ which amounts to $\Delta t_{Mach} = \frac{a^2 \Omega_0}{c^2}$.
We may now send both reradiated signals on their way to (4). The time offset* due to transforming from CM-1 to CM-4 amounts to $2 \frac{a^2 \Omega_0}{c^2}$ sec. The energy (frequency) received at (4) is in our approximation equal for both paths and the paths are of equal length; consequently the phase-shift $E \Delta t/\hbar$ is $\Delta\phi_{32} = 2\frac{M a^2}{\hbar}\Omega_0$ which is just expression 1 again. However the expression has just now been shown to be valid over the range from quite slow heavy particles to photons (or neutrinos!), since M stands for the relativistic mass.

One contemplates of course assembling a wave-packet to be split somehow at (1) and sent along the two paths; understandably then $\Delta\phi$ is not completely sharp but is fuzzed according as $M(k)$ reflects the distribution in k. In the extreme non relativistic domain $\Delta\phi$ does become sharp for the whole packet.

It can be shown that for any polygon, regular or otherwise, expression 1 holds with a^2 replaced by ~~projected~~ area. The factor two applies when the loop is traversed but <u>once</u>, thus half-way round for each signal as in our paradigm. Expression 1, restricting to nonrelativistic particles, neutrons, was given earlier (Page 1975) on the basis of some simple hand-waving. A more formal treatment (Anandan 1977) including gravitational as well as rotational effects yields a ~~the~~ relativistic correction XX (a first correction) to the rest-mass M_0 as ~~XXXX~~ $p^2/2M_0$ insofar as the rotation effect in first order is concerned. The factor two standing in expression 1 is reproduced in Anandan's results.

Before putting some sample numbers into expression 1 it might be

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* Reminiscent of other examples in Physics such as the Thomas precession we may not disregard this offset simply because we see c^2 in the denominator, even for extremely slow motion of apparatus or particle.

XX compatible with the present result.

of some interest to run through an extremely simple hand-waving argument by which one arrives at the correct relativistic expression for phase-shift.

<u>A free wave-packet in a slowly rotating system</u>

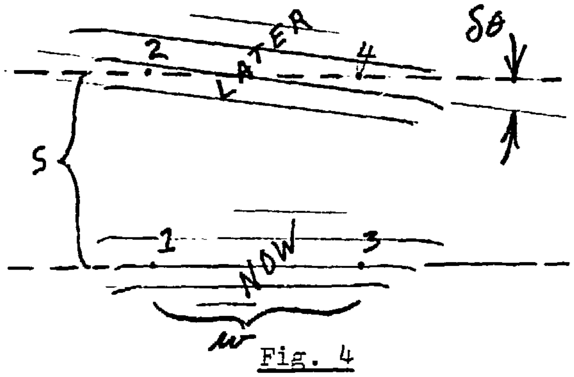

Fig. 4

We picture an unconstrained essentially plane wave-packet travelling in a quasi-inertial system which rotates very slowly at frequency Ω_0 with respect to the Mach system. We make the sensible requirement that if we were to follow the course of the packet for a time it must become aware of a slanting angle $\delta\theta$ (see Fig. 4) which evolves with time as $\delta\theta = 2\Omega_0 t$.

We relate nominal distance travelled s to time t via $t \sim s/\text{group speed} \sim \frac{s\tilde{M}}{\hbar k}$. We attribute the required veering of a given component of the group (component of wave-number k) to the fact that the phase difference Sta. 4 minus Sta. 3 viz ϕ_{43} exceeds the phase difference ϕ_{21}.

Thus $\phi_{43} - \phi_{21} \equiv \Delta\phi_k = k(w\,\delta\theta) \cong 2\frac{\tilde{M}}{\hbar}(ws)\Omega_0$, qed,

where $\tilde{M}$ stands for the relativistic average mass for the packet. The plausibility of this argument seems best when $(\hbar k/M_0 c)^2$ is not too large.

Some confidence in this kind of simple argument might be gained from this brief digression: Consider a charged particle bending mildly in a magnetic field B. The Larmor frequency is $qB/2Mc = \Omega_0$. In the language of Fig. 4, $\delta\theta = 2\Omega_0 t$ as before. Finally $\Delta\phi = (w\,\delta\theta)k = \frac{qB}{Mc}\frac{sM}{\hbar k}wk \Longrightarrow q/\hbar c \oint \vec{A}\cdot d\vec{r}$.

This forces us to conclude, at least in this gently bending situation, that the wave-number (times $\hbar$) has to be the normal kinetic momentum <u>plus</u> qA/c—the result of course being no surprise since we have in effect simply supplied the Lorentz force; but we do note that no 'factor of two' is lost or gained in this simple treatment. A corollary of this magnetic bending argument (running the argument backwards) is that a fast charged particle traversing magnetic matter has to have its (gentle) bending governed by field B and not H; this is generally assumed to be borne out experimentally— even though the particle may not enjoy physical access, so to say, to all or any of of the flux of B.

Size of the Rotational Phase-shift

We have seen that the size of the rotational phase-shift is proportional to (Relativistic Mass) times (Area) times (angular velocity with respect to the local Mach system). We may substitute photon mass in the same expression we use for particles.

One may compare just roughly the product $MA\Omega_0$ for ~~these~~ several experimental situations: Sagnac (S) who used 3 ev/c^2 photons around an area $\sim 10^3$ cm^2 with a reversible speed of ~ 10 rad/sec obtained a nominal 0.1 fringe shift; Michelson et al (MGP) went to 10^6 times the area of (S), used the unreversible angular speed of the earth and with difficulty achieved about twice the fringe shift of (S); the two experiments were compatible with each other and with known wavelengths, speed of light and known angular speed at the precision level of a couple per cent. (We can observe that the reciprocity between area A and Ω_0 has been well vindicated!) Neither experiment used Planck's constant explicitly of course. They could both be explained on the basis of a circulating "aether wind" if one so chose.

Considering now the possibility of rotating a Colella-Overhauser-Werner type of experiment using slow neutrons, the mass of 10^9 ev/c^2 is certainly favorable over the classic experiment (S) but there may be difficulties in rotating a spectrometer system substantially faster than the earth affords. Turning to electron diffraction the mass factor is some 10^5 times better than an optical photon; if a system of area one or two cm^2 could be rotated at a few revolutions per second one might hope for tens of fringe shifts.

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