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Short Note

The normal vertical gradient of gravity

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INTRODUCTION

Most gravity surveys are conducted to estimate subsurface density contrasts for one application or another. From large-scale crustal studies to relatively small exploration surveys, it is necessary to determine in some way what the normal gravity field should be in order to identify anomalous features. The anomalies then represent deviations to be interpreted in light of the original model. It is a central limitation of potential field methods that this model, sometimes representing a so-called "regional" field, is not unique. In the case of gravity, this model has traditionally involved geometrical approximations. It is generally assumed that variations in station elevations are small compared with the radius of the earth—an obviously excellent approximation, but one needs to be mathematically consistent.

For example, using the traditional infinite slab Bouguer effect as a first-order terrain correction and then applying topographic data as a second correction results in a station-dependent truncation at arbitrary distances (Danes, 1982). This planar geometry does not include more distant terrain and furthermore is not an approximation to the spherical earth to any order (Karl, 1971). I show that this distant terrain is, indeed, important.

Our approach is to avoid infinite planar geometry and to include distant terrain effects via spherical geometry. Locally, planar geometry can still be used to estimate the effects of nearby terrain.

THE VERTICAL GRADIENT

The two equations describing the gravitational field, in general, are

$$\nabla \cdot \mathbf{G} = 4\pi\gamma\rho(r)$$

and

$$\nabla \times \mathbf{G} = 0,$$

where positive $\mathbf{G}$ is radially inward. By introducing the regional field $\mathbf{G}_R$ and the anomalous field $\mathbf{G}_a$, the first equation above reads

$$\nabla \cdot (\mathbf{G}_R + \mathbf{G}_a) = 4\pi\gamma\rho(r).$$

Next I require that the regional field have only a radial component and average this equation over the surface of the earth at some reference radius R_0, giving

$$\frac{2\bar{g}_R}{R_0} + \overline{\frac{\partial g_R}{\partial r}} + \overline{\nabla \cdot \mathbf{G}_a} = 4\pi\gamma\overline{\rho(R_0)},$$

where the bar indicates this average and g_R is the radial component of the regional field. Because ρ represents the total averaged mass density, there is, on the average, no remaining source for the anomalous field, i.e.,

$$\overline{\nabla \cdot \mathbf{G}_a} = 0.$$

Thus the normal vertical gradient is

$$\overline{\frac{\partial g_R}{\partial r}} = -\frac{2\bar{g}_R}{R_0} + 4\pi\gamma\overline{\rho(R_0)}.$$

The first term is extremely close to the familiar elevation correction. The second term arises from the fact that the gravity stations are partially "inside" the earth because of a spherical shell partially occupied by mass from the topographic relief. This term is station-independent, substantial in effect, and easy to estimate. While realizing that the definition of the regional field is fully arbitrary, I argue that the two assumptions of (1) only radial components, and (2) its source is the total net mass are most suitable. In fact, these are the same assumptions made in the usual calculation of the free-air term. An estimate for $\bar{\rho}$ is obtained from hypsometric data. For example, for a reference datum near sea level about 20 percent of the earth's area has mass above that level (Wyllie, 1971). Thus $\bar{\rho} \cong 2.67 \times 20$ percent $= 0.534$ and $4\pi\gamma\bar{\rho} = 0.0447$ mgal/m. Thus near sea level, this additional effect produces about 14 percent lower vertical gradient than expected from the normal free-air term above. Approximately such a reduction has been reported in the Eastern Mediterranean by Hammer (1970). For large variations in station elevations, the hypsometric data may have to be integrated to give this correction sufficiently accurately for practical purposes.

This approach has made distant terrain corrections appear more like a free-air term. The proper terrain correction to use in this approach is the difference between the standard numerically integrated terrain effect within some volume, say out to Hammer's

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zone M and the contribution from $\bar{\rho}$ within the same volume (Hammer, 1939). Planar geometry can be used for this.

DISCUSSION

Because the gravitational force has infinite range, this additive $\bar{\rho}$ term is substantial and can profoundly affect interpretation both in large-scale crustal studies and smaller exploration surveys on the ground, in the air, and in boreholes. Its contribution is opposite to the normal free-air correction and hence it reduces the positive correlation between free-air anomalies and topography that have been observed worldwide. For example, Figure 1 shows gravity data from the Continental Divide (Woollard, 1962) along a line from Gypsum in Colorado to Hein in Nebraska. The free-air anomaly using our approach shows less correlation with topography and coincides more with the regional trend.

The term $4\pi\gamma\bar{\rho}$ also has significant implications for exploration targets in mountainous areas. For example, topography in the Appalachian ridge and valley province would produce 14 mgal from a 300 m relief due to this additional effect. The small anomalies commonly produced by thrusts of less dense Ordovician shales against more dense middle Mississipian units could be easily confused by improper elevation corrections. It would be interesting to compare the interpretation of gravity data from mountainous areas using both the standard data reduction method and the one suggested here. Certainly the results would be quite different. Unfortunately, the $\bar{\rho}$ effect cannot be avoided even by using direct forward modeling to interpret uncorrected observed data.

Density determinations by borehole gravity measurements respond directly to $\bar{\rho}$ modified by local variations caused by forma-

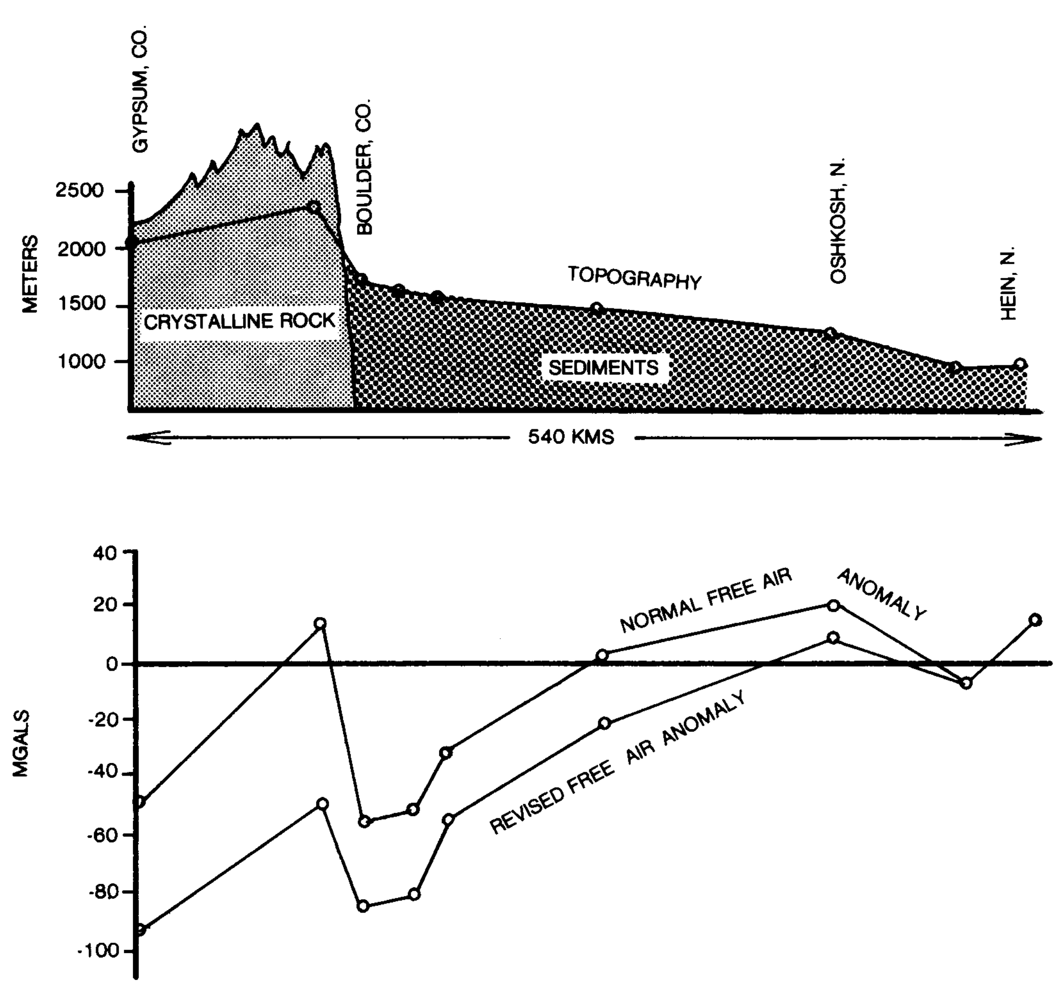

FIG. 1. Above: Section across the Rocky Mountain Front through Boulder, Colorado showing topography and station elevation. Below: The free-air anomaly both computed by the standard method and revised to include global terrain effects. This additional effect is a contribution $4\pi\gamma\bar{\rho}$ to the normal vertical gradient because of the spherical shell passing through the station location. This shell is partially occupied by mass from global topographic relief. For demonstration purposes, ρ has been approximated as 0.534. The original data are from Woollard (1962). The additional elevation effect reduces the correlation between free-air anomaly values and topography. This revised free-air curve also shows more agreement with regional gravity.

tion densities and terrain. Unless such measurements are corrected for this distant terrain effect, resulting formation densities will show systematic errors. Gibbs and Thomas (1980) reported such a discrepancy for two sets of measurements made in deep gold-mine shafts in the Archean Yellowknife greenstone belt in the Northwest Territories. On the other hand, McCulloh (1965) reported excellent agreement between density measurements made on core samples from a limestone mine in Ohio and values determined from gravimetry data. In any case, relative changes in densities over reasonable distances in a borehole are valid without any knowledge of the "normal" vertical gradient. The calculation of absolute formation densities from borehole gravity would, however, require a knowledge of $\bar{\rho}$.

The new airborne gravity (Hammer, 1982) could be used to shed further light on this $\bar{\rho}$ effect. In fact, for exploration purposes the greatest advantage of airborne gravity may turn out to be its ability to produce data from a fairly constant elevation, thereby reducing the $\bar{\rho}$ effect to a negligible amount. Direct forward modeling can be used in this case. On the other hand, long traverses made at different elevations might be used to estimate localized contributions to $\bar{\rho}$.

The normal vertical gradient of gravity presented here is the result of global effects, and in many problems more localized influences may have to be included. Regular gravity surveys made over areas of low topographic relief have been successful because anomalies have stood out against a regional field. Presumably, the same would be true of vertical gradient measurements. However, in cases where one wishes to use calculated vertical gradients as opposed to measured regional values, there is no obvious solution. Certainly, use of the traditional free-air term is difficult to justify.

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