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On Hertz's Invariant Form of Maxwell's Equations

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Abstract

The failure of Maxwell's equations to exhibit invariance under the Galilean transformation was corrected by Hertz through a simple, but today largely forgotten, mathematical trick. This involves substituting total (convective) time derivatives for partial time derivatives wherever the latter appear in Maxwell's equations. By this means Hertz derived a formally Galilean-invariant covering theory of Maxwell's vacuum electrodynamics - which, however, was not space-time symmetrical (in view of his tampering with the time but not space derivatives). Had Hertz's mathematical accomplishment received wider recognition, his invariant covering theory of Maxwell's could have furnished the formal key (almost two decades before Minkowski's "covariance") to unification of the "relativistic" properties of electrodynamics and Newtonian mechanics, explanation of the Michelson-Morley result, etc. The task of finding a viable physical interpretation of the Hertzian convective velocity parameter - which Hertz himself did not live to accomplish - remains for continuing research. We discuss this and related matters and give an explicit proof of invariance.

Key words: Hertzian electromagnetism, Galilean invariance, Maxwell's equations, covariance, total time derivatives

1. INTRODUCTION

There is considerable confusion in the literature about invariance and covariance of the equations of electromagnetism. For example, a paper by Jammer and Stachel(1) states that "if one drops the Faraday induction term from Maxwell's equations, they become exactly Galilei invariant." We shall demonstrate here, on the contrary, that (1) it is not necessary to drop the Faraday induction term, or any other, from Maxwell's vacuum electrodynamics in order to achieve exact Galilean invariance, provided one exploits a mathematical theme due to Hertz(2); (2) if one does drop the Faraday induction term in the manner of Jammer and Stachel, the resulting equations exhibited by them are not "exactly Galilei invariant," but might more properly be termed "Galilei covariant."

For another example, Miller,(3) in an otherwise valuable review of Hertz's contribution, states that "his axiomatic assertion of the form invariance of the electromagnetic field equations [or "covariance" as Minkowski described this mathematical property] led Hertz to predict new effects whose empirical confirmation could in turn serve to confirm his axiom of covariance." On the contrary. Hertz asserted *invariance* and meant what he said. He was referring, of course, to Galilean invariance, since the Galilean transformation was the only one envisioned in his day. It is the Galilean invariance of Hertz's equations that will be demonstrated in the present paper. Modern word usage reflects the received view of covariance as a just-as-good form of invariance. To dispute this universal opinion appears fruitless; but it is completely confusing to represent invariance as an equivalent form of covariance. For invariance is what covariance simulates, emulates, or aspires to be. There exists a true invariance, distinct from covariance, and the distinction is not merely formal. We shall presently show in the case of electromagnetism that the *physics* of true invariance is as different from that of covariance as is the mathematics.

The writer has never encountered formal definitions of "invariance" and "covariance" that satisfy all requirements of mathematical rigor - and none will be attempted here. Loosely, invariant quantities transform in place within the expressions containing them, without altering their mathematical character in any way. Covariant quantities transform by a rule of (generally linear) combination of related quantities, the rule being the same for all quantities within the expression containing them. The distinction is best shown by example. Since the reader is undoubtedly familiar with relativistic covariance, we need illustrate here only invariance to show the difference.

2. HISTORICAL BRIEF

Let us begin with an overview of the situation in electromagnetism toward the end of the nineteenth century. A dominant figure, in addition to Maxwell (1831-1879), was Heinrich Rudolf Hertz (1857-1894) - a physicist nowadays chiefly remembered for his experimental validation of Maxwell's theoretical prediction of electromagnetic waves. What is less widely recognized, but amply documented in his book,[(2)] is Hertz's strength as a theorist. In the last chapter of that book, which appeared in 1892, he treated the "electrodynamics of moving bodies" by an original set of equations comprising what we would call today an "invariant covering theory" of Maxwell's equations for vacuum electrodynamics. The new equations differed from Maxwell's through the inclusion of an extra velocity-dimensioned parameter, the components of which Hertz designated (α, ß, γ). The presence of this extra velocity parameter spoiled the space- time symmetry of Maxwell's equations, but caused them to become rigorously invariant under the Galilean transformation of coordinates. Hertz stated this invariance as a fact, but gave no proof in his book. We shall supply a proof in the next section.

Unfortunately, Hertz obscured his new formulation of electrodynamics through such baffling notation (all equations being written in component form, unsimplified by the use of vector notation and identities) that it is fair to say that few physicists or scholars from then to now have penetrated it. Indeed, it is clear from Einstein's reference to "Maxwell- Hertz equations" in his 1905 paper(4) that he was unaware of the import of Hertz's invariant mathematics (inasmuch as Maxwell's equations are space-time symmetrical, whereas Hertz's are not).

We may conclude this summary by explaining why Hertz's invariant covering theory never got into the textbooks. His theory did not become physics, because physics is never equations alone but equations plus physical interpretation. As often as not, and certainly in this case, interpretation proves the stumbling block. On the side of interpretation Hertz made a fatally bad guess. A modern mind would recognize that the discovery of a Galilean invariant formulation of electromagnetism implies a Galilean motional *relativity principle* for electromagnetism as well as for mechanics - and thus automatically accounts for the Michelson-Morley outcome and corresponding first-order observations by Mascart and others (all of which showed "relativity" to be an experimental fact) through formal attributes of the mathematics alone, without need to postulate a physical ether in some collective state of motion.

But Maxwell, Hertz, and most other late nineteenth- century physicists were fixated on ether... so, when Hertz saw a new velocity parameter unavoidably emerging from his invariant mathematics, he automatically identified it with ether velocity. In fact, he went one fateful step farther and identified his "ether" with the *converted ether* of George G. Stokes (1819-1903). Such an ether was hypothesized to be carried along (100% convected) by all material bodies. By this merciless racking of double-jointed hypothesis, Hertz's (α, ß, γ) became the velocity of just any "body" in the laboratory (whence his last chapter title,(2) perhaps unconsciously borrowed by Einstein,(4) (referring to the "electrodynamics of moving bodies"). One speculates that Hertz indulged in this dreadful definiteness of physical model because he distrusted intangibles and wanted to include only measurable quantities in his theory.

By boldly abandoning the ghostly qualities of nineteenth- century ether that protected it from empirical inquiry and making "ether velocity" operationally definable, Hertz's mathematics *plus his Stokesian interpretation* exposed themselves to crucial laboratory testing. For example, they predicted that a dielectric rotating in the laboratory would produce a magnetic field. This effect was looked for(5) soon after Hertz's untimely death and was not found. Hence Hertz's invariance was discarded and forgotten in favor of Einstein-Minkowski covariance. Note, however, that it was not Hertz's mathematics that was empirically discredited, but the *combination* of that and an obviously (in the modem view) unsound physical interpretation. An identical interpretational mistake (of staking all the physics on an ether mechanism) was made by Maxwell.

History, ever the joker, forgave Maxwell's errant physics and preserved - indeed, virtually sanctified - his mathematics (a noninvariant special case of the Hertz equations ... hence in formal terms a comparatively degraded breed of mathematics). Said Hertz himself, intending a put-down that can only be read now as supreme historical irony: "Maxwell's theory is Maxwell's equations." For history did not show the same courtesy to Hertz. Nobody had the inspired charity to say, "Hertz's theory is Hertz's equations." His invariant mathematical baby was ruthlessly thrown out with the ether-interpretational bathwater. Today we would possess no electromagnetic theory whatsoever if a fairness doctrine had decreed equal treatment for Maxwell.

Electromagnetic theory as developed by Maxwell's followers proved tougher and less frangible than Hertz's theory, precisely because they were persistently vague about the physical interpretational side. Maxwell's luminiferous ether faded like the Cheshire cat's smile and was replaced physically by ... *nothing* (the most infrangible substance known - with the possible exception of mathematical vectors, the material of which the present writer was taught that electric and magnetic fields are fabricated). There is little doubt that Hertz deliberately sought a frangible theory, because he knew that by the breaking of theories, science progresses most rapidly. The post-Maxwellians avoided frangible theory, perhaps (one speculates) because the goal of most rapid scientific progress was even in that day beginning to be supplanted by other objectives.

Although Hertz's invariant equations went down the drain, they depend only on certain immutable mathematical facts and thus are subject to continual rediscovery. (The modem independent rediscoverers include S. Kosowski, F.D. Tombe, C.I. Mocanu, and the present writer - all originally ignorant of Hertz's priority.) These mathematical facts will be examined in the next section.

3. HERTZ'S ELECTROMAGNETISM: PROOF OF FIRST-ORDER INVARIANCE

Considering a vacuum environment, so that "constitutive" relations do not enter, and choosing arbitrarily to designate the field vectors as $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, we can write Hertz's equations in Gaussian units in the form

(1a) $$\nabla \times \vec{\mathbf{B}} - \frac{1}{c}\frac{d\vec{\mathbf{E}}}{dt} - \frac{4\pi}{c}\vec{\mathbf{j}}_m = 0,$$

(1b) $$\nabla \times \vec{\mathbf{E}} + \frac{1}{c}\frac{d\vec{\mathbf{B}}}{dt} = 0,$$

(1c) $$\nabla \cdot \vec{\mathbf{B}} = 0,$$

(1d) $$\nabla \cdot \vec{\mathbf{E}} - 4\pi\rho = 0,$$

where all symbols have the same meanings (except as will be discussed presently) as in the counterpart Maxwell- Lorentz equations. (Actually, Hertz's equations were more general in that they allowed for nonzero divergence of B, but we are here more concerned with communicating ideas than with historical accuracy.) The total time derivative appearing in these equations is defined as

(2) $$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{\mathbf{v}}_d \cdot \nabla,$$

where v_d is a velocity-dimensioned vector [Hertz's (α, β, γ)] whose physical interpretation - a crucial issue, since it was what tripped up Hertz - will be addressed below.

It is obvious that Hertz's equations constitute a formal *covering theory* of Maxwell's equations for vacuum electromagnetism; for in the special case that the v_d parameter assumes the value $(v_{dx}, v_{dy}, v_{dz}) = (0, 0, 0)$, we see from Eq. (2) that the total derivative operator *d/dt* reduces to the partial operator *d/dt,* with the result that Eq. (1) becomes identical to Maxwell's equations. In that special case alone does space-time symmetry (in purely mathematical terms) obtain. The measured current source term $\vec{\mathbf{j}}_m$ in Eq. (la) differs by the effect of a Galilean transformation from the current source term $\vec{\mathbf{j}}$ in Maxwell's equations, but this difference also goes to zero in the special case mentioned.

When the new v_d parameter does not vanish, the resulting richer parametric content of Eq. (1) implies that it constitutes a more general or "covering" theory. It remains to show that in this case we are dealing with an *invariant covering theory.* Since the present section is limited to considerations of the first-order in *v*/c, the relevant coordinate transformation is the Galilean one.

(3) $$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{v}}t, \quad t' = t,$$

where we adopt a velocity sign convention opposite from that of Jammer and Stachel.[1]

As noted by Jammer and Stachel,[1] Eq. (3) implies that

(4) $$\nabla' = \nabla, \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{\mathbf{v}} \cdot \nabla$$

where primed and unprimed quantities refer to two inertial systems in uniform relative translatory motion. Observe that the velocity parameters v (inertial frame transformation velocity) and v_d (convection velocity) have nothing necessarily to do with each other - and both differ in general from any velocity parameter implicit in the source current density j. These distinctions being borne in mind, we need not concern ourselves at this prephysical stage with operation- ally defining these parameters.

We also borrow from Jammer and Stachel[1] the Galilean source transformation equations

(5) $$\rho'(\vec{\mathbf{r}}', t) = \rho(\vec{\mathbf{r}}, t)$$

and

(6) $$\vec{\mathbf{j}}'(\vec{\mathbf{r}}', t') = \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) - \rho(\vec{\mathbf{r}}, t)\vec{\mathbf{v}} ,$$

which are certainly valid at first order. We do not borrow the field vector transformations

(7) $$\vec{\mathbf{E}}' = \vec{\mathbf{E}}, \quad \vec{\mathbf{B}}' = \vec{\mathbf{B}} - \mathbf{v} \times \vec{\mathbf{E}}$$

proposed by them. The first of these satisfactorily connotes Galilean invariance, but the second involves a "scrambling" (linear combination) of electric and magnetic field quantities uniquely characteristic of covariance, hence properly termed "Galilean covariance." For consistency with the requirement of true invariance we demand, with Hertz, that

(8) $$\vec{\mathbf{E}}' = \vec{\mathbf{E}}, \quad \vec{\mathbf{B}}' = \vec{\mathbf{B}} .$$

The significance of Eq. (8) will emerge from inquiry into its physical meaning - which we postpone to Sec. 5. For the moment we simply take Eq. (8) as a formal ansatz and examine its consequences.

Regardless of what physical object the quantity v_d refers to, it must obey at first order the Galilean velocity composition law,

(9) $$\vec{\mathbf{v}}'_d = \vec{\mathbf{v}}_d - \vec{\mathbf{v}} .$$

We also take note of Galilean velocity reciprocity,

(10) $$\vec{\mathbf{v}}' = -\vec{\mathbf{v}} ,$$

although this will not feature in our invariance proof. Suppose we multiply Eq. (9) by charge density p and arbitrarily define a formal "current density" j_d by

(11) $$\vec{\mathbf{j}}_d = -\rho\vec{\mathbf{v}}_d ,$$

where a minus sign has been introduced for reasons to be discussed presently. Then $\mathbf{j_d}$ transforms as

$$\vec{\mathbf{j}}_d = -\rho\vec{\mathbf{v}}_d = -\rho(\vec{\mathbf{v}}'_d - \vec{\mathbf{v}}) = -\rho\vec{\mathbf{v}}'_d - \rho\vec{\mathbf{v}} = -(\rho\vec{\mathbf{v}}_d)' - \rho\vec{\mathbf{v}} = \vec{\mathbf{j}}'_d - \rho\vec{\mathbf{v}}$$

or

(12) $$\vec{j}'_d = \vec{j}_d + \rho\vec{v}_d \, .$$

We further define a "measured" current density j_m, the quantity appearing in Eq. (la), as the sum

(13) $$\vec{j}_m = \vec{j} + \vec{j}_d \, .$$

These are treated as formal definitions of source terms for purposes of the proof. They will be justified presently in physical terms. Finally, we assume invariance of the units ratio

(14) $$c' = c.$$

All ingredients of the invariance proof are now assembled. We show first the invariance of the total time derivative operator:

(15)

$$\left(\frac{d}{dt}\right)' = \left(\frac{\partial}{\partial t} + \vec{v}_d \cdot \nabla\right)' = \frac{\partial}{\partial t'} + \vec{v}'_d \cdot \nabla' = \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) + (\vec{v}_d - \vec{v}) \cdot \nabla = \frac{\partial}{\partial t} + \vec{v}_d \cdot \nabla = \frac{d}{dt}$$

which follows directly from Eqs. (2), (4). (9). Next, we establish invariance of the "measured" current density:

(16) $$\vec{j}'_m = (\vec{j} + \vec{j}_d)' = \vec{j}' + \vec{j}'_d = (\vec{j} - \rho\vec{v}) + (\vec{j}_d + \rho\vec{v}) = \vec{j} + \vec{j}_d = \vec{j}_m \, ,$$

as follows from Eqs. (6), (12), (13). With these preparations it becomes a mere matter of inspection to verify invariance of the field equations themselves. The simplest is Eq. (1c), which yields

(17) $$(\nabla \cdot \vec{\mathbf{B}})' = \nabla' \cdot \vec{\mathbf{B}}' = \nabla \cdot \vec{\mathbf{B}} = 0 \, ,$$

use being made of Eqs. (4), (8). This agrees with Eq. (1c') of Jammer and Stachel.[(1)] By means of Eqs. (4), (5), (8) we verify the invariance of Eq. (1d):

(18) $$\nabla' \cdot \vec{\mathbf{E}}' - 4\pi\rho' = \nabla \cdot \vec{\mathbf{E}} - 4\pi\rho = 0 \, .$$

This agrees with Eq. (la') of Jammer and Stachel. Using Eqs. (4), (8), (14), (15) we establish the invariance of Eq. (lb):

(19) $$\nabla' \times \vec{\mathbf{E}}' + \frac{1}{c'}\left(\frac{d}{dt}\right)' \vec{\mathbf{B}}' = \nabla \times \vec{\mathbf{E}} + \frac{1}{c}\frac{d\vec{\mathbf{B}}}{dt} = 0 \, .$$

Finally, application to Eq. (la) of Eqs. (4), (8), (14), (15), (16) yields similarly

(20) $$\nabla' \times \vec{\mathbf{B}}' - \frac{1}{c'}\left(\frac{d}{dt}\right)' \vec{\mathbf{E}}' - \frac{4\pi}{c'}\vec{j}'_m = \nabla \times \vec{\mathbf{B}} - \frac{1}{c}\frac{d\vec{\mathbf{E}}}{dt} - \frac{4\pi}{c}\vec{j}_m = 0 \, .$$

Thus the Galilean invariance of all four of the Hertz field equations, Eq. (1), in conformity with the ansatz of Eq. (8), is established. There is no need to drop the Faraday induction term or any other.

When (as above) each symbol appearing in a physical equation - here counting *d/dt* as one symbol - transforms directly into its primed counterpart, we may call this property of an equation "manifest invariance." So, we are entitled to speak of the Hertz equations as manifestly invariant at first order. This means that they obey a Galilean relativity principle: the laws of electromagnetism, like those of Newtonian mechanics, are invariant under transformations among different inertial systems (in uniform relative translator motion without rotation).

4. PHYSICAL INTERPRETATION: THE PARAMETER $\mathbf{v}_d$

To possess a set of formally attractive physical equations is not to possess a physics, as Hertz's experience confirms. Generally, the hard part is to find a physical interpretational scheme or appropriate set of operational definitions. This - one of those bona fide *physics* problems that can never be delegated to mathematicians - we now address ... with reference in this section to the Hertzian convective velocity parameter v_d and in the following section to our field- invariance ansatz, Eq. (8).

To draw inspiration from a master physicist of the past, we ask, How might Maxwell have argued concerning the physical interpretation of Hertz's parameter v_d? I speculate that he might have addressed this question from the stand- point of the *equation of continuity*... for this was never far from his thoughts, since it played a central role in his great discovery, the "displacement current" that led to one of the most dramatic predictions in the history of science - the existence of electromagnetic radiation. Making the total derivative substitution [for mathematical consistency with Eq. (1)] in the continuity equation, and applying Eqs. (4)., (5), (15), (16), we obtain

$$\text{(21)} \qquad \nabla' \cdot \vec{\mathbf{j}}'_m + \left(\frac{d}{dt}\right)' \rho' = \nabla \cdot \vec{\mathbf{j}}_m + \frac{d}{dt}\rho = 0 \, .$$

There is nothing abstruse or even specifically electromagnetic about Eq. (21) - it applies in hydrodynamics, acoustics, and any number of familiar areas of physical study. In ail cases the meaning of the "convective" velocity parameter v_d is the same: if we think of a small *detection volumes* moving within some sort of generic "flow," then v_d is *not the* velocity of this flow. It is *the velocity of the "detector" with respect to the observer.* The partial time derivative appearing in Eq. (2) describes time changes within the detector (or charge-density measuring device in this case) when it is held at rest (v = 0) in the flow at a point fixed with respect to the observer, and the convective part $\vec{\mathbf{v}}_d \cdot \nabla$ of the total time derivative operator describes any additional changes measurable within the detector due specifically to its motion ($\mathbf{v}_d \neq 0$) with respect to the observer.

Given such an interpretation, we can go back to our prior results and recognize the physical meaning of such a relation as Eq. (11): the "current density" j_d is that part of the total current density j_m measurable within a detector moving with velocity v_d with respect to the observer that is due specifically to such relative motion. It has a negative sign because detector motion in one direction is equivalent to a current (of electric charges) flowing in the opposite direction. Equation (13) states that the invariant total current j_m measured by the moving detector is the sum of the Maxwell source current j that would be observed if the detector were held stationary in our laboratory plus the motion-generated current j_d due to any proper motion of the detector with respect to our laboratory.

In this connection I believe Maxwell might have recognized that a detector's physical degrees of freedom require parametrization in any theory compatible with a motional relativity principle. Those degrees of freedom were not parametrized in his own equations, since his field detectors were always tacitly "frozen" at rest at a "field point" fixed with respect to the observer. Given such a recognition with regard to the charge-density detector or "ρ meter," he would certainly have applied the same reasoning to detectors of the field quantities - that is, "**E** meters" and "**B** meters." All such meters can be considered instantaneously comoving at the observer's field point (a stream of them being involved, over time, passing through the field point, if the field point does not share their state of motion). All field detectors are compositions of matter having degrees of translatory freedom that need to be explicitly parametrized if, as motional relativity implies, the same formal mathematics is to describe *in more than one inertial system* a unique event of "detection" occurring in one particular macroscopic detector.

Through such reasoning one arrives at an operational definition of the Hertzian v_d parameter quite different from Hertz's own. Whereas Hertz considered v_d the velocity with respect to the observer of an intangible ether, rendered (spuriously) tangible through the Stokesian convective "body" assumption, we suggest that Maxwell (or any comparably objective thinker) might have been led by considerations such as those just given to interpret Hertz's v_d as the velocity with respect to the observer not of any kind of flow but of a tangible object - namely, the "field detector," radiation absorber, or instrument that measures the numbers corresponding to the E and B field component values described by Hertzian field equations. (I trust that even the most fanatical of modern idealist-formalists will allow the student the *option* to think of E and B in terms of numbers and of numbers in terms of instrument readings and of material instruments in terms of specific states of motion subject to parametrization - without consigning him to perdition as a "discredited positivist.") When $v_d = 0$, the field detector becomes just the

Maxwellian detector fixed at the field point. Hence this fits exactly with Maxwell's equations being the special case of Hertz's equations in which the field detector is at rest in the observer's laboratory.

In summary, it would appear that the most plausible interpretation of the Hertzian velocity parameter is that $\mathbf{v}_d$ is the velocity of the field detector or **radiation absorber** with respect to the observer. This has the advantage of designating a tangible object and (unlike Hertz's Stokesian ether interpretation) of not being in conflict with observation. It has the minor disadvantage that no other rediscoverer of Hertz's invariant mathematics agrees with it. Probably a poll of such rediscoverers would favor an ether interpretation, but an intangible ether not convected by material bodies, hence not subject to hostile interrogation under strong lights.

5. THE CONTRASTING PHYSICS OF INVARIANCE AND COVARIANCE

At this point it is expedient to recognize that there is nothing intrinsically "right" or "wrong" about definitions in mathematics or science (although there may be overwhelming differences in fruitfulness) - and that Hertz's **E** and **B** are *by definition* different quantities from Maxwell's **E** and **B**. Hertz's field quantities are physically defined in a more general way - via measurements made by a detector that can move with respect to the observer and his chosen "field point." As a result of this parametrically more complicated definition of E and B, these field quantities acquire simple (manifestly invariant) mathematical transformation properties [Eq. (8)] under inertial motions. The Maxwell field quantities, by contrast, are physically defined very simply - by the readings of field detectors permanently *at rest* in the observer's laboratory. As a result, they have more complicated transformation properties, discovered by Lorentz and designated by Minkowski a few years later as "covariance," in contrast to "invariance." (Latterly the terminological distinction has been dropped by most physicists, but it remains mathematically significant ... and also physically significant, as the present definitional considerations confirm. Dropping the distinction is not in general mere carelessness, but serves as a propaganda trick to bolster covariance.

What is the physics behind Eq. (8), which asserts the true invariance of Hertzian **E** and **B** fields? Simply that observers in primed and unprimed inertial systems (or, indeed, arbitrarily moving observers) will read the *same numbers* of the digital readout of *a given* **E** meter or **B** meter at the instant it passes through an agreed field point. If two or more differently moving observers are involved, the field detection instrument in question can be at rest with respect to at most one of these observers - but the *same instrument,* the same readout, and the same numbers displayed on it are visible to all observers at the event of instrument passage through the field point. The fact that all observers thus read the same (field component) numbers is expressed by Eq. (8). That is the entire and entirely trivial *physical* meaning of true mathematical invariance.

The crucial point to keep in mind is: in Hertzian electromagnetism only one **E** meter and one **B** meter is involved no matter how many observers may be present. The instrument is *public property* - no one observer "owns" it, nor are different observers called upon to replicate it. Replication of experiments is not involved. And the states of observer motion are arbitrary, not necessarily inertial. The instrument itself - at least until more physics is known - must be considered at rest in an inertial system. (Whether detector acceleration generates specific observable effects is a question not yet experimentally addressed, as far as the writer knows.)

Covariance, by contrast, is subtler to conceptualize physically, in that we must picture as many instruments as there are inertial systems under consideration - and must have both **E** meters and **B** meters present - say, primed meters at rest in the primed system, unprimed at rest in the unprimed system, etc. Each observer reads only *his own* instruments at their instant of passage through an agreed field point... and linear combinations of the numbers so recorded on both E and B meters by the unprimed observer must be *calculated* in order to predict the numbers recorded by the primed observer. From this gedanken operational complexity and necessity to introduce a calculational step in addition to direct readings from measuring instruments, it is quite clear why *operational definitions*- despite a brave beginning by Einstein, as celebrated by Bridgman - have fallen into disuse by relativists. In Maxwell-Einstein electromagnetism field detection instruments are *private property* and must be replicated by each observer. Thus experiments must be *replicated* by differently moving observers. And observers must in all cases be *inertial.*

As to the crucial question of relative fruitfulness of the Hertzian and Maxwellian definitions of the field quantities, that can be judged only within the context of the whole of physics, past and future. It might be a mistake to judge even from the present pinnacle of historical enlightenment. In the era of Maxwell and Hertz nothing was known that could have decided the issue between the two definitions ... and nothing conclusive emerged for another quarter century, in fact, until the advent of quantum mechanics. The latter made it clear that events occurring at the quantum level within macroscopic apparatus are physically unique - so that only a theory parametrized to describe the motions of a given, *unique macroscopic detector* can work consistently at the quantum descriptive level. That is, the idea of detailed "replication" of measurements within macroscopic apparatus - an idea that had served perfectly well all during the nineteenth century and well past 1905 - was finally played out by 1925 and could thenceforth have nothing to do with physics at its more refined levels of inquiry.

As indicated above, it is just this forbidden concept of replication that tacitly underlies Maxwell's definition of the field quantities ... for in Maxwell-Einstein electromagnetism each inertial observer has his own laboratory, his own fixed field point, and his own field detector at rest at that field point. Lacking the descriptive (Hertzian velocity-dimensioned) parameters that allow field detectors to move with respect to observers or field points, each Maxwellian observer has to possess his own field detector and is forced by that circumstance to *replicate* the observations of other observers - so that in a (micro) sense each is measuring a different "field." Hence *no two Maxwell observers in relative motion can be talking about the same unique quantum detection event occurring in one particular piece of apparatus.*

Einstein's relativity treatment, applied to electromagnetic observations, took over Maxwell's formulation unmodified, hence had Maxwellian replication ineradicably built-in. After 1925 such replication could no longer be legitimately applied to single quantum events such as photon absorptions. (Curiously, this was not observed by noted investigators such as Bohr and Rosenfeld. A strange blindness seems to afflict all students of the "field." Perhaps they are blinded by science.) We who have learned the lesson of 1925 - that measurements cannot be replicated in micro detail - cannot fail to recognize that invariance and the Hertzian definitions of the field quantities are better adapted to describing the physics of the quantum world. However, aspects of Einstein's physics are certainly of permanent value.

Throughout the course of the twentieth century empirical evidence such as the CERN meson observations[6] has steadily accumulated testifying to the correctness of Einstein's identification of the timelike invariant of kinematics, the "proper time interval." Therefore, it is evident that Hertz's equations can be at best a first-order approximation. If they are to have physical validity at higher orders their total time derivative d/dt with respect to non-invariant Newtonian or "frame" time, appearing in Eq. (2), must be replaced by a total derivative with respect to the field detector's invariant proper time, $d/d\tau_d$. Such a *noncovariant* introduction of Einstein's timelike invariant preserves at higher orders the vital mathematical *invariance of the* Hertz equations as well as their operational definition in terms of a detector moving arbitrarily in the observer's laboratory. (At higher orders the Galilean transformation must be modified for the time coordinate, but need not be modified on the spatial side, since space-time symmetry has been jettisoned already at first order. Similar remarks apply to the higher-order modification of mechanics. Such considerations have been extensively treated elsewhere[7] under the rubric "neo-Hertzian electromagnetism." This need not detain us here.)

Despite the current vogue for covariance, it would thus be premature to conclude as we approach the twenty-first century that true invariance is for all time discredited and played out. This seems a message of hope for the future of physics - for if there is anything that can sap the vitality of a science, it is a total lack of "controversy" or intellectual challenge within its foundations.

6. COMMENTARY

There has existed in the published literature for one hundred years an invariant covering theory of Maxwell's electromagnetism due to Hertz. It should be emphasized that a covering theory really "covers." That is, the empirically validated electromagnetic "physics of one laboratory," wherein field detectors are at rest, is identical in Maxwell's and in Hertz's theories. By considering the equation of continuity within the framework of Hertz's formulation, we have been led to a view of the Hertzian velocity parameter quite different from Hertz's own interpretation. A different path- way to motional "relativity" thus opens up, based on invariant rather than covariant mathematics of electromagnetism. On this new pathway the **E** and **B** vectors transform invariantly [Eq. (8)], because observers in all states of motion read the same numbers from *the same instrument.* The two types of field remain both formally and operationally distinct, with no covariant "scrambling" of components.

To be sure, there are many twists of the actual historical process we have not attempted to bring into our discussion. For instance, the Lorentz force law, which emerged contemporaneously with Hertz's theory, fits with covariance rather than invariance. But present-day empirical evidence is mounting heavily against the Lorentz force law and in favor of Ampere's original law of forces between current elements (see Refs. 8 to 13 and further references given there), which honored Newton's third law but not covariance or space- time symmetry. Hence it may be premature to announce a final decision between those two similar-sounding, yet profoundly antithetical, mathematical approaches to the description of an external *reality:* invariance and covariance. Today, almost all physicists consider the issue completely and permanently settled in favor of covariance. But we have seen that the uniqueness of the quantum detection event within a macroscopic piece of apparatus accords perfectly with the Hertzian one-instrument approach and not at all with Maxwell-Einstein replication of measurements in different inertial systems. The way covariance got its start was as best available substitute for true or manifest invariance - the latter being represented as *unattainable because* Maxwell's equations did not attain it. That was a misrepresentation of fact.

Addendum

A referee has made the comment, "When the field vectors are truly invariant, are we not speaking of a Newtonian force field which could be made redundant by dropping the 'field' altogether and return to Newton's simultaneous far-actions? This would also remove all confusion regarding the ether." I concur with this thought and am personally partial to some Weber-type velocity-dependent potential(14) that allows the entire purpose of the "field" to be accomplished directly by a simple intercharge force law. This fits much better with operationalism (and with Ockham) than does field theory. The "field," as an extraneous agent, is actually as objectionable - and for the same reason - as "ether." It was not my intention in the present paper to defend electromagnetic field theory as physics, merely to get it right on its own terms.

Résumé

L'incapacité des équations de Maxwell de prouver qu'elles sont invariantes dans des transformations galiléennes a été corrigée par Hertz grâce à un artifice mathématique simple mais en grande partie oublié aujourd'hui. Get artifice consiste à remplacer les dérivés complètes (de convection) par rapport au temps avec les dérivés partielles par rapport au temps partout où ces dernières apparaissent dans les équations de Maxwell. Par ce moyen, Hertz a derivé une théorie de recouvrement invariante formellement galiléenne de l'électrodynamique du vide de Maxwell laquelle théorie n 'était toutefois pas symétrique dans l'espace-temps (au égard à son tripotage des dérivés par rapport au temps mais pas ceux par rapport à l'espace). Si l'accomplissement mathématique de Hertz avait bénéficié d'une reconnaissance plus large, sa théorie de recouvrement invariant de la théorie de Maxwell aurait pu fournir la clé formelle (presque vingt ans avant la "covariance" de Minkuwski) pour unifier les propriétés "relativistes" de l'électrodynamique et la mécanique newtonienne, expliquer le résultat de Michelson-Morley, etc. La tâche de trouver une interprétation physique viable du paramètre de vitesse de convection hertzienne, que Hertz lui-même n'a pas accompli quand il était vivant, est un sujet de recherche continue. Nous discutons cette question et des autres connexes et donnons une preuve explicite de l'invariance.

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