SOME CONSIDERATIONS ON THE BALLISTICS OF A GUN OF SEVENTY-FIVE-MILE RANGE.[1]

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On the afternoon of March 23, 1918, the civilized world was astounded by the news that the Germans were bombarding Paris. Inasmuch as it was known that the nearest point at which the German lines approached Paris was over seventy miles distant, curiosity was universal as to how this result was accomplished, and the most unlikely and absurd hypotheses were suggested.

The writer, among others, was asked whether or not the result was likely to be due to an aërial torpedo. The next day revealed the truth, which was, simply, that the Germans had really built a gun which carried a projectile this hitherto unheard-of distance.

It has since been determined that the gun is situated in the Forest of St. Gobain in the neighborhood of Laon, at a distance of about 120 kilometers or seventy miles.

At this distance the curvature of the earth causes one end of such a line to be about a half mile below the horizon at the other end, so that it is impossible to see the target from the gun or vice versa; there being no mountains of any such height in the whole region, visual aiming would be quite out of the question.

I wish, first, to call the attention not only to the remarkable ballistic achievement of the Germans in so far surpassing previous ranges but also to the unique opportunity possessed. It is obvious that precision of aim at such a distance is well-nigh impossible and that the only hope of effecting any damage lies in the possession of a very large and valuable target. Little has been allowed to come through the cable as to the damage done by these long-range shots, but enough has been learned in order to see their terrible potential-

[1] Contribution from the Ballistic Institute, Clark University, No. 2.

ity. I shall remind you by a few lantern slides of the concentration of monuments of civilization to such a degree as probably exists nowhere else in the world.

The gun was obviously aimed to strike the Cathedral of Notre Dame, cathedrals being the German specialty in this war. The church in which eighty people were killed on Good Friday is easily identified as the church of St. Gervais, which is across the street from the Hotel de Ville, the slides of which shown are taken from the top of that church. Other objects nearly in the line of the "*axe*" are the Louvre, the Sorbonne, the Pantheon, the Bon Marché (a department store); I will not fatigue you with others.

Even if the so-called ellipse of dispersion should be very large it is easily seen that, if the shot should fall anywhere within a circle of perhaps twenty-five miles in diameter, the moral effect would be very great.

The longest previous range used during the war was about twenty-two miles with the gun with which the Germans bombarded Dunkirk. In an article in *Nature,* March 28, 1918, which I have just seen this morning, my friend Sir George Greenhill, the author of the article on Ballistics in the Encyclopedia Brittanica, says:

> "In the language of sport the German gunner has 'wiped the eye' of our artillery experts and defied all the timid preconceived notions of our old-fashioned traditions. The Jubilee long-range artillery experiments of 30 years ago were considered the *ne plus ultra* of our authorities, and we were stopped at that as they were declared of no military value. Today we have the arrears to make up of those years of delay, but the Germans watched our experiments with great interest, resumed them where we had left off and carried the idea forward until it has culminated today in his latest achievement of artillery of a gun to fire 75 miles and bombard Paris from the frontier."

The Jubilee gun, referred to, fired a shell weighing 380 pounds at an elevation of 40° with a muzzle velocity of 2,400 feet per second, giving a range of 22,000 yards or 12½ miles.[2]

[2] It is a singular coincidence that as I left my laboratory to attend the meeting of the American Philosophical Society I remarked to my assistant, "Nothing is likely to 'queer' this paper except the fact that Sir George Greenhill may have calculated the trajectory and published it in *Nature.*" As I entered the room, the Secretary, Dr. Hays, called my attention to Sir George's paper and I was greatly relieved to find that it contained no figures of the trajectory.

Inasmuch as the only thing that has made possible this great ballistic achievement is the taking advantage of the decrease in resistance due to the decreasing density of the air in high altitudes, it is first necessary to make some assumptions as to the law of decrease with the height. I have here assumed that the temperature is constant, giving the isothermal law of Laplace. I accordingly put

$$(1) \qquad \delta = \delta_0 e^{-ky},$$

where $\delta_0 = 1.2932$kg./meter^3, and k is equal to .1251, the altitude y being expressed in kilometers. As the shot rises to a height of about forty kilometers, or twenty-five miles, this results in a diminution of density of about sixteen times. While it is true that for the upper part of the trajectory the form is practically that of the parabola as investigated by Gallileo for the vacuum, it is by no means true, as certain discussions that I have seen would seem to indicate, that there is a region about two miles high at which the effect of the atmosphere suddenly stops.

The second thing that must be known is the so-called ballistic coefficient of the projectile, involving its mass and diameter, the resistance of the air being supposed to be proportional to the square of the diameter while the acceleration is inversely proportional to the mass.

For lack of sufficient information at the time that this paper was prepared, it was assumed that the mass of the projectile was 300 kilograms and its diameter twenty-two cenitmeters, or eight and one half inches. It is also necessary to know the form factor, which depends upon the sharpness of the projectile. In the calculations made here the number .9, which is that of old-fashioned, rather blunt projectiles, is used. As, however, reports on the shell have shown that it is furnished with a long pointed cap of sheet metal, the form factor should be considerably reduced. If, however, we take a mass of 180 kilograms, or 396.8 pounds, the results given here will be exact if we assume a form factor of .54, which is undoubtedly much nearer the correct value. Finally, if we assume the mass to be 120 kilograms, this will give the same trajectory with a form factor of .36, which is smaller than that of any shot with which I am acquainted.

The method of calculation is as follows: The first approximation by successive arcs indicated by Gen. Siacci is used. The distance of 120 kilometers is divided into twelve equal parts, and the chord is drawn, making use of the velocity obtained, at the end of the preceding chord. We make use of the familiar equations ot ballistics,

$$ds = \rho d\theta, \tag{2}$$

$$v = \frac{ds}{dt}, \tag{3}$$

$$a_n = v^2/\rho = -g\cos\theta, \tag{4}$$

$$\frac{dv_x}{dt} = -cf(v), \tag{5}$$

in the last of which, which is the only dynamical equation involved, we put

$$dt = -\frac{vd\theta}{g\cos\theta}, \qquad \frac{d(v\cos\theta)}{d\theta} = \frac{c}{g}vf(v). \tag{6}$$

Instead of differentials we use finite differences. The table shows the computation required for Curve 2, Fig. 4.

COMPUTATION FOR CURVE 2

r No.	V_r	V_r^3	θ_r DEG.	$V^2\cos^2\theta$	Δy	$\Delta\theta$ DEG.	$\Delta\theta_R$ RAD.	$\bar{y}$	$R=\delta_{(y)}f_2v$ KG./MET.	ΔV_x	Δt SEC.	Dx	$V_x-\Delta V_x$	$\cos\theta_{r+1}$	V_r+1
1	1300	169.10^3	52°	640.10^3	765	3.3°	.058	6.10^3	190	138.3	12.5	862	661.7	.660	1002.0
2	1002	1009	48.7	438	1120	5.5	.097	16	40	37.5	15 1	282	624.2	.729	856.5
3	865.5	744	43.2	389	1260	7.7	.133	25.5	12	13.2	16.0	105 5	611.0	.814	749.0
4	749	561	35.5	373	1310	10.0	.175	32.5	5.5	6.88	16.35	56 3	604.1	.903	669.0
5	669	447	25.5	365	1340	12.55	.219	37.0	2.0	2.83	16.55	23 3	601.3	.974	617.0
6	617	380	13.0	360	1360	14.75	.258	39.0	1.0	1.54	16.65	12.8	599.8	.999	600.0
7	600	360	−1.75	360	1360	15.6	.272	38.25	1.0	1.58	16.66	13.15	598.2	.954	627.0
8	627	392	−17.35	359	1365	14.35	.250	36.0	2.0	3.03	16.60	25 1	595 2	.851	698.0
9	698	487	−31.7	355	1380	11.5	201	26.5	6.0	8.01	16 80	67.2	587.2	729	805.0
10	805	649	−43.2	345	1420	8.65	.151	18.0	22.0	25 8	17 0	219.0	561.4	.618	908.0
11	908	825	−51.8	315	1550	6.8	.118	8.0	105.0	108.5	17 7	952	453.0	.522	867.0

MASS = 300 KG. INITIAL VELOCITY = 1300 $\frac{\text{MET.}}{\text{SEC.}}$ $\theta_0 = 52°$ $\Sigma\Delta t = 178$ SEC. APPROX.

The meaning of the symbols is as follows:

The number of the chord is denoted by r; the velocity of the projectile is denoted by v; and its x-component, by v_x. The inclination of the tangent at the beginning of the arc is θ_r. The drop away from the tangent, due to gravity, which is of the second order of smallness with respect to the length of the arc, is denoted by Δy. The mean ordinate of a given element is indicated by y. The resistance R is found from the graphical table (Fig. 2). The change in the horizontal component of the velocity is Δv_x, the time taken for the projectile to traverse one arc is Δt. D_x represents the "set-back" computed from the resistance, representing the amount by which the projectile falls short of the assumed horizontal distance owing to the resistance of the air. The height reached at the end of the arc is

$$y = (x - D_x) \tan \theta - \Delta y.$$

The most important thing in a ballistic calculation is the knowledge of the ballistic, or resistance, function $f(v)$. We have here made use of the famous result of the greatest of recent ballisticians,

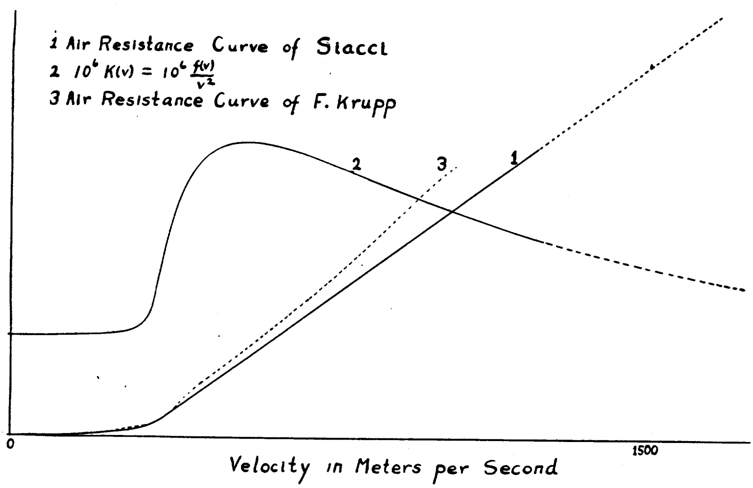

FIG. 1.

General Siacci, in his papers in the *Rivista di Artiglieria e Genio* (1896), from which it appears from the results of thousands of shots made by Bashforth in England, Mayevski in Russia (shots made at Krupp's), Hojel in Holland, and Krupp in Germany, that for velocities of over 300 meters per second the law is represented with great exactness by a straight line. Although experimental results do not go above velocities of 1,200 meters/sec., I have felt no hesitation in extrapolating for such values as are here used. In Fig. 1 are shown, in Curve 1, the values given by Siacci. In Curve 2, the values of the function $K(v) = f(v)/v^2$, and in Curve 3 the values of $K(v)$ as given by Krupp. It is only fair to say that the results of the French Commission de Gâvre more nearly resemble Curve 3 than Curve 1.

The use of the linear law (originally suggested by Chapel) has been recommended by the Comte de Sparre in a paper published in 1901.

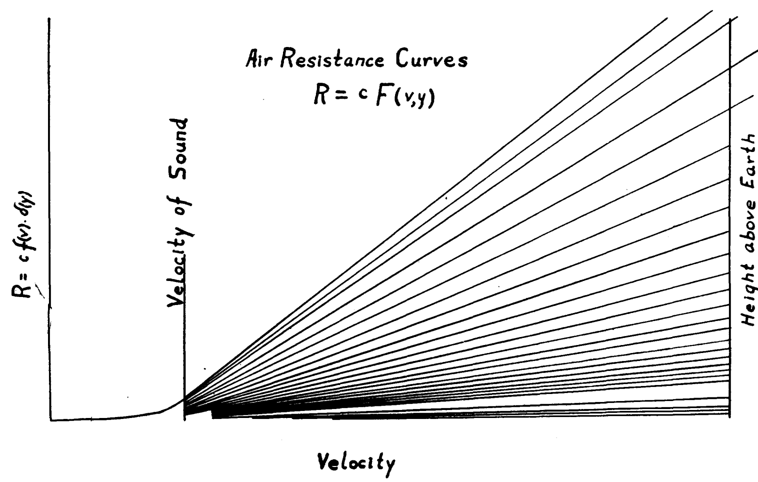

FIG. 2.

In order to expedite matters, a graphical chart was prepared (Fig. 2) with abscissas denoting the velocity in meters per second, containing straight lines and inclinations proportional to the densities of the air at different heights. On the right is a scale giving the height above the earth and by following the line corresponding the resistance R is read off on the scale at the left.

The equations used are as follows:

$$(7) \qquad \Delta y = \frac{g}{2x_0'^2}(\Delta x)^2, \qquad (8) \qquad \Delta v_x = \frac{c_y}{g} v f(v) \Delta\theta,$$

$$(9) \qquad \Delta t = \frac{\Delta x}{v_x{}^r}, \qquad (10) \qquad \Delta\theta = -\frac{g\Delta x}{v_r{}^2},$$

$$(11) \qquad \frac{v_x{}^r - \Delta v_x{}^r}{\cos\theta_{r+1}} = v_{r+1}, \qquad (12) \qquad D_x = \frac{\Delta v \cdot \Delta t}{2}.$$

The calculation is made as follows: An arbitrary value of x is selected, the drop Δy is calculated from (7), the change in angle from (10), the "set-back" from (12), and, finally, the corrected value of y corresponding to the given $x-D_x$ is obtained. One row of values is obtained for each element of the trajectory.

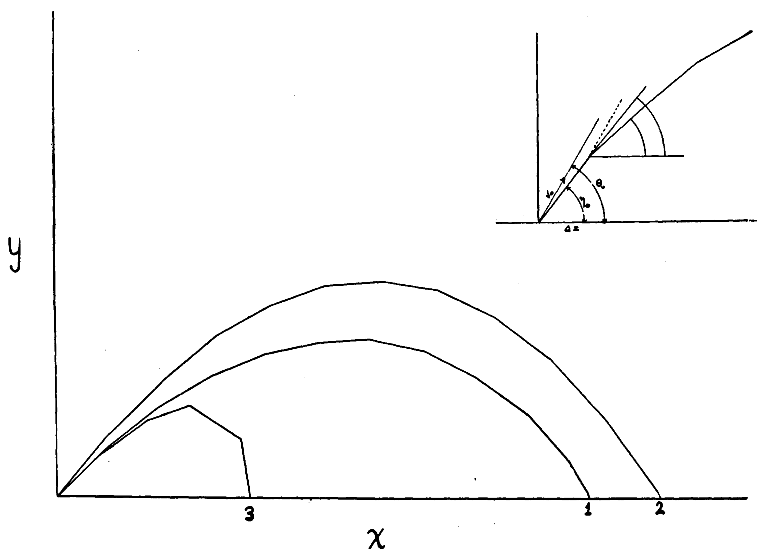

Fig. 3.

Fig. 3 shows the details of the graphical method. After the chords are drawn by means of a flexible ruler, the trajectory is drawn through the vertices of the polygon constructed.

In Fig. 4 are shown some of the most striking results. The heavy black curve shows the surface of the earth and the change of range, amounting to about a mile, that it would cause (neglecting the conversions of the verticals).

The method of choosing was frankly one of guessing. At first, an angle of departure of 45° was assumed with an initial velocity of 1,300 meters/sec. This gave Curve 1. The angle of departure was then increased to 52°, which gave Curve 2, with a range of 120 kilometers. In order to show the enormous effect of the resistance

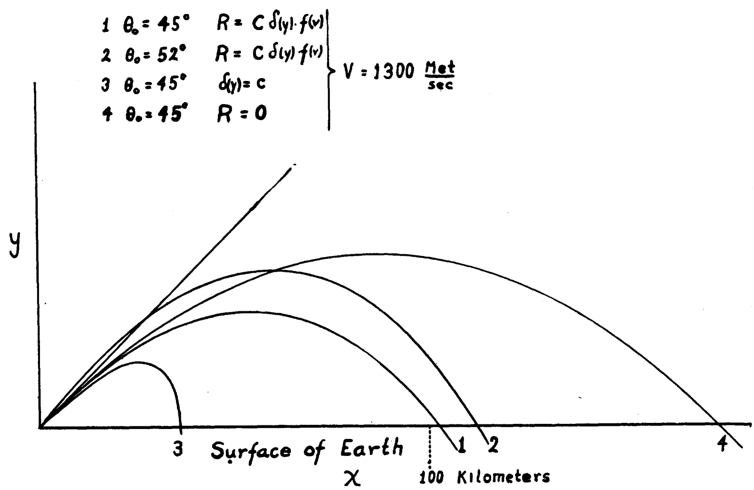

FIG. 4.

of the air, No. 4 is drawn showing the parabolic or vacuum trajectory, with a range of 174.4, and, finally, No. 3, on the assumption that the air has the constant density found at the surface of the earth. Further, the trajectory that we should have in case the density had the constant value, taken at a height two thirds of the maximum, as suggested by Colonel Ingalls for shots nearer the surface of the earth, was calculated. The range obtained was approximately 56 kilometers, quite different from the correct value of Curve 1.

Note, May 2, 1919.—The foregoing paper, which was read over a year ago, has, of course, lost the timeliness that it had at that moment. In fact, I have been advised by a high ballistic authority not to publish it, as such calculations are now "a matter of routine."

To this I have replied, that, although they may be *now,* they were not when I read the paper, and even now are not so in the Navy. In fact, when I inquired of a high naval authority how long it would take to calculate such a trajectory, he replied, "About two days." I said that we did it with two men in an hour.

Since then far better methods have been developed, but as I have seen only one publication of a trajectory, viz., by Major J. Maitland-Addison, R. A. (*Journal of the Royal Artillery,* Vol. XLV., No. 4), which confirms my results, I publish the paper as read, in the interest of the history of the matter, regretting that more pressing matters have so delayed the publication.