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Samuel A. Werner

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Neutron interferometry

Coherent beams of neutrons, split and recombined by Bragg diffraction in a perfect single crystal of silicon, demonstrate effects on the phase of the wavefunction due to gravity and other phenomena.

Samuel A. Werner

Diffraction effects at wavelengths on the order of angstroms have been known since Max von Laue's demonstrations of x-ray diffraction in 1912. Interference between well-separated, coherent beams, however, is much more difficult to arrange.

In 1965 Ulrich Bonse and Michael Hart (then at Cornell University) were able to obtain interference effects between beams of x rays with a wavelength of about 1 Å and spatially separated by about 1 cm. They used a perfect single crystal of pure silicon to split an x-ray beam into two coherent parts by Bragg reflection, and then used further Bragg reflections to recombine the beams. When they varied the optical path of the x rays in one of the beams they found oscillations in the intensity of the recombined beam. This remarkable achievement opened up the field of interferometry to the region of angstrom wavelengths and raised the question of whether one could use the same principles to obtain interference effects between coherent beams of thermal neutrons, which are diffracted by crystals in the same way as x rays.

Helmut Rauch, Wolfgang Treimer and Bonse finally demonstrated neutron interferometry in 1974 in a series of experiments at a small reactor at the Austrian Atomic Institute in Vienna.[1] An earlier attempt, in 1968, by Heinz Maier-Leibnitz and Tasso Springer to construct a neutron interferometer based on diffraction by a slit and subsequent deflection by a biprism proved only partially successful.

Obviously, the principles upon which neutron interferometry is based are very different from those applied in optical interferometry, in part because we are dealing with much smaller wavelengths and in part because we are dealing with neutrons. For these short wavelengths one generally uses Bragg reflection from crystal planes to split and recombine the beam; to ensure the coherence of the recombined beams the crystals must be large and almost perfectly free from lattice imperfections. Obviously it is not possible to polish and align the optical surfaces to fractions of a wavelength (in this case fractions of an angstrom) as one does for visible light. However, the use of Bragg reflection from sections of large, perfect crystals circumvents this difficulty and permits ready observation of interference between the two beams. With neutron interferometry the phase of neutron's wavefunction, ψ, becomes directly accessible to measurement, whereas earlier only the amplitude—or, rather the probability density, $|\psi|^2$—was directly measurable.

Since 1974 a number of experiments have been carried out that use the Bonse–Hart type of interferometer to probe the phase of the neutron wavefunction. Figure 1 shows the interferometer and a typical experimental arrangement. Among these experiments have been:

- demonstrations of the effects of the Earth's rotation and gravitational field on the neutron phase, as predicted by the Schrödinger equation
- measurements of the neutron–nucleus interaction potential
- demonstration that a fermion wavefunction reverses its sign after a rotation by 2π and is only restored to its initial phase by a rotation through 4π.
- searches for nonlinear variants of a Schrödinger equation.

In this article I will give an overall review of these experiments in an attempt to convey the beauty and simplicity of this technique in probing certain fundamental aspects of quantum physics. I will also speculate on future experiments to give some flavor of the scope of new applications.

A neutron interferometer

Various schemes have been proposed, and to some extent realized, for obtaining interference effects between spatially separated coherent thermal neutron beams having a wavelength in the angstrom range. I will limit myself here to considering the sort of arrangement shown in figure 1 and schematically depicted in figure 2. The interferometer consists of three identical, perfect slabs of silicon, cut perpendicular to a set of strongly reflecting lattice planes, typically the (220) planes; the slabs are machined from a single silicon crystal to ensure the perfect alignment of the crystal planes from slab to slab. The distances between slabs are usually a few centimeters and must be equal to within about a micron. A nominally collimated, monochromatic beam is directed from the source to the first slab of the interferometer (point A in figure 2), where it is coherently split by Bragg reflection. The two resulting beams are split by the second silicon slab in the regions near points B and C. The central two of these four beams overlap, in the region of point D on the third silicon slab. The two beams are each partly reflected, so that the "G" and "O" beams leaving the third slabs are coherent superpositions of the beams, I and II, that have passed through the interferometer.

If the beam traversing path I is shifted in phase by an amount β with respect to the beam along path II—by some interaction that increases the optical path length—the intensities measured by the detectors C_2 and C_3 will change. The expected intensities at these two detectors are of the form

$$I_2 = \gamma - \alpha \cos\beta \quad (1)$$

$$I_3 = \alpha(1 + \cos\beta) \quad (2)$$

where β is the difference in the relative phase of the wave functions between paths I and II, and α and γ are constants that depend upon the incident flux, the crystal structure and the neutron–nuclear scattering length of silicon. Equations 1 and 2 predict that the neutron current is "swapped" back and forth between C_2 and C_3 as the phase shift β is varied. Note that the con-

Samuel Werner is professor of physics at the University of Missouri, Columbia campus.

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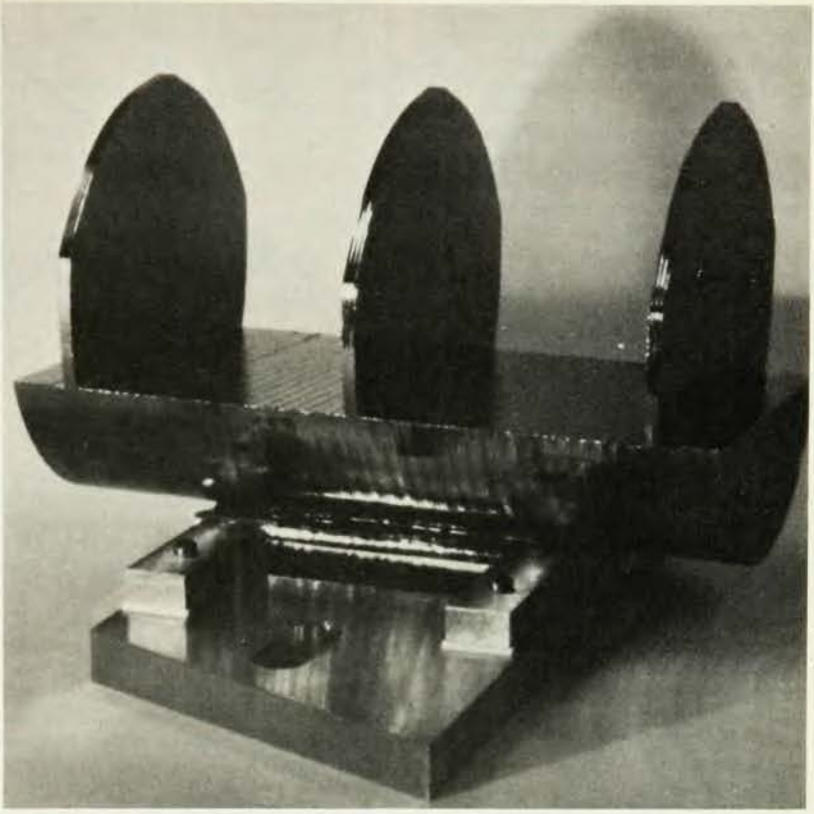
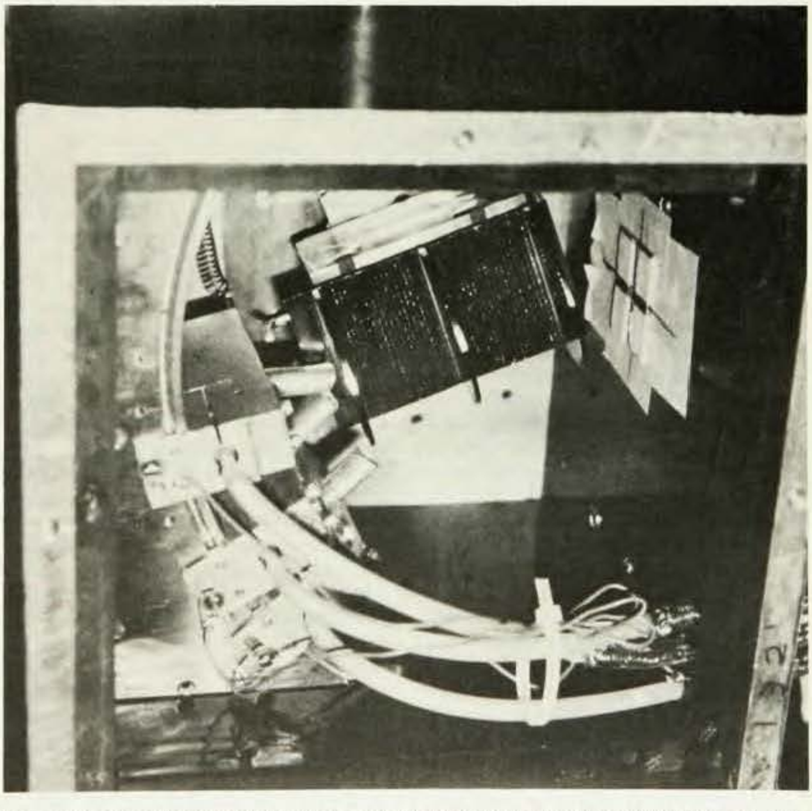
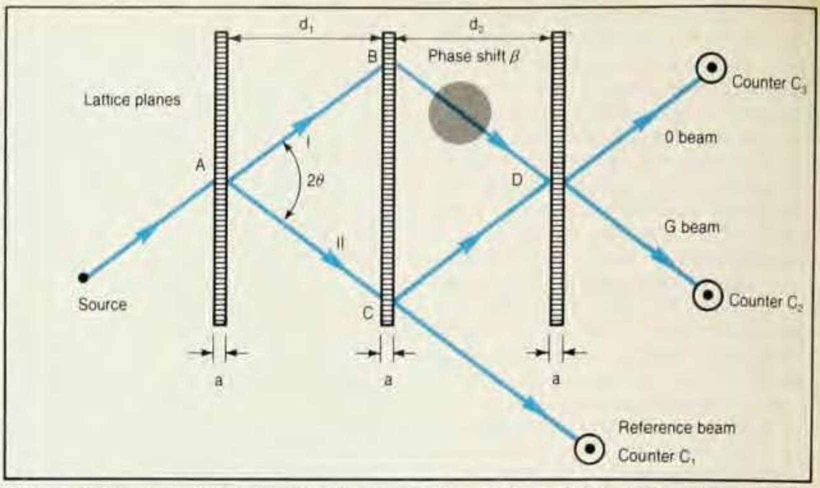
strast of the interference observed by counter C_3 is, in principle, 100%, while the contrast in I_2 can never be 100% (α is always less than γ). One generally records the output from both counters to enhance the rate at which significant data are collected. (Our graphs, however, will only show the experimental points from one counter.) The detector C_1 counts the neutrons in the non-interfering beam directed along the line AC: it serves as a reference.

Although the basic principle of this interferometer seems simple enough, we must remember that there are perhaps 10^9 oscillations of the neutron wave on each of the paths. Thus, there are clearly very stringent requirements on the thermal and microphonic stability of the apparatus. It is necessary to isolate the interferometer from the vibrations in the reactor hall, and to maintain an isothermal enclosure around it. The effect of vibrations is much more important for neutrons than for x rays because the transit time of neutrons across the interferometer (typically on the order of 50 microsec) is much longer than that of x rays, which travel with the speed of light. Thus, for a neutron interferometer, if the length of path I varies (because of vibrations, say) relative to path II by as much as a fraction of an angstrom during the 50 microsec transit time, the interference fringes are wiped out. To preserve the Bragg reflection condition for a given wavelength, the three crystals must be aligned to within the "Darwin width" of the beams. (This width is the angular width of the incident beam over which strong Bragg reflection occurs; for neutrons it is on the order of 0.1 sec of arc.) Bonse and Hart achieved this alignment in a simple and ingenious way: they cut all three slabs from a large, monolithic single crystal. As a consequence of great advances in crystal growth techniques, prompted by the needs of the solid-state electronics industry, it is possible today to purchase (at a modest cost) silicon crystals of the required perfection with typical dimensions of 5 to 10 cm from commercial manufacturers. The accuracy with which the surfaces of the slabs need to be polished is not as severe as might be anticipated. The reason for this is that the index of refraction of silicon (or most any other material) for thermal neutrons differs from 1 by only a few parts in 10^6. Calculation shows that a step of two microns on the surface of one of the slabs causes a phase shift of only $^1/_{100}$ of a fringe for 1.4-Å neutrons. Thus, the requirements on polishing are very similar to the requirements in ordinary optical interferometry. Finally, there is the question of the extent to which the incident beam must be monochromatic. Because the interferometer is based on Bragg reflection, the





The neutron interferometer used in the author's laboratory at the University of Missouri. The upper photo shows the silicon slabs machined from a single crystal of high-purity silicon to maintain alignment of the crystal planes from slab to slab. The lower photo shows such an interferometer in place in its housing at the reactor. The entrance slit is at the left and the three counters C_1, C_2 and C_3 (see figure2) are on the right. Figure 1 wavelength along a given trajectory (ray line) must be defined to within about one part in 10^6. However, this definition is accomplished by the interferometer itself, and not through the preparation of the incident beam. Thus one can carry out the experiments with beams for which $\Delta\lambda/\lambda$ is on the order of 1%, which can be provided by nuclear reactors with standard techniques for producing monochromatic beams. For a complete understanding of the quantitative performance of the interferometers, one must analyze the diffraction of neutrons within slabs of perfect single crystals using the dynamical theory of diffraction.[2]



Schematic diagram of the neutron interferometer shown in figure 1. The lattice planes, usually the (220) planes, are continuous from slab to slab, and the dimensions a, d_1 and d_2 are machined to optical precision. The phase shift in path I can be produced by inserting a gas cell, a slab of solid material or a magnetic field into the marked region. Figure 2

Coherent scattering

The scattering of thermal neutrons by nuclei is s-wave scattering because the neutron wavelength is much larger (by a factor of about 10^4) than the radius of a typical nucleus. Another way of seeing this is as follows: The angular momentum of an incident neutron (mass m, speed v) is on the order of mvR_0, where R_0 is the radius of the nucleus. Because v is on the order of 2200 m/sec and R_0 is around 10^{-12} or 10^{-13} cm, the angular momentum is much smaller than $\hbar\sqrt{l(l+1)}$ for $l=1$ or larger. Thus only the s-wave ($l=0$) component of the incident beam interacts with the nuclear potential. The outgoing wave is therefore a spherical wave, of the form

$$be^{ikr}/r$$

The parameter b is called the coherent scattering length; it is proportional to the strength of the neutron–nucleus interaction. The total scattering cross section is $4\pi b^2$, so that (except for the factor of 4, which is a quantum-mechanical effect) b is an effective radius of the scattering nucleus; for repulsive potentials b is positive while for attractive potentials it is negative.

Traditionally, one has determined the scattering lengths of nuclei from diffraction data from single crystals or from polycrystals; some experimenters have also used mirror-reflection or refraction of neutron beams to determine scattering lengths. These experiments typically have a precision of few percent. The neutron interferometer provides us with a new and very precise technique for determining scattering lengths (sometimes to within one part in 10^4), free from the usual uncertainties of the Debye–Waller and extinction effects always present in crystal diffraction experiments. (The Debye–Waller effect is the change in the intensity of Bragg reflections due to the thermal motions of the nuclei; the extinction effect is the change in the Bragg reflections due to absorption and multiple scattering).

Over the past five years a group of collaborators from the Austrian Atomic Institute in Vienna and the University of Dortmund in Germany have carried out a series of precise experiments at the Institut Laue-Langevin in Grenoble, France, to measure scattering lengths of nuclei, using neutron interferometry. The principle of the technique is straightforward: If one of the coherent beams traverses a sample of thickness t and refractive index n in one leg of the interferometer, then the two beams will differ in phase by

$$\beta = 2\pi(1-n)t/\lambda \qquad (3)$$

when they recombine. Here λ is the neutron wavelength. The index of refraction n, is given by

$$n = 1 - Nb\lambda^2/2\pi \qquad (4)$$

where N is the density of atoms, so the phase shift is proportional to the scattering length:

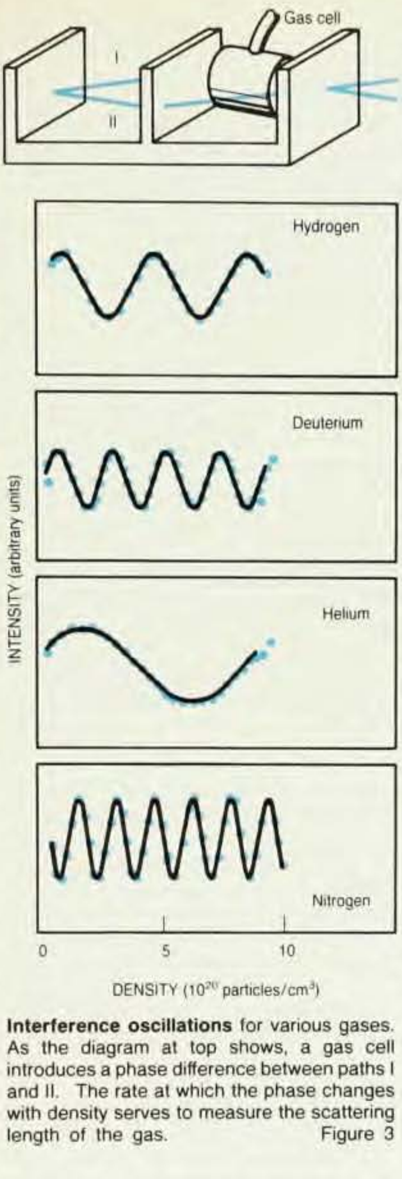
$$\beta = Nt\lambda b$$

For solids one can insert a slab of material in one of the arms and rotate it about an axis perpendicular to the plane of the interferometer, thus changing the effective thickness t and inducing interference oscillations in the counting rates observed in detectors C_2 and C_3. For gases one inserts a gas cell into one of the arms and observes the interference oscillations as one varies the gas pressure.

Figure 3 shows a few of the beautiful results[3] that the Vienna–Dortmund group have recently obtained at the Institut Laue-Langevin. These experiments were performed with a gas cell in one of the interferometer arms; one can compute the density of atoms from the (measured) pressure, with corrections from the known temperature dependence of the virial coefficients. The larger the scattering length, the more rapid the interference oscillations. For example, for helium-4 the scattering length is 3.26×10^{-13} cm (or 3.26 fermi) with a precision of 1%. For deuterium the scattering length is 6.55 fm, and the frequency of oscillation with atom density is clearly more rapid. The scattering length for hydrogen is negative. This is known from other experiments, but the sign of the scattering length can also be determined in an interferometer experiment by using a quarter-wave plate, made, say, of aluminum (0.05 mm thick), for which the scattering length is known to be positive and observing whether the intensity oscillations shift to the left or to the right by a phase of 90° in graphs like the ones shown in figure 3.

To calculate the scattering length from first principles one must solve the problem of the interaction of the neutron and all the nucleons in the scattering nucleus. This can be solved exactly only for two bodies, that is for scattering from hydrogen. There are some very useful results for the three-body problem (neutron–deuterium scattering), and recently developed methods for calculations of the few-body problem to allow a more fundamental treatment of the four-body problem. V. F. Katchenko and V. P. Levashere in Russia have carried out a detailed analysis using the Faddeev–Yacubovsky equations with a charge-independent separable central potential. Neutron scattering from helium-3 and from tritium are experimental realizations of the four-body problem. In the case of He^3, a strong effective attraction exists in the state where the compound nucleus has total spin of zero; it depends markedly on the details of the nuclear force. Because of the large cross section for neutron absorption by helium-3, measurement of the scattering length is quite difficult. However, the group at Grenoble has carried out a careful neutron-interferometry experiment and has obtained reasonable agreement with the theory. They are currently involved in measuring the scattering length for tritium, with the idea of gaining insight into the charge dependence of nuclear forces.

The neutron interferometer measures the average neutron–nuclear potential of the sample through which one of the beams passes. Because the interferometer is very sensitive to small changes in scattering length, it is also sensitive to small changes in the composition of the scattering material. Thus, for example, by comparing the interference pattern obtained from a metal sample containing hydrogen with that from a pure metal sample, the Grenoble group has been able to measure the hydrogen content in samples of various transition metals to a precision of about 0.05 atomic percent—a sensitivity competitive with the best analytical-chemical techniques.



Interference oscillations for various gases. As the diagram at top shows, a gas cell introduces a phase difference between paths I and II. The rate at which the phase changes with density serves to measure the scattering length of the gas. Figure 3

Quantum interference due to gravity

In most phenomena of interest in physics, gravity and quantum mechanics do not simultaneously play an important role. However, the neutron interferometer is sufficiently sensitive to detect the tiny changes in the phase of the wave function that arise from changes in gravitational potential energy. Here at the University of Missouri Research Reactor a group consisting of Jean-Louis Staudenmann (now at Iowa State University), Roberto Colella and Albert Overhauser (both of Purdue University) and myself has recently carried out[4] a precision experiment whose outcome depends necessarily upon both the gravitational constant and Planck's constant. Preliminary experiments were carried out five years ago at the University of Michigan.

Figure 4 shows the overall setup at the Missouri reactor. The thermal-neutron beam emerges from the reactor through a helium-filled tube. A pair of pyrolytic graphite crystals serves as a monochromator for the beam; this double crystal monochromator allows one to carry out experiments at various neutron wavelengths. The direction of the incident beam is fixed along the local North–South axis of the Earth; we will see that this is important in these experiments.

The experimental procedure involves turning the interferometer, including the entrance slit and the three detectors C_1, C_2 and C_3, about the incident beam, as shown in Figure 5. At each angular setting ϕ, neutrons are counted for a preset length of time. Paths I and II each have a horizontal segment and a sloping segment. The average gravitational potential of the sloping segments is the same, but depending on the angle ϕ, the horizontal segment for path II will be either above or below that for path I. The difference in the Earth's gravitational potential between these two levels causes a quantum-mechanical phase shift of the neutron wave on path I relative to path II. To calculate the phase shift one simply uses the de Broglie relationship between the momentum p and wavelength λ of the neutrons:

$$p = h/\lambda \qquad (5)$$

The momentum depends on the height, z, of the neutrons because energy is conserved:

$$E = p^2/2m_i + m_g gz \qquad (6)$$

where m_i is the inertial mass and m_g is the gravitational mass of the neutron; g is the acceleration due to gravity. From these relationships it is straightforward to show that the phase shift is

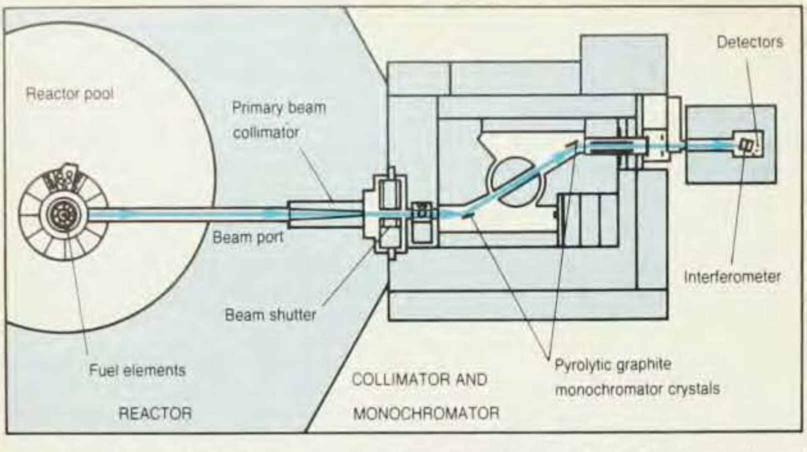
$$\beta_g = -2\pi m_i m_g (g/h^2)\lambda A \sin\phi \qquad (7)$$

where A is the area enclosed by the two beam trajectories. Thus, as we turn the interferometer through various angles ϕ about the incident beam direction, always maintaining the Bragg condition, we expect to see oscillations, induced by the Earth's gravitational field, in the outputs from detectors C_2 and C_3.

The results of an experiment carried out with incident neutrons of wavelength $\lambda = 1.419$ Å is shown in figure 5b. The contrast of the interference pattern dies out with increasing rotation angle because the interferometer warps and bends slightly under its own weight (on the scale of angstroms) as it is rotated about the axis of the incident beam, which is not an axis of elastic symmetry of the device. This bending effect also causes a small change of the period of the main oscillations. These effects have been studied and quantitatively measured with *in-situ* x-ray experiments, in which x rays are directed along the same incident beam path and the interfering x-ray beams are observed as a function of rotation angle ϕ. The effect of gravity (gravitational red shift) on x rays over the distances involved in the interferometer is negligible; one consequently assumes that the phase shifts observed are due only to the bending of the interferometer. The frequency of oscillation due to bending as measured by x rays can be subtracted from the frequency of oscillation measured with neutrons, leaving only the effects of gravitationally induced quantum interference. We have carried out an extensive series of measurements at various neutron wavelengths, the agreement of our most recent results with theory is at the 0.1% level.

This interference experiment demonstrates that a gravitational potential coherently changes the phase of a neutron wavefunction. Furthermore Daniel Greenberger (City College of New York) and Overhauser have derived[5] a result like equation 7, but with m_i^2 instead of $m_i m_g$, for the situation when the neutron source, beam slits and the interferometer have a uniform acceleration g. In comparing our experimental results with the theoretical results we have used the neutron mass as measured in a mass spectrometer (again, essentially m_i^2 instead of the combination of $m_i m_g$ that appears in equation 7). Thus, the agreement of our experiment with equation 7 provides the first verification of the equivalence principle in the quantum limit.

In 1925, Albert Michelson, Henry Gale and Fred Pearson carried out an



Overall arrangement of the neutron-interferometry experiment at the University of Missouri reactor in Columbia. The small box at right contains the equipment shown in figure 1. Figure 4

heroic experiment designed to detect the effect of the Earth's rotation on the speed of light. They constructed an interferometer in the form of a rectangle 2010 ft×1113 ft and were able to detect a retardation of light due to the Earth's rotation corresponding to about 1/4 of a fringe, in agreement with the theory of relativity. The French scientist Maurice Sagnac had demonstrated in 1913 that rotational motion (unlike rectilinear motion) can be detected with optical interferometry. The physical principle involved in the Sagnac effect is, in fact, the basis for the modern ring-laser gyroscope.

Neutron Sagnac effect

In view of the differences in the way light waves and matter waves behave under coordinate transformations, it is not obvious that an analogous quantum-mechanical effect should exist for neutrons. Because the gravitationally induced quantum interference experiment is carried out on the surface of our rotating Earth, a noninertial frame, the Hamiltonian governing the neutron's motion contains a term dependent on the angular velocity of the Earth, ω, and the angular momentum $\mathbf{L}$, of the neutron's motion about the center of the Earth. This term, $-\omega\cdot\mathbf{L}$, gives rise to the classical Coriolis force. Although this force has an exceedingly small effect of changing the neutron's trajectory in the interferometer, its effect on the neutron phase is not small. Calculation shows that the phase shift in the interferometer due to the Earth's rotation is expected to be[6]

$$\beta_r = 2m_i\,\omega\cdot\mathbf{A}/\hbar \qquad (8)$$

Here $\mathbf{A}$ is the normal vector for the area enclosed by the beam trajectories in the interferometer. The magnitude of β_r is about 2% of β_g, which should have been easily observable in the precision experiments described in the last section. The effects of gravity and rotation can readily be separated because the two phase shifts change in very different ways as one changes the interferometer's axis (the incident-beam direction) from the local north–south direction. In fact, as one can see from equation 8, β_r vanishes if the area vector is normal to the Earth's axis of rotation. On the other hand, if the interferometer axis is vertical, the gravitational shift remains constant as the interferometer rotates, because of the symmetry of the situation.

Staudenmann, Collella and I performed an experiment with a vertical incident beam at Missouri, measuring the phase shift as a function of the interferometer's orientation angle about the vertical direction. Because β_g remains constant, this experiment is a direct test of equation 8. The result is shown in figure 6. When the normal area vector, $\mathbf{A}$, points east or west, the phase shift vanishes, while if it points north or south the phase shift is $+95°$ or $-95°$, respectively. This result is in reasonable agreement with theory which predicts it should be $\pm 92°$.

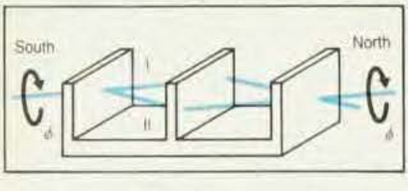
When we began this experiment we did not know the location of the north–south axis of the Earth relative to the reactor hall. Because the building contains a great deal of steel and magnetite concrete, a compass is useless in determining true north. We finally solved this problem by using a telescope to sight on the star Polaris outside the reactor building, and then using precision surveying techniques to carry this line of sight into the reactor hall (which is below ground level).

Because the results of this experiment depend only on the inertial mass of the neutron m_i, and the results of the gravity experiment depend on the product $m_i m_g$, one can interpret the combination of the two experiments as independent measurements of the inertial and gravitational neutron masses in a quantum-mechanical experiment.

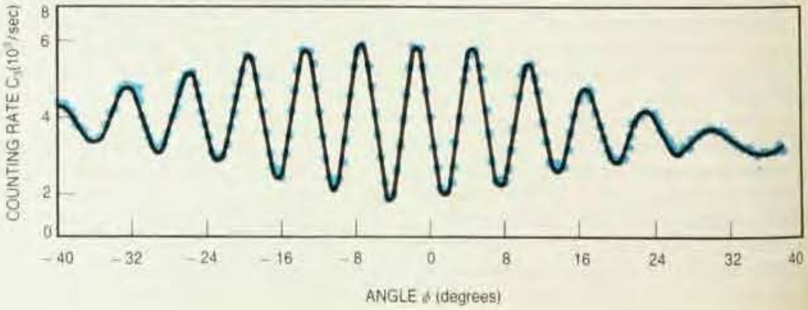
Recently Max Dresden and Chen-Ning Yang (SUNY at Stony Brook) have given[7] an interesting derivation of equation 8 by considering the phase shift of the rotating interferometer as arising from the Doppler shift due to a moving source and moving reflecting crystals. According to the general theory of relativity there are phase shifts of *very* small magnitude in addition to β_g and β_r which become larger as the neutron velocity increases. In particular, there is a phase shift due to the coupling of the neutron spin to the curvature of space-time. These effects have been studied theoretically[8] by Jeeva Anandan (University of Maryland) and, independently, by Leo Stodolsky (Max Planck Institut, Munich). Whether or not experiments can be carried out in this velocity regime is an open question; its answer will probably require new technology.

Magnetic effects

The operator for rotation through 2π radians causes a reversal of sign of the



Gravity-induced interference pattern, found by rotating the interferometer about a horizontal axis. (Data obtained by Werner, Staudenmann, Colella and Overhauser. Figure 5



wavefunction for a fermion. Although this principle is well known and is deeply imbedded in quantum theory, it had not been directly tested before the development of neutron interferometry. An experiment to observe this effect with the neutron interferometer was suggested in 1967 by Herbert Bernstein[9] (then at the Institute for Advanced Study, Princeton). In 1975, nearly simultaneously, both the Grenoble interferometry group in France and our group in the US performed the experiment.[10] Anthony Klein and Geoffrey Opat (University of Melbourne) later demonstrated[11] the same principle very nicely using a novel Fresnel-diffraction interferometer.

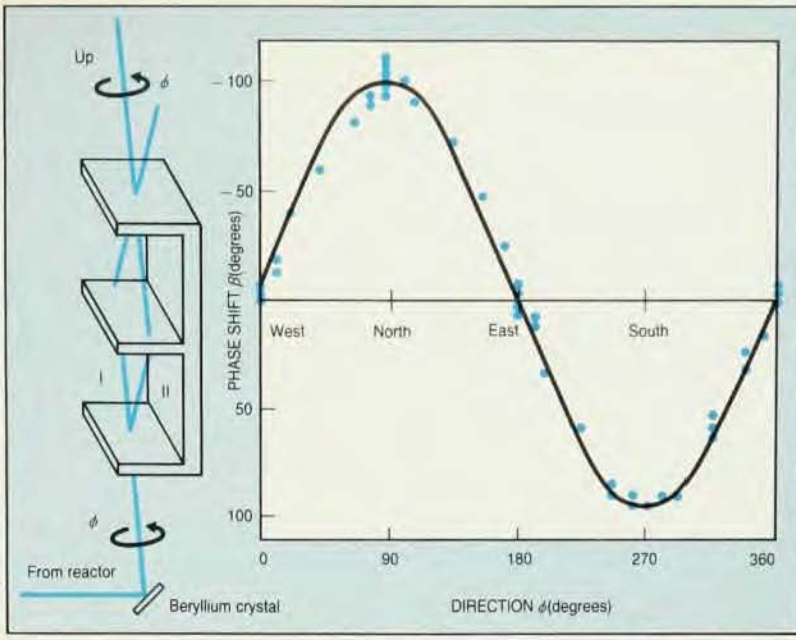
The basic idea of the experiment is that after a neutron (or other fermion) rotates through an angle of 2π, the sign of the wave function changes, or, in other words, the quantum phase shifts by π. We use a magnetic field to change the orientation of the neutrons in one arm of our interferometer and observe the resulting phase shift. The geometry of the experiment is shown in figure 7. If the neutron travels for a distance l through a magnetic field B, one expects its phase to shift by an amount

$$\beta_m = \pm 2\pi g_n \mu_n m\lambda Bl/h^2 \qquad (9)$$

Here the $\pm$ signs are for spin-up and spin-down neutrons; g_n is the neutron magnetic moment in nuclear magnetons (-1.91), μ_n is the nuclear magneton and m is the neutron mass. Putting in the numerical values, we expect that for a precession of 4π (or a phase shift of 2π)

$$B\lambda l = 272 \text{ gauss cm Å}$$

We show in figure 9 the data from the original Grenoble experiment, in which the neutron wavelength was 1.82 Å. The graph clearly shows that a phase shift of 2π corresponds to a precession of 4π radians. The neutron wavefunction thus is indeed a spinor. In classical physics, of course, no such behavior is possible: a rotation through 2π exactly restores the original state.



Neutron Sagnac effect: phase shift due to the Earth's rotation. The graph shows the phase difference between paths I and II as a function of the orientation of the normal vector for the area enclosed by the paths. The data were obtained by Staudenmann, Werner and Colella. Figure 6

Nonlinear Schrödinger equations

Last year, Abner Shimony (Boston University) suggested that the neutron interferometer offers sufficient sensitivity to test the physical reality of certain nonlinear variants of the Schrödinger equation, in which an additional term of the form $F(|\psi|^2)$ appears in the Hamiltonian. In earlier theoretical studies, Iwo Bialynicki-Birula and J. Myscielski had concluded that, unlike the linear Schrödinger equation, some of these nonlinear equations can have solutions in which traveling wave packets do not spread out in space with time. These nonlinear equations, however, allow us to retain many of our usual interpretations of quantum theory, in particular, the Born interpretation of the wavefunction, Galilean invariance, and the conservation of probability. Furthermore, they concluded that the function F should be logarithmic to obtain physically attractive correlations between non-interacting particles. Thus, in the case of a single-particle Schrödinger equation they proposed that a term of the form

$$-b\psi \ln(a^3|\psi|^2) \qquad (10)$$

should be added to the usual linear Schrödinger equation. The length a need not be a universal constant, because changing its value is equivalent to adding an unobservable constant potential to the Hamiltonian. The energy constant b, however, must be a universal constant, the same for all systems. They estimated from experimental data on the Lamb shift in hydrogen that b must be less than 4×10^{-10} eV.

Shimony pointed out that merely repositioning an intensity-attenuating plate downstream in a particle beam should cause a change of phase not predicted by a linear Schrödinger equation: The factor $-b\ln(a^3|\psi|^2)$ in equation 10 can be simply viewed as a small intensity-dependent potential, and because all potentials cause phase shifts in an interferometer experiment, this one should also. A straightforward calculation shows that if one positions an attenuating plate first at a point in one of the beams of an interferometer and then downstream a distance l from that point, the integrated effect of the intensity-dependent potential on the neutron phase is

$$\beta_{nl} = b(l/vh)\ln|\alpha|^2 \qquad (11)$$

Here v is the neutron velocity, and α^2 is the intensity attenuation of the absorbing plate.

Clifford G. Shull and his collaborators at MIT have recently carried out two beautiful experiments to test these ideas, using a two-crystal interferometer and absorbing plates of lithium fluoride and cadmium. They detected no phase shift. This experiment places an upper limit on the value of the fundamental constant b of 3.4×10^{-13} eV. A Fresnel-diffraction experiment carried out by R. Gähler, A. G. Klein and A. Zeilinger at the Institut Laue-Langevin placed an even lower limit of 3.3×10^{-15} eV on the value of b. This is more than five orders of magnitude lower than the limit previously inferred from Lamb shift data![12]

Speculations on future applications

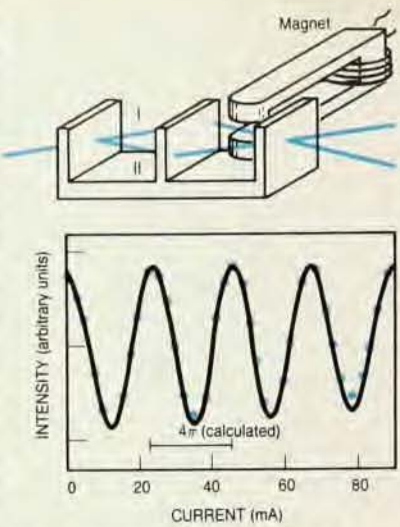
In a field as new as neutron interferometry it is, of course, difficult to predict the future. Over the past several years a large number of "back-of-the-envelope" ideas for experiments have been suggested. An international workshop on neutron interferometry[13] was held in Grenoble two years ago, at which time some of these ideas were put forward. In the way of a conclusion, I will briefly discuss some of these suggestions.

Quantum theory rests on the principle of superposition, which states that if ψ_1 and ψ_2 are two possible states of a system, and c_1 and c_2 are arbitrary numbers, then $c_1\psi_1 + c_2\psi_2$ is also a possible state of the system. It is usually taken for granted that the coefficients c_1 and c_2 are complex numbers. However, it is possible to imagine a quantum theory based on quaternions (numbers of the form $a + \mathbf{i}b + \mathbf{j}c + \mathbf{k}d$). Asher Peres (Technion, Israel) has proposed a simple test (at least in principle) to see if the neutron–nuclear scattering lengths are strictly complex numbers or whether they have an additional "dimension" allowed by a quaternion quantum theory. The idea is to use two flat plates of two different materials, say A and B. Placing material A in one leg of the interferometer will cause a phase shift, say Δ_A, while placing material B in the interferometer will cause a phase shift Δ_B. Placing both slabs in one of the interferometer beams together will cause a phase shift $\Delta_{AB} = \Delta_A + \Delta_B$, if the neutron scattering lengths are complex numbers. However, if they have a small "quaternion" component, Δ_{AB} will not equal $\Delta_A + \Delta_B$; furthermore, because quaternion multiplications do not commute, we expect $\Delta_{AB} \neq \Delta_{BA}$ so we should expect a difference if we interchange the order of A and B with respect to the direction of the beam. It is obvious that the effects to be searched for are very small; otherwise they would already have been detected in neutron diffraction crystallography.

In 1853 Fizeau carried out an experiment designed to measure the velocity of light in a moving fluid. The result was found to be in agreement with Fresnel's calculation, based on elastic vibrations of a stationary ether. We now know that this result can be derived from conventional Maxwell electromagnetic theory, and that it is therefore consistent with the special theory of relativity. Several people, among them Michael Horne (Stonehill College), Anton Zeilinger (Vienna) and Anthony Klein (Melbourne), have suggested carrying out the analogous quantum-mechanical "Fizeau experiment" using the neutron interferometer to measure the phase shift in a moving medium. In fact, experiments along these lines are under way in Grenoble. Of course, we think we know how to predict the result, but maybe we are over-confident.

Richard Deslattes (National Bureau of Standards) has proposed constructing a Michelson-type interferometer and repeating the Michelson–Morley experiment using neutrons.

Greenberger and I have suggested that carrying out an experiment to measure gravitationally induced quantum interference with ultra-cold neutrons could give surprising results, because the neutron trajectory would no longer be uniquely defined, and the usual WKB techniques for calculating phase shifts fails in the limit of zero velocity. If one could construct an interferometer that works with these very slowly moving neutrons, it is conceivable that it would also be useful in a precision search for an electric dipole moment on the neutron.



Demonstration of spinor rotation. The magnetic field induces a rotation of the neutron spin and changes the phase of the neutron wave function; a rotation through 4π produces phase shift of 2π. Figure 7

There has been considerable interest in recent years on coherent parity violations. Because the neutron participates in weak interactions, which do not conserve parity, one might expect that in the forward scattering of neutrons through matter there is a weak parity violating spin dependence. F. C. Michel and independently, Stodolsky have estimated the rotary power of matter composed of heavy atoms at normal densities to be about 10^{-8} radians/cm. This small rotation appears to be beyond the limits of sensitivity of the interferometers that we are currently using. However, Gabriel Karl has suggested that the effect can be enhanced if the phase-shifting medium is composed of twisted molecules with a given sense of helicity.

With pulsed neutron-spallation sources on the horizon, it is clear that energy dependence of neutron scattering lengths through the epithermal Breit–Wigner resonances of many isotopes will need to be measured. It is also clear that in the future one will be able to carry out experiments using both polarized neutrons and polarized targets (probably dynamically polarized), so that one can directly measure the scattering lengths for both spin states. Neutron interferometry will be useful for both kinds of experiments.

Finally, there is the possibility of using the two coherent beams available in a neutron interferometer to help solve the "phase problem" in certain crystallographic studies. Although this possibility has been discussed for some time, it still appears to require levels of perfection in the sample crystals and stability in the sample position that are as yet difficult to achieve.

I have not attempted to be all-inclusive in mentioning the wide variety of new ideas which have come to my attention, but only to give the reader a glimpse of the exciting possibilities for future experiments.

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References

1. H. Rauch, W. Treimer, U. Bonse, Phys. Lett. **47A**, 425 (1974).
2. See, for example, U. Bonse, W. Graeff, in *X-Ray Optics*, Topics in Applied Physics Vol. 22, H.-J. Queisser, ed., Springer, Berlin (1977).
3. H. Kaiser, H. Rauch, G. Badurek, W. Bauspiess, U. Bonse, Z. Physik **A291**, 231 (1979).
4. J.-L. Staudenmann, S. A. Werner, R. Colella, A. W. Overhauser, Phys. Rev. **A21**, 1419 (1980).
5. D. M. Greenberger, A. W. Overhauser, Rev. Mod. Phys. **51**, 43 (1979).
6. L. A. Page, Phys. Rev. Lett. **35**, 543 (1975).
7. M. Dresden, C. N. Yang, Phys. Rev. **D20**, 1846 (1979).
8. For general recent discussion of interference, gravity and gauge fields see, J. Anandan, Nuovo Cimento **53A**, 221 (1979).
9. H. J. Bernstein, Phys. Rev. Lett. **18**, 1102 (1967).
10. S. A. Werner, R. Colella, A. W. Overhauser, C. Eagen, Phys. Rev. Lett. **35**, 1053 (1975). H. Rauch, A. Zeilinger, G. Badurek, A. Wilting, W. Bauspiess, U. Bonse, Phys. Lett. **54A**, 425 (1975).
11. A. G. Klein, G. I. Opat, Phys. Rev. Lett. **37**, 238 (1976).
12. C. G. Shull, D. K. Atwood, J. Arthur, M. A. Horne, Phys. Rev. Lett. **44**, 765 (1980). R. Gahler, A. G. Klein, A. Zeilinger, submitted to Phys. Rev. A (1980).
13. *Neutron Interferometry*, Proceedings of an International Workshop at Institut Laue-Langevin, Grenoble, France, U. Bonse, H. Rauch, eds., Oxford U. P. (1979). □