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Trouble with Maxwell's Electromagnetic Theory: Can Fields Induce Other Fields in Vacuum?

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Abstract

The purpose of this article is to point out that Maxwell's electromagnetic theory, believed by the majority of scientists a fundamental theory of physics, is in fact built on an unsupported assumption and on a faulty method of theoretical investigation. The result is that the whole theory cannot be considered reliable, nor its conclusions accurate descriptions of reality. In this work it is called into question whether radio waves (and light) travelling in vacuum, are indeed composed of mutually inducing electric and magnetic fields.

Introduction

This study is addressed to that small percent of students and researchers who suspect that there is something wrong with the way in which we understand nowadays *how radio waves are generated and how they propagate in space*.

I know that there is always a feeling of distrust amongst students when university professors obtain the equation of a wave from the four Maxwell's equations. I felt that myself as a student and I have seen it again in the open courses made available on the Internet by prestigious universities of the world. Students ask pertinent questions but the professor fails to address the issue.

[See http://www.youtube.com/watch?v=JJZkjMRcTD4&feature=endscreen, min. 0:35:00].

When still a student I promised myself that, someday, I will get back to the subject of radio waves and analyze it piece by piece, statement by statement, equation by equation, and I will not declare myself in agreement with the theory if I discover unfounded assumptions, guesswork, or things contrary to experimental observations. I can say that I have found each of these.

What I consider most controversial in all the present conception regarding radio waves is the belief that the electric and magnetic fields produced in and around the antenna by the charges moving in it induce each other and create new fields at other points of space, *even in regions of space such as vacuum where there are no electric charges*, and that these fields become self-sustaining 'electromagnetic waves'. The majority of physicists and engineers agree with this description. No wonder, since they were good students and learnt what they could from their teachers and the textbooks available to them, all expounding the same doctrine.

In this work I will argue that the *idea of electric and magnetic fields inducing each other without the mediation of electrical charges* is false because it is not based on experimental evidence. Pure electric fields, varying or not, are not known to produce pure magnetic fields in regions of space where electrical charges do not exist. Neither pure magnetic fields are known to produce, in regions of space where electrical charges do not exist, pure electric fields. It is only through the mediation of electric charges and currents that these phenomena can happen. I will take excerpts from the works (mainly textbooks) of authors who support the present day theory and I will point out where their argument fails.

What produce radio waves is known – rapidly changing electric currents in a conductor. But what is not known with certainty is *how exactly radio waves are generated* from these changing electric currents, how the waves *detach themselves from the antenna* and *what radio waves really are* when traveling through space. These, I contend, are problems still open for argument and will be discussed here.

My alternative explanation is that radio waves in vacuum are simply mechanical waves in the aether filling the vacuum and produced by the charges (electrons) surging in the antenna. This view contradicts that purporting that radio waves (and light) are composed of electric and magnetic fields that oscillate and induce (create) each other in vacuum.

But this article is about Maxwell's theory and about the fact that it contains faulty methods of theoretical investigation and claims unsupported by experiment. I hope that what I have to say about this theory will make you eager to study the subject yourself with more attention than you did when you were a student and had to accept it because you needed credits to graduate the University. And, preferably, develop a personal opinion on what is believed to be one of the most important theories of physics.

If I have offered some alternative ideas throughout the paper it was without any other intention than to show that there are other possible ways to look at the problems discussed. Since this work is a critique of Maxwell's theory, the reader should not dismiss the latter just because he does not agree with the former. In fact, I warmly invite anyone interested to discuss whether the objections I raise are founded or not in the hope that, through this kind of debate, Maxwell's theory will either come out strengthened or be replaced altogether by another that makes more sense.

Mainstream science considers these matters settled beyond question and I do not expect great interest in this work from professional scientists. My hope is only that the young student, the young researcher at the beginning of his career or scientists who want to remain true to their profession will have enough time to ponder on these questions. My intention is not to demolish something that is valuable, but to find the true answers to the questions posed above and avoid the perpetuation of false ideas and flawed reasoning in physical science by turning a blind eye to what I believe is inaccurate. I consider it my duty as an educator, towards science itself, and towards present and future scientists.

SECTION I. What standard textbooks say

I know that it may be some time since you have graduated the high school, but I want to remind you how little standard textbooks for secondary grades have to say about *how* radio waves are generated. So I will start with some excerpts that deal with this topic.

A. First category : GCSE (and IGCSE) textbooks

These textbooks are written for secondary students (Grades 9 and 10). The two examples chosen below give, in one single sentence, some information about *what* produces the radio waves. Nothing is said about *how* these waves are generated.

1. Tom Duncan, Heather Kennett, *GCSE Physics*, 4th Ed., Hodder Murray, 2001, p. 52:

Radio waves

Radio waves have the longest wavelengths in the electromagnetic spectrum. They are radiated from aerials and used to 'carry' sound, pictures and other information over long distances.

"They [radio waves] are radiated from aerials […]."

2. Stephen Pople, *Complete Physics for IGCSE*, Oxford University Press, 2007, p. 162:

Radio waves

Stars are natural emitters of radio waves. However, radio waves can be produced artificially by making a current oscillate in a transmitting aerial (antenna). In a simple radio system, a microphone controls the current to the aerial so that the radio waves 'pulsate'. In the radio receiver, the incoming pulsations control a loudspeaker so that it produces a copy of the original sound. Radio waves are also used to transmit TV pictures.

"[…] radio waves can be produced artificially by making a current oscillate in a transmitting aerial (antenna)."

B. Second category : Advanced Level (A-Level) Physics and IB Physics textbooks

These textbooks are written for secondary students (Grades 11 and 12) taking a Physics course after finishing GCSE. They discuss more technicalities but are still silent about *how* are the *waves* generated by the current (or the charges) oscillating in the antenna.

1. M. Nelkon and P. Parker, *Advanced Level Physics*, 3rd Ed., Heinemann Educational Books, 1970, p. 986:

Radiation of Electromagnetic Waves into Space

Consider an oscillator with a connected transmission line, and suppose the transmission line is bent as shown in Fig. 39.23 (i).

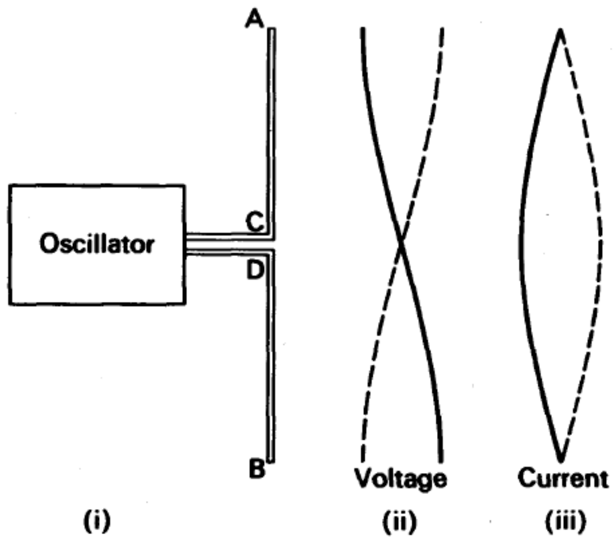


FIG. 39.23. Aerial. Half-wave dipole.

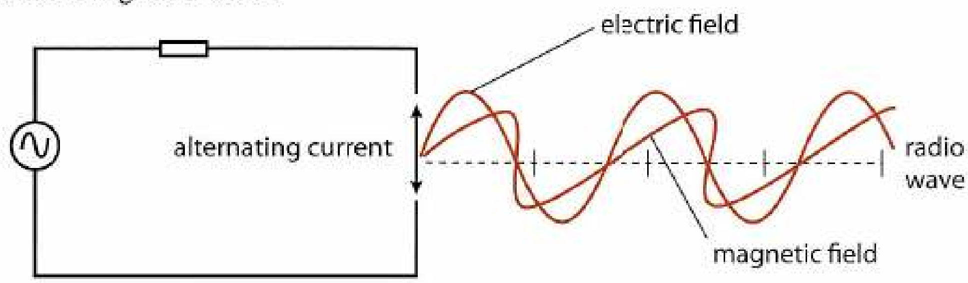
The charges moving along **AB** are forced up to **A** during one half cycle of oscillation and then down to **B** during the next half cycle. The charge, therefore, oscillates between **A** and **B**. *This accelerating charge radiates energy in the form of electromagnetic waves.* In contrast, charges moving in a line with a steady speed create a static magnetic field, and no electromagnetic wave is radiated.

"*This accelerating charge radiates energy in the form of electromagnetic waves*."

2. Chris Hamper, Keith Ord, *Standard Level Physics Developed Specifically for the IB Diploma*, Pearson Education Limited, 2007, p. 387:

Creating an electromagnetic wave

An electromagnetic wave can be created by passing an alternating current through a wire as shown in Figure 15.2. Waves created in this way are called *radio waves.* James Maxwell found that it was not the moving charge that caused the magnetic field but the changing electric field that caused the charge to move. This explains how electromagnetic waves can travel through a vacuum: the changing fields induce each other. Maxwell also calculated that the speed of the wave in a vacuum was approximately $3 \times 10^8\ \mathrm{m\,s^{-1}}$. This value was about the same as the measured value for the speed of light, so close in fact, that Maxwell concluded that light was an electromagnetic wave.



Here we find, for the first time, two statements that seem to me *inconsistent* with one another.
- the first is:
"An electromagnetic wave can be created by passing an alternating current through a wire […]. Waves created in this way are called *radio waves*."
- the second is:
"James Maxwell found that it was not the moving charge that caused the magnetic field, but the changing electric field that caused the charge to move."

What is the student supposed to understand from these statements? It seems that the electric currents are not seen any more as the primary cause producing the radio waves. The primary cause for the production of radio waves has been shifted to the *changing electric field* that produces the oscillating current.

Thus the textbook tells the student something new: that *a changing electric field generates a changing magnetic field*. But is this true? Can a field produce another field? The textbook says that this was "found" by Maxwell. But did Maxwell prove what the textbook says he "found"? Sadly, the answer turns out to be no. Not only that Maxwell did not prove it by any experiment but nobody proved it experimentally in the 150 years that have passed since then. What Maxwell did was a mathematical manipulation, which we shall discuss later.

Why is this important? It is important because Maxwell's "finding" is then used to explain why 'electromagnetic waves' can travel through vacuum, or even exist as a system of electric and magnetic fields in such regions of space as vacuum where there are no electric charges whatsoever. The explanation is: "the changing fields induce each other". It is meant that, after being created by the original charges that oscillated in the antenna, the electric and magnetic fields continue to create (induce) each other and exist even in regions of space far from the antenna, *where there are no electrical charges at all*. In the following section, I will argue that this picture is inaccurate.

In closing this section and before we discuss what mathematical manipulation Maxwell did and why he did it, there is an obvious fact that shows that electromagnetic waves are not produced by changing electric fields. Look at the antennas that we use: they are all conductors. If the primary source of radio waves would be the varying electric fields (which would then induce magnetic fields, which would then in their turn induce another new electric field further away, and so on) we would use for our antennas huge *capacitors* and not conductors. Our antennas would look like two huge metal plates separated by a dielectric (air) and connected to a source of oscillating high voltage. But this is not the case in practice: even since the times of Hertz and Marconi, radio waves have been produced by *discharges* (sparks) between the knobs of the induction coil. [See, J. J. Fahie, *A History Of Wireless Telegraphy 1838-1899*, William Blackwood and Sons, 1899]. All past experimentation comes to demonstrate that if an *electric current* is not made to move violently in a conductor, no radio waves can be released into space.

SECTION II. Changing fields do not induce each other

A. Where is Maxwell not correct?

Since the previous textbook did not say *how James Clerk Maxwell found that a changing electric field can produce a magnetic field*, we will take another, more advanced, textbook, designed for undergraduate students: David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999. It is a well-known standard textbook and many physics students have used it in their studies. This section makes heavy reference to it.

We discover from this textbook that Maxwell introduced the idea that a changing electric field can produce a changing magnetic field *by modifying the experimentally found Ampere's law*. At pages 321 and 326, we read:

7.3 Maxwell's Equations

7.3.1 Electrodynamics Before Maxwell

So far, we have encountered the following laws, specifying the divergence and curl of electric and magnetic fields:

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \quad \text{(Gauss's law)},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \quad \text{(no name)},$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday's law)},$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{(Ampère's law)}.$$

These equations represent the state of electromagnetic theory over a century ago, when Maxwell began his work.

This set of equations has been changed by Maxwell into:

7.3.3 Maxwell's Equations

In the last section we put the finishing touches on Maxwell's equations:

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \quad \text{(Gauss's law)},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \quad \text{(no name)},$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday's law)},$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \text{(Ampère's law with Maxwell's correction)}.$$

Observe that Ampere's original law $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J}$, which was a mathematical description of *experimental findings* relating the magnetic field $\mathbf{B}$ to the current density $\mathbf{J}$ producing it, has been changed by Maxwell by *adding a supplementary term* to the right-hand side of the equation $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \varepsilon_0 \cdot \dfrac{\partial \mathbf{E}}{\partial t}$.

Maxwell's addition, $\mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$, has received the name "Maxwell's displacement current".

Ampere's original law allows the calculation of the magnetic field $\mathbf{B}$ produced at a point in space by currents $\mathbf{J}$ flowing along other curves in space. It has its experimental roots in Oersted's great discovery that an electric current produces a magnetic field in the space around it. If another term is added to this equation, it follows that the magnetic field can be produced also in the manner described by this new term. Adding $\mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$ to Ampere's original equation is equivalent to saying that a changing electric field $\mathbf{E}$ can produce a magnetic field $\mathbf{B}$. Maxwell's modification of Ampere's law by the addition of this supplementary term is not correct.

B. Why is Maxwell not correct?

Maxwell is not correct for the following reasons:

(i) Such an effect (that a changing electric field $\mathbf{E}$ can produce a magnetic field $\mathbf{B}$) has *not been observed experimentally*. Therefore, adding the term $\mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$ to Ampere's original equation is pseudo science.
To see how absurd the matters can get, observe that you obtain a magnetic field even if there are no electric currents at all. For $\mathbf{J} = 0$, Ampere's law (modified by Maxwell) becomes:

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Since the electric charges, static or in motion, do not appear in the equation, this equation says that a pure electric field $\mathbf{E}$ varying in time can create a pure magnetic field $\mathbf{B}$.

This is pseudo science because experiments show that fields are created by charges. An electric field $\mathbf{E}$ is created by a static charge and a magnetic field $\mathbf{B}$ by a moving charge. Every time there is a field, this field can be traced to an electrical charge, at rest or in motion. The equation above, however, implies that charges and currents are not necessary for the creation of fields and that one field (time-varying electric field $\mathbf{E}$) can, directly, by itself, without other means, without the aid or mediation of something else other than itself, create another field (magnetic field $\mathbf{B}$).

According to Coulomb's law $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$, the electric field $\mathbf{E}$ can change in time only if the charge density ρ changes in time, but this is not apparent any more in Maxwell's modification of Ampere's law.

(Note: Faraday's law seems to indicate another way in which the electric field $\mathbf{E}$ can be changed, but this is only apparent. As will be discussed later, Maxwell has modified Faraday's law by making the same conceptual mistake as he did when he modified Ampere's original law. For what Faraday observed was that a changing magnetic field $\mathbf{B}$ induces *an electric current* $\mathbf{J}$ and not an electric field $\mathbf{E}$. So the mathematical rendering of Faraday's law is also questionable and will be discussed later.)

(ii) Maxwell's "displacement current" is not an electric current. If there are supplementary currents to be added in Ampere's law (and we will see later that one supplementary current must indeed be added), *these currents must be added as currents*, not as something else (such as varying electric fields), because this is what observations show: moving electric charges produce a magnetic field around them. A current (more accurately, *current density*, because Ampere's law is written in terms of $\mathbf{J}$ - the current density) is defined as

$$\mathbf{J} = \rho \cdot \mathbf{v}$$

where ρ is the charge density and $\mathbf{v}$ is the velocity of the charges.

C. How should Maxwell have corrected Ampere's law?

Maxwell introduced his "displacement current" in Ampere's law in an attempt to make it more general, i.e. to make it comply with the equation of continuity for the electric charge. Look at the explanations below, which will start with a repetition of the excerpt from David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999, shown above:

7.3 Maxwell's Equations

7.3.1 Electrodynamics Before Maxwell

So far, we have encountered the following laws, specifying the divergence and curl of electric and magnetic fields:

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \quad \text{(Gauss's law)},$$

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$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday's law)},$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{(Ampère's law)}.$$

These equations represent the state of electromagnetic theory over a century ago, when Maxwell began his work. They were not written in so compact a form in those days, but their physical content was familiar. Now, it happens there is a fatal inconsistency in these formulas. It has to do with the old rule that divergence of curl is always zero. If you apply the divergence to number (iii), everything works out:

$$\nabla \cdot (\nabla \times \mathbf{E}) = \nabla \cdot \left(-\frac{\partial \mathbf{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \cdot \mathbf{B}).$$

The left side is zero because divergence of curl is zero; the right side is zero by virtue of equation (ii). But when you do the same thing to number (iv), you get into trouble:

$$\nabla \cdot (\nabla \times \mathbf{B}) = \mu_0(\nabla \cdot \mathbf{J}); \tag{7.35}$$

the left side must be zero, but the right side, in general, is *not*. For *steady* currents, the divergence of $\mathbf{J}$ is zero, but evidently when we go beyond magnetostatics Ampère's law cannot be right.

[…]

Of course, we had no right to *expect* Ampère's law to hold outside of magnetostatics; after all, we derived it from the Biot-Savart law. However, in Maxwell's time there was no *experimental* reason to doubt that Ampère's law was of wider validity. The flaw was a purely theoretical one, and Maxwell fixed it by purely theoretical arguments.

7.3.2 How Maxwell Fixed Ampère's Law

The problem is on the right side of Eq. 7.35, which *should be* zero, but *isn't*. Applying the continuity equation (5.29) and Gauss's law, the offending term can be rewritten:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\epsilon_0 \nabla \cdot \mathbf{E}) = -\nabla \cdot \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right).$$

It might occur to you that if we were to combine $\epsilon_0(\partial \mathbf{E}/\partial t)$ with $\mathbf{J}$, in Ampère's law, it would be just right to kill off the extra divergence:

$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.} \tag{7.36}$$

[…]

Such a modification changes nothing, as far as magneto*statics* is concerned: when $\mathbf{E}$ is constant, we still have $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$. In fact, Maxwell's term is hard to detect in ordinary electromagnetic experiments, where it must compete for recognition with $\mathbf{J}$; that's why Faraday and the others never discovered it in the laboratory. However, it plays a crucial role in the propagation of electromagnetic waves, as we'll see in Chapter 9.

In my opinion there is a correct and honest way in which Ampere's law can be "fixed" and make it comply with the equation of continuity; it starts from the following idea: how must the *current density* $\mathbf{J}$ in the original Ampere's law $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J}$ be changed so that $\nabla \cdot (\nabla \times \mathbf{B})$ equals zero? The change of the *current density* $\mathbf{J}$ can be made by adding another current density $\mathbf{J}'$, so that Ampere's law becomes:

$$\nabla \times \mathbf{B} = \mu_0 \cdot (\mathbf{J} + \mathbf{J}')$$

Then the vector calculus identity used by Maxwell, which says that, for *any* vector $\mathbf{B}$, the expression $\nabla \cdot (\nabla \times \mathbf{B})$ must be zero, gives:

$$\nabla \cdot (\nabla \times \mathbf{B}) \equiv 0 \Rightarrow \mu_0 \cdot \nabla \cdot (\mathbf{J} + \mathbf{J}') = 0 \Rightarrow \nabla \cdot (\mathbf{J} + \mathbf{J}') = 0 \Rightarrow \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}' = 0$$

The equation of continuity $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$ and the above result then show that the extra current $\mathbf{J}'$ that must be added to Ampere's law must be such that $\nabla \cdot \mathbf{J}' = \frac{\partial \rho}{\partial t}$. This expression shows that the additional term $\mathbf{J}'$ depends on the time-derivative of the charge density ρ, i.e. $\mathbf{J}' = \mathbf{J}'\left(\frac{\partial \rho}{\partial t}\right)$.

With this expression for $\mathbf{J}'$, Ampere's law becomes $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \cdot \mathbf{J}'\left(\frac{\partial \rho}{\partial t}\right)$ and $\nabla \cdot (\nabla \times \mathbf{B})$ equals zero, as required by the vector calculus identity.

D. How is this modification different from Maxwell's?

The above modification is different from Maxwell's in that Ampere's law still contains currents and only currents (current densities, actually), as observed experimentally. I consider it correct because no other physical quantities are added artificially – only currents.

Also, observe that for vacuum, where there are no charges ($\rho = 0$) and no currents ($\mathbf{J} = 0$), $\mathbf{J}'$ becomes also zero because it depends on the existence of electric charges. Ampere's law for vacuum becomes $\nabla \times \mathbf{B} = 0$, which is completely different from that obtained by Maxwell, $\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$.

In Maxwell's modification of Ampere's law, the supplementary current $\mathbf{J}' = \mathbf{J}'\left(\frac{\partial \rho}{\partial t}\right)$ is not left as above, but it is expressed further through *purely mathematical manipulations*. This was shown in the previous page in the excerpt from David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999, p.323, but I rewrite it here:

Maxwell starts from Coulomb's law, $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$, which he uses for the case in which *there are* charges, i.e. $\rho \neq 0$ (note this because Maxwell afterwards claims that the modified Ampere's law that he obtains through its use is valid also for vacuum, where $\rho = 0$). I will mention the condition $\rho \neq 0$ with a vertical bar $\big|_{\rho \neq 0}$ throughout the calculations made by Maxwell to remind the reader that the calculations are performed with *this* condition ($\rho \neq 0$) and that the calculations are not possible if $\rho = 0$.

$$\nabla \cdot \mathbf{E}\Big|_{\rho\neq 0} = \frac{\rho}{\varepsilon_0}\Big|_{\rho\neq 0} \Rightarrow \rho\Big|_{\rho\neq 0} = \varepsilon_0 \cdot \nabla \cdot \mathbf{E}\Big|_{\rho\neq 0} \Rightarrow \frac{\partial \rho}{\partial t}\Big|_{\rho\neq 0} = \varepsilon_0 \cdot \frac{\partial}{\partial t}(\nabla \cdot \mathbf{E})\Big|_{\rho\neq 0} \Rightarrow$$

$$\Rightarrow \frac{\partial \rho}{\partial t}\Big|_{\rho\neq 0} = \varepsilon_0 \cdot \nabla \cdot \left(\frac{\partial \mathbf{E}}{\partial t}\right) \Rightarrow \frac{\partial \rho}{\partial t}\Big|_{\rho\neq 0} = \nabla \cdot \left(\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\right)\Big|_{\rho\neq 0}$$

Comparing $\nabla \cdot \mathbf{J}' = \frac{\partial \rho}{\partial t}$ with $\frac{\partial \rho}{\partial t}\Big|_{\rho\neq 0} = \nabla \cdot \left(\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\right)\Big|_{\rho\neq 0}$ found above, Maxwell observed that $\mathbf{J}' = \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\Big|_{\rho\neq 0}$.

He then *introduced it directly in Ampere's law*, obtaining:

$$\nabla \times \mathbf{B} = \mu_0 \cdot (\mathbf{J} + \mathbf{J}') \Rightarrow \nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\Big|_{\rho\neq 0}$$

The difference between Maxwell's modification of Ampere's law, and the one which I consider correct, is summarized in the table below:

Maxwell's modification of Ampere's law	Amperes' law modified correctly
$\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\Big\|_{\rho\neq 0}$ where, as the vertical bar $\Big\|_{\rho\neq 0}$ indicates, $\mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\Big\|_{\rho\neq 0}$ was obtained from $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$, with $\rho \neq 0$	$\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \cdot \mathbf{J}'\left(\frac{\partial \rho}{\partial t}\right)$ where $\nabla \cdot \mathbf{J}' = \frac{\partial \rho}{\partial t}$

The difference between the two is enormous because, in general, in physics the equations connecting different physical quantities are interpreted phenomenologically, that is, they must correspond to effects observed experimentally.

As stated above, Maxwell's version of Ampere's law implies that a magnetic field $\mathbf{B}$ can be produced by a changing electric field $\mathbf{E}$, and this is not observed experimentally.

For vacuum, Maxwell's modification of Ampere's law and Amperes' law modified correctly differ significantly, as shown in the table below:

Maxwell's modification of Ampere's law for vacuum ($\rho = 0$, $\mathbf{J} = 0$)	Amperes' law modified correctly for vacuum ($\rho = 0$, $\mathbf{J} = 0$)
$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\Big\|_{\rho\neq 0}$ where $\mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}\Big\|_{\rho\neq 0}$ still corresponds to the situation with $\rho \neq 0$, so the equation in reality does not correspond to vacuum.	$\nabla \times \mathbf{B} = 0$

The difference is due to the fact that Maxwell was inconsistent in his calculations: he used Coulomb's law *with charges* ($\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \rho \neq 0$) to modify Ampere's law and then claimed that the modified Ampere's law obtained from it is valid for vacuum (*no charges*, i.e. $\rho = 0$), too. In fact, this claim cannot be true because, if $\rho = 0$, Coulomb's law becomes $\nabla \cdot \mathbf{E} = 0$, and cannot be used any more to find the expression for $\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t} \right)$ and for $\mathbf{J}' = \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$, as shown in the above derivation.

That Ampere's law for vacuum must be $\nabla \times \mathbf{B} = 0$ and not $\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$ can be seen also from another fact. Remember that Ampere's original law read $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J}$ and that the term $\mu_0 \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$ was added by Maxwell to make it comply with the equation of continuity $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$ and with the fact that always $\nabla \cdot \nabla \times \mathbf{B} = 0$. Obviously, these equations referred to regions of space where there were charges and currents ($\rho \neq 0$, $\mathbf{J} \neq 0$).

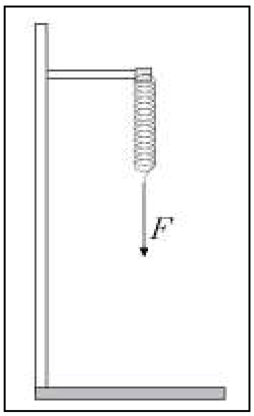
However, if we refer to vacuum ($\rho = 0$, $\mathbf{J} = 0$), we observe that the equation of continuity $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$ becomes identical zero while Ampere's original law $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J}$ becomes $\nabla \times \mathbf{B} = 0$ and satisfies the vector calculus identity $\nabla \cdot \nabla \times \mathbf{B} = 0$. So, in the case of vacuum, it is not necessary at all to modify Ampere's original law $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J}$ in any way since it already satisfies all the necessary requirements invoked for its modification: $\nabla \cdot \nabla \times \mathbf{B} = 0$ and $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$.

Going back to what experimental evidence say, Ampere's law states that there must always be *electric currents* to produce a magnetic field: even if $\mathbf{J} = 0$, it is the supplementary current $\mathbf{J}' \neq 0$ that produces a magnetic field $\mathbf{B}$. This supplementary current $\mathbf{J}'$ is produced through the change of charge density ρ, such that $\nabla \cdot \mathbf{J}' = \frac{\partial \rho}{\partial t}$. The equations always *link the fields with the charges producing them* and never omit them as important intermediaries between the fields. The correctly modified Ampere's law does not predict absurd, never observed, phenomena such as that according to which a magnetic field $\mathbf{B}$ can be produced by a changing electric field $\mathbf{E}$. Even if there is an equality of magnitude between $\mathbf{J}'$ and $\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$ (as Maxwell showed), this does not mean that $\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$ can be replaced in the equations where $\mathbf{J}'$ appears and expect that a changing electric field $\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$ replaces the physical effects of $\mathbf{J}'$.

As a conclusion, Maxwell is not correct because, in science, the equations we write should not be correct only dimensionally and quantitatively, but they must also correspond to observed phenomena. *Substituting* $\varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t}$ *for* $\mathbf{J}'$ *in Ampere's law, although correct mathematically and dimensionally, is not correct phenomenologically, because the interpretation of the law thus modified leads to absurdities not observed in* *real world*.

There are many situations in physics when we replace physical quantities in different equations, obtain other equations that are correct dimensionally and quantitatively, and use them to calculate unknown physical quantities. But we cannot expect these manipulated equations to make sense phenomenologically, to see in them a true, direct cause-effect relationship between the physical quantities that appear in it. As is the case with Maxwell's modification of Ampere's law, the equations manipulated by mathematical operations, even if correct, bring together mathematical expressions corresponding to physical phenomena that have no direct cause-effect relationship and turn out to be absurd statements if interpreted phenomenologically.

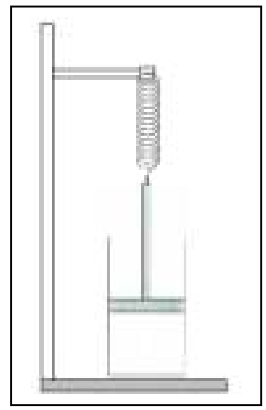
To give you an example, consider a spring hung vertically. We know experimentally that the spring stretches because there is a force F acting on it and we express the extension of the spring x in terms of the force F as



$$x = \frac{1}{k} \cdot F$$

But we can apply a force to the spring in another way. For example, consider a piston attached to the spring and the cylinder fixed to the ground.

The gas in the cylinder contracts when cooled and the effect is that the piston moves downwards, pulling the spring with a force.



So besides pulling forces F that may act on the spring, we have to consider another force F' that produces the same effect. The original formula giving the extension of the spring becomes

$x = \frac{1}{k} \cdot (F + F')$, according to the law of addition of forces, verified experimentally.

Then we can measure experimentally how F' changes with the temperature. Suppose that experiments yield:

$$F' = -R \cdot \Delta T$$

where R is a constant and ΔT is the change in the temperature of the gas in the piston, showing that a negative temperature change produces a positive force F' that stretches the spring.

Now, equations $x = \frac{1}{k} \cdot F$, $x = \frac{1}{k} \cdot (F + F')$, and $F' = -R \cdot \Delta T$ have been obtained experimentally and can be interpreted phenomenologically.

But if we replace F' in the equation for extension x, we obtain

$$x = \frac{1}{k} \cdot (F - R \cdot \Delta T)$$

which, although correct mathematically (quantitatively and dimensionally), leads to absurdities when interpreted phenomenologically, for it says that *a spring can be stretched by a decrease in the temperature*.

Maxwell's modification of Ampere's law has been obtained by a similar *false method of theoretical investigation* and this is why it cannot be considered correct.

E. How does this affect Maxwell's electromagnetic theory?

Maxwell's theory remains unchanged if the equations are not used for vacuum, but for regions of space where *there are* charges and currents. Coulomb's law with charges leads indeed to the system of four equations,

(i) $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ (iii) $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

(ii) $\nabla \cdot \mathbf{B} = 0$ (iv) $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \cdot \varepsilon_0 \cdot \dfrac{\partial \mathbf{E}}{\partial t}$

and, as it will be seen later, they yield the equations of waves. Also, it will be discussed in what follows that equation (iii), corresponding to Faraday's law, agrees with experimental observations and therefore it can be written in such a form *provided that due account is given to the fact that charges and currents must always be present* (i.e. $\rho \neq 0$, $\mathbf{J} \neq 0$).

However, Maxwell's theory changes dramatically if the equations are written for vacuum, because of Ampere's law:

(i) $\nabla \cdot \mathbf{E} = 0$ (iii) $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

(ii) $\nabla \cdot \mathbf{B} = 0$ (iv) $\nabla \times \mathbf{B} = 0$

It can be seen that Faraday's law $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ remains the only law claiming that a changing field (magnetic) creates another field (electric) in vacuum. But is it true?

F. Trouble with Faraday's law. When a magnet is inserted in a coil, what does the changing magnetic field of the magnet induce in the coil, an electric field or an electric current?

As mentioned earlier, I will discuss in this second section of the study another serious logical inaccuracy that I observed in the accepted laws of electricity and magnetism. It refers to the interpretation of Faraday's law of electromagnetic induction.

Let us refer this time to another well-known textbook: John David Jackson, *Classical Electrodynamics*, John Wiley and Sons, 1962, designed, at its time, for beginning graduate students. At page 170, we read:

170 *Classical Electrodynamics*

6.1 Faraday's Law of Induction

The first quantitative observations relating time-dependent electric and

> magnetic fields were made by Faraday (1831) in experiments on the behavior of currents in circuits placed in time-varying magnetic fields. It was observed by Faraday that a transient current is induced in a circuit if (*a*) the steady current flowing in an adjacent circuit is turned on or off, (*b*) the adjacent circuit with a steady current flowing is moved relative to the first circuit, (*c*) a permanent magnet is thrust into or out of the circuit. No current flows unless either the adjacent current changes or there is relative motion. Faraday interpreted the transient current flow as being due to a changing magnetic flux linked by the circuit. The changing flux induces an electric field around the circuit, the line integral of which is called the *electromotive force*, $\mathscr{E}$. The electromotive force causes a current flow, according to Ohm's law.

It is clear, I think, to everyone, that the sentence 'Faraday interpreted the transient current flow as being due to a changing magnetic flux linked by the circuit.' means that the observed cause-effect is that *a changing magnetic flux causes an electric current*.

Then, what is the reason for which it is invoked the existence of an electric field and an electromotive force?

Quote again from the excerpt above: "The changing flux induces an *electric field* around a circuit […]. The *electromotive force* causes a current to flow, according to Ohm's law."

So the production of an electric field is invoked to account for the movement of charges. To this it may be asked: But has not Faraday discovered a *completely new effect* in which the changing magnetic field pushes the charges? Why invoke the creation of an imaginary electric field to account for the movement of charges? Is not this an *unfounded assumption*?

What was done here was pseudo science because instead of faithfully encoding in mathematical formulas the effects as they were observed in reality, guesswork made its way into the explanation of what is happening in that observed process. The key point here is that Maxwell did not recognize that Faraday, with his changing magnetic field, found out another *new* way to make the electric charges start moving in a conductor: Faraday proved that electric charges can be made to move by varying magnetic fields and not exclusively by electric fields, as it had been believed before him. Maxwell considered that the electric charges in a conductor could be made to move *only* if the charges were under the influence of an electric field $\mathbf{E}$, so he assumed that the changing magnetic field $\mathbf{B}$ must create an electric field equivalent to $\nabla \times \mathbf{E}$ first.

In fact, it can be seen from his works that Maxwell denied the existence of any force acting on a charge other than that due to an electric field. Maxwell claimed, for example, that the force acting on a current-carrying conductor placed in a magnetic field does not act on the moving charges but on the conductor itself. By stating this, it can be said that Maxwell *denied even the existence of Lorentz's force*, whose derivation is based precisely on this very experimental finding. See below the relevant excerpt from James Clerk Maxwell, *A Treatise on Electricity and Magnetism*, Vol. II, Oxford, Clarendon Press, 1873, p. 144-145:

501.] It must be carefully remembered, that the mechanical force which urges a conductor carrying a current across the lines of magnetic force, acts, not on the electric current, but on the conductor which carries it. If the conductor be a rotating disk or a fluid it will move in obedience to this force, and this motion may or may not be accompanied with a change of position of the electric current which it carries. But if the current itself be free to choose any path through a fixed solid conductor or a network of wires, then, when a constant magnetic force is made to act on the system, the path of the current through the conductors is not permanently altered, but after certain transient phenomena, called induction currents, have subsided, the distribution of the current will be found to be the same as if no magnetic force were in action.

The only force which acts on electric currents is electromotive force, which must be distinguished from the mechanical force which is the subject of this chapter.

Observe the stark contradiction of Maxwell's words…

"[…] if the current be free to choose any path through a fixed solid conductor or a network of wires, then, when a constant magnetic force is made to act on the system, the path of the current through the conductors is not permanently altered […]"

…with J. J. Thomson's later experiments showing that electron beams can be deviated in vacuum by magnets or even the obvious contradiction between Maxwell's conception and the observations of Hall effect.

Due to this erroneous type of reasoning, I think it is fair to say that Maxwell spoiled Faraday's law and the mathematical equation called Faraday's law is not an accurate description of the observed phenomena.

Exactly the same ideas as those propounded in John David Jackson, *Classical Electrodynamics*, John Wiley and Sons, 1962, can be found in the more recent work of David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999.

At pages 301-302, we find the flagrant: the author admits that Faraday observed an *electric current* induced in the circuit and that, before it was codified mathematically, the law was *interpreted* in terms of *electric field*:

▪.2 Electromagnetic Induction

7.2.1 Faraday's Law

In 1831 Michael Faraday reported on a series of experiments, including three that (with some violence to history) can be characterized as follows:

Experiment 1. He pulled a loop of wire to the right through a magnetic field (Fig. 7.20a). A current flowed in the loop.

Experiment 2. He moved the *magnet* to the *left*, holding the loop still (Fig. 7.20b). Again, a current flowed in the loop.

Experiment 3. With both the loop and the magnet at rest (Fig. 7.20c), he changed the *strength* of the field (he used an electromagnet, and varied the current in the coil). Once again, current flowed in the loop.

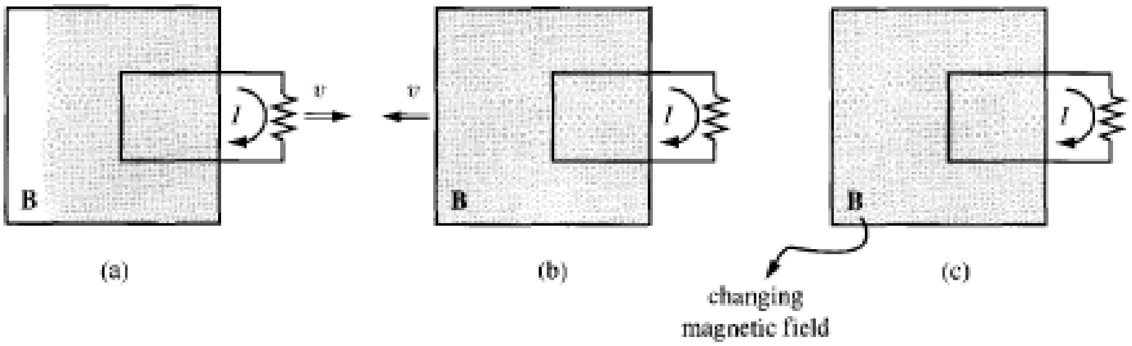


Figure 7.20

The first experiment, of course, is an example of motional emf, conveniently expressed by the flux rule:

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

I don't think it will surprise you to learn that exactly the same emf arises in Experiment 2—all that really matters is the *relative* motion of the magnet and the loop. Indeed, in the light of special relativity is *has* to be so. But Faraday knew nothing of relativity, and in classical electrodynamics this simple reciprocity is a coincidence, with remarkable implications. For if the *loop* moves, it's a *magnetic* force that sets up the emf, but if the loop is *stationary*, the force *cannot* be magnetic—stationary charges experience no magnetic forces. In that case, what *is* responsible? What sort of field exerts a force on charges at rest? Well, *electric* fields do, of course, but in this case there doesn't seem to be any electric field in sight.

Faraday had an ingenious inspiration:

A changing magnetic field induces an electric field.

It is this "induced" electric field that accounts for the emf in Experiment 2.[6]

[6]You might argue that the magnetic field in Experiment 2 is not really *changing*—just *moving*. What I mean is that if you sit at a *fixed location*, the field *does* change, as the magnet passes by.

Observe the sentence "[…] but if the loop is *stationary*, the force *cannot* be magnetic – stationary charges experience no magnetic forces." in which the author, Mr. David J. Griffiths, contradicts Faraday's very experimental finding which says precisely this: *that stationary charges do experience magnetic forces – they are set in motion by a changing magnetic field*.

Note also two other facts:
α) that, even to this day, the fact that a changing magnetic field produces an electric field *has not been proven experimentally* – it has the same status of assumption.

β) that the author, Mr. David J. Griffiths, is not specific on whether the magnetic field $\mathbf{B}$ in cases (a) and (b) is uniform or non-uniform; this is important because if $\mathbf{B}$ is not uniform then the gradient of *some of its components* $\nabla \mathbf{B}_i$ may not be zero; this implies that such gradients are present *in certain directions of space* and if $\nabla \mathbf{B}_i$ is in the plane in which the loop/magnet moves, then a force $\mathbf{f} = \mathbf{m} \cdot \nabla \mathbf{B}_i$ (where $\mathbf{m}$ is the magnetic moment of the electron) will act on the charges in the wire and cause the production of the induced electric current in it. *Even if the magnetic field is assumed to be uniform*, there is still the possibility that the movement of the loop of wire to the right through the magnetic field or of the magnet to the left holding the loop fixed may create a magnetic field gradient $\nabla \mathbf{B}_i$ locally, along the wire.

The explanations given in other textbooks resemble those in the excerpts given above, with the difference that they use unscientific terminology to explain why Faraday's law is written in terms of induced electric field (or induced e.m.f.) instead of induced current; or offer no explanation at all, mixing the notions together, as if an induced electric current were equivalent with an electric field. See the two examples given below:

1. Raymond A. Serway, John W. Jewett, Jr., *Physics for Scientists and Engineers with Modern Physics, 8th Edition*, Cengage Learning, 2010, p. 894-895:

> As a result of these observations, Faraday concluded that an electric current can be induced in a loop by a changing magnetic field. The induced current exists only while the magnetic field through the loop is changing. Once the magnetic field reaches a steady value, the current in the loop disappears. In effect, the loop behaves as though a source of emf were connected to it for a short time. It is customary to say that an induced emf is produced in the loop by the changing magnetic field.

"As a result of these observations, Faraday concluded that an electric current can be induced in a loop by a changing magnetic field. […]
It is customary (*sic!*) to say that an induced emf is produced in the loop by the changing magnetic field."

2. Walter Greiner, *Classical Electrodynamics,* Springer-Verlag, NewYork. 1998, p. 237-238:

The first quantitative studies of time-dependent electric and magnetic fields were performed by Faraday in 1831. He discovered that an electric current arises in a closed wire loop when it is moved through a magnetic field

[…]

The induced voltage is proportional to the rate of change of the magnetic flux. The sign is fixed by the *Lenz law*, implying that the induced currents and the magnetic flux associated with it are directed such that they oppose the change of the external flux. The law is generally valid.

In this second example no explanation at all is offered as to why the observed electric current was changed into an induced voltage.

In my opinion, rather that trying to explain the above experiments by invoking the magnetic force for the case moving loop / stationary magnetic field and the creation of an electric field for the case stationary loop / moving magnetic field, they should have been translated in mathematical language in a form that expressed the fact that *any relative motion of the magnetic field and the charges creates a force on the charges*.

A possible way to achieve this would be to generalize the formula for the magnetic force

$$\mathbf{f} = q \cdot \mathbf{v} \times \mathbf{B}$$

through the addition of new terms that account for these phenomena. Its generalization can be done by observing that $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ and that it can be rewritten as $\mathbf{f} = q \cdot \frac{d\mathbf{r}}{dt} \times \mathbf{B}$.

The generalized formula would be of the form

$$\mathbf{f} = q \cdot \frac{d}{dt}(\mathbf{r} \times \mathbf{B}).$$

It can be seen that

$$\mathbf{f} = q \cdot \frac{d}{dt}(\mathbf{r} \times \mathbf{B}) = q \cdot \mathbf{v} \times \mathbf{B} + q \cdot \mathbf{r} \times \frac{d\mathbf{B}}{dt}$$

and that it is composed of two terms: the original one ($q \cdot \mathbf{v} \times \mathbf{B}$) and that corresponding to Faraday's changing magnetic field ($q \cdot \mathbf{r} \times \frac{d\mathbf{B}}{dt}$).

The term $\frac{d\mathbf{B}}{dt}$ does not necessarily have the direction of $\mathbf{B}$ because $\frac{d\mathbf{B}}{dt} = \frac{\partial \mathbf{B}}{\partial t} + \frac{\partial \mathbf{B}}{\partial \mathbf{r}} \cdot \frac{\partial \mathbf{r}}{\partial t}$, which can be rewritten as $\frac{d\mathbf{B}}{dt} = \frac{\partial \mathbf{B}}{\partial t} + \frac{\partial(B_x, B_y, B_z)}{\partial(x, y, z)} \cdot \mathbf{u}$.

The expression $\frac{\partial(B_x, B_y, B_z)}{\partial(x, y, z)}$ is the Jacobian matrix:

$$\frac{\partial(B_x, B_y, B_z)}{\partial(x, y, z)} = \begin{bmatrix} \frac{\partial Bx}{\partial x} & \frac{\partial Bx}{\partial y} & \frac{\partial Bx}{\partial z} \\ \frac{\partial By}{\partial x} & \frac{\partial By}{\partial y} & \frac{\partial By}{\partial z} \\ \frac{\partial Bz}{\partial x} & \frac{\partial Bz}{\partial y} & \frac{\partial Bz}{\partial z} \end{bmatrix}$$

whose rows are the gradients of the components of the magnetic field $\nabla \mathbf{B}_i$ $(i = x, y, z)$.

It follows that the acceleration of charges by time-changing magnetic fields can be explained either:

- by the term $q \cdot \mathbf{r} \times \frac{d\mathbf{B}}{dt}$, whose significance is, at present, not known;

or

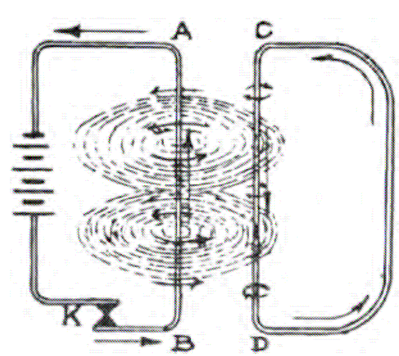
- in a more straightforward way, based on the idea that a time-changing magnetic field **B** might produce a gradient of magnetic field $\nabla \mathbf{B}_i$ *along* the path of the charges. This implies that the charges accelerate not because they are electrically charged, but because they have an intrinsic magnetic moment $\mathbf{m}$. The force responsible for the acceleration of charges in a time-varying magnetic field $\frac{d\mathbf{B}}{dt}$ is then the magnetic force that acts on any magnetic dipole placed in a non-uniform magnetic field and is given by the expression:

$$\mathbf{f} = \mathbf{m} \cdot \nabla \mathbf{B}_i$$

The possibility that in electromagnetic induction the electrical charges are set in motion by the magnetic field gradient $\nabla \mathbf{B}_i$ caused by the varying magnetic field $\frac{d\mathbf{B}}{dt} = \frac{\partial \mathbf{B}}{\partial t} + \frac{\partial(B_x, B_y, B_z)}{\partial(x, y, z)} \cdot \mathbf{u}$ (see the Jacobian matrix above) receives a very striking support from the fact that it gives, in its turn, a consistent (and unexpected) explanation of the empirically derived – but never accounted for satisfactorily- rule of Lenz regarding the direction of the induced currents.

For consider two parallel conductors (1) and (2) with extremities AB and CD, respectively.

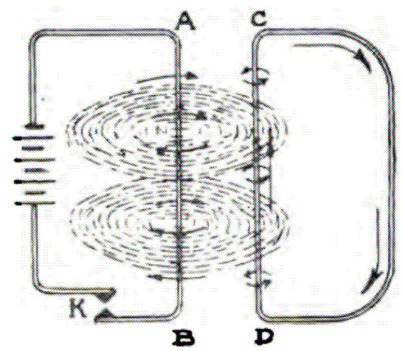
When an electric current of intensity increasing from zero to I is sent through conductor (1) from B to A, the magnetic field produced in its vicinity will have a gradient from C to D because at any instant of time the field at D is greater than at C so the magnetic field increases in the direction from C to D. Therefore, the charges in conductor (2) will, under the action of $\mathbf{f} = \mathbf{m} \cdot \nabla \mathbf{B}_i$, be pushed from C to D, which is opposite to the inducing current I and in accord with observations and with Lenz's rule.



When the current in conductor (1) is of constant intensity I, there is no gradient of magnetic field produced in its vicinity in the direction of the conductor (2), so no current is induced in conductor (2).

When the current in conductor (1) is reduced from intensity I to zero, the gradient of the magnetic field in its vicinity is from D to C so the charges in conductor (2) will be pushed in the direction from D to C, i.e. in the direction of the inducing current. This is also in accord with the observations and with Lenz's rule.

[Figures adapted from S. S. Robison, *Manual Of Wireless Telegraphy 1838-1899*, Ford Baltimore Press, 1911]



Even if the circuit is broken, the movement of charges still takes place. The charges moving in a broken circuit under the action of the changing magnetic field causes them to separate and gather at the ends of the gap in the circuit: the electrons gather at one end making it negative and leave the other end charged positively. It is this movement that creates a momentary electrostatic field **E** inside the conductor.

The mechanism may be detailed as follows:

1. When the current $\mathbf{J}^{(1)}(t)$ and charge density $\rho^{(1)}(t)$ change in conductor (1), Ampere's law $\nabla \times \mathbf{B}^{(2)}(t) = \mu_0 \cdot \mathbf{J}^{(1)}(t) + \mu_0 \cdot \mathbf{J}^{(1)\prime}\left(\frac{\partial \rho^{(1)}(t)}{\partial t}\right)$ gives the magnetic field $\mathbf{B}^{(2)}(t)$ in conductor (2).

2. The time-varying $\mathbf{B}^{(2)}(t)$ creates a gradient $\nabla \mathbf{B}^{(2)}{}_i(t)$ along the conductor (2) because

$$\frac{d\mathbf{B}^{(2)}}{dt} = \frac{\partial \mathbf{B}^{(2)}}{\partial t} + \frac{\partial\left(B^{(2)}{}_x, B^{(2)}{}_y, B^{(2)}{}_z\right)}{\partial(x, y, z)} \cdot \mathbf{u}$$

3. This gradient $\nabla \mathbf{B}^{(2)}{}_i(t)$ causes the charges in the conductor (2) to move due to the magnetic force $\mathbf{f}^{(2)}(t) = \mathbf{m} \cdot \nabla \mathbf{B}^{(2)}{}_i(t)$

4. The conductivity σ of the conductor (2) affects the resultant force acting on each charge in conductor (2); the resulting acceleration of each charge is proportional to $\mathbf{m} \cdot \nabla \mathbf{B}^{(2)}{}_i(t) - \frac{K}{\sigma}$ and the distance traveled by each charge along the conductor (2) will affect the final charge density $\rho^{(2)}(t)$ at the ends of the conductor (2).

5. The charge density $\rho^{(2)}(t)$ at the ends of the conductor (2) produces an electrostatic field $\mathbf{E}^{(2)}(t)$ in the conductor (2) that can be found by Coulomb's law $\nabla \cdot \mathbf{E}^{(2)}(t) = \frac{\rho^{(2)}(t)}{\varepsilon_0}$.

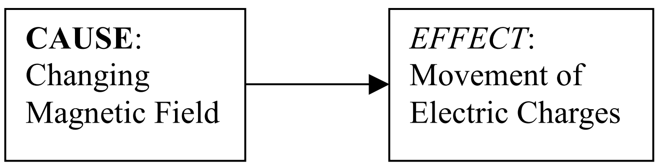
It can be seen therefore that the production of the electric field $\mathbf{E}^{(2)}(t)$ in the conductor (2) can be related to the time-varying magnetic field $\mathbf{B}^{(2)}(t)$ so an equation of the type

$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (equation (iii) in Maxwell's set of equations) can be *written provided that due account is given to the fact that charges and currents must always be present* (i.e. $\rho \neq 0$, $\mathbf{J} \neq 0$).

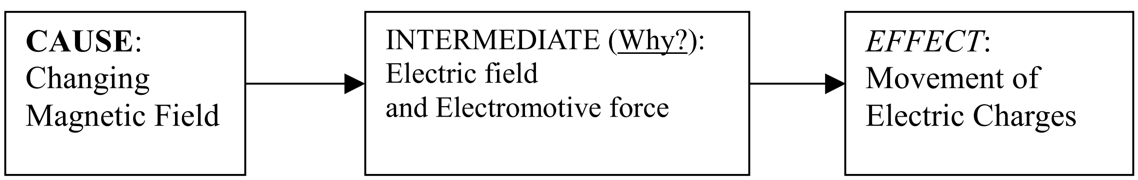
In conclusion, Faraday's observations, strictly speaking, can be summarized in the statement that *there is a force acting on an electric charge whenever there is a relative motion between the charge and a magnetic field*.

Below I have tried to show diagrammatically the difference between Faraday's original discovery and its mathematical rendering:

Faraday's discovery:



Faraday's discovery was reinterpreted by the *artificial insertion* of an electric field and e.m.f. in the cause-effect chain of the observed phenomenon:



The equation below (David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999, p. 302) corresponds to the interpreted version of Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Observe that no induced current appears in this equation. Faraday's law has been transformed into an equation between two fields, it does not mention or require the existence of any static or moving charges and this is not related to what was observed experimentally.

G. A recourse to history

I have tried to track back the origin of the confusion as to what is being induced in a conductor by a changing magnetic field – an electric current or an electric field – and I have found that this confusion is indeed due to Maxwell.

It is known that Faraday has not written any mathematical equation describing his observations related to electromagnetic induction (to be more precise, Faraday called the effect he discovered *magneto-electric* or *magnelectric* induction, as can be seen in *Experimental Researches in Electricity*, 2nd Ed., Vol. I, 1849, p.16), and has always stated in his works that what is induced is an electric current.

For example, Faraday stated one of his quantitative observations in terms of *induced currents*, as follows (Michael Faraday, *Experimental Researches in Electricity*, 2nd Ed., Vol. I, 1849, p.62):

213. These results tend to prove that the currents produced by magneto-electric induction in bodies is proportional to their conducting power.

Another example is Faraday's enunciation of the condition in which electromagnetic induction takes place (Michael Faraday, *Experimental Researches in Electricity*, 2nd Ed., Vol. I, 1849, p.73-74):

256. Although it will require further research, and probably close investigation, both experimental and mathematical, before the exact mode of action between a magnet and metal moving relatively to each other is ascertained; yet many of the results appear sufficiently clear and simple to allow of expression in a somewhat general manner.—If a terminated wire move so as to cut a magnetic curve, a power is called into action which tends to urge an electric current through it; but this current cannot be brought into existence unless provision be made at the ends of the wire for its discharge and renewal.

"If a terminated wire move so as to cut a magnetic curve, a power is called into action which tends to urge an electric current through it; […]." The use of the term "power" here implies that Faraday thought of a force tending to make the charges move, thus forming an electric current; this is indeed the case, as he uses the term force in one of the entries that follow (258 below):

257. If a second wire move in the same direction as the first, the same power is exerted upon it, and it is therefore unable to alter the condition of the first: for there appear to be no natural differences among substances when connected in a series, by which, when moving under the same circumstances

relative to the magnet, one tends to produce a more powerful electric current in the whole circuit than another (201. 214.).

258. But if the second wire move with a different velocity, or in some other direction, then variations in the force exerted take place; and if connected at their extremities, an electric current passes through them.

"[…] then variations in the force exerted take place; […]"

Faraday's numerous experiments, published in a series of three volumes titled *Experimental Researches in Electricity*, constituted the reference material Maxwell consulted when building his theory.

For example, Maxwell's first article on electricity and magnetism was titled *On Faraday's Lines of Force* (1855) and in it he attempted a mathematical description of the effect of electromagnetic induction observed by Faraday. In this article Maxwell referred to the following excerpt from Faraday's works (Michael Faraday, *Experimental Researches in Electricity*, Vol. III, 1855, p.331):

3077. The general principles of the development of an electric current in a wire moving under the influence of magnetic forces, were given on a former occasion, in the First and Second Series of these Researches (36. &c.); it will therefore be unnecessary to do more than to call attention, at this time, to the special character of its indications as compared to those of a magnetic needle, and to show how it becomes a peculiar and important addition to it, in the illustration of magnetic action.

Observe that Faraday spoke again in terms of *induced current*: "The general principles of the development of an electric current in a wire moving under the influence of magnetic forces […]."

Below I reproduce the relevant excerpt from Maxwell's article (*On Faraday's Lines of Force*, 1855, p. 185) in which reference in made to the above idea of Faraday:

On Electric Currents produced by Induction.

Faraday has shewn‡ that when a conductor moves transversely to the lines of magnetic force, an electro-motive force arises in the conductor, tending to produce a current in it. If the conductor is closed, there is a continuous current, if open, tension is the result. If a closed conductor move transversely to the lines of magnetic induction, then, if the number of lines which pass

* *Exp. Res.* (3122). See Art. (6) of this paper. † Art. (13).
‡ *Exp. Res.* (3077), &c.

Observe that Maxwell writes the subtitle “On Electric Currents produced by Induction.” but in the text erroneously claims that “Faraday has shewn (reference to Faraday’s *Exp. Res. (3077), &c.)* that when a conductor moves transversely to the lines of magnetic force, an electro-motive force arises in the conductor, tending to produce a current in it.” This is clearly *an unfounded assumption* on behalf of Maxwell, because it is clear from the paragraph of Faraday’s work to which Maxwell refers (number 3077 shown above) that Faraday made no such a statement. As you can see, Faraday did not claim that *an electro-motive force* arises in the conductor, but merely that an *electric current* is produced in it.

H. Maxwell’s equations and his wave equations – with honesty

It is often stated that Maxwell’s equations yield the equations of electromagnetic waves in vacuum.
By this is meant that Maxwell’s equations are valid for regions of space where there are no charges or currents and that the electric and magnetic fields that compose the electromagnetic wave are not produced by any charges whatsoever. In other words, that the electric and magnetic fields that compose the electromagnetic wave induce each other in vacuum, without the mediation of electric charges static or in motion.

In truth, the said equations can be obtained for regions where *there are* charges and currents, and no reason can be given why they should be valid for vacuum as well.
Since textbooks never mention the fact that Maxwell’s famous equations for electromagnetic waves can be obtained even without the conditions $\rho = 0$ and $\mathbf{J} = 0$ for vacuum, few students suspect that they are being *lied by omission*.

Look at the derivation of the equations of electromagnetic waves as given by one of the standard textbooks (David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999, p. 375):

9.2 Electromagnetic Waves in Vacuum
9.2.1 The Wave Equation for E and B

In regions of space where there is no charge or current, Maxwell’s equations read

$$\left.\begin{array}{ll} \text{(i)} \quad \nabla \cdot \mathbf{E} = 0, & \text{(iii)} \quad \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}, \\ \text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, & \text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}. \end{array}\right\} \qquad (9.40)$$

They constitute a set of coupled, first-order, partial differential equations for **E** and **B**. They can be *de*coupled by applying the curl to (iii) and (iv):

$$\begin{aligned}\nabla \times (\nabla \times \mathbf{E}) &= \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) \\ &= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \\ \nabla \times (\nabla \times \mathbf{B}) &= \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \nabla \times \left(\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \\ &= \mu_0 \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.\end{aligned}$$

Or, since $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$,

$$\boxed{\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.} \tag{9.41}$$

We now have *separate* equations for **E** and **B**, but they are of *second* order; that's the price you pay for decoupling them.

In vacuum, then, each Cartesian component of **E** and **B** satisfies the **three-dimensional wave equation**,

$$\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}.$$

You can interpret the two differential equations for **E** and **B** in any way you wish. The ambiguity is so great that you can consider them to be the vibrations of a line of electric or magnetic field fixed at its ends, or of a line with one free end, or even without ends (closed loops); or you can consider that they are waves that travel in space at infinite distances. What criteria should we use when we choose between these possibilities?

The fact that no experimental evidence exists that the electric and magnetic fields induce each other in vacuum *where there are no electric currents and no electric charges*, would prevent an honest scientist from interpreting them as being waves propagating freely in empty space.

However, the significance of the expression that yields the speed of light in vacuum

$$c = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}$$

is not lost. This is *because the wave equations for* **E** *and* **B** *can be obtained even in regions where there are charges and currents*. Here is the proof, following the same method as shown in the above excerpt from David J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall, 1999, p. 375:

We consider a region of space in which there is a charge density ρ and a current density $\mathbf{J}$. The equations are:

(i) $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ (iii) $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

(ii) $\nabla \cdot \mathbf{B} = 0$ (iv) $\nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} + \mu_0 \cdot \varepsilon_0 \cdot \dfrac{\partial \mathbf{E}}{\partial t}$

We proceed in the same way as in the said textbook and apply curl to (iii) and (iv).

Curl of (iii) yields:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \cdot (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\frac{\partial}{\partial t}\left(\mu_0 \cdot \mathbf{J} + \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t} \right)$$

So we have, $\nabla \cdot (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \cdot \dfrac{\partial \mathbf{J}}{\partial t} - \mu_0 \cdot \varepsilon_0 \cdot \dfrac{\partial^2 \mathbf{E}}{\partial t^2}$

or, by using (i),

$$\frac{1}{\varepsilon_0} \cdot \nabla \rho - \nabla^2 \mathbf{E} = -\mu_0 \cdot \frac{\partial \mathbf{J}}{\partial t} - \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad \text{(Eq. M1)}$$

Curl of (iv) yields:

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla \cdot (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \nabla \times \left(\mu_0 \cdot \mathbf{J} + \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t} \right)$$

or,

$$\nabla \cdot (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu_0 \cdot \nabla \times \mathbf{J} + \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = \mu_0 \cdot \nabla \times \mathbf{J} - \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

So we have, $\nabla \cdot (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu_0 \cdot \nabla \times \mathbf{J} - \mu_0 \cdot \varepsilon_0 \cdot \dfrac{\partial^2 \mathbf{B}}{\partial t^2}$

or, by using (ii)

$$\nabla^2 \mathbf{B} = -\mu_0 \cdot \nabla \times \mathbf{J} + \mu_0 \cdot \varepsilon_0 \cdot \frac{\partial^2 \mathbf{B}}{\partial t^2} \qquad \text{(Eq. M2)}$$

Observe that Eq.M2 becomes the equation of a wave $\nabla^2 \mathbf{B} = \mu_0 \cdot \varepsilon_0 \cdot \dfrac{\partial^2 \mathbf{B}}{\partial t^2}$ for the magnetic field **B** if $\nabla \times \mathbf{J} = 0$ *without being necessary to use the condition for free space with no charge and no current* ($\rho \neq 0$ and $\mathbf{J} \neq 0$). Since the equation was obtained from the normal set of Maxwell's equation *with* charges and currents, it follows that even in Maxwell's theory *we cannot say that this is a wave corresponding to vacuum*.

Also observe that Eq.M1 becomes the equation of a wave $\nabla^2 \mathbf{E} = \mu_0 \cdot \varepsilon_0 \cdot \dfrac{\partial^2 \mathbf{E}}{\partial t^2}$ for the electric field **E**, if $\nabla \rho = 0$ and $\dfrac{\partial \mathbf{J}}{\partial t} = 0$. These equations tell that there can be charges ($\rho \neq 0$) but no charge gradient ($\nabla \rho = 0$) and there can be currents ($\mathbf{J} \neq 0$) but no time changing currents ($\dfrac{\partial \mathbf{J}}{\partial t} = 0$) for this wave equation to obtain.

It can be seen that the "electromagnetic wave equations" are valid for matter containing charges and currents and *no reason can be given for considering that they represent waves in vacuum*.

What is then the significance of the speed $c = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}$?

Since the vibratory behavior of the magnetic field **B** and of the electric field **E** are obtained from Maxwell's equations with currents and charges, the significance of c cannot be other than that it is the celerity with which an electric charge acts on another electric charge in its vicinity.

Summary

In conclusion, in this article it was shown that Maxwell's theory of electromagnetic waves contains an *unfounded assumption*, a *faulty method of theoretical investigation* and makes a *prediction* that is contrary to observations.

These are:

(i) the *unfounded assumption* that a changing magnetic field **B** creates (induces) an electric field **E** (a.k.a. Faraday's law of electromagnetic induction). In fact, a changing magnetic field **B** is observed to produce an electric current **J**, not an electric field **E** and there is a great difference between an electric current **J** and an electric field **E**.

(ii) the assumption that a changing electric field **E** creates (induces) a magnetic field **B** (a.k.a. Maxwell's correction to Ampere's Law). This was derived by Maxwell through a *faulty method of theoretical investigation*, no such effect was known in Maxwell's time and no experiment has been made since then that proves this assumption.

(iii) the *prediction* that radio waves and light are composed of entangled electric and magnetic waves that create (induce) one another in vacuum. No experiment revealed that radio waves and light have a structure containing electric and magnetic fields.

Although it seemed an easy and straightforward matter to accomplish, Faraday failed in his attempt to change the plane of polarization of light travelling in vacuum by the application of strong electric and magnetic fields. Only when the polarized beam of light passed through glass of great density could this be accomplished, and even then by the application of a magnetic field only.

Furthermore, Faraday initially applied the magnetic field *perpendicular* to the ray, believing that this would change the direction of the plane of polarization. Not obtaining any positive result, he then placed the magnetic field *parallel* to the direction of the ray, and he finally obtained the change he was looking for. But then how can this result be reconciled with the theory in which light is considered to be composed of two transverse magnetic and electric fields? It does not seem that the magnetic field applied by Faraday and the magnetic field of the light-ray vibrating perpendicular to it give a resultant in a different plane.

It was shown in this article that Maxwell's theory is valid only for regions of space containing electric charges and currents and fails to give any account whatsoever of the nature of the waves travelling in vacuum at great distances from their original source, where neither charges nor currents exist.

With these missing parts, Maxwell's theory cannot be considered an established scientific theory. The author, himself a physics teacher, considers that his duty is not only teaching the syllabus and asking the students to believe theories, but also to look for the proofs that exist and support the theories propounded in the textbooks. With all that has been discussed in this article, can an honest teacher stand in front of a student and teach him that Maxwell's theory is proven beyond doubt?