## A look inside Feynman's route to gravitation

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In this contribution we report about Feynman's approach to gravitation, starting from the records of his interventions at the Chapel Hill Conference of 1957. As well known, Feynman was concerned about the relation of gravitation with the rest of physics. Probably for this reason, he promoted an unusual, field theoretical approach to general relativity, in which, after the recognition that the interaction must be mediated by the quanta of a massless spin-2 field, Einstein's field equations should follow from the general properties of Lorentz-invariant quantum field theory, plus self-consistency requirements. Quantum corrections would then be included by considering loop diagrams. These ideas were further developed by Feynman in his famous lectures on gravitation, delivered at Caltech in 1962-63, and in a handful of published papers, where he also introduced some field theoretical tools which were soon recognized to be of general interest, such as ghosts and the tree theorem. Some original pieces of Feynman's work on gravity are also present in a set of unpublished lectures delivered at Hughes Aircraft Company in 1966-67 and devoted primarily to astrophysics and cosmology. Some comments on the relation between Feynman's approach to gravity and his ideas on the quantum foundations of the fundamental interactions are included.

*Keywords*: Gravitation; Quantum field theory; Loop diagrams.

# 1. Introduction: a timeline

Among the many scientific interests that Feynman had in the 1950s and in the 1960s, a prominent place was taken by the understanding of the relation of gravitation to the rest of physics, $^{\alpha}$  $^{\alpha}$  $^{\alpha}$  and in particular the assessment of its consistency with the uncertainty principle.<sup>[b](#page-0-1)</sup> Feynman likely began to seriously think about gravity in

<span id="page-0-0"></span><sup>a</sup>"Next we shall discuss the possible relation of gravitation to other forces. There is no explanation of gravitation in terms of other forces at the present time. It is not an aspect of electricity or anything like that, so we have no explanation. However, gravitation and other forces are very similar, and it is interesting to note analogies." (Ref. [1](#page-10-0), Vol. 1, Sec. 7-7); "My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another." (Ref. [2,](#page-10-1) p. 697)

<span id="page-0-1"></span>b"...it would be important to see whether Newton's law modified to Einstein's law can be further modified to be consistent with the uncertainty principle. This last modification has not yet been completed."(Ref. [1](#page-10-0), Vol. 1, Sec. 7-8)

the early 1950s, just after completing his work on quantum electrodynamics. This was attested by Murray Gell-Mann<sup>[3](#page-10-2)</sup> in a paper written for a Physics Today special issue devoted to Feynman's lega[c](#page-1-0)y,<sup>c</sup> as well as by Bryce S. DeWitt<sup>[4](#page-10-3)</sup>, in a letter to Agnew Bahnson written in November 1955 [5](#page-10-4) . Feynman's efforts began in a period in which general relativity, after a stagnation which lasted about thirty years, was gradually emerging as a mainstream research area, giving rise to a process known as the *renaissance* of general relativity<sup>[6](#page-10-5)</sup>.

A crucial event to consider, in order to reconstruct Feynman's approach to grav-itation, is the 1957 Chapel Hill conference<sup>[4](#page-10-3)</sup> on The Role of Gravitation in Physics, which was also pivotal to the renaissance of general relativity. At that conference, the gravitational physics community delineated the tracks along which subsequent work would develop in the subsequent decades. The main threads were the fol-lowing<sup>[7](#page-10-6)</sup>: classical gravity, quantum gravity, and the classical and quantum theory of measurement (as a bridge between the previous two topics). In that conference Feynman's work on gravity, of which nothing had been published yet, was pre-sented for the first time, and put on paper in the records<sup>[4](#page-10-3)</sup>. In fact these written records testify that by the time of the conference he had already deeply thought, and performed computations, about each of the above listed three topics, focusing in particular on classical gravitational waves, on the arguments in favor of quantum gravity from fundamental quantum mechanics, and finally on quantum gravity itself. Thus, in this contribution, we take as starting point for our reconstruction Feynman's interventions at Chapel Hill and follow the development of his work in the subsequent years, until the late sixties, where he apparently lost interest in the subject.

The ultimate goal of Feynman's work was the development of a quantum field theory of gravitation, which led him to face deep conceptual as well as mathematical issues, such as divergent integrals and the lack of unitarity beyond the tree level approximation. This task accompanied Feynman for some years, as stated in a letter<sup>[8](#page-10-7)</sup> he wrote to Viktor F. Weisskopf in 1961 ("As you know, I am studying the problem of quantization of Einstein's General Relativity. I am still working out the details of handling divergent integrals which arise in problems in which some virtual momentum must be integrated over"), as well as reported by William R. Frazer<sup>[9](#page-10-8)</sup> in a short summary of the talks given at the La Jolla conference, later in the same year.<sup>[d](#page-1-1)</sup> A first comprehensive account of Feynman's results on quantum gravity can

<span id="page-1-0"></span>c Indeed here Gell-Mann remembered his visit to Caltech during the Christmas vacation of 1954- 55, the discussions with Feynman about quantum gravity, and the fact that Feynman "had made considerable progress".

<span id="page-1-1"></span><sup>&</sup>lt;sup>d</sup>The International Conference on the Theory of Weak and Strong Interactions was held in June 14-16, 1961, at the University of California, San Diego, in La Jolla. Here, Geoffrey F. Chew gave his celebrated talk on the  $S$ -matrix  $^{10}$  $^{10}$  $^{10}$ , while an afternoon session was devoted to the theory of gravitation, where Feynman reported on his work on the renormalization of the gravitational field, and recognized non-unitarity as the main difficulty, which was shared also by Yang-Mills theory.

b[e](#page-2-0) found in the talk he gave in  $1962$  at the Warsaw conference,<sup>e</sup> whose written version was later published as a regular paper in Acta Physica Polonica<sup>[2](#page-10-1)</sup>. Further details were given by Feynman much later, in a couple of papers  $11,12$  $11,12$  published in 1972 in the Festschrift for John A. Wheeler's 60th birthday[13](#page-11-3), which were written in a period in which he was already deeply absorbed in the study of partons and strong interactions.

Among the main sources which contribute to offer a clear account of Feynman's work on gravity issues, it is mandatory to include the famous Caltech Lectures on Gravitation<sup>[14](#page-11-4)</sup>, delivered in 1962-63 and aimed to advanced graduate students and postdocs. Finally, there is some unpublished material, included in two sets of lectures, given in the 1960s at the Hughes Aircraft Company, which only recently have been made available on the web.<sup>[f](#page-2-1)</sup> In particular, the 1966-67 set of lectures  $^{16}$  $^{16}$  $^{16}$ , which were devoted to astronomy, astrophysics and cosmology, contains an introductory treatment of general relativity, with an emphasis on applications to the main sub-jects. While sharing many similarities with the above quoted Caltech lectures<sup>[14](#page-11-4)</sup>, the Hughes treatment offers to the attentive reader several original points. In those same years Feynman had succeeded in finding a new derivation of Maxwell's equations [17](#page-11-6)[,18](#page-11-7), and a generalization of this approach to gravity is suggested (but not pursued) in several places in the Hughes lectures on astrophysics, as well as in the set of lectures given in the following year and devoted to electromagnetism<sup>[19](#page-11-8)</sup>.

After outlining the main steps and sources which helped us to reconstruct the full development of Feynman's work on [g](#page-2-2)ravitation,<sup>g</sup> in this contribution we focus on two key issues: the formulation of quantum gravity as a quantum field theory of a massless spin-2 field, the graviton, in whole analogy with quantum electrodynamics, which is the content of Section 2, and the unitarity and renormalization issues arising beyond the tree level approximation, presented in Section 3. Our concluding remarks close the paper.

## 2. Gravity as a quantum field theory

Feynman's strategy in approaching gravity was firstly outlined at Chapel Hill conference in a series of critical comments (Ref. [4](#page-10-3), pp. 272-276), in which a non-geometric and field theoretical line of attack is put forward. His starting point was an hypothetical, counterfactual situation, in which scientists would discover the principles of Lorentzian quantum field t[h](#page-2-3)eories before general relativity.<sup>h</sup> The main concern

<span id="page-2-0"></span><sup>&</sup>lt;sup>e</sup>The International Conference on General Relativity and Gravitation was held in Jablonna and Warsaw in July 25-31, 1962, with Leopold Infeld as the chairman of the local organizing committee. The discussion focused on three main topics: the general properties of gravitational radiation, the quantization of gravity and the exact solutions of the Einstein field equations.

<span id="page-2-1"></span><sup>f</sup>See Ref. [15](#page-11-9) for a brief account of Feynman's involvement in teaching at the Hughes Aircraft Company.

<sup>&</sup>lt;sup>g</sup>See Ref. [20](#page-11-10) for a comprehensive account of this work.

<span id="page-2-3"></span><span id="page-2-2"></span>hIn fact, in Ref. [4,](#page-10-3) p. 273, Feynman said: "Instead of trying to explain the rest of physics in terms of gravity I propose to reverse the problem by changing history. Suppose Einstein never existed

in such a situation would then be to include a new force, the gravitational one, in the framework of quantum field theory. This approach would be later completed in the first part of the Caltech lectures  $14$ . Feynman's reasoning was the following: on the basis of the general principles of quantum field theory and of experimental results it is possible to conclude that gravity, as any other force, has to be mediated by exchanges of a virtual particle, which in this case is a massless neutral spin-2 quantum, the graviton. Thus, by constructing a Lorentz invariant quantum field theory of the grav[i](#page-3-0)ton<sup>i</sup> and by imposing certain consistency requirements, full general relativity should be recovered. Clearly, by following the same procedure for the spin-1 case Maxwell's equations are obtained (in this case it is much simpler, being the theory linear). Such an approach testifies his ideas about fundamental interactions as manifestations of underlying quantum theories [18](#page-11-7), which were expressed by him several times, for example in the Hughes Lectures<sup>[19](#page-11-8)</sup>:

I shall call conservative forces, those forces which can be deduced from quantum mechanics in the classical limit. As you know, Q.M. is the underpinning of Nature (Ref. [19](#page-11-8), p.35).

Let us describe the steps in more detail. First of all, one has to establish the spin of the mediating quantum. Both in the Caltech<sup>[14](#page-11-4)</sup> and in the 1966-67 Hughes lectures  $16$ , the choice of a spin-2 mediator was justified by the observation that energy, which is the source of the gravitational force, grows with the velocity. The same observation ruled out the possibility of a spin-0 field, because the associated charge would decrease with the velocity. This result can be traced back to an old argument by Einstein (never published but recalled in Ref. [26,](#page-11-11) pp. 285-290), according to which the vertical acceleration of a body would depend on its horizontal velocity, and in particular would be zero for light, making light deflection impossible. Once the spin of the graviton is established, one can easily construct the linearized theory of the associated field, which is a massless spin-2 field. Nonlinearity then comes into play because the graviton has to couple with anything carrying energy-momentum, then also with itself, and this coupling must be universal. The resulting nonlinear theory is general relativity. General covariance and the geometric interpretation of general relativity are finally recovered as a byproduct of the gauge invariance of the theory [j](#page-3-1) . For Feynman, this was only half of the whole story. In fact, by pushing calculations beyond tree level, quantum gravity effects would be taken into account. This was Feynman's ultimate goal, i.e. obtaining a quantum theory of gravity, which in this approach amounts to the quantization of another field.

 $[...]$ ".

<span id="page-3-1"></span><span id="page-3-0"></span><sup>i</sup>The linear theory for a massless spin-2 field and its massive counterpart was completely worked out by Markus Fierz and Wolfgang E. Pauli in Ref. [21](#page-11-12), on the basis of a previous work by Paul A. M. Dirac  $2^2$ , while iterative arguments similar to Feynman's ones were later put forward by Suraj N. Gupta  $^{23}$  $^{23}$  $^{23}$  and Robert H. Kraichnan  $^{24}$  $^{24}$  $^{24}$  in an attempt to generate infinite nonlinear terms both N. Gupta  $^{23}$ in the Lagrangian and in the stress-energy tensor. See for details Ref. [25](#page-11-16). <sup>j</sup>See Ref. [14,](#page-11-4) p. 113.

Let us describe how Feynman sketched the above procedure at the Chapel Hill conference (Ref. [4,](#page-10-3) pp. 272-276). In whole analogy with electrodynamics, he wrote down the following action:

$$
\int \left(\frac{\partial A_{\mu}}{\partial x_{\nu}} - \frac{\partial A^{\nu}}{\partial x^{\mu}}\right)^{2} d^{4}x + \int A_{\mu}j^{\mu}d^{4}x + \frac{m}{2} \int \dot{z}_{\mu}^{2}ds + \frac{1}{2} \int T_{\mu\nu}h^{\mu\nu}d^{4}x + \int (\text{second power of first derivatives of } h),
$$
 (1)

where  $h_{\mu\nu}$  is the new field associated with the graviton, i.e. a symmetric secondorder tensor field, which satisfies second order linear equations of the kind:

$$
h^{\mu\nu,\sigma}{}_{,\sigma} - 2\overline{h}^{\mu}{}_{\sigma}{}^{\nu\sigma} = \overline{T}^{\mu\nu},\tag{2}
$$

where the *bar* operation is defined on a generic second rank tensor  $X_{\mu\nu}$  as:

<span id="page-4-4"></span>
$$
\overline{X}_{\mu\nu} = \frac{1}{2} \left( X_{\mu\nu} + X_{\nu\mu} \right) - \frac{1}{2} \eta_{\mu\nu} X^{\sigma}{}_{\sigma}.
$$
 (3)

The equations of motion for particles also follow from the above action:

<span id="page-4-0"></span>
$$
g_{\mu\nu}\ddot{z}^{\nu} = -\left[\rho\sigma,\mu\right]\dot{z}^{\rho}\dot{z}^{\sigma},\tag{4}
$$

where  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ,  $\eta_{\mu\nu}$  is the Minkowski metric, and  $[\rho\sigma, \mu]$  are the Christoffel symbols of the first kind.

However, when the field  $h_{\mu\nu}$  is coupled to the matter according to Eq.[\(4\)](#page-4-0), the corresponding stress energy tensor  $T_{\mu\nu}$  does not obey to a conservation law, leading to a consistency problem. This happens at variance with electromagnetism, where Maxwell equations guarantee conservation of the current  $j^{\mu}$ . Instead here  $T_{\mu\nu}$  does not ta[k](#page-4-1)e into account the effect of gravity on itself, which requires nonlinearity.<sup>k</sup> Thus a suitable  $T_{\mu\nu}$  has to be found in order to satisfy the condition  $\partial_{\nu}T^{\mu\nu} = 0$ . The so[l](#page-4-2)ution to this consistency issue<sup>l</sup> can be obtained by adding to the action a nonlinear third order term in  $h_{\mu\nu}$ , which leads to the following equation for  $T_{\mu\nu}$ :

<span id="page-4-3"></span>
$$
g_{\mu\lambda}T^{\mu\nu}_{\quad;\nu} = -\left[\rho\nu,\lambda\right]T^{\rho\nu}.\tag{5}
$$

One can then go on to higher order approximations, until the procedure converges. But finding the general solution of Eq. [\(5\)](#page-4-3) is a really difficult task in the absence of a standard procedure. Feynman's idea was to look for an expression for the  $T_{\mu\nu}$ , and hence for the action, that is invariant under the following infinitesimal transformation of the whole tensor field  $g_{\mu\nu}$ :

$$
g'_{\mu\nu} = g_{\mu\nu} + g_{\mu\lambda} \frac{\partial A^{\lambda}}{\partial x^{\nu}} + g_{\nu\lambda} \frac{\partial A^{\lambda}}{\partial x^{\mu}} + A^{\lambda} \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}}.
$$
 (6)

where the 4-vector  $A^{\lambda}$  is the generator. This is a geometric transformation in a Riemannian manifold with metric, hence one can say that geometry gives the

<sup>k</sup>Feynman noticed that nonlinearity was necessary in order to explain the precession of the perihelion of Mercury (as discussed in Ref. [14,](#page-11-4) p. 75).

<span id="page-4-2"></span><span id="page-4-1"></span><sup>&</sup>lt;sup>1</sup>See for details Refs. [4](#page-10-3), p. 274 and [14,](#page-11-4) pp. 78-79.

metric  $g_{\mu\nu}$ . As such, in Feynman's approach geometry comes into play at the end and not at the beginning. By working out calculations, the full nonlinear Einstein gravitational field equations are obtained.

Lectures 3-6 of Feynman's graduate course on gravitation (Ref. [14\)](#page-11-4) contain all the details of the above procedure. Interestingly, a proof is also given of the ability of this field theory based formalism to reproduce key physical effects of curved spacetime geometry. For instance, in Lecture 5 (Ref. [14,](#page-11-4) pp. 66-69) it is shown that, in the action of a scalar field, the time dilation  $t \to t' = t\sqrt{1+\epsilon}$  exactly reproduces the effect of a constant weak gravitational field described by the tensor  $g_{44} = 1 + \epsilon$ ,  $g_{ii} = -1$ ,  $i = 1, 2, 3$ . Incidentally such an effect plays a pivotal role in producing the right result for the precession of Mercury's perihelion. Before moving to applications, Feynman devoted some lectures to the discussion of the usual geometric approach to gravity and of its link with the field theory based approach (Ref. [14](#page-11-4), Lectures 7-10):

Let us try to discuss what it is that we are learning in finding out that these various approaches give the same results (Ref. [14,](#page-11-4) p. 112).

Despite advocating one approach over another, Feynman was in fact intrigued by the double nature of gravity, which has both a geometrical interpretation and a field interpretation, and in Section 8.4 of Ref. [14](#page-11-4) he recognized how an explanation may be provided by gauge invariance. Indeed a viable procedure may be established in order to obtain the invariance of the equations of physics under space dependent variable displacements. This amounts to looking for a more general Lagrangian, to be obtained by adding to the old one new terms, involving a gravity field. The net result is a new picture of gravity, as the field corresponding to a gauge invariance with respect to displacement transformations.

Summing up, Feynman succeeded in obtaining the full nonlinear Einstein equations by means of a consistency argument applied to a Lorentz invariant quantum field theory of the graviton. According to John Preskill and Kip S. Thorne[25](#page-11-16), it is likely that he was completely unaware of the earlier work by Gupta<sup>[23](#page-11-14)</sup> and Kraich- $\tan^{24}$  $\tan^{24}$  $\tan^{24}$ , which as mentioned had been developed along a similar line of attack, but was still incomplete. So he would have developed his approach independently, besides getting more complete results. A Lorentz-invariant field theoretical approach to gravity would have been later pursued also by Steven Weinberg<sup>[27](#page-11-17)[,28](#page-11-18)</sup>, albeit quite different fro[m](#page-5-0) Feynman's one,<sup>m</sup> as well as by Stanley Deser<sup>[29](#page-11-19)[–32](#page-11-20)</sup>. Indeed Deser's approach, while being similar to Feynman's one, was more elegant and general and led to completion the whole program started with Kraichnan and Gupta. Finally, a rigorous and general analysis of the relation between spin-2 theories and general

<span id="page-5-0"></span><sup>m</sup>Weinberg's approach relied on the analyticity properties of graviton-graviton scattering amplitudes, and it was quite more general, since Weinberg actually proved that a quantum theory of a massless spin-2 field can only be consistent if this field universally couples to the energy-momentum of matter, hence the equivalence principle has to be obeyed.

covariance was carried out by Robert M. Wald<sup>[33](#page-11-21)</sup>.

# 3. Fighting with loops: the renormalization of gravity

After obtaining the Einstein-Hilbert action and the full nonlinear Einstein gravitational field equations, Feynman's efforts were mainly directed towards discussing quantum field theory issues beyond the tree level approximation, i.e. loop diagrams, unitarity and renormalization.

To the best of our knowledge, these issues were publicly addressed for the first time by Feynman in 1[9](#page-10-8)61 at the already mentioned La Jolla conference<sup>9</sup>, where nonlinearity was recognized as the very source of difficulty within both gravitation and Yang-Mills theories. Indeed the sources of the gravitational field are energy and momentum, and the gravitational field carries energy and momentum itself. In the same way, the source of a Yang-Mills field is the isotopic spin current, and the Yang-Mills field carries isotopic spin itself. This means that both the gravitational field and the Yang-Mills field are self-coupled, resulting in nonlinear field theories. A difficulty that nonlinearity brings about is the fact that loop diagrams seem to clash with unitarity.

As recalled in the introduction, Feynman's results on quantum gravity can be found in a report of the talk given at the 1962 Warsaw conference, later published in Acta Physica Polonica<sup>[2](#page-10-1)</sup>,<sup>[n](#page-6-0)</sup> with many details discussed much later in the two 1972 Wheeler Festschrift papers<sup>[11](#page-11-1)[,12](#page-11-2)</sup>.

Also in this case Feynman followed his original strategy, leaving aside quantization of space-time geometry, constructing a quantum field theory for the graviton, and working out results at different perturbative orders. Since the goal was the quantum theory, the Einstein equations and the corresponding Lagrangian were assumed as a starting point, rather than derived from scratch. The theory was coupled to a scalar field, and perturbative calculations up to the next to leading order were pursued. This implied the inclusion of loop diagrams, which make quantum corrections to enter the game. In Feynman's own words:

I started with the Lagrangian of Einstein for the interacting field of gravity and I had to make some definition for the matter since I'm dealing with real bodies and make up my mind what the matter was made of; and then later I would check whether the results that I have depend on the specific choice or they are more powerful (Ref. [2,](#page-10-1) p. 698).

The metric was split in the following way:

<span id="page-6-1"></span>
$$
g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu};\tag{7}
$$

<span id="page-6-0"></span><sup>&</sup>lt;sup>n</sup>As pointed out by Trautman in some recently published memories  $(^{34}$  $(^{34}$  $(^{34}$ , p. 406), the text of Feynman's plenary lecture became available too late to be included in the proceedings, therefore it was published only in 1963 as a regular paper.

where the Minkowski metric is here denoted by  $\delta_{\mu\nu}$  and  $\kappa$  is a dimensionful coupling constant. Substituting [\(7\)](#page-6-1) and expanding, the Lagrangian for gravity coupled with a scalar field can be cast in the form:

<span id="page-7-0"></span>
$$
L = \int \left( h_{\mu\nu,\sigma} \overline{h}_{\mu\nu,\sigma} - 2 \overline{h}_{\mu\sigma,\sigma} \overline{h}_{\mu\sigma,\sigma} \right) d\tau + \frac{1}{2} \int \left( \phi_{,\mu}^2 - m^2 \phi^2 \right) d\tau
$$
\n
$$
+ \kappa \int \left( \overline{h}_{\mu\nu} \phi_{,\mu} \phi_{,\nu} - m^2 \frac{1}{2} h_{\sigma\sigma} \phi^2 \right) d\tau + \kappa \int \left( h h h \right) d\tau + \kappa^2 \int \left( h h \phi \phi \right) d\tau + \dots
$$
\n(8)

where the bar operation has been defined in [\(3\)](#page-4-4) and a schematic notation has been adopted for the highly complex higher order terms. The first two terms are simply the free Lagrangians of the gravitational field and of matter, respectively. Before considering radiative corrections the classical solution of the problem was worked out, which involved the variation of Eq.  $(8)$  with respect to h and, then, to  $\phi$ , giving rise to the following equations of motion with a source term:

<span id="page-7-1"></span>
$$
h_{\mu\nu,\sigma\sigma} - \overline{h}_{\sigma\nu,\sigma\mu} - \overline{h}_{\sigma\mu,\sigma\nu} = \overline{S}_{\mu\nu} (h,\phi), \qquad (9)
$$

$$
\phi_{,\sigma\sigma} - m^2 \phi = \chi(\phi, h). \tag{10}
$$

A close inspection revealed that Eq. [\(9\)](#page-7-1) was singular, so that Feynman was forced to resort to the invariance of the Lagrangian under the transformation:

$$
h'_{\mu\nu} = h_{\mu\nu} + 2\xi_{\mu,\nu} + 2h_{\mu\sigma}\xi_{\sigma,\nu} + \xi_{\sigma}h_{\mu\nu,\sigma},
$$
\n(11)

 $\xi_{\mu}$  being arbitrary. This meant that the source  $S_{\mu\nu}$  had to be divergenceless in order to make Eq. [\(9\)](#page-7-1) consistent. Finally, by making the gauge choice  $\overline{h}_{\mu\sigma,\sigma} = 0$ , the law of the gravitational interaction of two systems by means of the exchange of a virtual graviton was obtained. Feynman then went on computing other processes, such as an interaction vertex coupling two particles and a graviton and the gravitational analog of gravitational Compton effect (i.e. with a graviton replacing the photon).

After these preliminary calculations, Feynman went to the next to leading order approximation, thus encountering diagrams with closed loops:

However the next step is to take situations in which we have what we call closed loops, or rings, or circuits, in which not all momenta of the problem are defined (Ref. [2](#page-10-1), pp. 703-704).

He realized that working out closed loop diagrams required the solution of a number of conceptual issues, and he succeeded in showing that any diagram with closed loops can be expressed in terms of sums of on shell tree diagrams, which is the content of his celebrated tree theorem (which was treated in detail in Ref. [11\)](#page-11-1). Further details on the statement of the tree theorem and, in particular, on the nature of the proof for the one-loop case were given by Feynman in the discussion section (Ref. [2,](#page-10-1) pp. 714-717), while answering some related questions by DeWitt. But the main problem to face in carrying out one-loop calculations was the lack of unitarity, due to the presence of contributions arising from the unphysical longitudinal polarization states of the graviton, which did not cancel as they should. Following a suggestion

by Gell-Mann,<sup>[o](#page-8-0)</sup> Feynman considered the simpler Yang-Mills case (his results in this case were summarized in Ref. [12\)](#page-11-2) and found the same pathological behavior:

But this disease which I discovered here is a disease which exist in other theories. So at least there is one good thing: gravity isn't alone in this difficulty. This observation that Yang-Mills was also in trouble was of very great advantage to me. [...] the Yang-Mills theory is enormously easier to compute with than the gravity theory, and therefore I continued most of my investigations on the Yang-Mills theory, with the idea, if I ever cure that one, I'll turn around and cure the other (Ref. [2](#page-10-1), p. 707).

The solution to this issue was obtained by expressing each loop diagram as a sum of trees and then computing the trees. This worked even if the process of opening a loop by cutting a graviton line implies the replacement of a virtual graviton with a real transverse one. Finally, in order to guarantee gauge invariance the sum of the whole set of tree diagrams corresponding to a given process has to be taken.

The same results, according to Feynman, could be obtained by direct integration of the closed loop. In the last case a mass-like term has to be added to the Lagrangian to avoid singularity but at the price of breaking gauge invariance. At the same time a contribution has to be subtracted, which is obtained by making a ghost particle (with spin-1 and Fermi statistics) to go around the loop and artificially coupled to the external field. In this way both unitarity and gauge invariance would be restored. This procedure was worked out also for Yang-Mills theory, but in that case the ghost particle has spin- $0^{12}$  $0^{12}$  $0^{12}$ .

Once successfully solved the one-loop case, Feynman's efforts pointed toward a further generalization of the above procedure to two or more loops:

Now, the next question is, what happens when there are two or more loops? Since I only got this completely straightened out a week before I came here, I haven't had time to investigate the case of 2 or more loops to my own satisfaction. The preliminary investigations that I have made do not indicate that it's going to be possible so easily gather the things into the right barrels. It's surprising, I can't understand it; when you gather the trees into processes, there seems to be some loose trees, extra trees (Ref. [2,](#page-10-1) p. 710).

But, in Feynman's words, preliminary attempts seem to suggest that novel difficulties enter the game when dealing with two or more loops, as also mentioned in the last of his published Lectures on Gravitation (Ref. [14,](#page-11-4) Lecture 16, pp. 211-212). Here, once again, he recognized in the lack of unitarity of some sums of diagrams the main source of the observed pathological behavior and pointed out that a similar feature was shared also by Yang-Mills theory. Finally, while hinting at the

<span id="page-8-0"></span><sup>&</sup>lt;sup>o</sup>"I suggested that he try the analogous problem in Yang-Mills theory, a much simpler nonlinear gauge theory than Einsteinian gravitation." (Ref. [3](#page-10-2), p. 53).

problem of finding ghost rules for high order diagrams, he argued in favor of the non-renormalizability of gravity as a consequence of these difficulties:

I do not know whether it will be possible to develop a cure for treating the multi-ring diagrams. I suspect not – in other words, I suspect that the theory is not renormalizable. Whether it is a truly significant objection to a theory, to say that it is not renormalizable, I don't know (Ref. [14,](#page-11-4) Lecture 16, pp. 211-212).

It is not clear whether Feynman was suggesting a link between non-unitarity and non-renormalizability issues. But in any case Feynman's results played a prominent role in the development of gauge theory and quantum gravity. Feynman's rules for ghosts were later generalized to all orders by  $DeWitt^{35-37}$  $DeWitt^{35-37}$  $DeWitt^{35-37}$ , while Ludvig D. Faddeev and Viktor N. Popov<sup>[38](#page-12-0)</sup> derived them in a much simpler way, by means of functional integral quantization, setting the standard for all subsequent work in the field. In particular, DeWitt proved that Yang-Mills theory and quantum gravity are in fact unitary at two  $35$  and arbitrarily many loops  $36,37$  $36,37$ . However, while Yang-Mills theory was later shown to be renormalizable (cf. Refs. [39-](#page-12-1)42), gravity presented divergences which could not be renormalized (cf. Refs. [43](#page-12-3)- [46\)](#page-12-4), confirming Feynman's suspect. It should be mentioned that, in subsequent years, modified theories of gravity, characterized by an action quadratic in the curvature, have been put forward. Unlike ordinary general relativity, these theories are renormalizable but not unitary<sup>[47](#page-12-5)</sup>.

It is worth mentioning that, unlike most of his contemporaries, Feynman did not think about non-renormalizability as a signature of inconsistency of a theory, as also recalled by  $Gell-Mann$ <su[p](#page-9-0)>p</sup>, and claimed by Feynman himself in one of his last interviews, given in January 1988:

The fact that the theory has infinities never bothered me quite so much as it bother others, because I always thought that it just meant that we've gone too far: that when we go to very short distances the world is very different; geometry, or whatever it is, is different, and it's all very subtle ( [48](#page-12-6), p. 507).

In fact, within the modern view on quantum field theory, which was developed in 1970s, non-renormalizability is considered only a signature of the fact that the theory loses its validity at energies higher that a certain scale. Nevertheless, one has an effective field theory, which works and can be useful to make predictions under that scale (interesting historical discussions can be found in Refs. [49](#page-12-7) and [50](#page-12-8)). This is true also for gravity<sup>[51](#page-12-9)</sup>. But, as remembered by John P. Preskill in a recent talk (Ref. [52](#page-12-10), slide 37), although he anticipated this view, apparently Feynman was not really at ease with it:

<span id="page-9-0"></span><sup>&</sup>lt;sup>p</sup>"He was always very suspicious of unrenormalizability as a criterion for rejecting theories" (Ref. [3,](#page-10-2) p. 53).

I spoke to Feynman a number of times about renormalization theory during the mid-80s (I arrived at Caltech in 1981 and he died in 1988). I was surprised on a few occasions how the effective field theory viewpoint did not come naturally to him". [...] Feynman briefly discusses in his lectures on gravitation (1962) why there are no higher derivative terms in the Einstein action, saying this is the "simplest" theory, not mentioning that higher derivative terms would be suppressed by more powers of the Planck length.

As a further remark, let us notice that in the last years Feynman's tree theorem has spurred a renewed interest in researchers working in the context of advanced perturbative calculations and generalized unitarity (cf. Refs. [53](#page-12-11) - [57\)](#page-12-12).

### 4. Concluding remarks

In this paper we focused on Feynman's contributions to the research in quantum gravity, starting from his interventions at Chapel Hill conference in 1957 and ending with the Wheeler festschrift papers. His approach was field theoretical rather than geometric, reflecting his strong belief in the unity of Nature, which is quantum at the deepest level. Quantization of gravity, according to him, simply had to be considered as the quantization of another field, the spin-2 graviton field. In this way full general relativity would be recovered at leading order, while the inclusion of loop diagrams brought into the picture a bunch of new difficulties. In this respect, Feynman's struggle against loops, while succeeding at one-loop order, failed with two- and higher-loop diagrams. Nevertheless, his results triggered further efforts and some tools he developed, such as the tree theorem, have recently become of widespread use among people working on scattering amplitudes.

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