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Four Mistakes in the Original Paper of Einstein in 1915 to Calculate the Precession of Mercury's Perihelion

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Abstract

Based on the gravitational field equations of general relativity, Einstein proposed a formula to calculate the precession of Mercury's perihelion in 1915. It is pointed out in this paper that there were four mistakes in Einstein's original paper. The first was the mistake of integral calculation discovered by Huadi in 2015. If this mistake did not exist, the precession value of Mercury's perihelion was 71.7" a century. The second was the mistake of the expansion coefficient of integral function. If this mistake did not exist, the precession value of Mercury's perihelion was 14.3" a century.The third was to treat the perihelion and aphelion of Mercury's elliptical orbit as poles of integrand expansion, resulted in that the correction term of general relativity should be equal to zero. The fourth was Einstein's assumption that the constant terms in the formula to calculate the precession of Mercury's perihelion was equal to those in the Newton's formula of gravity. However, by precise calculations based on the Schwarzschild metric and the geodesic equation of Riemann geometric, this constant term must be equal to zero. The result means that general relativity can only describe the parabolic orbits (with minor corrections) of planets in the solar system. It can not describe the elliptical and hyperbolic orbits of celestial bodies. So Einstein did not prove that general relativity could explain the precession of Mercury's perihelion with the value of 43” a century. Since the fourth mistake was fatal which made the equations of planetary motion of general relativity untenable, Einstein's gravity theory of curved space-time does not hold.

KeyWords: General relativity, Newton's theory of gravity, Mercury's perihelion precession, Riemann geometry, Geodesic equation, Parabolic orbits, Elliptical orbit, Hyperbolic orbit

1. Introduction

Einstein gave a formula to calculate the precession of Mercury's perihelion and obtained the result of 43”a century in his original paper in 1915 based on the equation of gravitational field. This result was considered as one of the most important theoretical achievements of general relativity.

As early as 2005, Dr. Ji Hao in China found that Einstein's calculation on the precession of Mercury's perihelion was wrong [1]. In February 2016, Dr. Huadi, a member of the Russian Academy of Astronautics, sent a manuscript to the author of this paper and said that he found an integral mistake in the Einstein's original paper to calculate the precession of Mercury’s perihelion [2]. By the correct calculation, the precession value should be $71.7''$ a century, not be $43''$. The error rate was up to 67 percent.

So the author examined the Einstein's original paper carefully and found that Huadi's calculations were correct and that Einstein’s calculation was certainly wrong.

In addition, the author also found other three mistakes in the Einstein's calculation.

1. To be able to do the calculation, Einstein introduced a transform of the integrand function, but the transformation was wrong. With the correct transformation, the precession value of Mercury's perihelion became $14.3''$ a century.

2. Einstein took the perihelion and aphelion of the Mercury ellipse orbit as the poles of integrand, but this was impossible. If the perihelion and aphelion of the Mercury elliptical orbit were the poles of the integrand, what described was ellipse orbit, the the correction term of the motion equation of general relativity could not exist. This was a fundamental mathematical error, not the problem of physics approximation. Many physicists also had made

the same mistake in this problem since Einstein did it [3].

3. Based on the Schwarzschild metric and the geodesic equation of Riemann geometric, the author proved that the constant term in the Einstein's formula for calculating the precession of Mercury's perihelion must be equal to zero. In indicated that general relativity can only describe the parabolic motions of celestial bodies in the solar system (with minor corrections). It can not describe the elliptic and hyperbolic orbital motions of celestial bodies [3].

Of there mistakes, last one is fatal. It not only shows that Einstein did not prove that precession value of Mercury's perihelion is 43 " a century, but also shows that the planetary motion equation of general relativity is wrong. Because describing the elliptical orbit motion of planets is the minimum requirement for a qualified theory of gravity, general relativity can not describe the elliptical orbit motion, which means that Einstein's gravity theory of curved space-time is invalid.

2. The mistake of integral calculation

The calculation in this section is taken from Huadi's unpublished online papers [2]. When Einstein calculated the precession of Mercury's perihelion based on his gravitational field equation in 1915, the Schwarzschild solution had not yet been discovered. By calculating the motion of a particle along geodesics, Einstein obtained the following formula [4]

$$\left(\frac{dx}{d\varphi}\right)^2 = \frac{2A}{B^2} + \frac{\alpha}{B^2}x - x^2 + \alpha x^3 \qquad (1)$$

Here $x = 1/r$, $\alpha = 2GM/c^2$, M is the mass of the sun , A and B are integral constants determined by the conservation formulas of energy and angle momentum.

Set $\Delta\varphi$ being the increased angle while the Mercury moves from the aphelion to the perihelion of elliptical orbit, the integral of Eq.(1) is

$$\Delta\varphi = \int_{\alpha_1}^{\alpha_2} \frac{dx}{\sqrt{2A/B^2 + (\alpha/B^2)x - x^2 + \alpha x^3}} \qquad (2)$$

Here $\alpha_1 = 1/r_1$ and $\alpha_2 = 1/r_2$, r_1 was the aphelion of orbit and r_2 was the perihelion of orbit . So α_1 and α_2 are the roots of following equation

$$f(x) = \frac{2A}{B^2} + \frac{\alpha}{B^2}x - x^2 + \alpha x^3 = 0 \qquad (3)$$

Then, Einstein wrote Eq.(2) as

$$\Delta\varphi = [1+\alpha(\alpha_1+\alpha_2]\int_{\alpha_1}^{\alpha_2}\frac{dx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)(1-\alpha x)}} \tag{4}$$

Due to $\alpha x = \alpha / r << 1$, so $(1-\alpha x)^{-1/2} \approx (1+\alpha x/2)$, Eq.(4) became

$$\Delta\varphi = [1+\alpha(\alpha_1+\alpha_2)]\int_{\alpha_1}^{\alpha_2}\frac{(1+\alpha x/2)dx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)}} \tag{5}$$

The calculation result of Einstein was

$$\Delta\varphi = \pi\left[1+\frac{3}{4}\alpha(\alpha_1+\alpha_2)\right] \tag{6}$$

For the Mercury's precision, take $r_1 = 6.9818\times10^{10}m$, $\alpha_1 = 1.4323\times10^{-11}/m$, $r_2 = 4.6004\times10^{10}m$ and $\alpha_2 = 2.1737\times10^{-11}/m$. The mass of the sun is $M = 1.9892\times10^{30}Kg$, the gravity constant $G = 6.6732\times10^{-11}N\cdot m^2/Kg^2$ and the speed of light is $c = 2.9979\times10^{8}m/s$. According to Eq.(6), the precession angle for the Mercury moves a circle around the sun (one Mercury year is equal to 88 Earth days) is

$$\varepsilon = 2(\Delta\varphi-\pi) = \frac{3}{2}\pi\alpha(\alpha_1+\alpha_2) = \frac{3\pi GM}{c^2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right) = 5.0197\times10^{-7}rad \tag{7}$$

The precession angle of Mercury's perihelion every 100 Earth years is

$$5.0350\times10^{-7}\times415.28 = 2.0846\times10^{-4}rad = 43'' \tag{8}$$

The result was deemed to be consistent with actual observations, and Einstein's theory of gravity was accepted by physicists due to this result. For a hundred years, physicists have not doubts for Einstein's calculation.

However, Wadi pointed out that Einstein's calculation was wrong and the correct result should be [2]

$$\Delta\varphi = \pi\left[1+\frac{5}{4}\alpha(\alpha_1+\alpha_2)+\frac{1}{4}\alpha^2(\alpha_1+\alpha_2)^2\right] \approx \pi\left[1+\frac{5}{4}\alpha(\alpha_1+\alpha_2)\right] \tag{9}$$

According to Eq. (9), the precession angle of Mercury's perihelion is $71.7''$ a century, rather than 43 ". The correct calculation is below

$$\int\frac{dx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)}}=\int\frac{dx}{\sqrt{\alpha_1\alpha_2+(\alpha_1+\alpha_2)x-x^2}}=\arcsin\frac{2x-(\alpha_1+\alpha_2)}{\sqrt{-(\alpha_1-x)(\alpha_2-x)}} \tag{10}$$

$$\int\frac{xdx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)}}=-\sqrt{-(x-\alpha_1)(x-\alpha_2)}+\frac{(\alpha_1+\alpha_2)}{2}\arcsin\frac{2x-(\alpha_1+\alpha_2)}{\sqrt{-(x-\alpha_1)(x-\alpha_2)}} \tag{11}$$

So it can be obtained from Eq.(5)

$$\int_{\alpha_1}^{\alpha_2}\frac{(1+\alpha x/2)dx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)}}=\left[1+\frac{\alpha}{4}(\alpha_1+\alpha_2)\right]\cdot\left[\arcsin\frac{\alpha_2-\alpha_1}{\alpha_2-\alpha_1}-\arcsin\frac{\alpha_1-\alpha_2}{\alpha_2-\alpha_1}\right]$$

$$=\left[1+\frac{\alpha}{4}(\alpha_1+\alpha_2)\right](\arcsin 1-\arcsin(-1))=\pi\left[1+\frac{\alpha}{4}(\alpha_1+\alpha_2)\right] \tag{12}$$

Eq.(6) becomes

$$\Delta\varphi=\pi(1+\alpha(\alpha_1+\alpha_2))\left[1+\frac{\alpha}{4}(\alpha_1+\alpha_2)\right]\approx\pi\left[1+\frac{5}{4}\alpha(\alpha_1+\alpha_2)\right] \tag{13}$$

Eq.(13) is different from Eq.(6), and Einstein's calculation is obviously wrong. Eq. (7) becomes:

$$\varepsilon=2(\Delta\varphi-\pi)\approx\frac{5\pi}{2}\alpha(\alpha_1+\alpha_2)=8.3662\times10^{-7}\,rad \tag{14}$$

The precession angle of Mercury's perihelion every 100 Earth years is

$$8.3662\times10^{-7}\times415.28=3.4743\times10^{-4}\,rad=71.7'' \tag{15}$$

It is 1.67 times greater than Einstein's calculation.

3. The mistake in the expansion of integrand

The second mistake in Einstein's calculation was that the integrand expansion of Eq.(4) did not hold. Due to $\alpha(\alpha_1+\alpha_2)\sim10^{-8}<<1$, we write the intergrand of Eq.(4) as

$$\frac{1+\alpha(\alpha_1+\alpha_2)}{\sqrt{-(x-\alpha_1)(x-\alpha_2)(1-\alpha x)}}\approx\frac{1}{\sqrt{-(1-2\alpha(\alpha_1+\alpha_2))(x-\alpha_1)(x-\alpha_2)(1-\alpha x)}} \tag{15}$$

Comparing with Eq.(2), we get

$$-[1-2\alpha(\alpha_1+\alpha_2)](x-\alpha_1)(x-\alpha_2)(1-\alpha x)=\frac{2A}{B^2}+\frac{\alpha}{B^2}x-x^2+\alpha x^3 \tag{16}$$

By expanding the left side of Eq. (16) , we have

$$-[1-2\alpha(\alpha_1+\alpha_2)]\left[\alpha_1\alpha_2-(\alpha_1+\alpha_2+\alpha\alpha_1\alpha_2)x+\left(1+\alpha(\alpha_1+\alpha_2)\right)x^2-\alpha x^3\right] \quad (17)$$

Comparing with the right hand side of Eq. (16), we have

$$\frac{2A}{B^2}=-[1-2\alpha(\alpha_1+\alpha_2)]\alpha_1\alpha_2 \quad (18)$$

$$\frac{\alpha}{B^2}=[1-2\alpha(\alpha_1+\alpha_2)](\alpha_1+\alpha_2+\alpha\alpha_1\alpha_2) \quad (19)$$

$$1=[1-2\alpha(\alpha_1+\alpha_2)][1+\alpha(\alpha_1+\alpha_2)]=[1-\alpha(\alpha_1+\alpha_2)+\alpha^2(\alpha_1+\alpha_2)^2] \quad (20)$$

$$\alpha=[1-2\alpha(\alpha_1+\alpha_2)]\alpha \qquad \text{或} \qquad 1-2\alpha(\alpha_1+\alpha_2)=1 \quad (21)$$

To make two sides of Eq.(20) and (21) be equal each other, the items $\alpha(\alpha_1+\alpha_2)$ and $\alpha^2(\alpha_1+\alpha_2)^2$ must be omitted. Because $\alpha\sim 10^3$, $\alpha_1\sim\alpha_2\sim 10^{-11}$, $\alpha(\alpha_1+\alpha_2)\sim 10^{-8}<<1$, $\alpha\alpha_1\alpha_2\sim 10^{-19}<<1$, these neglects are acceptable. So we get

$$\frac{2A}{B^2}=-\alpha_1\alpha_2 \qquad \frac{\alpha}{B^2}=\alpha_1+\alpha_2 \quad (22)$$

But the problem is that if the item $\alpha(\alpha_1+\alpha_2)$ is omitted, Eq.(4) becomes

$$\Delta\varphi=\int_{\alpha_1}^{\alpha_2}\frac{dx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)(1-\alpha x)}} \quad (23)$$

Therefore, the factor before the integral sign of Eq.(4) does not exist. If this fact is not considered, we let

$$\frac{2A}{B^2}+\frac{\alpha}{B^2}x-x^2+\alpha x^3=-(x-\alpha_1)(x-\alpha_2)(1-\alpha x)$$

$$=-\alpha_1\alpha_2+(\alpha_1+\alpha_2+\alpha\alpha_1\alpha_2)x+[1+\alpha(\alpha_1+\alpha_2)]x^2+\alpha x^3 \quad (23)$$

By comparing the parameters on the two sides of Eq.(23) and ignoring the high order factors $\alpha(\alpha_1+\alpha_2)$ and $\alpha\alpha_1\alpha_2$, we have

$$\frac{2A}{B^2}=-\alpha_1\alpha_2 \qquad \frac{\alpha}{B^2}=(\alpha_1+\alpha_2+\alpha\alpha_1\alpha_2)\approx\alpha_1+\alpha_2$$

$$1 = 1 + \alpha(\alpha_1 + \alpha_2) = 1 \qquad \alpha = [1 - 2\alpha(\alpha_1 + \alpha_2)]\alpha = \alpha \tag{24}$$

The same result are obtained as shown in Eq.(22), so the Einstein's formula is wrong. The factor before the integral sign of Eq.(4) should be equal to 1. The result of integral is

$$\Delta\varphi = \int_{\alpha_1}^{\alpha_2} \frac{dx}{\sqrt{-(x-\alpha_1)(x-\alpha_2)(1-\alpha x)}} = \pi\left[1 + \frac{\alpha}{4}(\alpha_1 + \alpha_2)\right] \tag{25}$$

The perihelion precession angle of Mercury moving around the sun for a circle becomes

$$\varepsilon = 2(\Delta\varphi - \pi) = \frac{1}{2}\pi\alpha(\alpha_1 + \alpha_2) = \frac{\pi GM}{c^2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = 1.6732 \times 10^{-7} rad \tag{26}$$

The precession angle in a century is

$$1.6732 \times 10^{-7} \times 415.28 = 6.9487 \times 10^{-5} rad = 14.3'' \tag{27}$$

It is only a third of Einstein's calculation.

Anatoli Andrei Vankov, a Russian scholar, wrote an unpublished article in 1947. In this paper, Vankov points out 17 errors in Einstein's original paper on the calculations of the precession of Mercury's perihelion, of which the 16th one is that the coefficient $1 + \alpha(\alpha_1 + \alpha_2)$ in front of the integral sign is wrong. It should be replaced by the factor $1 + \alpha(\alpha_1 + \alpha_2)/2$. to obtain the result of 43" a century[5].

But why made this change, Vankov had no any explained. In fact, according to the method of Vankov, we can also multiply a factor $1 + Q\alpha(\alpha_1 + \alpha_2)$ before Eq.(25) and taking arbitrary value for Q , for example, to take $Q = 100$, so that the precession angle of Mercury's perihelion becomes 4300", but this is meaningless.

4. The mistake of integrand's poles.

Einstein was wrong to take the perihelion and aphelion of Mercury's elliptical orbit as the poles of Eq.(3) . The orbit poles of palatial motions is described by $dx/d\varphi = 0$. By considering Eq.(16), Eq.(3) becomes

$$-[1 - 2\alpha(\alpha_1 + \alpha_2)](x - \alpha_1)(x - \alpha_2)(1 - \alpha x) = 0 \tag{28}$$

Eq. (28) has three roots. They are $x_1 = \alpha_1$, $x_2 = \alpha_2$ and $x_3 = 1/\alpha$. The problem was that

Einstein did not calculate the exact values of these three roots by solving the cubic equation (3) of one variable. In fact, since the constants A and B sum were unknown, Einstein could not actually obtain the roots of equation (3). However, Einstein assumed that the third root had a known and definite value $x_3 = 1/\alpha$, which was simply impossible.

Meanwhile, in his final calculations, Einstein took α_1 and α_2 as the aphelion and perihelion of Mercury's elliptical orbit, which was also impossible. Because if the aphelion and perihelion of Mercury's elliptical orbit were regarded as the roots of equation (3) , what the equation described was the elliptical orbit of Newtonian gravity, the correction term αx^3 of motion equation of general relativity cannot exist.

In fact, for a quadratic equation of one variable, if its two roots are α_1 and α_2, we have

$$x^2 + bx + c = (x-\alpha_1)(x-\alpha_2) = 0 \tag{29}$$

The relation between roots and the coefficients of equation are

$$\alpha_1 + \alpha_2 = -b \qquad\qquad \alpha_1\alpha_2 = c \tag{30}$$

By ignoring the correction of general relativity, Eq.(3) becomes

$$x^2 - \frac{\alpha}{B^2}x - \frac{2A}{B^2} = 0 \tag{31}$$

If α_1 and α_2 are the roots of Eq.(31), it should have

$$\alpha_1 + \alpha_2 = \frac{\alpha}{B^2} \qquad\qquad \alpha_1\alpha_2 = -\frac{2A}{B^2} \tag{32}$$

Eq.(32) and Eq.(22) are the same, so the correction item of Eq.(30) must not exist, otherwise α_1 and α_2 can not be the roots of Eq.(3).

Einstein made a primary mathematical mistake here. This is not a matter of physical approximation, which made Eq.(4) invalid. Many physicists since Einstein have made the same mistake in calculating the precession of Mercury's perihelion[ ].

5. The mistake of integral constants in the motion equation of planet

It has been more than 100 years since Einstein proposed his theory of gravity in curved spacetime, but the constants in the most important motion equations of planetary and light in general relativity have not been rigorously calculated. Einstein directly assumed that the constant term in Eq.(1) was not equal to zero, and calculated the precession of Mercury's perihelion based on it.

It is shown below that the constant term on the right-hand side of Eq.(1) should be equal to zero strictly according to general relativity and the geodesic equations of Riemannian geometry. Therefore, general relativity can only describe the parabolic orbital motion of celestial bodies (with minor correction), can not describe the elliptical and hyperbolic orbital motions, so it cannot be used to calculate the precession of Mercury's perihelion[3]. This was the most fatal error for general relativity, invalidating Einstein's gravity theory of curved space time.

By solving the Einstein's equation in the spherically symmetric gravitational field, the Schwarzschild metric represented in four-dimensional space-time was obtained:

$$ds^2 = c^2 A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (33)$$

Here

$$A(r) = 1 - \alpha / r \qquad\qquad B(r) = (1 - \alpha / r)^{-1} \qquad (34)$$

Einstein assumed that an object moved in a gravitational field along a geodesic and the geodesic equations were calculated by using Riemann geometry. Therefore, the derivation of the orbital equations of planets in the solar system required two sets of independent equations, namely, the Einstein's equation of gravitation field and the geodesic equation of Riemann geometric.

According to the standard method, from the Schwarzschild metric of Eq.(33), four geodesic equations can be obtained. By taking $\theta = \pi / 2$, both sides of equal sign of one equation are equal to zero, and the remaining three independent equations are [6] [7] :

$$\frac{d^2 r}{ds^2} + \frac{B'}{2B}\left(\frac{dr}{ds}\right)^2 - \frac{r}{B}\left(\frac{d\varphi}{ds}\right)^2 + \frac{A'}{2B}\left(\frac{dx^0}{ds}\right)^2 = 0 \qquad (35)$$

$$\frac{d^2\varphi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds} = 0 \qquad (36)$$

$$\frac{d^2 x^0}{ds^2} + \frac{A'}{A}\frac{dr}{ds}\frac{dx^0}{ds} = 0 \qquad \text{或} \qquad \frac{d}{ds}\left(A\frac{dx^0}{ds}\right) = 0 \qquad (37)$$

Here $x^0 = ct$, $B' = dB(r)/dr$, $A' = dA(r)/dr$. The integrals of Eq.(36) and (37) are

$$r^2 \frac{d\varphi}{ds} = J \tag{38}$$

$$\frac{dx^0}{ds} = \frac{K}{A(r)} \tag{39}$$

Here J and K are integral constants. By considering Eq. (38) and (39), the integral of Eq.(35) is [7]

$$B\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 - \frac{K^2}{A} = -E \tag{40}$$

or

$$\frac{dr}{ds} = \frac{1}{B^{1/2}}\sqrt{-E - \frac{J^2}{r^2} + \frac{K^2}{A}} \tag{41}$$

Based on Eqs.(38), (39) and (40), it can be proved that the time-independent orbit equation of planet of general relativity is [3]

$$\left(\frac{du}{d\varphi}\right)^2 + u^2 = \frac{K^2 - E}{J^2} + \frac{E\alpha}{J^2}u + \alpha u^3 \tag{42}$$

The time-dependent orbit equation of planet of general relativity is

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2\left(1 - \frac{E}{K^2}\right) + c^2\left(\frac{3E}{K^2} - 2\right)\frac{\alpha}{r}$$

$$+ c^2\left(1 - \frac{3E}{K^2}\right)\frac{\alpha^2}{r^2} + \frac{c^2 E}{K^2}\frac{\alpha^3}{r^3} + \frac{\alpha c^2 J^2}{K^2 r^3}\left(1 - \frac{\alpha}{r}\right)^2 \tag{43}$$

To determine the integral constant, it is necessary to compare equation (43) with the of Newton's equation of gravity. Let L be the angle momentum of unit mass with $cJ = L \sim rV$, as well as $V/c << 1$ in the weak gravity field of the sun, we have

$$\frac{\alpha c^2 J^2}{r^3} = \frac{\alpha L^2}{r^3} \sim \frac{2GMr^2V^2}{c^2 r^3} = \frac{2GM}{r}\frac{V^2}{c^2} \tag{44}$$

By ignoring the the high order correction terms to contain α^2/r^2 and V^2/c^2 in Eq.(43), we can obtain

$$\frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2\right]-\left(\frac{3E}{K^2}-2\right)\frac{GM}{r}=\frac{c^2}{2}\left(1-\frac{E}{K^2}\right) \qquad (45)$$

If Eq.(45) is the motion equation of Newtonian gravity, the coefficient before the term GM/r must be equal to zero. Let

$$\frac{3E}{K^2}-2=1 \qquad \text{or} \qquad \frac{E}{K^2}=1 \qquad (46)$$

Eq.(45) becomes

$$\frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2\right]-\frac{GM}{r}=0 \qquad (47)$$

On the other hand, according to the Newton's theory of gravity, the energy conservation formula of a celestial body with unit mass in the solar gravity field is

$$\frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2\right]-\frac{GM}{r}=E' \qquad (48)$$

When the constant (total mechanical energy) $E'=-GM/2b<0$, the celestial body moves in the elliptical orbit (b is the major axis of elliptical orbit). When $E'=GM/2a>0$, the celestial moves in the hyperbolic orbit (a is the parameter of hyperbolic orbit). When $E'=0$, the celestial body moves in the parabolic orbit. So Eq.(47) only describes the parabolic orbit of celestial body, can not describe the elliptical and hyperbolic orbits.

On the other hand, substituting Eqs.(38), (39) and (41) in Eq.(33), we can get $ds^2=Eds^2$, so constant $E=0,\ 1$ [3]. For the motion of an object with mass, it takes $E=1$. For the motion of light, Einstein assumed $E=0$. Therefore, according to Eq.(46), we have $K=1$. Up to now, all integral constants in the geodesic equations are determined. Eq.(43) becomes

$$\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2=\frac{c^2\alpha}{r}-\frac{2c^2\alpha^2}{r^2}+\frac{c^2\alpha^3}{r^3}+\frac{\alpha L^2}{r^3}\left(1-\frac{\alpha}{r}\right)^2 \qquad (49)$$

We write Eq.(49) in the standard form with

$$\frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2\right]+U(r)=0 \qquad (50)$$

The potential energy of unit mass in the gravitational field is

$$U(r) = -\frac{GM}{r}\left[1 - \frac{2\alpha}{r} + \frac{\alpha^2}{r^2} + \frac{L^2}{c^2 r^2}\left(1 - \frac{\alpha}{r}\right)^2\right] \tag{51}$$

According to Eq.(50) and (51), general relativity can only describe the parabolic orbital motions (with minor corrections). It can not describe the elliptical and hyperbolic motions.

If it is not, assuming that the Newtonian approximation of general relativity can describe the elliptical orbital motion, the constant term on the right side of Eq.(48) should be

$$1 - \frac{E}{K^2} = -\frac{GM}{c^2 b} \quad \text{or} \quad \frac{3E}{K^2} - 2 = 1 + \frac{3GM}{c^2 b} \tag{52}$$

Eq.(45) becomes

$$\frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right] - \left(1 + \frac{3GM}{c^2 b}\right)\frac{GM}{r} = -\frac{GM}{2b} \tag{53}$$

It means that when a planet moves in an elliptical orbit, the Newtonian gravity becomes

$$\vec{F} = -\left(1 + \frac{3GM}{c^2 b}\right)\frac{GM\,\vec{r}}{r^3} \tag{54}$$

If a celestial body moves in a hyperbolic orbit, the constant term on the right side of Eq.(48) should be

$$1 - \frac{E}{K^2} = \frac{GM}{c^2 a} \quad \text{or} \quad \frac{3E}{K^2} - 2 = 1 - \frac{3GM}{c^2 a} \tag{55}$$

Eq.(45) becomes

$$\frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right] - \left(1 - \frac{3GM}{c^2 a}\right)\frac{GM}{r} = \frac{GM}{2a} \tag{56}$$

The Newtonian gravity becomes

$$\vec{F} = -\left(1 - \frac{3GM}{c^2 a}\right)\frac{GM\,\vec{r}}{r^3} \tag{57}$$

The formulas (54) and (57) are obviously impossible, because according to the Newtonian theory of gravity, the basic form of gravity is the same whether a body moves in parabolic, elliptical or hyperbolic orbits. The basic formula of gravity can only contain the basic constant of physics. It can not contain the special factors a and b of individual orbits. The factors

$3GM/(c^2 b)$ and $-3GM/(c^2 a)$ can not exist in Eq.(54) and (57), otherwise physics would have no regularity and unity.

Substituting $E=1$ and $K=1$ in Eq.(42) we get

$$\left(\frac{du}{d\varphi}\right)^2 = \frac{\alpha}{J^2}u - u^2 + \alpha u^3 \qquad (58)$$

Let $x=u$ and $B=J$ in Eq.(58) and comparing it with Eq.(1), it can be seen that Eq.(1) has an extra constant term $2A/B^2$ which was added by Einstein artificially and not exist actually according the strict calculation. Since Eq. (58) does not describe an elliptical orbit, it is impossible using it to calculate the precession of Mercury's perihelion.

Take the derivatives of Eqs.((1) and (58) with respect to φ ($x=u$), the result are the same with

$$\frac{d^2u}{d\varphi^2} + u = \frac{2\alpha}{J^2} + \frac{3\alpha}{2}u^2 \qquad (59)$$

Physicists after Einstein used Eq.(59) to calculate the precession of Mercury's perihelion and obtained the same value 43 " . If the correction of general relativity is not considered, Eq.(59) becomes

$$\frac{d^2u}{d\varphi^2} + u = \frac{2\alpha}{J^2} \qquad (60)$$

Eq.(60) describes the conic orbit motion with the solution

$$u = \frac{2\alpha/J^2}{1+e\cos(\varphi-\varphi_0)} \qquad (61)$$

In Eq.(61), $e = C\alpha/J^2$ is the the eccentricity in which C is an integral constant. When $e<1$, Eq.(61) describes elliptical orbit. When $e=1$, it describers parabolic orbit and when $e>1$, it describes hyperbolic orbit. Exactly what kind of motion is made depends on the total mechanical energy of celestial body. According to Eq. (47), we should take $e=1$. Thus Eq.(59) of general relativity can only describes the parabolic orbit (with minor corrections), not the elliptic and hyperbolic orbits.

6. Conclusion

Einstein proposed a formula to calculate the precession of Mercury perihelion and obtained the result of 43 " a century based on the gravitational field equation of curved space-time. The result was considered was considered to be one of the most important theoretical support for general relativity.

In this paper, four mistakes in Einstein's original paper are pointed out. One was a miscalculation of the integral, discovered by Mr. Wadi. The other three are found by the author of this paper, including the calculation error of the coefficient of integral expansion, the error of taking the aphelion and perihelion of mercury's elliptical orbit as the poles of the integrand, and the calculation error of the constant term in the equation of planetary motion in general relativity.

Among them, the calculation error of the constant term in the planetary motion equation is the most fatal and irreparable. Because the constant term should be zero, general relativity can describe only the parabolic orbital motions of objects in the solar system (with minor corrections), not the elliptical and hyperbolic orbital motions of objects. Since describing the elliptical orbits of planets is the minimum requirement for a theory of gravity, Einstein's gravity theory of curved space-time can only be considered wrong.

In fact, Einstein's curved space-time gravity theory also leads to a number of serious problems, such as the definition of energy momentum tensor of gravitational field, the singularity of space-time, the interchangeability of time and space in black holes and so on [11, 12, 13]. Non-baryonic dark matter and dark energy in cosmology are also the results of regarding gravity as the curvature of space-time [14,15].

Therefore, the conclusion of this paper is that modern physics must completely abandon the geometrical description of gravity in curved space-time and return to the dynamic description in flat space-time through reinventing the Newton's theory of gravity.

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