From Fresnel's ether to special relativity.

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Summary. I present a review of the study of optical phenomena in moving bodies during the 19th century. I show how H.A. Lorentz, using a step-by-step approach, succeeded in explaining these phenomena, and how he arrived at his relativistic formulas. I highlight H. Poincaré's important role as a critic of existing theories. In particular, I discuss his synchronization of remote clocks, which clarifies the meaning of Lorentz's "local time".

Abstract. I present a review of the study of optical phenomena in moving bodies during the 19th century. I show how Lorentz's progressive stages approach successfully explained these phenomena and how he finally got his relativistic formulae. The important role of H. Poincaré as a critic reviewer of existent theory is underlined. I discuss with some details the Poincaré's distant clocks synchronisation, which clarifies the meaning of Lorentz's local time.

Introduction.

The contrast between the following two quotations clearly illustrates the difference in appreciation of the difficulty of achieving a conceptual change, depending on whether we judge it before or long after it has taken place.

"Much research has been done on the influence of earth movement. The results have always been negative. But if these

In fact, according to the prevailing theories, compensation would only be approximate, and it was to be expected that precise methods would yield positive results. (.). We
carried out experiments that should have detected first-order terms; the results were negative; could this be due to chance? No one accepted this; a general explanation was sought, and Lorentz found it; he showed that first-order terms should destroy themselves, but second-order terms were not. Then more precise experiments were carried out; they too were negative; it couldn't be the effect of chance either; an explanation was needed; one was found; one is always found; hypotheses are the least lacking fund. But that's not enough; who doesn't feel that it's still leaving too great a role to chance? Could it not also be a coincidence that a certain circumstance comes just at the right moment to destroy the terms of the first order, and that another circumstance, quite different, but just as opportune, takes on the task of destroying those of the second order? No, we must find the same explanation for both, and then everything leads us to believe that this explanation will also apply to higher-order terms, and that the mutual destruction of these terms will be rigorous and absolute."

Henri Poincaré, "Science and Hypothesis", (1902).

When I asked him how he had learned of the Michelson-Morley experiment, he told me that he had become aware of it through the writings of H.A. Lorentz, but *only after 1905* had it come to his attention![S.'s italics] "Otherwise", he said, "I would have mentioned it in my paper." He continued to say that experimental results which had influenced him most were the observations on stellar aberration and Fizeau's measurements on the speed of light in moving water. "They were enough," he said.'

R.S. Shankland, "Conversations with A. Einstein", (4 Feb.1950).

My presentation aims to follow, step by step, albeit in summary form, the long road that led 19th-century physicists from an approach to optics with the ether playing an essential role as an absolute reference point to a relativistic theory in which the ether no longer plays a kinematic role.

1. From mechanical ether to electromagnetic ether[1]

The early 19th century saw a rapid transition from the corpuscular theory of light proposed by Newton (1675) to a wave theory proposed by Huygens (1690). In 1801, Th. Young (1773-1829) provided the first convincing evidence of this, notably by showing that adding light to light can cause the absence of light (interference).

At the time, a wave theory could only be a mechanical theory. By accepting this theory, we also admit the existence of a universal medium, present in the best vacuums (notably the interstellar void) as well as in all transparent, heavy bodies, whose vibrations we perceive as light. Augustin Fresnel (1788-1827) championed this idea. In an impressive series of works from 1815 to his death, he established so many results that the wave theory was later considered definitively proven. After Fresnel, physicists believed they could assert that light was merely a certain mode of vibration of a universal fluid: the ether. This reduced the task of optics to studying the physical properties of this fluid.

However, this study was to reveal some very strange properties, such as the exclusively transverse vibration of the ether, as indicated by the polarization of light. It also raised important questions about the relationship between aether and heavy matter:

- was the ether of the vacuum exactly the same as the ether of transparent bodies?
- do dispersive phenomena indicate a diversification of the ether in transparent bodies, depending on the frequency of vibrations?
- did transparent moving bodies drag the ether along with them?

It is, of course, the latter question that will be examined in today's talk, since it was of crucial importance for the conceptual change that took place at the beginning of the 20th century: the birth of special relativity.

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(1) For the preparation of this first paragraph, I made extensive use of E.T. Whittaker: "A History of Aether and Electricity", [Wh-51].

When, around 1865, the progress of electricity and magnetism was masterfully synthesized by J.C. Maxwell (1813-1879), it became clear that the ether of optics could be identified with that of electromagnetism (Faraday ether). Maxwell (1813-1879), it became clear that the ether of optics and the ether of electromagnetism (Faraday ether) could be identified. This solved certain problems, such as that of exclusively transverse polarization. Others arose, such as deciding whether Fresnel's polarization vector corresponded to electric or magnetic vibration. Theories diversified on the basis of these questions, and the problem of ether entrainment in the motion of matter reappeared as one of the distinguishing elements between these theories.

Fresnel had already solved the problem of ether entrainment as part of his mechanical theory of optics, proposing a partial entrainment solution as early as 1822:

- when a transparent body of refractive index n has a velocity v relative to the immobile ether, the ether contained in this body is partially entrained in this movement, in proportion:

$$\alpha = 1 - \frac{1}{n^2}\ ; \tag{1}$$

- Consequently, the velocities of the material body in the ether (v) and of light in the material body when at rest in the ether (c/n, c being the speed of light in a vacuum and n being the refractive index of the body[2]) only partially add up. For a propagation of light parallel to the speed of the material body, we thus obtain:

$$V = \frac{c}{n} \pm \left(1 - \frac{1}{n^2}\right)v \tag{2}$$

Fresnel's theoretical reasoning is presented in Appendix 1 (1a). Here, the ether is considered a genuine fluid with mechanical properties. In contrast, I also present in this Appendix the

(2) This of course refers to the wave theory of light. The speed would be nc in the case of a corpuscular theory. The question was settled experimentally by Foucault and Fizeau in 1850: light travels faster in air than in water and, consequently, light propagation is wave-like.

Lorentz's reasoning (1886) where the electromagnetic ether, which has lost its mechanical properties and is perpetually at rest, appears to be entrained à la Fresnel (1b). I also present (1c) the purely kinematic reasoning of von Laue [La-07] where, without ether, a pseudo-drive simply results from Einstein's relativistic law of velocity composition.

In the second half of the 19th century, Fresnel's result was verified quite satisfactorily by various experiments, which we will now review:

- Fizeau experiment (1851). A monochromatic beam is split in two by a semi-transparent mirror m, and these two beams travel along the same path but in opposite directions. Part of this path takes place in a stream of water, which is assumed to partially entrain its ether (Fig.1).

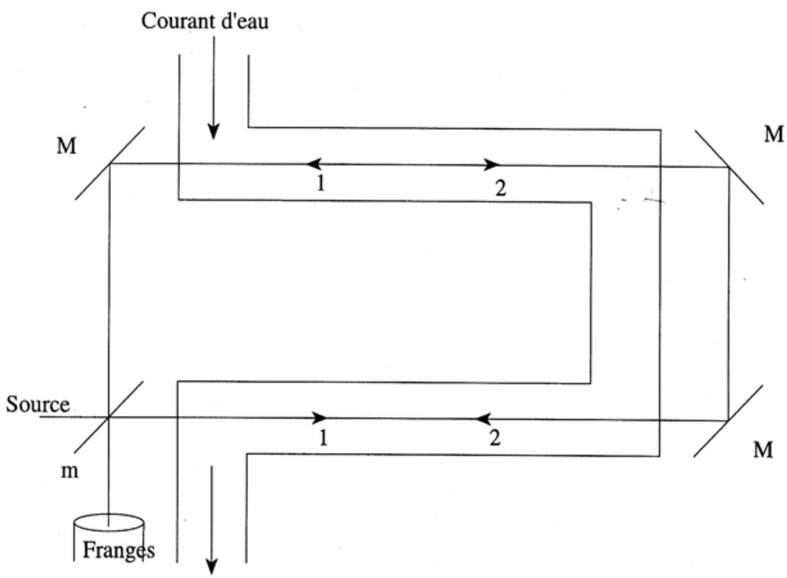

Fig. 1: Schematic diagram of the Fizeau experiment; m: semi-transparent mirror, M: mirror(s).

The bangs are due to the phase difference $\Delta\varphi$ between the two light paths 1 and 2, corresponding to the difference in the travel times of these paths when the aether is entrained by the water current: $\Delta\varphi = 2u\,\nu\,\Delta t$, where ν is the frequency of light. For the direction of the water current shown in the figure, we have:

$$\Delta t = t_1 - t_2 = 2L\left[\frac{1}{\frac{c}{n} + \alpha v} - \frac{1}{\frac{c}{n} - \alpha v}\right] = -4\frac{L}{c}\alpha\, n^2 \beta + O\left(\beta^2\right) \quad (3)$$

where: 2L is the path length of light in water, n is the refractive index of water, v is the current velocity ($\beta = v/c$) and α is the ether entrainment coefficient. To preserve only the effect of ether entrainment by the current, we observe the displacement of the bangs when the direction of the current is reversed. The α coefficient is thus measured; the experimental result is compatible with the value predicted by Fresnel.

- Hoek experiment (1868). A monochromatic beam is split in two by a semi-transparent mirror m, and the two beams travel along the same path but in opposite directions. Part of the path is through water (or quartz)(Fig.2). As the apparatus is driven by the movement of the Earth in the ether, we hope to demonstrate the partial ether drive of the water (or quartz) from a displacement of the interference bangs when the whole apparatus is rotated 180° around m.

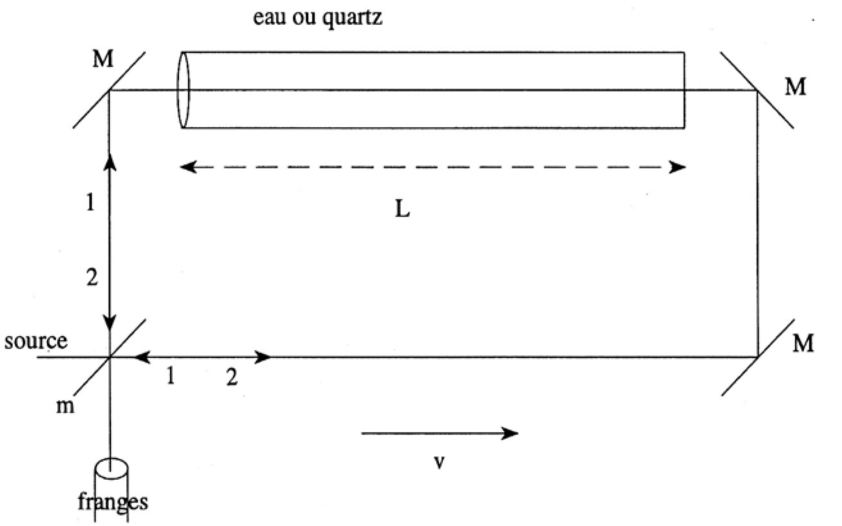

Fig. 2: Schematic diagram of Hoek's experiment.

The experiment shows no displacement of the bangs, thus confirming the value of the entrainment coefficient predicted by Fresnel! And indeed:

- as the device is driven by the Earth at speed v, light propagates through it at speed c ± v in the empty part, and at speed (c/n) ± (α -1)v in the part containing water (or quartz);
- if we disregard the travel time in identical sections for both beams (of no interest), we have:

beam 1:
$$t_1 = \frac{L}{\frac{c}{n} + (\alpha - 1)v} + \frac{L}{c + v}, \tag{4}$$

beam 2:
$$t_2 = \frac{L}{\frac{c}{n} - (\alpha - 1)v} + \frac{L}{c - v}, \tag{5}$$

i.e. a difference in journey time equal to:

$$\Delta t = t_2 - t_1 = 2\,\frac{L}{c}\,n^2 \beta \left[\alpha - 1 + \frac{1}{n^2} \right] + O(\beta^2)\,; \qquad (6)$$

this difference is effectively zero (to first order in β) if a α the value proposed by Fresnel.

- Airy experiment (1871). The aim of this experiment is to demonstrate the partial entrainment of the aether by the Earth's motion, by observing a difference in the angle of astronomical aberration, depending on whether the telescope is empty or full of water. A word of explanation about aberration is useful. We know that it's the opening angle of a small apparent annual circular motion of the so-called fixed stars, around a mean position. This apparent motion is easily understood by the following velocity composition diagram (Fig.3):

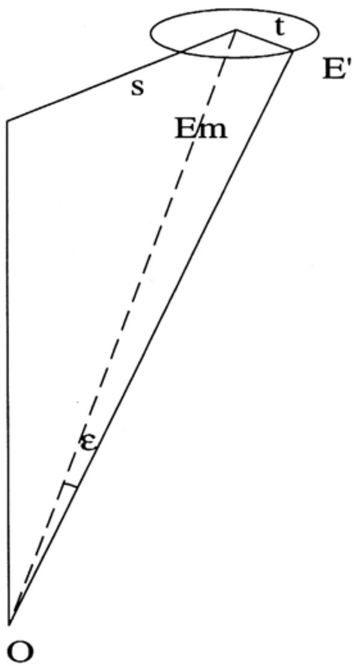

Fig. 3: Velocity composition diagram for aberration.

- Let's imagine an observer O observing the fixed star E, and let's carry the vector OE of length c, the speed of light, in the direction from O to E. This is the speed of light;
- if the Sun is moving at speed s relative to this star, an observer at rest relative to the Sun will see the star in the OEm direction;
- as the Earth itself performs an annual revolution around the Sun, the image is still displaced at each instant by the vector t of the Earth's instantaneous speed relative to the Sun; the terrestrial observer therefore sees the star in the direction OE', which annually describes a small cone around the "mean" direction OEm;
- the angle of aberration is the angle ε of aperture of this cone.

To estimate the angle ε, we assume that the relative speeds of the celestial bodies are very small compared to the speed of light (this is of course the case for the Earth/Sun relative speed, t ≅30 km/s, and we assume the same for the Sun/star relative speed, s << c). Under these conditions, we have: OE ≅ OEm ≅ OE' , and $\varepsilon \cong t/c \cong 10^{-4}$. If we now use a telescope full of water to observe the star, the angle measured is obviously the angle after refraction in the telescope , from which we reconstruct the angle of incident aberration by Descartes' law (for such small angles, i = n r); but this angle of refraction itself corresponds to an aberration coefficient determined by new velocities: c/n instead of c , and (1- α) t instead of t, to account for the partial entrainment of the ether by the water in the terrestrial telescope. This gives us a new value for the angle of aberration:

$$\varepsilon' = n\frac{(1 - \alpha)t}{(c/n)} = n^2 (1 - \alpha)\ \varepsilon. \qquad (7)$$

The experiment gives $\varepsilon' \approx \varepsilon$, which shows that the entrainment coefficient has α a value very close to that proposed by Fresnel.

These experiments have given Fresnel's entrainment coefficient (often called Fizeau's coefficient, because of its first experimental determination) the status of an experimental truth that we have to live with anyway. In the remainder of this talk, I'll show how it conditioned the long march towards relativity, following the efforts of two of the world's leading physicists.

major participants in this collective effort:

- H.A. Lorentz (1853-1928), Doctor of Physics in 1875, with a thesis on a microscopic dynamic theory of the reflection and refraction of light.(3)
- J.H. Poincaré (1854-1912), Polytechnicien and Doctor of Mathematics in 1875, with a thesis on partial differential equations(4) **.**

2. Lorentz's electrodynamics, from 1875 to 1904.

The central idea of Lorentz's theory is that matter is ultimately nothing more than an empty medium (or, more accurately, ether at absolute rest) in which electrified particles, called "ions" and later "electrons", move, creating and being subjected to electromagnetic fields(5) Lorentz can thus provide a qualitative and often even quantitative dynamic explanation of most phenomena concerning the interaction of matter and light for matter considered "at rest" in the ether: emission, absorption, refractive index, dispersion, light scattering, .. etc.(6) .

Lorentz's model of a perpetually quiescent aether seems likely to conflict with Fresnel's partially entrained aether. Lorentz therefore sought to justify the existence of an "apparent" Fresnel entrainment. In 1886, he succeeded in justifying the Fresnel coefficient within the framework of his model; he even improved the Fresnel formula by introducing a dispersion correction due to the Doppler effect:

$$\alpha = 1 - \frac{1}{n^2} - \frac{1}{n}\frac{dn}{d\lambda} ; \qquad (8)$$

this Doppler correction will be demonstrated experimentally by

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(3) "Over der terugkaatsing en breking van het licht", Universiteit Leiden.

(4) " On the integration of partial differential equations with any number of unknowns", University of Paris.

(5) The basic formulas of Maxwell's and Lorentz's theories are given in Appendix 2.

(6) A notable exception is, of course, the photoelectric effect, which requires t h e introduction of Planck's quantum of action (Einstein 1905).

Zeeman in 1911. Lorentz's reasoning is presented in Appendix 1, following Fresnel's (1b, 1d).

Lorentz thus arrives at an important conclusion: it seems that the appearance of a drive, given by the Fresnel coefficient, makes it impossible to demonstrate the motion of material bodies in relation to the ether, at least to first order in the velocity ratio $\beta = v/c$. We would therefore have to turn to experiments demonstrating effects of order β^2. This is precisely the case with A. Michelson (1881), which we will now consider (Fig.4).

A monochromatic beam is split in two by a semi-transparent mirror m, and these two beams travel back and forth along two different paths of equal length L, one (//) parallel to the movement of the earth in the ether, the other ($\perp$) perpendicular to this movement. We hope to demonstrate the movement of the apparatus in the ether by the displacement of the interference bangs when the whole apparatus is rotated 90º around m.

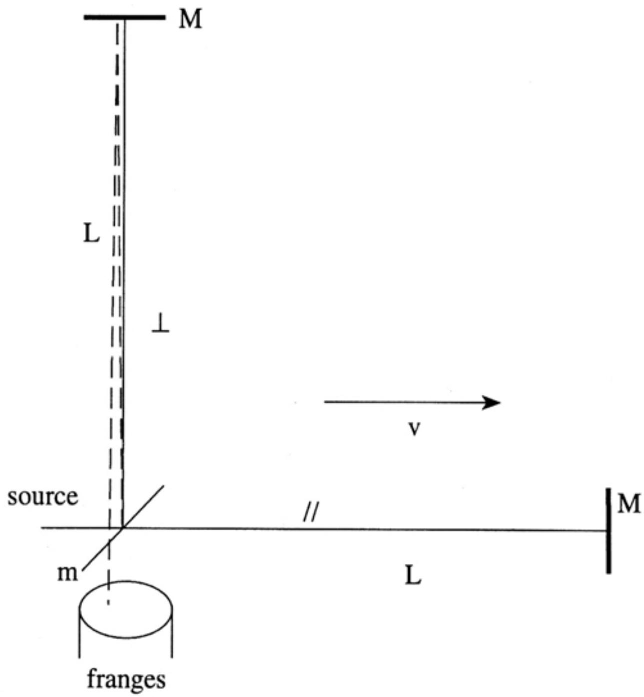

Fig.4: Diagram of the Michelson experiment.

We do have journey times:

$$t_{//} = L\left[\frac{1}{c-v} + \frac{1}{c+v}\right] = 2\frac{L}{c}\frac{1}{1-\beta^2} \ , \tag{9}$$

$$t_{\perp} = \frac{2}{c}\sqrt{L^2 + (v\,t_{\perp})^2} = 2\frac{L}{c}\frac{1}{\sqrt{1-\beta^2}} \ , \tag{10}$$

i.e. a difference in travel time:

$$\Delta t = t_{//} - t_{\perp} = \frac{2L}{c}\left(\frac{1}{1-\beta^2} - \frac{1}{\sqrt{1-\beta^2}}\right) \approx \frac{L}{c}\beta^2 + O(\beta^4)\ . \quad (11)$$

Michelson's 1881 experiment showed no displacement of the bangs. Michelson concludes that the ether is totally entrained, in accordance with Stokes' theory, and in contradiction with Fresnel's theory (ether partially entrained), and also with Lorentz's alternative theory (ether not entrained + dynamic effect giving the appearance of partial entrainment). Lorentz reacted only to point out a minor error in his reasoning, which, by reducing Michelson's expected effect by half[(7)] , rendered the experiment inconclusive because it was at the limit of experimental error. He therefore maintained his view of an untrained ether. But a few years later, the experiment having become much more precise thanks to a considerable lengthening of the light paths (A. Michelson and E. Morley, 1887), and the experimental result remaining negative, Lorentz had to find a solution. He came up with the astonishing proposal of a contraction of the longitudinal arm caused by the ether wind[(8)] ! In fact, the ether wind would cause a contraction in the direction of motion of all material bodies, making the arm contraction undetectable by direct measurement (since all bodies are affected in the same way). The contraction of the arm can only be demonstrated indirectly, by the negative result of Michelson and Morley's optical experiment. The ad hoc contraction must then be :

$$\delta L = L_{\perp} - L_{//} \approx \frac{1}{2} L\,\beta^2\ . \quad (12)$$

This is a typical example of interpreting an experimental result for the sole purpose of saving a theory.

In fact, Lorentz is becoming aware of the need to change something about his

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[(7)] Michelson had not taken into account the displacement of the equipment during the outward journey.
return of light on the perpendicular arm, giving $t_{\perp} = 2\ L/c$ and therefore $\Delta t = 2\ (L/c)\ \beta^2$.

[(8)] The same proposition was made simultaneously and quite independently by G.F. FitzGerald (1892).

electrodynamics. In an important 1895 paper entitled "Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern" [Lo-95], he introduced two concepts which were to prove essential for the subsequent history of the birth of special relativity:

- local time:

$$t' = t - \frac{v\,x}{c^2} \tag{13}$$

to ensure Dalembertian invariance (to terms of order β^2 after a Galilean transformation of velocity v along x ;

- the corresponding states, i.e., an ad-hoc transformation of the electric and magnetic fields so as to ensure the identity of electro-optical phenomena (always within terms of order β^2) for bodies moving in the ether.

If necessary, the theory is completed by Lorentz-FitzGerald contraction corrections. This ensures that the Earth's motion in the ether cannot be demonstrated up to second order.

These early Lorentz transformations and the modifications introduced in 1904 are described in Appendix 3.

The year 1896 saw the triumph of Lorentz's microscopic electrodynamics, with his calculation of the Zeeman effect and Zeeman's remarkable experimental verification of details predicted by the theory (line polarization under different observational circumstances). Remember that this is a "classical" electrodynamics approach based on the perturbation of atomic electron motion by a constant, homogeneous magnetic field. The theory contains only one parameter, the ratio e/m of charge to electron mass(9) , and Zeeman's experimental work actually measured this ratio (in magnitude and sign) a year before J.J. Thomson's determination (Cf. [Lo-02], [Ro-65]). Lorentz and Zeeman

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(9) It was an extraordinary stroke of luck that the first Zeeman observations concerned "normal" Zeeman lines, for which the action quantum h disappears from quantum calculations, allowing a classical approach. Subsequently, Lorentz tried in vain to understand the "anomalous" Zeeman effect within the framework of his model. The "anomalous" Zeeman effect only became comprehensible after the introduction of the electron' s spin and magnetic moment (Uhlenbeck and Goudsmit, 1925).

would see their work rewarded with the 1902 Nobel Prize in Physics.

Following Poincaré's criticisms and Larmor's work, around 1900 Lorentz gave "the little push" needed to obtain the covariance of electro-optical phenomena, including terms in β^2 . Finally, in 1904, he published "Electromagnetic phenomena in a system moving with any velocity less than that of light" [Lo-04], which can be considered the crowning achievement of his work on the electrodynamics of moving bodies. It includes (see Appendix 3):

- the correct Lorentz transformation;
- the corresponding state theorem for all orders in v/c;
- his formulation of electron dynamics, in which the electron, whose mass would be of purely electromagnetic origin, is deformed by its motion in the ether (Lorentz-FitzGerald contraction). Lorentz's inertia of the electron is manifested by two "masses"(10) varying differently with speed:

- transverse mass: $$m_\perp = \frac{m_0}{(1-\beta^2)^{1/2}}, \qquad (14)$$

- longitudinal mass: $$m_{//} = \frac{m_0}{(1-\beta^2)^{3/2}} \qquad (15)$$

These results were confirmed the following year by the relativistic dynamics of the electron founded by H. Poincaré.

To conclude this brief overview of Lorentz's work, we can state that:

- Lorentzian electrodynamics lies at the very heart of our modern relativistic and quantum concepts. It is the (classical) prototype of a microscopic approach to matter-light interaction phenomena, to be succeeded by quantum electrodynamics (see, for example, Heitler's book,

(10) To avoid confusion with our modern (relativistic) ideas, let's remember that mass is here the coefficient of acceleration (i.e. the second derivative of position with respect to time) in an equation of motion whose second member is the Lorentz force.

[He-54]). It revealed the need to correct the Galilean transformation formulas, and even, after much trial and error, to obtain a true relativistic covariance.

- however, Lorentz cannot be considered the first of the "relativists". Not only because his thinking rests firmly on a conception of the ether as a privileged medium, but above all, because his basic transformation remains Galileo's transformation, which he then arranges with his theorem of corresponding states. In so doing, this theorem comes back to the old idea of "saving appearances". Once appearances had been saved, Lorentz did not bother to reconsider the role of the ether.

This revision of the ether's role was made by Poincaré and Einstein, each in his own way:

- Poincaré retained the idea of the existence of an ether as an active medium, the seat of electromagnetic phenomena. But he discovered the group structure of Lorentz transformations and skilfully used it to reduce the ether to being kinematically just one of an infinite number of equivalent inertial reference frames. In this way, he eliminates the ether "de facto", removing any possibility of studying its specific properties through electro-optical phenomena [Po-05], [Po-06].
- Einstein, much more radical, eliminates the ether "de jure" by constructing relativistic kinematics without any reference to absolute space [Ei-05].

3. J. H. Poincaré physicist.

Henri Poincaré is without doubt one of the greatest mathematicians of the late 19th century. His immense mathematical output covers all the classical fields of mathematics: arithmetic, geometry, algebra, analysis, differential and partial differential equations, analysis situs (topology). Importantly, Poincaré was one of the founding masters of continuous group theory (1884 - 1901), and between 1899 and 1901, he devoted two large memoirs to the general exposition of this theory. Poincaré was also a great mechanic: analytical mechanics, continuum mechanics, celestial mechanics (King of Sweden Prize for his revolutionary approach to the 3-body problem in 1889). All these mathematical tools would be of great importance in his approach to the principle of relativity.

We often forget these days that Poincaré was also a great physicist. He is the founding father of mathematical physics, which aims to put all the rigor of mathematical reasoning at the service of physics. If we refer to the corresponding part of the analysis of his scientific work, which he wrote in 1901, we find for the period 1887-1901 (Cf. [Po-01]):

- 18 dissertations (1887-1892) on differential equations in mathematical physics;
- 9 memoirs (1890-1894) on Hertzian waves;
- 36 memoirs (1889-1901) on the criticism of physical theories;
- numerous (printed) lectures on mathematical physics, often containing a critical review of current theories (Cf. in particular, his "Théorie mathématique de la lumière" of 1889, and his "Électricité et Optique" of 1890, which presents and discusses the theories of electromagnetism and electro-optics, according to Maxwell's "successors").

In 1895, Poincaré published under the general title "À propos de la théorie de M. Larmor", a series of four articles (published between April and November 1895 in L'Éclairage électrique), 57 pages in all [Po-95]. These are reflections (and calculations) on the theories of electro-optics, i.e., on the adaptation of the "mechanical" theories of optics by Fresnel, Neumann and Mac Cullagh, to a "Maxwellian" vision by Larmor, Helmholtz, Lorentz, J.-J. Thomson, and Hertz. Poincaré proposes three criteria for these attempts at adaptation to constitute an acceptable theory. At the very least, they must:

1) to account for Fizeau's drive coefficient;
2) guarantee the conservation of electricity and magnetism;
3) guarantee the validity of the principle: action = reaction.

He notes that none of the proposed theories satisfies all three criteria simultaneously: for example, Hertz's theory satisfies criteria 2) and 3).

but not the first; Lorentz's theory satisfies criteria 1) and 2) but not the third; ... etc. And Poincaré states his "provisional conclusions":

"(...). We must therefore give up trying to develop a perfectly satisfactory theory and provisionally stick to the least defective of all, which appears to be Lorentz's (...). It seems to me very difficult to admit that the principle of reaction is violated, even in appearance, and that it is no longer true if we consider only the actions undergone by weightable matter and if we leave aside the reaction of this matter on the ether. At some point, therefore, we will have to modify our ideas in some important respect, and break the framework into which we seek to fit both optical and electrical phenomena. But even if we confine ourselves to optical phenomena proper, what we have said so far to explain the partial entrainment of waves is not satisfactory. Experience has revealed a host of facts which can be summed up in the following formula: it is impossible to make manifest the absolute motion of matter, or better still the relative motion of matter in relation to the ether; all that can be demonstrated is the motion of weightable matter in relation to weightable matter...(...)."

From then on, Poincaré followed Lorentz's work very closely. In an article entitled "La théorie de Lorentz et le principe de réaction" (Lorentz's theory and the principle of reaction), published in 1900 in a tribute to the twenty-fifth anniversary of Lorentz's thesis [Po-00], he did not hesitate to return to the difficulty pointed out and proposed a solution: if we consider electromagnetic energy as a fictitious fluid endowed with inertia(11) , there is conservation of momentum in emission and absorption, at least to first order in β(12) . But the compensation is not simple:

"... For compensation to take place, phenomena must be related, not to true time t, but to a certain *local time* t' defined as follows (...).", and Poincaré explains that "local Lorentz time" (formula 13) corresponds to the synchronization of clocks at a distance by

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(11) In our current notation, if we denote the electric and magnetic fields of the electromagnetic wave by E and H and the Maxwellian energy density by ρ: $\rho = (E^2+H^2)/8u$, then the inertial mass density proposed by Poincaré is $\mu = \rho/c^2$.

(12) But not to higher orders, "... unless we make a certain complementary hypothesis that I won't discuss for the moment." (this refers to Lorentz's "nudges" and in particular to the Lorentz-FitzGerald contraction).

the exchange of light signals, under the illusion that the speed of light is the same in both directions, irrespective of movement relative to the ether. In fact, he invented the idea of synchronizing the clocks of distant observers by exchanging light signals(13) . Poincaré doesn't go into detail about how this was achieved, but it's easy to reconstruct his reasoning. I'll devote Appendix 4 to this question of local Lorentz-Poincaré time.

Keeping a close eye on the evolution of Lorentz's ideas, Poincaré slowly moved towards a fundamental revision of the Galilean principle of relative motion, while maintaining the idea of the existence of an electro-optical ether. He spoke about this at the St. Louis Conference (USA, 1904), where he stated for the first time the "Principle of Relativity" [Po-04]: the laws of physical phenomena are the same for a fixed observer and for an observer driven in uniform translational motion (in other words,... relative to the ether). After the St-Louis Conference, he wrote to Lorentz(14) to point out that, in their last form, the Lorentz transformations form a group and that, if the second parameter f(15) of these transformations must depend only on the first (i.e. the relative velocity of the reference frames), this second parameter is necessarily equal to one. This letter contains explicitly, but without comment, the relativistic formula for the addition of velocities. Clearly, Poincaré was on the verge of creating a new dynamic based on the principle of relativity he had enunciated at the Saint-Louis Conference.

It may come as a surprise to some that the third major player in the creation of special relativity, Albert Einstein, has not been given a prominent place in this lecture, the only one whose name has gone down in history (at least, the history told to our students). This was because the talk was devoted to the long march towards new ideas, and Einstein, born in 1879, clearly had no opportunity to take part. Brought up to speed by the gigantic efforts of his predecessors, he easily climbed to the final summit, bringing with him the new idea of relativistic kinematics,

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(13) The problem of setting remote clocks was already discussed by Poincaré in 1898, in an article entitled "La mesure du temps" [Po-98].

(14) Letter discovered fairly recently by Miller (Cf. Mi-81); the date is uncertain, but somewhere between autumn 1904 and spring 1905.

(15) This parameter is designated by the letter l by Lorentz and Poincaré; I write f to avoid typographical confusion with the number 1 in the calculations presented in the Appendices.

while sherpas Lorentz and Poincaré (who reached the summit at the same time as Einstein) persisted in building an essentially dynamic approach. There has often been debate about the importance of the respective roles played by Lorentz, Poincaré and Einstein in the construction of special relativity. Most authors agree in attributing it to Einstein alone, with preparatory but not yet relativistic work by Lorentz and Poincaré (Cf. for example: [To-71], [Mi-81], [Pa-82]). For an alternative view, with a detailed comparison of Einstein's and Poincaré's relativistic constructions in 1905, see references [Pi-99] and [Re-02].

4- Appendices.

Appendix1-Fresnel-Fizeau drag coefficient. 1a) according to Fresnel (1822).

The basic idea was Young's: the refractive index reflects the "concentration" of ether in matter. Fresnel clarified the hypothesis by admitting that the density of ether is proportional to the square of the refractive index. If ρ_0 and ρ_1 are the ether densities, respectively in vacuum and in a transparent substance of index n, then we have:

$$\rho_1 = \rho_0 n^2 . \tag{16}$$

If the transparent body has a velocity v relative to the aether, Fresnel considers that <u>only the excess aether is entrained</u>, so that the aether's center of gravity moves at speed:

$$w = \frac{\rho_1 - \rho_0}{\rho_1} v = \frac{n^2 - 1}{n^2} v \tag{17}$$

This speed must be added to (or subtracted from) the speed c/n of light in the transparent body at rest:

$$V = \frac{c}{n} \pm \left(1 - \frac{1}{n^2}\right) v . \tag{18}$$

1b) according to Lorentz (1886).

Light propagation in a dielectric is described b y Maxwell's equations (without charge or current):

$$\text{div } \vec{D} = 0 \qquad \text{div } \vec{B} = 0$$

$$\text{rot } \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = 0 \qquad \text{rot } \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \qquad (19)$$

completed by the constituent relations:

$$\vec{B} = \vec{H} \quad \text{(magnetic permeability } \mu = 1) \qquad (20)$$

$$\vec{D} = \varepsilon\vec{E} = \vec{E} + 4\pi\vec{P} \quad \text{(dielectric constant } \varepsilon = n^2,\ n = \text{refractive index)} \qquad (21)$$

Lorentz hypothesis:

- the constitutive relationship (21) is to be considered as follows:

$$4\pi\vec{P} = (\varepsilon - 1)\vec{E} = (n^2 - 1).\ \text{[force exerted on the load unit]}.$$

- if the dielectric is in motion, the force exerted on the charge unit is (Lorentz force):

$$\vec{E} \rightarrow \vec{E}' = \vec{E} + \frac{\vec{v}}{c} \times \vec{H}, \qquad (22)$$

and the polarization should be replaced by:

$$4\pi\vec{P} = (\varepsilon - 1)\vec{E}' = (n^2 - 1)\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}\right). \qquad (23)$$

Let's consider a dielectric moving relative to the ether at speed v along the z axis, and let's study a plane wave propagating in this dielectric along the same z direction:

$$\vec{E} = \vec{E}_{1x} \exp\left[i(kz - \omega t)\right]. \qquad (24)$$

By substituting this solution into Maxwell's equations, we can easily calculate the wave's propagation speed,

$$V = \omega / k , \tag{25}$$

and we find, according to the different cases and dynamic hypotheses:

- if n=1 (no medium polarization) : $V = \pm c$, (26)

- if n>1 and the dielectric at rest in the ether: $V = \pm c/n$, (27)

- if n>1 and the dielectric has a velocity v relative to the ether:

a) according to Maxwell : $4u\vec{P} = (n^2 - 1)\vec{E}$

$$\Rightarrow V = \pm \frac{c}{n} + \frac{1}{2}(1 - \frac{1}{n^2})v + O(c\beta^2) ; \tag{28}$$

b) according to Lorentz: $4u\vec{P} = (n^2 - 1)(\vec{E} + \frac{\vec{v}}{c} \times \vec{H})$

$$\Rightarrow V = \pm \frac{c}{n} + (1 - \frac{1}{n^2})v + O(c\beta^2) ; \tag{29}$$

Note that this dynamic calculation leads to an "appearance" of ether entrainment. Polarization current" plays an essential role here, and the "relativistic" correction of the Lorentz force is required to obtain the "correct" Fizeau entrainment coefficient.

1c) according to von Laue (1907).

The Fresnel coefficient results from a simple application of the Einsteinian relativistic rule of velocity composition: that of light in the dielectric when the latter is considered at rest (c/n), and that of the moving dielectric (v):

$$V = \frac{\pm c/n + v}{1 \pm \frac{(c/n)v}{c^2}} = \pm\frac{c}{n} + \left(1 - \frac{1}{n^2}\right)v + O(c\beta^2). \qquad (30)$$

The ether has disappeared; speeds are those measured in the inertial laboratory where we work.

1d) dispersion correction (Lorentz 1886).

For Fresnel, this correction is non-existent; we would have to imagine that each frequency corresponds to a different density of ether.

For Lorentz, and also for von Laue, it's a Doppler correction that needs to be made when the refractive index is dispersive. Indeed, the frequency perceived by the observer ω_0 differs from the frequency in the moving body ω_1 due to the Doppler effect:

$$\omega_0 = \omega_1\left(1 \pm \frac{v}{c}\right), \qquad (31)$$

(exactly for Lorentz, to order β^2 for von Laue). The result is that the index used in the above formulas must be corrected:

$$n = n(\omega_1) = n(\omega_0) + \left(\frac{dn}{d\omega}\right)(\omega_1 - \omega_0); \qquad (32)$$

the correction is of order β and must therefore be made only on the first term c/n :

$$\frac{c}{n} \to \frac{c}{n} + v\frac{\omega}{nn}\frac{dn}{d\omega} = \frac{c}{n} - v\frac{1}{nn}\frac{dn}{d\lambda}. \qquad (33)$$

This is Lorentz's dispersive correction, verified by Zeeman in 1911.

Appendix 2- Maxwell's and Lorentz's theories.

Maxwell: a 4-field theory (electric field E, magnetic field H, electric displacement D and magnetic induction B), with given external electricity sources (electricity density ρ and current density j). The differential equations are supplemented by phenomenological constitutive equations between the fields.

- Maxwell's equations :

$$\operatorname{div} \vec{D} = 4\pi\rho, \qquad \operatorname{div} \vec{B} = 0, \tag{34}$$

$$\operatorname{rot} \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{j}, \qquad \operatorname{rot} \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0, \tag{35}$$

- constitutive equations:

$$\vec{B} = \mu \vec{H} \quad \text{(magnetic permeability } \mu\text{)} \tag{35a}$$

$$\vec{D} = \varepsilon \vec{E} = \vec{E} + 4\pi \vec{P} \quad \text{(dielectric constant } \varepsilon\text{)} \tag{35b}$$

Lorentz: this is electrodynamics with 2 microscopic fields (electric field e, magnetic field h) and charged particles. These particles participate in the sources (if necessary, known external sources can be added, as in Maxwell), but they are also dynamically subjected to a force F created by the fields. The constitutive equations are those of vacuum, i.e., with Lorentz units: $\varepsilon_0 = 1$, $\mu_0 = 1$.

$$\operatorname{div} \vec{e} = 4\pi \sum_{i=1}^{N} \rho_i, \qquad \operatorname{div} \vec{h} = 0, \tag{36}$$

$$\operatorname{rot} \vec{h} - \frac{1}{c}\frac{\partial \vec{e}}{\partial t} = \frac{4\pi}{c}\sum_{i=1}^{N} \rho_i \vec{u}_i \qquad \operatorname{rot} \vec{e} + \frac{1}{c}\frac{\partial \vec{h}}{\partial t} = 0,$$

$$\vec{F}_i = \rho_i \left(\vec{e}(i) + \frac{\vec{u}_i}{c} \times \vec{h}(i) \right). \tag{37}$$

In formula (37) giving the Lorentz force exerted on the particle i, the notation e(i), h(i) represents the electric and magnetic fields at the location of particle i at the moment in question.

Lorentz's aim was to derive macroscopic properties from these microscopic equations.

Appendix 3 - The Lorentz transformation and corresponding states.

1) We go from the motionless aether to another frame of reference in uniform rectilinear motion (MRU) by a Galilean transformation:

- contact details:

$$
\begin{aligned}
x &\rightarrow x_r = x - vt, \\
y &\rightarrow y_r = y, \\
z &\rightarrow z_r = z,
\end{aligned}
\qquad (38)
$$

$$t \rightarrow t_r = t \,;$$

- speeds add up like Galileo's;
- fields are transformed scalarly, but the partial derivative with respect to time becomes the "material derivative" of continuous media:

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - v\frac{\partial}{\partial x} \,. \qquad (39)$$

2) A change in the kinematic variables and a change in the electromagnetic state are effected; this is what Lorentz calls "passing to the corresponding states".

<u>Change in kinematic variables:</u>

$$
\begin{aligned}
x' &= f\gamma\, x_r &&= f\gamma\,(x - vt), \\
y' &= f\, y_r &&= f\,y, \\
z' &= f\, z_r &&= f\,z, \\
t' &= f\, t_r/\gamma - f\gamma\, v\, x_r / c^2 &&= f\gamma\,(t - v\,x / c^2),
\end{aligned}
\qquad (40)
$$

with, in his 1895 work (Versuch) :

$$f = 1 \text{ and } \gamma = 1 , \qquad (41)$$

and in his final work of 1904:

$$\text{any } f \text{ and } \gamma = (1 - \beta^2)^{-1/2} . \qquad (42)$$

The transition from variables (x,y,z,t) to variables (x',y',z',t') is the actual Lorentz transformation when we choose (42) with f = 1. Note that, for Lorentz, this transformation results from a rather complex approach: the basic transformation remains the Galilean transformation, and we then "save appearances" by an ad hoc transformation that constitutes the kinematic part of the corresponding state theorem.

Lorentz uses a highly complex dynamic reasoning to show that, within the framework of his electron theory, where the parameter f depends only on the velocity v (principle of relative motion), then this parameter necessarily has the value 1. Poincaré wrote to him (late 1904 or early 1905?) to point out that this result results more simply from the requirement that the transformations (40) and (42), restricted to the single parameter v by the hypothesis f = f(v), continue to form a group.

t' is a "local time" in the following sense: an observer A located in a x-coordinate point x_A in the ether, but in motion because linked to the MRU system, shifts its time by the indicated amount, whatever the event it is considering. As early as 1900, Poincaré showed that local Lorentz time was the natural result of adjusting the clocks in the MRU system, under the illusion that light propagates with the same speed in all directions (see Appendix 4).

Change of electromagnetic state :

For fields: (43)

$$E'_x = \frac{1}{f^2} E_x \qquad E'_y = \frac{\gamma}{f^2}(E_y - \beta H_z) \qquad E'_z = \frac{\gamma}{f^2}(E_z + \beta H_y)$$

$$H'_x = \frac{1}{f^2} H_x \qquad H'_y = \frac{\gamma}{f^2}(H_y + \beta E_z) \qquad H'_z = \frac{\gamma}{f^2}(H_z - \beta E_y)$$

and for charge (ρ) and current (ρu) densities: (44)

$$\rho' = \frac{1}{\gamma f^3}\rho, \qquad u'_x = \gamma^2 (u_x - v), \qquad u'_y = \gamma u_y, \qquad u'_z = \gamma u_z .$$

These corresponding states ensure, according to Lorentz, the identity of the description of electromagnetic phenomena in both systems (to the first order in β in the "Versuch ..." of 1895 and to all orders in β in the final work of 1904). In reality, there is still a small error in the transformation of charge and current densities. This point was eventually corrected by Poincaré [Po-05, Po-06] and Einstein [Ei-05], who were thus the first to correctly write down the transformation of Maxwell's dynamics.

Appendix 4- Local time according to Poincaré.

In his tribute to Lorentz published in 1900 [Po-00], Poincaré explains how the principle of relative motion (to first order in v/c) can be rescued in Lorentz's 1895 theory, thanks to certain term compensations, and writes the following sentence:

"For compensation to take place, phenomena must be related, not to true time t, but to a certain *local time* t' defined as follows. I suppose that observers placed at different points, set their watches by means of luminous signals; that they try to correct these signals for the time of transmission, but that ignoring the translational movement of which they are animated and believing consequently that the signals are transmitted equally fast in both directions, they limit themselves to crossing observations by sending a signal from A to B, then another from B to A. The local time t' is the time marked by the watches set in this way.

If then V = 1/ € $_{K0}$ is the speed of light, and v is the translation of the earth, which I assume to be parallel to the positive x axis, we have:

$$t' = t - vx / V^2 ."$$

In 1904, at the St. Louis Conference, he used essentially the same formulation [Po-04], with the difference that the principle of relative motion was then saved at all orders in v/c by Lorentz's very recent work [Lo-04], which ad-hoc introduces the Lorentz-FitzGerald contraction into his transformations, so Poincaré presents this contraction as an additional hypothesis(16) . The local Lorentz time is then (see formulas 40 and 42):

$$t' = f\gamma\ (t - vx / c^2), \text{ any f and } \gamma = (1 - \beta)^{2-1/2} . \quad (45)$$

In his 1904 paper [Lo-04], Lorentz establishes (rather painfully) that, in his theory of the electron, the scaling factor f that must be considered dependent on v (principle of relative motion) reduces to unity.

Poincaré left no calculations showing how he arrived at his interpretation of local Lorentz time. The first person to show explicitly by calculation the link between the times used by observers in relative motion was Einstein [Ei-05]. He thus explained the method of setting clocks by exchanging light signals. In his presentation of local time, Poincaré insists on the need to cross signals, which Einstein does by using the instantaneous return to A (by reflection) of the signal received by B. This is indeed the simplest method for crossing signals, because we can then easily control the instants of sending and receiving, both in the moving frame of reference and in the ether. I used it in a study on the question of "Poincaré's third hypothesis" (Cf. ref. Re-00). But signals can just as easily be crossed without immediate reflection. So let's see how Poincaré must have reasoned.

In 1900, Poincaré came across the following formulas from Lorentz, which link a frame of reference at rest in the ether, where the "true time" of mechanics is used (frame of reference x, t), to another frame of reference in uniform motion at speed v relative to the first, where local time is used (frame of reference

(16) Some have severely criticized this "third hypothesis" of Poincaré's, without taking into account either the time when it was first formulated, or the simplifications that it is sometimes good to make in presentations aimed at a public not specialized in these questions (Cf. for example [Pa-82]). I believe I have done justice to these criticisms in my article [Re-00]. See also the rest of this Appendix.

x', t')[17]

$$x' = x - v\,t, \qquad (46\text{ a})$$

$$t' = t - vx\,/\,c^2. \qquad (46\text{ b})$$

Formula 46a is clearly the Galilean formula of uniform translation, with the conventions of coinciding coordinate system origins at time zero and the use of the same units in both reference frames. Formula 46b defines the local Lorentz time, i.e. the time t'(x') required by an observer whose coordinate is x' in the moving reference frame, so that electro-optical phenomena appear to him as if he were at rest in the ether, at least to first order in v/c. Here, too, we apply the conventions of coincidence of origins and the same running of clocks.

In 1904, Lorentz introduced an ad-hoc factor into his formulas (see formulas 40 and 42), this time with the aim of achieving invariance of electro-optical phenomena at all orders in v/c. In Galilean formula 47a, it is clear that this factor corresponds to a change of length scale in the moving frame of reference: if a rigid bar of length L in the moving frame of reference appears contracted (or dilated) by a factor g in the frame of reference at rest, i.e. if its measurement there is gL, we must effectively write the Galilean transformation in the form:

$$x' = g^{-1}\,(x - vt). \qquad (47\text{ a})$$

Now we need to understand how the process of synchronizing clocks by means of crossed optical signals will lead Poincaré to time.

-1

(46b) in 1900 and at local time (47b) with the same g factor in 1904:

$$t' = g^{-1}\,(t - vx\,/\,c^2). \qquad (47\text{ b})$$

---

[17] The other two space coordinates y and z are omitted here for simplicity of discussion. I'll come back to these coordinates later.

Formulas 46 and 47 define coordinate changes $(x, t) \rightarrow (x', t')$ that are linear (corresponding to the homogeneity of space and time in the two reference frames), and homogeneous (thanks to the origin conventions). Formulas 46 can be considered as a special case of 47 when lengths are not modified by motion relative to the ether (g = 1). We therefore need to show how synchronization by cross-exchange of optical signals determines the coefficients a and b of the linear form:

$$t' = at + bx, \tag{48}$$

and leads to answer 47b. Recall that the synchronization rule defined by Poincaré can be translated as follows:

- observers A and B are linked to the moving frame of reference, whose speed in the ether they do not know. A can be placed at the origin of the coordinates and B at the point with x' coordinates$_B$ = L; distance AB is is equal to L in the moving frame of reference, and is equal to gL in the ether frame of reference.

- when A's watch indicates time $t'_A = 0$, A sends a signal to B, and asks B to adjust his watch to receive the signal. over time $t'_B = L/c$; (having no idea of its movement relative to the ether, A believes in good faith that the speed of light in his frame of reference is c);

- but as a precaution, the two observers are going to carry out a check by the reverse operation: when B's watch indicates the time $t'_B = t'_0$, B sends an optical signal to A, and invites A to check that his watch indicates time $t'_A = t'_0 + L/c$ on signal reception. Watches are then synchronized, as if the inertial frame of reference were at rest in the ether.

We'll see that these two operations define two equations that allow us to calculate the coefficients a and b of transformation 48. The result is the transition from an a priori synchronized chronology in the ether (true time t) to a synchronized chronology in the frame of reference in motion relative to the ether, under the illusion that this system is at rest, since the speed of light is the same in both directions.

The signal from A reaches B after a path of length D_1, estimated at in the ether frame of reference, is equal to the distance AB (i.e., gL) increased by the displacement of the moving frame of reference vt_1, where t_1 is the time taken by light to make this journey:

$$D_1 = gL + vt_1 = ct_1 . \qquad (49)$$

This gives us the instant t_1:

$$t_1 = \frac{gL}{c - v}, \qquad (50)$$

and the $x_B(t_1)$ coordinate of B when it receives the signal from A (in fact, $x_B(t_1) = D_1$):

$$x_B(t_1) = gL \frac{c}{c - v}. \qquad (51)$$

B's watch then marks time $t'_1 = L/c$. By replacing these given in equation 48, we obtain an initial relationship for determining the coefficients a and b:

$$\frac{L}{c} = (a + b\,c) \frac{gL}{c - v}. \qquad (52)$$

For the crossed signal, we need to proceed in the same way. At local time t'_0 of B corresponds to the true ether time t_0 given by (48):

$$t'_0 = a\,t_0 + b\,(gL + v\,t_0). \qquad (53)$$

The light now travels against the direction of motion of the moving frame of reference, and the travel time in real time is (see equation 50):

$$t_2 - t_0 = \frac{gL}{c + v}. \qquad (54)$$

At true time t_2 of the arrival of the signal at A, it has the coordinate vt_2 in the ether frame of reference, and his watch must mark the local time $t'_0 + L/c$. So we have a second equation:

$$t'_0 + \frac{L}{c} = a\,t_2 + b\,v\,t_2, \tag{55}$$

which becomes by replacement according to (53) and (54):

$$\frac{L}{c} = (a - b\,c)\frac{gL}{c + v}. \tag{56}$$

Equations 52 and 56 determine the coefficients a and b of the local synchronization time in the moving frame of reference:

$$t' = \frac{1}{g}(t - vx / c^2). \tag{57}$$

Equations 46, 47 and 57 show that Poincaré was right when, in 1900 and without the contraction hypothesis (i.e. with g = 1), he associated local time (46b) with Galileo's transformation of space (46a). And he was right again when, in 1904 and with the contraction hypothesis, he introduced the same factor $g = \gamma^{-1} = (1 - v^2/c^2)^{-1/2}$ in local time 47b and in the Lorentz 47a space transformation. But this demonstration also shows that, at this stage, the contraction parameter g is arbitrary. We therefore need to introduce a new element to remove this ambiguity.

We know that the ambiguity can be removed when we also consider the transformation of y and z coordinates, perpendicular to motion. Einstein [Ei-05] shows that these coordinates are transformed by a factor f(v)[18] and that what we call g is equal to $\phi(v)(1-\beta^2)^{-1/2}$.
Using the relativistic principle, which allows the roles of the two frames of reference to be reversed, Einstein demonstrated that $\phi(v)\,\phi(-v) = 1$. He then proceeded to synchronize the optical signals with immediate feedback for two observers, A and B, located respectively at the origin and at a point of reference.

---

[18] In fact, this is Lorentz's factor f (eq. 42); in the Einsteinian approach, it can only depend on v by virtue of the principle of relative motion.

coordinate y' = L on the moving axis y'. It then follows, by symmetry, that ϕ(v) is necessarily an even function of v. Hence, ϕ(v) = 1. Hence, ϕ(v) = 1. Clearly, consideration of the other two coordinates and the relativistic principle of reversing the role of the two reference frames (there's no ether in Einstein 1905!) demonstrates that the contraction factor of Lorentz g can only be $(1-\beta^2)^{1/2}$.

The ambiguity is also removed by Poincaré [Po-05], who notes that Lorentz transformations (eqs. 40 and 42) form a group(19) , of which simple transformations (along x and t only) on the one hand, and space rotations on the other, are subgroups (subgroup of boosts, and subgroup of rotations). Making skilful use of these subgroups, Poincaré demonstrated that Lorentz's factor f, when constrained to depend only on the relative velocity v (principle of relative motion!), is an even function of v, and ultimately necessarily has the value 1.

It is possible to come very close to this conclusion simply by considering the transformations according to x and t. If we impose that these transformations form a group(20) (boosts group), and that the contraction factor g depends only on the relative velocity of the reference frames, we obtain an important restriction on the explicit form of g. Let's show that if frame of reference (x' , t') is linked to the ether frame of reference by a velocity transformation v_1, and if frame of reference (x'' , t'') is also linked to the ether frame of reference by a velocity transformation v_2, then the two moving reference frames are linked by a velocity transformation v_3. The calculation is a simple substitution in the formulas; it gives effectively:

$$x''(t',x') = g_3^{-1}\,(x' - v_3 t'), \qquad \text{(53a)}$$

$$t''(t',x') = g_3^{-1}\,(t' - v_3 x' / c^2), \qquad \text{(58b)}$$

with:

(19) Today referred to as LΠ+ .

(20) This is tantamount to denying the ether any privileged kinematic role. Any observer bound to an inertial frame of reference can rightly believe himself to be at rest in the ether.

- the well-known rule of relativistic velocity composition:

$$v_3 = \frac{v_1 - v_2}{1 - (v_1 v_2)/c^2}; \tag{59}$$

- a composition law for the parameter g :

$$g_3 = \frac{g_2}{g_1}\frac{1 - (v_1/c)^2}{1 - (v_1 v_2)/c^2}, \tag{60}$$

which should lead to its value being set as a function of speed.

Thus, to satisfy the relativistic principle "à la Poincaré" (... everyone can, quite rightly, believe themselves to be at rest in the ether...), the contraction coefficient g must satisfy the equation with two independent variables x and y :

$$g\left(\frac{y - x}{1 - xy/c^2}\right) = \frac{g(y)}{g(x)}\frac{1 - (x/c)^2}{1 - xy/c^2}. \tag{61}$$

For y = x , we naturally find g(0) = 1; then taking y = x + ε , where ε is infinitely small, and expanding equation 61 to first order in ε, we obtain the differential equation:

$$g'(x) = g(x)\frac{g'(0) - x/c^2}{1 - (x/c)^2}, \tag{62}$$

which contains the arbitrary parameter g'(0) . The solution corresponding to the initial condition g(0) = 1 is :

$$g(x) = \sqrt{1 - (x/c)^2}\left[\frac{1 + x/c}{1 - x/c}\right]^{cg'(0)/2}. \tag{63}$$

We can see that the requirement that the transformations (x, t) form a group ("boosts" group) is not enough to completely determine the contraction factor g (v). We still need an additional "little push", even if this push reduces to a very weakened postulate: either that g(v) is an even function of v (as in Poincaré's letter to Lorentz; footnote 14), or even (even more weakly) that the parameter g'(0) is zero. In Poincaré's approach prior to his great 1905 paper, the Lorentz contraction factor therefore represents an independent postulate.

Bibliography.

[Ei-05] A . Einstein, *"Zur Elektrodynamik bewegter Körper"* , Annalen der Physik **17** (1905) 891-921.

[He-54] W. Heitler, *"Quantum theory of radiation"*, Oxford, 3rd edition, (1954).

[La-07] M. von Laue, "Die Mitführung des Lichtes durch bewegter Körper nach dem Relativitätsprinzip", Annalen der Physik, 23 (1907) 989-990.

[Lo-95] H.A. Lorentz, "Versuch einer r Theeloerkiteris cdheen und optischen Erscheinungen in bewegten Körpern", Leiden: E.J. Brill (1895).Reprint in Collected Papers **5** , 1-137.

[Lo-02] H.A. Lorentz, "The theory of electrons and the propagation of light", Nobel Lecture, Dec. 1902. Reprint in "Nobel Lectures: Physics", Elsevier (1967).

[Lo-04] H.A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity less than that of light", Proc. R. Acad. Amsterdam **6** (1904) 809; Reprint in Collected Papers **5** , 172-197.

[Lo-09] H.A. Lorentz, "The theory of electrons and its applications to the phenomena of light and radiant heat", Teubner, Leipzig (1909); reprint by Dover (1952).

[Mi-81] A.I. Miller, "Albert Einstein's special theory of relativity. Emergence (1905) and early interpretation(1905-1911)" Addison-Wesley Pub. Cy., New-York, (1981).

[Pa-82] A. Pais, "Subtle is the Lord... The science and life of Albert Einstein", Oxford Univ. Press, (1982).

[Pi-99] Y. Pierseaux, "La " structure fine " de la théorie de la relativité restreinte", Thèse de Doctorat, Université Libre de Bruxelles, (1998); L'Harmattan, Paris, (1999).

[Po-95] H. Poincaré, "À propos de la théorie de M. Larmor", L'éclairage électrique (1895). Reprinted in "Oeuvre" Tome IX, 369-426.

[Po-98] H . Poincaré, "La mesure du temps", Rev. de Métaphysique et de Morale **6** (1898) 1.

[Po-00] H. Poincaré, "La théorie de Lorentz et le principe de réaction", Archives Néerlandaises des Sciences Exactes et Naturelles, 2nd series, **5** (1900) 252-278. Reproduced in "Oeuvre" Tome IX, 464-488.

[Po-01] H. Poincaré, "Analyse de ses travaux scientifiques (1901)", Acta Mathematica, **38** (1921) 116-125. Reprinted in "Oeuvre" Tome IX, 1-14 .

[Po-02]. Poincaré, "La Science et l'Hypothèse", Flammarion (1902).

[Po-04] H . Poincaré, "Les principes de la physique-mathématique", Conférence de

St-Louis (1904). Reprint in "Physics for a New Century", Am. Inst. of Phys. 281-299.

[Po-05] H. Poincaré, "Sur la dynamique de l'électron", Comptes Rendus de l'Acad. des Sc. **14** (1905)1504-1508. Reprinted in "Oeuvre" Tome IX, 489-493.

[Po-06] H. Poincaré, "Sur la dynamique de l'électron", Rendiconti del Circolo matematico di Palermo **21** (1906) 129-176. Reproduced in "Oeuvre" Tome IX, 494-586.

[Re-00] J. Reignier, "À propos de la naissance de la relativité restreinte. Une suggestion concernant la troisième hypothèse de H.Poincaré", Bull. Acad. Roy. Belgique, Cl des Sc. 6º série , Tome XI, (2000) 63-77.

[Re-02] J. Reignier, "The birth of special relativity. An alternative account", Communication and Cognition, (2002), forthcoming.

[Ro-65] L. Rosenfeld, "Theory of electrons" , Dover Publ. NY(1965).

[Sh-63] R.S. Shankland, "Conversations with A. Einstein", Am. J. Phys. **31** (1963) 47-57.

[To-71] M.A. Tonnelat, "Histoire du principe de relativité", Flammarion, Paris, (1971).

[Wh-51] E.T. Whittaker, "A History of Aether and Electricity", (2 Vol. 1951/1953), reprinted by The American Institute of Physics (1987).