

# Dynamics of the core, geodynamo

Paul H. Roberts

Institute of Geophysics and Planetary Physics, University of California, Los Angeles

## 1. Introduction

"The mechanism for generating the geomagnetic field remains one of the central unsolved problems in geoscience." So states the report on the *National Geomagnetic Initiative* (NGI) prepared by the *U.S. Geodynamics Committee, et al.* [1993], with advice from the NGI Workshop held in Washington D.C. in March 1992. All analyses of the geomagnetic data point to the core as containing the source of the field and "The basic premise that virtually everyone accepts is that the Earth's magnetism is created by a self-sustaining dynamo driven by fluid motions in Earth's core" (NGI, p.135). Dynamical questions at once arise, such as "What is the energy source driving those motions?" *Jacobs* [1953] proposed that the solid inner core (SIC) is the result of the freezing of the fluid outer core (FOC). *Verhoogen* [1961] noticed that the release of latent heat at the inner core boundary (ICB) during freezing would help drive thermal convection in the FOC, and *Braginsky* [1963] pointed out that the release of the light alloying elements during fractionation at the ICB would provide compositional buoyancy. These two sources suffice to supply the geodynamo with energy throughout geological time, even in the absence of dissolved radioactivity in the core [*Braginsky and Roberts*, 1994a; *Kuang et al.*, 1994]. *Stevenson* [1991] argues that potential differences on the core-mantle boundary (CMB) of electrochemical origin may be partially responsible for the geomagnetic field.

Whatever the driving mechanism, it is clear that the magnetohydrodynamics (MHD) of the core must be understood. This has proved to be a challenging task; progress has been slow. The directional property of the magnetic compass needle demonstrates that Coriolis forces play an essential role. Because the molecular diffusivities of heat and composition are so small, these sources of buoyancy must instead be transported across the core by turbulence. The phenomena of interest arise from slight deviations in the FOC from a well-mixed adiabatic state, and theory must consistently disentangle these from the large "background." In short, it is far from obvious what equations best describe large scale core MHD [*Braginsky and Roberts*, 1994a].

We shall concentrate below on the research of US scientists, even when it was carried out abroad. We shall add the work of foreign scientists who have long

standing affiliations with US Institutions. US theoreticians have played a major role in elucidating fast dynamos, which amplify fields on the same time scales as the flows. In contrast, slow dynamos act on a diffusive timescale, based on the magnetic diffusivity,  $\eta$ , of the conductor. Interestingly, *McFadden and Merrill* [1993] have recently derived  $\eta \sim 1 \text{ m}^2 \text{ s}^{-1}$  from the paleomagnetic data, but we shall take  $\eta = 3 \text{ m}^2 \text{ s}^{-1}$ . The diffusive time scale of the core is therefore about  $10^4$  years, and the geodynamo problem is to understand how the geomagnetic field is maintained over times that are substantially longer than this. Fast dynamo theory has no obvious contributions to make, and will therefore not be considered here. The core is a slow dynamo.

## 2. Core Dynamics (Microscale)

There has been increasing interest in the role played by small-scale motions in transporting and mixing thermal and chemical inhomogeneities in the core. When the heavy constituents (mainly Fe) of core fluid freeze onto the ICB, latent heat and the light constituents are released. *Moffatt* [1989] proposed that this light, hot fluid would congregate into "blobs" at the ICB that would, when large enough, break away from the ICB and rise through the core, stirring it as it did so, and possibly retaining their identity until they reach the CMB, where some may remain to form a light layer (see §4 below). This idea has been pursued by *Ruan and Loper* [1993], *Loper and Moffatt* [1993] and *Moffatt and Loper* [1994]; see also *Bush et al.* [1992, 1994], *Loper et al.* [1994]

*Braginsky and Meytlis* [1990] argued that core turbulence is totally unlike classical turbulence of Kolmogoroff type, in which energy 'cascades' from large to small eddies. Nor is it related to classical MHD turbulence, where a reverse cascade may create large-scale magnetic fields by turbulent dynamo action, possibly through a turbulent  $\alpha$ -effect. [The  $\alpha$ -effect is the creation of a mean electromotive force (emf) parallel to the mean magnetic field.] They argue that the microscale fields are so tiny that they produce neither a significant turbulent  $\alpha$ -effect, nor an enhancement in the mean field diffusivity. Nevertheless, the turbulent diffusivities of the mean thermal and chemical inhomogeneities are enormously greater than their molecular counterparts, at least in some directions. Because of the strong influence of Coriolis and Lorentz forces on motions of all scales, the turbulence is highly anisotropic, forming 'plate-like' eddies that have their long dimensions parallel to the rotation axis ( $Oz$ ) and to the mainly zonal ( $\phi$ -)direction of the prevailing toroidal field; the short

Copyright 1995 by the American Geophysical Union.

Paper number 95RG00735.  
8755-1209/95/95RG-00735\$15.00

dimension is in the  $s$ -direction, away from the rotation axis. Turbulent diffusion is represented by one tensor, the same for both heat and composition. The elements of that tensor corresponding to diffusion in the  $z$  and  $\phi$  directions are large, of the same order as the molecular magnetic diffusivity,  $\eta$ ; turbulent diffusion in the  $s$ -direction is comparatively weak. Braginsky–Meytlis theory has recently been taken further by *Braginsky and Roberts* [1994a]; it still contains *ad hoc* elements.

It is the highly dispersive character of rotating fluids that led to this unusual picture of core turbulence. The effect of that dispersion on the Moffatt mechanism has recently been studied by *St. Pierre* [1994a, b] who argues, on the basis of computer simulations, that a blob released at the ICB will be stretched, laminated in plates, and absorbed into its surroundings before it can rise far into the FOC; see also *St. Pierre and Roberts* [1994]. It is difficult in the laboratory to mimic core conditions in which the effective diffusivities of heat and composition are (see above) the same. Experiments have however been performed by *Cardin and Olson* [1992].

### 3. Core Dynamics (Macroscale)

Nearly all existing models of the geodynamo are axisymmetric. Since the axisymmetric part of the geomagnetic field cannot, according to Cowling's theorem, be self-maintained, the emf created by the non-axisymmetric components of fluid flow and magnetic field must be retained. For simplicity, it is usually parameterized by an  $\alpha$ -effect. Since turbulent induction is unimportant (see §2), the  $\alpha$ -effect is created by asymmetric waves/instabilities of global scale. Recent work on axisymmetric geodynamo models is reported in §5 below; here we describe studies of waves/instabilities.

Viewed from the inertial frame, a contained rotating fluid is filled with vortex lines parallel to the rotation axis that impart the "elasticity" that inertial waves require. In a non-rotating electrically conducting fluid, the "elasticity" of the lines of force of the prevailing magnetic field is responsible for Alfvén waves. In a rapidly rotating conductor, the Alfvén and inertial waves are replaced by 'fast' and 'slow' waves. The former resemble the inertial waves and have a timescale of the order of a day; the latter act on timescales of order  $\tau_s = 2\Omega\mu_0\rho R^2/B_\phi^2$ , where  $\Omega \approx 7 \times 10^{-5} \text{ s}^{-1}$  is the angular velocity of Earth,  $\mu_0 \approx 4\pi \times 10^{-7} \text{ H m}^{-1}$  is the magnetic permeability,  $\rho \approx 10^4 \text{ kg m}^{-3}$  is the core density,  $R \approx 3.5 \times 10^6 \text{ m}$  is the radius of the core and  $B_\phi$  is a characteristic strength of the (zonal) field. If  $B_\phi \approx 30 \text{ mT}$ ,  $\tau_s \approx 750 \text{ years}$ , which is similar to timescales observed in the secular variation.

Slow waves are sometimes called 'MC waves', because the primary dynamical balance is between Magnetic and Coriolis forces. In some circumstances MC waves become MC instabilities; these are much studied in the hope of deriving constraints on the structure and strength of the field in the core. It has also been argued that MC instabilities play a significant role in the po-

larity reversal mechanism. Some MC instabilities are of short time scale, of the order of  $10^3$  years. These are the so-called ideal instabilities, where "ideal" refers to the fact that, unlike the so-called resistive instabilities, they do not rely on the finite resistivity of the fluid. Ideal and resistive instabilities are the counterparts of similar instabilities that arise in laboratory plasmas but, because of the importance of Coriolis forces in the core, they evolve there on longer time scales (see above). *London* [1992a, b] examined MC waves and instabilities, supposing that the prevailing magnetic field,  $B$ , is zonal and proportional in strength to distance,  $s$ , from the rotation axis,  $Oz$ . Assuming a geostrophic dynamical balance of the kind used in atmospheric dynamics, he developed a uniform approximation for waves that have a short wavelength in the  $s$ -direction, i.e. away from the rotation axis. He showed [*London*, 1992b] that these waves propagate in a westward direction. In a later work [*London*, 1993], he generalized to other zonal fields; see also *London* [1994].

Stable field configurations may become unstable when the fluid is top heavy, and there is much interest therefore in MAC waves and instabilities, where the added 'A' stands for 'Archimedean' (i.e. buoyancy) forces. Simple examples of MC and MAC instabilities have been analyzed by *Kuang and Roberts* [1991, 1992], *Lan, Kuang and Roberts* [1993]. *Fearn and Kuang* [1994] and *Kuang* [1994] stress the importance of the conductivity of the boundaries on the instabilities.

*Bergman and Madden* [1993] studied core convection, paying particular attention to the steady mean poloidal circulation in the core, for which they argued equatorial upwelling would occur. Such a circulation has a profound effect on the functioning of an  $\alpha\omega$ -dynamo; see §5. Compositional buoyancy is important for driving core convection in the considerations of *Kuang et al.* [1994], and *Bergman et al.* [1994]. The surface of the inner core is probably constitutionally supercooled, so that a mushy layer exists there. If no magnetic field is present, chimneys form in such a layer through which the light fluid, released during fractionation inside the layer, is ejected into the FOC. *Bergman et al.* investigate how this mechanism is affected by the prevailing magnetic field.

For simplicity, many investigations of core dynamics and the geodynamo ignore the SIC, by assuming that the entire core is fluid. At first sight, this unrealism seems not too serious: the SIC is only 4% of the volume of the core and 5% of its mass. Nevertheless, the SIC may have a disproportionate effect on core flows and field generation. It has long been known [*Stewartson*, 1966] that, because of the rapid rotation of Earth, differential rotation between inner core and mantle, in a non-magnetic core, exerts a profound influence on the dynamics of the FOC. *Ruzmaikin* [1993] and *Hollerbach* [1993] pointed out that the same is likely to be true in corresponding MHD situations. The dynamics of the FOC has a different character inside and outside the tangent cylinder (TC), that is the circular cylinder drawn around the rotation axis and tangent to the SIC at its equator. As *Stewartson* showed, the TC is itself

surrounded by a thin “shear layer” in which the flows inside and outside the TC adjust themselves to one another. *Hollerbach* [1993] showed how an axisymmetric magnetic field alters the structure of this layer. *Hollerbach and Proctor* [1993] observed that the significance of the TC and its adjustment layer may be even greater for the asymmetric fields; see also *Hollerbach* [1994]. *Glatzmaier and Olson* [1993] studied non-magnetic convection in a rotating sphere and showed that the amplitude of the convective motions is greatest outside the TC; see also *Cardin and Olson* [1994a]. In contrast, for the corresponding MHD situation, where a zonal magnetic field was imposed, *Olson and Glatzmaier* [1993, 1994] and *Glatzmaier and Olson* [1994] found that the convection was strongest inside the TC, the Taylor columns outside that cylinder being suppressed by the Lorentz force; see also *Jones et al.* [1994], *Cardin and Olson* [1994b] and §5 below.

#### 4. Effects of the Mantle on Core MHD

The core is only one component of the coupled core-mantle system. Each component profoundly affects, and is affected by, the other. Strictly, the core cannot be considered in isolation from the mantle but, when it is, the mantle is replaced by a set of conditions on the CMB. The resulting theoretical simplification is enormous, but sometimes is an over-simplification. In particular, to suppose that the form, and the physical state, of the CMB are uniform in space and unvarying in time is simplistic. *Larson and Olson* [1991] argue that variations in the convective regime in the mantle, and in particular the changing configuration of mantle plumes, control the rate of geomagnetic field reversals. It has recently been realized that lateral variations in the temperature of the CMB will have a strong effect on core motions and therefore on core-mantle coupling; see *Sun et al.* [1994]. A new type of geodynamo is also possible; see §5. In this section we shall ignore lateral variations on the CMB, apart from topography.

The assumption (§1) of an adiabatic well-mixed core becomes suspect near the CMB, and several authors have argued that a layer of comparatively light fluid exists adjacent to the CMB. *Braginsky* [1993] has christened this “the hidden ocean of the core” and has argued that waves propagating in this stable layer may be partially responsible for the short period geomagnetic secular variation. It may also strongly affect core-mantle coupling, particularly topographic coupling. Waves in a stratified layer at the top of the core have been studied by *Bergman* [1993], who developed a theory of magnetic Rossby waves based on a generalization of Laplace’s tidal equation in which the Lorentz force is included and the induction equation is added. He also developed  $\beta$ -plane solutions analytically and showed that the magnetic field can release equatorially trapped Rossby waves.

*Love and Bloxham* [1994a] have recently investigated a new idea which may lead to the abandonment of magnetic core-mantle coupling in comparison with other,

and especially topographic coupling, mechanisms. Their *reductio ad absurdum* argument is based on an inverse problem: assuming that core-mantle coupling is electromagnetic, they seek the time-varying toroidal field,  $B_T(R)$ , at the CMB that creates an electromagnetic torque that best fits the length of day data. They make three demands which they find cannot be simultaneously met: (a)  $B_T(R)$  does not exceed the upper limit provided by dynamo theory (see also *Levy and Pearce* [1991] who argue that  $B_T(R)$  is less than 10mT, and is probably less than 1mT), (b) the poloidal electric currents which generate that toroidal field and which leak into the mantle do not exceed bounds on the electric field inferred from measurements at Earth’s surface [*Lanzerotti et al.*, 1992, 1993, 1994], (c) the ohmic dissipation in the mantle caused by those currents does not exceed the heat flux from the Earth. They conclude that magnetic stresses cannot be the main factor in core-mantle coupling. Their treatment of flux diffusion in the analysis leading to their conclusion merits further study. *Love and Bloxham* [1994b] have recently proposed a second application of their idea.

Diffusion of flux plays an important role in the study of *Braginsky and Le Mouél* [1993], who are particularly interested in the inductive effects of high shears in a “ $\Delta$ -layer” at the top of the core. *Kuang and Bloxham* [1993] analyze how magnetic field in the upper core affects topographic core-mantle coupling. They find that the field may change the strength of the coupling by several orders of magnitude but, for parameters appropriate to the core, the stress is of order  $10^{-1} \text{N m}^{-2}$ , which is adequate to account for the decadal variations in Earth’s rotation. Angular momentum exchange between core and mantle is also discussed by *Bloxham and Kuang* [1994].

Malkus has long urged that precessional driving of the core is an important and, possibly, the dominant source of energy for core motions and the geodynamo; e.g. see *Malkus* [1994]. Interest in this idea has been revitalized by the discovery that flows with elliptical streamlines, somewhat similar to flows driven by the luni-solar precession, are unstable. Malkus has provided experimental demonstrations of the instability in an elliptically distorted cylinder of fluid. Experiments have also been performed by *Vanyo* [1991], *Vanyo et al.* [1992, 1994b] and *Wilde and Vanyo* [1994]; see also *Vanyo et al.* [1994a], *Vanyo and Lods* [1994]. So far, all studies have been non-magnetic, but it is hoped that they will provide stepping stones to the corresponding MHD situations. The  $\alpha$ -effect created by precessionally-driven flows has already been estimated by *Barenghi et al.* [1994]. The effects of the SIC on the forced nutation of the Earth have been studied theoretically, and the results have been compared with observational data by *Mathews et al.* [1991a, b] and *Herring et al.* [1991]. Cognate issues are analyzed by *de Vries and Wahr* [1991].

The exchange of the  $z$ -component of angular momentum between core and mantle is accomplished via geostrophic motions in the core; these are zonal flows

that depend only on distance  $s$  from the rotation axis,  $Oz$ , and on time  $t$ . By analyzing the field extrapolated downwards to the CMB, *Jault and Le Mouél* [1988] estimated the geostrophic flow in the recent past and could therefore monitor the angular momentum of the core as a function of time. They showed that its variations are roughly equal but opposite to those of the angular momentum of the mantle over the same period, as determined by changes in the length of day; the net angular momentum of the core–mantle system as a whole is constant. *Jackson et al.* [1993] have developed this theme and have used their analysis of the core geostrophic flow to predict, with encouraging results, variations in the length of day.

## 5. Geodynamo Modeling

Dynamo models that solve the induction equation alone are called ‘kinematic’, and several such geodynamos models have been integrated. The main challenge today is to solve the fully dynamic dynamo problem, sometimes also called ‘the MHD dynamo problem’ or ‘the fully self-consistent dynamo problem’, in which the induction equation is solved *and* the equation of motion for the fluid. This nonlinear problem raises formidable difficulties. Because of Cowling’s theorem, a true MHD dynamo model should be 3D, but a supercomputer is then required to integrate it numerically. Axisymmetric (2D) models can be solved on workstations, but the magnetic field will decay to zero unless the emf generated by the omitted asymmetric fields and flows is reintroduced in some way, through an  $\alpha$ -effect.

Axisymmetric  $\alpha$ -effect models are of two extreme types,  $\alpha^2$ - and  $\alpha\omega$ -dynamos, together with a range of  $\alpha^2\omega$ -models between them. In an  $\alpha^2$ -model, zonal field creates meridional field and *vice versa*; in an  $\alpha\omega$ -model, the  $\alpha$ -effect creates meridional field from zonal field, but the zonal field is created by an  $\omega$ -effect, i.e. by the inductive effects of zonal shearing motions. Sometimes  $\alpha\omega$ -models are called “strong field dynamos” since the zonal field, which is locked inside the conductor, is large compared with the observed meridional field, in contrast to the “weak field” dynamos of  $\alpha^2$ -type where the strength,  $B_M$ , of the meridional field is characteristic of the strength of the entire field. A strong field dynamo functions only if the product of  $\alpha$  and  $\omega$ , as measured by the so-called “dynamo number”,  $D$ , exceeds in magnitude a certain marginal value,  $D_m$ ; for an  $\alpha^2$ -model to function the  $\alpha$ -effect magnetic Reynolds number,  $R_\alpha$ , a dimensionless measure of  $\alpha$ , must be large enough. While  $\alpha^2$ -dynamos are usually steady,  $\alpha\omega$ -dynamos tend to be oscillatory, but they too may become steady, and more efficient (as judged by a smaller value of  $D_m$ ), when a sufficiently strong meridional flow is present. Meridional flow is produced by Lorentz forces or by core–mantle coupling; see also *Bergman and Madden* [1993].

Zonal shearing motion is comparatively easily excited in rotating fluids, for example by pole–equator temperature differences; the  $\omega$ -effect and the zonal field may

therefore be strong. It used to be said that an appeal to an invisible zonal field is a return to armchair science, but galaxies are transparent to observation. Their fields are predominantly toroidal and seem to be produced by an  $\alpha\omega$ -mechanism; see §7 of *Krause et al.* [1993]. It is in principle possible to detect a zonal field in Earth’s core through the electric fields it creates outside the core, in particular the potential difference between the two ends of a trans-oceanic cable [*Lanzerotti et al.*, 1992, 1993, 1994]. In practice, the obscuring, long period, inductive effects of ocean currents have so far prevented a convincing demonstration [*Runcorn and Winch*, 1991].

The axisymmetric force balance in an MHD dynamo is not easily understood. Many studies of 2D “intermediate” dynamo models have been launched to elucidate it. These are so named because, while they do not address the full MHD problem, they take a step beyond kinematic models. An  $\alpha$ -source is invoked to maintain the axisymmetric field. Whether an  $\alpha^2$ -,  $\alpha\omega$ - or  $\alpha^2\omega$ -dynamo results depends on the dynamical balance. In an  $\alpha^2$ -dynamo the primary balance is geostrophic, i.e. between Coriolis and pressure forces; the magnetic field strength,  $B$ , is determined by a secondary balance, e.g. between the Lorentz and viscous forces, which gives  $B \sim (\mu_0\rho\nu\eta)^{1/2}/R$ , where  $\nu$  is core viscosity. In a strong field dynamo, the magnetic field plays a role in the primary balance and therefore  $\bar{B} = \sqrt{(B_\phi B_M)} \sim (2\Omega\eta\mu_0\rho)^{1/2} \sim 2.4\text{mT}$ , a relation confirmed by *Benton* [1992]. *Benton* argued further that a typical zonal flow velocity would be  $(2\Omega\eta^2/R)^{1/3} \sim 7 \times 10^{-4}\text{m s}^{-1}$ , which is similar to the speed with which some features of the geomagnetic field drift westward.

*St. Pierre* [1993b] demonstrated that, when a strong field branch exists, weak field solutions are likely to be nonlinearly unstable. Although his plane layer model is geometrically too simple to represent the geodynamo, it is a convective MHD dynamo of similar physical type. It clearly demonstrates the existence of a strong field branch, one that also operates subcritically, i.e. at smaller thermal forcing than that at which kinematic dynamo action is first possible. The model of *St. Pierre* is fully 3D, as is the spherical 3D model of *Glatzmaier and Roberts* [1994] described below.

The axisymmetric force balance in intermediate models is so dominated by magnetic and Coriolis forces that inertial forces are often omitted. One of two extreme scenarios arise, or perhaps some intermediate scenario. At one extreme is the model- $Z$  state [*Braginsky*, 1975, 1991, 1994] which relies on the coupling of core to mantle and in which the geostrophic motions in the core are large. At the other extreme is the Taylor state [*Taylor*, 1963] in which core–mantle coupling is insignificant, but in which a certain integral demand (the Taylor constraint) must be satisfied. Sometimes models of either type can exist under the same conditions of excitation. Model- $Z$  is energetically the more expensive to run, because of core–mantle friction, and the external fields it produces therefore tend to be smaller than in the corresponding Taylor-type model. There are there-

fore two contenders for the geodynamo, a strong field (model- $Z$ ) mechanism and a very strong field (Taylor-type) mechanism. In trying to decide between these, it is usually supposed for simplicity that core-mantle coupling is viscous — it is the existence of this coupling rather than its precise nature that is significant. *St. Pierre* [1993a] has examined the stability of Taylor states.

This then is the background against which much of the recent work on intermediate geodynamos may be viewed. *Hollerbach and Ierley* [1991] analyzed an intermediate dynamo of  $\alpha^2$ -type and showed that, as  $R_\alpha$  exceeds its marginal value,  $R_{\alpha m}$ , the solution is at first viscously controlled. As  $R_\alpha$  is further increased, a second critical value,  $R_{\alpha T}$ , is reached at which Taylor states appear. When *Hollerbach et al.* [1992] carried out a parallel study for an  $\alpha\omega$ -model, they uncovered a more complex situation. Despite very simple choices of  $\alpha$  and  $\omega$ , they found that, as the dynamo number  $D$  increases beyond  $D_m$ , the solution is at first viscously controlled but that, as  $D$  increases through a second critical value  $D_z$ , oscillations arise in which the Taylor balance is struck during part of each cycle but in which viscous coupling is essential during the remainder.

*Braginsky and Roberts* [1994b] continued earlier investigations of one particular model. They observed a transition from Taylor-type behavior to model- $Z$ -type behavior as  $D$  increases. An  $\alpha\omega$ -dynamo model integrated by *Glatzmaier and Roberts* [1993] developed an interesting bifurcation as  $D$  was increased. Against the background of an approximately steady dipole component, an oscillatory quadrupole field causes the meridional field lines to bunch up alternately in one hemisphere and the other. The role of the dipole and quadrupole families of solutions of the geodynamo equation in geomagnetic field reversals is an oft recurring theme, and was raised again in a novel way by *Hoffman* [1991]. Questions of parity coupling in  $\alpha^2$ -models also arose in the work of *Hollerbach* [1991].

In integrating a kinematic  $\alpha\omega$ -geodynamo model, *Braginsky* [1964] found that the fields induced outside the TC differed substantially from those generated inside it. Dynamic (intermediate) models have recently been studied by *Hollerbach and Jones* [1993a]. They find that most dynamo action takes place outside the TC, a conclusion that may depend on their choices of  $\alpha$  and  $\omega$  since it was not confirmed by the recent 3D integrations of *Glatzmaier and Roberts* [1994]. The model of *Hollerbach and Jones* was used to benchmark that of *Glatzmaier and Roberts* [1993]; the agreement was nearly perfect. *Hollerbach and Jones* [1993b, 1994] argued that the SIC plays a potent role in the reversal mechanism; its electromagnetic inertia diminishes chaos in the FOC. *Glatzmaier and Roberts* [1994] agreed. The effects of conducting boundaries were investigated by *Hirsching and Busse* [1993].

Although the emphasis of the subject has moved towards MHD models, kinematic geodynamos are still being profitably studied. In particular, *Hagee and Olson* [1991] have suggested an interesting connection between the observed secular variation and certain types

of anisotropic  $\alpha\omega$ -models; see also *Kono and Roberts* [1994]. A general method for solving weak field MHD models when conditions for kinematic dynamo action are only marginally exceeded, has been explored by *Kono and Roberts* [1991, 1992].

Lateral variations in the temperature of the CMB bring about concomitant changes in the electrical conductivity of the lower mantle so that new current paths are allowed and old ones forbidden. The axisymmetry assumed in most geodynamo modeling is destroyed and with it the applicability of Cowling's theorem [*Busse, 1992*]. A zonal shear can readily create zonal magnetic field from meridional field through the  $\omega$ -effect, but it is incapable, in an axisymmetric system, of creating meridional field from zonal field. This, however, is no longer true when longitudinal inhomogeneities destroy the axial symmetry. A zonal shear can then produce zonal field from meridional field and vice versa. This fact enabled *Busse and Wicht* [1991] and *Wicht and Busse* [1993] to construct new, simple models of dynamo action that make use of the broken symmetry and which work through zonal shear alone.

An unusual approach to the geodynamo problem was initiated by *Ruzmaikin et al.* [1993]. They divide the fluid domain into fixed cells, each of which randomly amplifies or destroys field by dynamo action; nonlinearity, diffusion and correlations between cells are then added. An initially smooth field becomes intermittent, the field concentrating mainly in a few cells, the location of which changes with time, a phenomenon they liken to the motion of geomagnetic field anomalies. *Roberts* [1992] introduced a "mapping method" that has been successfully tested against axisymmetric [*Nakajima and Roberts, 1994a*] and asymmetric [*Nakajima and Roberts, 1994b*] dynamo models; see also *Nakajima et al.* [1993a].

The resources of the *NSF Pittsburgh Supercomputing Center* were enlisted to generate the first 3D time-dependent, fully self-consistent numerical solution of the MHD equations that describes thermal convection and magnetic field generation in a low-viscosity rapidly-rotating spherical shell with a solid conducting inner core. The resulting solution, reported by *Glatzmaier and Roberts* [1994], serves as a crude simulation of the geodynamo, crude because because the truncation was too severe and because geophysically realistic values for some parameters were not numerically accessible (e.g.  $\nu$  was several orders of magnitude too large, though still apparently not very influential). The heat flux from the core was taken to be  $4 \times 10^{13}$  W and the integration was continued over approximately three magnetic diffusion times, during which the field showed no signs of disappearing. Field generation takes place mainly within and near the TC. The pattern and amplitude of the radial magnetic field at the CMB is qualitatively similar to that of the Earth. The toroidal field energy is rather larger than the poloidal field energy but the maximum amplitudes attained by the two fields are comparable ( $\sim 0.05$  T); the maximum fluid velocity is of order  $4 \times 10^{-3}$  m s $^{-1}$ . An irregular exchange of field between hemispheres takes place, similar to that found

in the 2D model of Glatzmaier and Roberts [1993]. Its timescale is about 10% of the magnetic diffusion time of the FOC. Excitingly, the dynamo sometimes reverses its polarity spontaneously. Preliminary to doing so, the poloidal field in the SIC has to reverse; if it does not do so, the reversal is aborted (as in geomagnetic excursions). It is hard not to be excited by such similarities between the computed model and the real Earth. Perhaps the answer to the challenging sentence that opened this review is at last within sight?

## References

- Barenghi, C.F., R. Hollerbach and R.R. Kerswell, Alpha-effects produced by precessionally induced flows, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 3: Core Dynamics and Thermodynamics, 1994.
- Benton, E.R., Hydromagnetic scale analysis at Earth's core-mantle boundary, *Geophys. Astrophys. Fluid Dynam.*, **67**, 259-272, 1992.
- Bergman, I., Magnetic Rossby waves in a stably stratified layer near the surface of the Earth's inner core, *Geophys. Astrophys. Fluid Dynam.*, **68**, 151-176, 1993.
- Bergman, M.I., D.R. Fearn, P. Shannon and J. Bloxham, The effect of a magnetic field on chimney convection in a solidifying, metallic alloy melt, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 3: Core Dynamics and Thermodynamics, 1994.
- Bergman, M.I., and T.R. Madden, Steady, finite amplitude, rotating magnetoconvection and the mean poloidal circulation of the Earth's core, *Phys. Earth planet. Int.*, **77**, 267-284, 1993.
- Bloxham, J., and W. Kuang, Core-mantle coupling and the exchange of angular momentum, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 2: Core Mantle Interactions and the CMB, 1994.
- Braginsky, S.I., Structure of the F-layer and reasons for convection in the Earth's core, *Doklady Akad. Nauk SSSR*, **149**, 1311-1314, 1963.
- Braginsky, S.I., Kinematic models of the Earth's hydromagnetic dynamo, *Geomagn. & Aeron.*, **4**, 572-583, 1964.
- Braginsky, S.I., Nearly axisymmetric model of the hydromagnetic dynamo of the Earth, *Geomagn. & Aeron.*, **15**, 122-128, 1975.
- Braginsky, S.I., Towards a realistic theory of the geodynamo, *Geophys. Astrophys. Fluid Dynam.*, **60**, 89-134, 1991.
- Braginsky, S.I., MAC oscillations and the hidden ocean of the core, *J. Geomag. Geoelec.*, **45**, 1517-1538, 1993.
- Braginsky, S.I., The nonlinear dynamo and model-Z, in *Lectures on Solar and Planetary Dynamos*, edited by M.R.E. Proctor and A.D. Gilbert, pp. 267-304, University Press, Cambridge U.K., 1994.
- Braginsky, S.I., and J.L. Le Mouél, Two-scale model of a geomagnetic field variation, *Geophys. J. Int.*, **112**, 147-158, 1993.
- Braginsky, S.I., and V.P. Meytlis, Local turbulence in the Earth's core, *Geophys. Astrophys. Fluid Dynam.*, **55**, 71-87, 1990.
- Braginsky, S.I., and P.H. Roberts, Equations governing convection in Earth's core and the geodynamo, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994a.
- Braginsky, S.I., and P.H. Roberts, From Taylor state to model-Z, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994b.
- Bush, J.W.M., H.A. Stone and J. Bloxham, The motion of an inviscid drop in a bounded rotating fluid, *Phys. Fluids A4*, 1142-1147, 1992.
- Bush, J.W.M., H.A. Stone and J. Bloxham, Axial drop motion in rotating fluids, *J. Fluid Mech.*, to appear, 1994.
- Busse, F.H., Theory of the geodynamo and core-mantle coupling, in *Chaotic Processes in the Geological Sciences*, edited by D.A. Yuen, pp. 281-292, Springer, Heidelberg, 1992.
- Busse, F.H., and J. Wicht, A simple dynamo caused by conductivity variations, *Geophys. Astrophys. Fluid Dynam.*, **64**, 135-144, 1992.
- Cardin, P., and P. Olson, An experimental approach to thermochemical convection in the Earth's core, *Geophys. Res. Letts.*, **19**, 1995-1998, 1992.
- Cardin, P., and P. Olson, Chaotic thermal convection in a rapidly rotating spherical shell: consequences for flow in the outer core, *Phys. Earth planet. Int.*, **82**, 235-259, 1994a.
- Cardin, P., and P. Olson, The influence of toroidal magnetic field on thermal convection in the core, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 3: Core Dynamics and Thermodynamics, 1994b.
- de Vries, D., and J.M. Wahr, The effects of the solid inner core on the Earth's forced nutations and Earth tides, *J. Geophys. Res.*, **96**, 8275-8293, 1991.
- Fearn, D.E., and W. Kuang, Resistive instabilities in a rapidly rotating fluid without critical layers, *Geophys. Astrophys. Fluid Dynam.*, **94**, 181-206, 1994.
- Glatzmaier, G.A., and P. Olson, Highly supercritical thermal convection in a rotating spherical shell: Centrifugal vs radial gravity, *Geophys. Astrophys. Fluid Dynam.*, **70**, 113-136, 1993.
- Glatzmaier, G.A., and P. Olson, A study of the structure of convection in the Earth's liquid core, *Abstract S4.18, IASPEI 27<sup>th</sup> General Assembly*, Wellington, New Zealand, August 1994.
- Glatzmaier, G.A., and P.H. Roberts, Intermediate dynamo models, *J. Geomag. Geoelec.*, **45**, 1605-1616, 1993.
- Glatzmaier, G.A., and P.H. Roberts, A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and outer mantle, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994.
- Hagee, V.L., and P. Olson, Dynamo models with permanent dipole fields and secular variation, *J. Geophys. Res.*, **96**, 11673-11687, 1991.
- Herring, T.A., B.A. Buffett, P.M. Mathews and I.I. Shapiro, Forced nutations of the Earth: influence of inner core dynamics. 3. Very long baseline interferometry data analysis, *J. Geophys. Res.*, **96**, 8259-8273, 1991.
- Hirsching, W., and F.H. Busse, A numerical study of dynamos in spherical shells with conducting boundaries, in *Solar and Planetary Dynamos*, edited by M.R.E. Proctor, P.C. Matthews and A.M. Rucklidge, pp. 137-144, University Press, Cambridge U.K., 1993.
- Hoffman, K.A., Long-lived transitional states of the geomagnetic field and the two dynamo families, *Nature* **354**, 273-277, 1991.
- Hollerbach, R., Parity coupling in  $\alpha^2$ -dynamos, *Geophys. Astrophys. Fluid Dynam.*, **60**, 245-260, 1991.
- Hollerbach, R., Magneto-hydrodynamic Ekman and Stewartson layers in a rotating spherical shell, *Proc. R. Soc. Lond.*, **A444**, 333-346, 1993.
- Hollerbach, R., Imposing a magnetic field across a non-axisymmetric shear layer in a rotating spherical shell, *Phys. Fluids*, to appear, 1994.
- Hollerbach, R., C.F. Barenghi and C. A. Jones, Taylor's constraint in a spherical  $\alpha\omega$ -dynamo, *Geophys. Astrophys. Fluid Dynam.*, **67**, 3-25, 1992.

- Hollerbach, R. and G.R. Ierley, A modal  $\alpha^2$ -dynamo in the limit of asymptotically small viscosity, *Geophys. Astrophys. Fluid Dynam.*, **56**, 133-158, 1991.
- Hollerbach, R., and C. A. Jones, A geodynamo model incorporating a finitely conducting inner core, *Phys. Earth planet. Int.*, **75**, 317-327, 1993a.
- Hollerbach, R., and C. A. Jones, Influence of the Earth's inner core on geomagnetic fluctuations and reversals, *Nature*, **365**, 541-543, 1993b.
- Hollerbach, R., and C. A. Jones, On the magnetically stabilizing role of Earth's inner core, *Phys. Earth planet. Int.*, to appear, 1994.
- Hollerbach, R., and M.R.E. Proctor, Non-axisymmetric shear layers in a rotating spherical shell, in *Solar and Planetary Dynamos*, edited by M.R.E. Proctor, P.C. Matthews and A.M. Rucklidge, pp. 145-152, University Press, Cambridge U.K., 1993.
- Jackson, A., J. Bloxham and D. Gubbins, Time dependent flow at the core surface and conservation of angular momentum in the coupled core-mantle system, in *Dynamics of Earth's Deep Interior and Earth Rotation*, edited by J.-L. Le Mouél, D.E. Smylie and T. Herring, pp. 97-107. *AGU Monograph 72*, American Geophysical Union, Washington DC., 1993.
- Jacobs, J.A., The Earth's inner core, *Nature* **172**, 297-298, 1953.
- Jault, D. and J.-L. Le Mouél, Westward drift, core motions and exchanges of angular momentum between core and mantle, *Nature* **333**, 353-356, 1988.
- Jones, C.A., A.W. Longbottoms, K. Zhang and R. Hollerbach, Convection in the outer core of the Earth, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994.
- Kono, M., and P.H. Roberts, Small amplitude solutions of the dynamo problem. 1. The adjoint system and its solutions, *J. Geomag. Geoelec.*, **42**, 839-862, 1991.
- Kono, M., and P.H. Roberts, Small amplitude solutions of the dynamo problem. 2. The case of the  $\alpha^2$ -dynamoes, *Geophys. Astrophys. Fluid Dynam.*, **67**, 65-85, 1992.
- Kono, M., and P.H. Roberts, Kinematic dynamoes with anisotropic  $\alpha$ -effect, *Geophys. Astrophys. Fluid Dynam.*, to appear, 1994.
- Krause, F., K.-H. Rädler and G. Rüdiger (editors), *The Cosmic Dynamo*, IAU Symposium 157, Kluwer, Dordrecht, 1993.
- Kuang, W., Effect of boundary conditions on resistive instabilities in rapidly rotating fluids, *Geophys. Astrophys. Fluid Dynam.*, to appear, 1994.
- Kuang, W., and J. Bloxham, On the effect of the boundary topography on flow in the Earth's core, *Geophys. Astrophys. Fluid Dynam.*, **72**, 161-195, 1993.
- Kuang, W., J. Bloxham and D.E. Fearn, Thermal and compositional convection in Earth's fluid outer core, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 3: Core Dynamics and Thermodynamics, 1994.
- Kuang, W., and P.H. Roberts, Resistive instabilities in rapidly rotating fluids: linear theory of the  $g$ -modes, *Geophys. Astrophys. Fluid Dynam.*, **60**, 295-333, 1991.
- Kuang, W., and P.H. Roberts, Resistive instabilities in rapidly rotating fluids: linear theory of the convective modes, *Geophys. Astrophys. Fluid Dynam.*, **67**, 129-162, 1992.
- Lan, S., W. Kuang and P.H. Roberts, Ideal instabilities in rapidly rotating MHD systems that have critical layers, *Geophys. Astrophys. Fluid Dynam.*, **69**, 131-160, 1993.
- Lanzerotti, L.J., How strong is the invisible component of the magnetic field in Earth's core? *Geophys. Res. Ltr.*, **21**, 2339-2340, 1994.
- Lanzerotti, L.J., C.H. Sayres, L.V. Medford, J.S. Kran and C.G. MacLennan, Earth potential over 4000km between Hawaii and California, *Geophys. Res. Ltr.*, **19**, 1177-1180, 1992.
- Lanzerotti, L.J., R.A. Langel and A.D. Crane, Geoelectromagnetism, in *Encyclopedia of Applied Physics 7*, edited by G.L. Trigg, pp. 109-123, VCH Publishers, New York, 1993.
- Larson, R.L., and P. Olson, Mantle plumes control magnetic reversal frequency, *Earth planet. Science Ltr.*, **107**, 437-447, 1991.
- Levy, E.H., and S.J. Pearce, Steady state toroidal magnetic field at Earth's core-mantle boundary, *J. Geophys. Res.*, **96**, 3935 - 3942, 1991.
- London S.D., Hydromagnetic waves in a rotating sphere, *Geophys. Astrophys. Fluid Dynam.*, **63**, 91-109, 1992a.
- London S.D., A uniform asymptotic approximation for unstable waves in a rotating sphere, *Geophys. Astrophys. Fluid Dynam.*, **67**, 185-198, 1992b.
- London S.D., Stability and trapping of slow hydromagnetic waves in rotating cylindrical geometry, *J. Geomag. Geoelec.*, **45**, 1553-1561, 1993.
- London S.D., Stability and trapping of slow hydromagnetic waves in rotating spherical geometry, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994.
- Loper, D.E., and H.K. Moffatt, Small-scale hydromagnetic flow in the Earth's core: Rise of a vertical buoyant plume, *Geophys. Astrophys. Fluid Dynam.*, **68**, 177-202, 1993.
- Loper, D.E., H.K. Moffatt and H. Shimizu, Magnetostrophic dynamics of localized upwellings near the CMB, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 2: Core Mantle Interactions and the CMB, 1994.
- Love, J.J., and J. Bloxham, Electromagnetic coupling and the toroidal magnetic field at the core-mantle boundary, *Geophys. J. Int.*, **117**, 235-256, 1994a.
- Love, J.J., and J. Bloxham, Dynamoes by inverse theory, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994b.
- Malkus, W.V.R., Energy sources for planetary dynamoes, in *Lectures on Solar and Planetary Dynamos*, edited by M.R.E. Proctor and A.D. Gilbert, pp. 161-179, University Press, Cambridge U.K., 1994.
- Mathews, P.M., B.A. Buffett, T.A. Herring and I.I. Shapiro, Forced nutations of the Earth: influence of inner core dynamics. 1. Theory, *J. Geophys. Res.*, **96**, 8219-8242, 1991a.
- Mathews, P.M., B.A. Buffett, T.A. Herring and I.I. Shapiro, Forced nutations of the Earth: influence of inner core dynamics. 2. Numerical results and comparisons, *J. Geophys. Res.*, **96**, 8243-8257, 1991b.
- McFadden, P.L., and R.T. Merrill, Inhibition and geomagnetic field reversals, *J. Geophys. Res.*, **98**, 6189-6199, 1993.
- Moffatt, H.K., Liquid metal MHD and the geodynamo, in *Liquid Metal Magneto-hydrodynamics*, edited by J. Liel-peteris and R. Moreau, pp. 403-412. Kluwer, Dordrecht, 1989.
- Moffatt, H.K., and D.E. Loper, The magnetostatic rise of a buoyant parcel in the Earth's core, *Geophys. J. Int.*, **117**, 394-402, 1994.
- Nakajima, T., and P.H. Roberts, A mapping method for solving dynamo equations, *Proc. R. Soc. Lond.*, to appear, 1994a.
- Nakajima, T., and P.H. Roberts, An application of the mapping method to the asymmetric dynamo equations, *Abstract, 4<sup>th</sup> SEDI Symposium, Whistler*, Session 4: The Geodynamo and Observational Constraints, 1994b.
- Nakajima, T., P.H. Roberts and M. Kono, An application of a mapping method to the geodynamo problem. *Abstract 01.01.02, 7<sup>th</sup> IAGA Scientific Assembly*, Buenos Aires, 1993.

- Olson, P., and G.A. Glatzmaier, Magnetoconvection in a rotating sphere and the structure of flow in the liquid core, *EOS*, 74, Supp., 79, 1993.
- Olson, P., and G.A. Glatzmaier, Magnetoconvection in a rotating sphere and the structure of flow in the liquid core, *Phys. Earth planet. Int.*, to appear, 1994.
- Proctor, M.R.E., P.C. Matthews and A.M. Rucklidge (editors), *Solar and Planetary Dynamos*, University Press, Cambridge U.K., 1993.
- Proctor, M.R.E., and A.D. Gilbert (editors), *Lectures on Solar and Planetary Dynamos*, University Press, Cambridge U.K., 1994.
- Roberts, P.H., A mapping method for integration of the geodynamo equations, Abstract, 3<sup>rd</sup> SEDI Symposium, Mizusawa, 1992.
- Ruan, K. and D. Loper, On small-scale hydromagnetic flow in the Earth's core: motion of a rigid cylinder parallel to its axis, *J. Geomag. Geoelec.*, 45, 1539–1552, 1993.
- Runcorn, S.K., and D.E. Winch, The Pacific cable experiment, *IAGA Program and Abstracts*, Vienna, p. 607, 1991.
- Ruzmaikin, A., On the role of rotation of the inner core relative to the mantle, in *Solar and Planetary Dynamos*, edited by M.R.E. Proctor, P.C. Matthews and A.M. Rucklidge, pp. 265–270, University Press, Cambridge U.K., 1993.
- Ruzmaikin, A.A., P.C. Liewer and J. Feynman, Random cell dynamo, *Geophys. Astrophys. Fluid Dynam.*, 73, 163–117, 1993.
- St. Pierre, M.G., The stability of the magnetostrophic approximation I: Taylor state solutions, *Geophys. Astrophys. Fluid Dynam.*, 67, 99–115, 1993a.
- St. Pierre, M.G., The strong field branch of the Childress–Soward dynamo, in *Solar and Planetary Dynamos*, edited by M.R.E. Proctor, P.C. Matthews and A.M. Rucklidge, pp. 295–302, University Press, Cambridge U.K., 1993b.
- St. Pierre, M.G., On the local nature of fluid flow in the Earth's outer core, submitted, 1994a.
- St. Pierre, M.G., The stability of buoyant structures in the outer core, Abstract, 4<sup>th</sup> SEDI Symposium, Whistler, Session 3: Core Dynamics and Thermodynamics, 1994b.
- St. Pierre, M.G., and P.H. Roberts, Direct simulations of the local turbulence in the Earth's core, Abstract, 4<sup>th</sup> SEDI Symposium, Whistler, Session 2: Core Mantle Interactions and the CMB, 1994.
- Stevenson, D.J., The core–mantle electrochemical cell as an explanation for part of the geomagnetic field, *EOS*, 72, Supp., 133, 1991.
- Stewartson, K., On almost rigid rotations, *J. Fluid Mech.*, 26, 131–144, 1966.
- Sun, Z.-P., G. Schubert and G.A. Glatzmaier, Numerical simulations of thermal convection in a rapidly rotating spherical shell cooled inhomogeneously from above, *Geophys. Astrophys. Fluid Dynam.*, to appear, 1994.
- Taylor, J.B., The magneto-hydrodynamics of a rotating fluid and the Earth's dynamo problem, *Proc. R. Soc. Lond.*, A274, 274–283, 1963.
- U.S. Geodynamics Committee, Board on Earth Sciences and Resources, Commission on Geosciences and Resources, and National Research Council, *The Geomagnetic Initiative*, National Academy Press, Washington D.C., 1993.
- Vanyo, J.P., A geodynamo powered by luni-solar precession, *Geophys. Astrophys. Fluid Dynam.*, 59, 209–234, 1991.
- Vanyo, J.P., and D. Lods, MCB relative velocities in precessionally driven flows, *EOS*, 75 Supp., 120, 1994.
- Vanyo, J., D. Lods and P. Wilde, Precessing mantle, liquid core and solid core interactions, Abstract, 4<sup>th</sup> SEDI Symposium, Whistler, Session 2: Core Mantle Interactions and the CMB, 1994b.
- Vanyo, J., P. Wilde, P. Cardin and P. Olson, Flow structures induced in a liquid core by precession, *EOS*, 73 Supp., 63, 1992.
- Vanyo, J., P. Wilde, P. Cardin and P. Olson, Experiments on precessing flows in the Earth's liquid core, *Geophys. J. Int.*, to appear, 1994a.
- Verhoogen, J., Heat balance of the Earth's core, *Geophys. J.*, 4, 276–281, 1961.
- Wicht, J., and F.H. Busse, Dynamo action induced by lateral variation of electrical conductivity, in *Solar and Planetary Dynamos*, edited by M.R.E. Proctor, P.C. Matthews and A.M. Rucklidge, pp. 329–337, University Press, Cambridge U.K., 1993.
- Wilde, P.D., and J.P. Vanyo, Flow observations from spin-up to fully developed precession, Abstract, 4<sup>th</sup> SEDI Symposium, Whistler, Session 2: Core Mantle Interactions and the CMB, 1994.

---

Paul H. Roberts, Institute of Geophysics and Planetary Physics, University of California, Los Angeles, CA90095.

(Received June 13, 1994; revised January 18, 1995; accepted January 23, 1995.)