Modern replication of Eratosthenes' measurement of the circumference of Earth

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Modern replication of Eratosthenes' measurement of the circumference of Earth

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Abstract

Twenty-two hundred years ago, the Greek scientist Eratosthenes measured the circumference of the Earth. This paper describes an experiment to replicate Eratosthenes' experiment with observers located in Australia and New Zealand. The most accurate circumference produced in the experiment described in this paper is 38 874 km, measured at Rosebud, Victoria, Australia, and Jimboomba, Queensland, Australia with an error of 2.9%. This exceeds the accuracy of Eratosthenes, although not of the modern recreation of his experiment between Syene and Alexandria. The experiment described in this paper might form a useful model for cooperation between schools in different countries.

1. Introduction

The Greek scientist and head librarian of Alexandria, Eratosthenes (276–194 BC), noted that the Sun's rays fell in parallel lines. In Ptolemaic Egypt, where Eratosthenes resided, it was known that the Earth was round. It was said that at the temple of Syene (modern day Aswan), at midday on the summer solstice, the Sun cast no shadow, and the bottom of a well at Syene was fully illuminated. Since the Sun's rays fall in parallel lines, these lines must be pointing to the centre of the Earth at this location. Since the Earth is round, Eratosthenes postulated that the shadow cast at Alexandria on the summer solstice must be longer.

At the appointed time, Eratosthenes measured the length of a shadow of an obelisk at Alexandria. Knowing that the length of a shadow at Syene would be zero at the same time, he could see that the angle the shadow cast by the obelisk is the same as the angle made by projecting the axis of the Well of Syene and the Obelisk of Alexandria down to the centre of the Earth. He found that this angle is approximately one fiftieth of a full circle. After pacing the distance between the two ancient cities by camel (or, in some accounts, by getting a slave to do it), he multiplied this length by 50 and obtained a measure for the circumference of the Earth.

The key assumptions Eratosthenes made were that the two cities, Alexandria and Syene, both lie directly north–south from each other (i.e. on the same meridian), that the distance between the two cities is 5000 Attic stadia (5000 × 185 m = 925 km) and that the Earth is perfectly spherical.

The north–south distance between two locations at different longitudes can be calculated in various ways, for example the triangulation technique used by surveyors. Eratosthenes must have used some other means equivalent to counting

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steps to measure the north–south distance between Alexandria and Syene. In this paper, north–south distances between locations were calculated using GPS (global positioning system) coordinates.

It is now known that the Earth is not perfectly spherical, the Earth is 'squashed' at the poles and 'bulging' at the equator [1] due to the rotation of the Earth and the tidal forces exerted on it by the Moon. The average radius of the Earth is known today to be 6371 km, giving a circumference of 40 030 km. The circumference of the Earth as given by Eratosthenes is 46 620 km, which is 16% above the accepted value. Some historical sources say he measured the Earth using Egyptian stadia instead of Attic stadia, making the circumference of the Earth 39 690 km, a smaller error of 1.6%. Given modern knowledge and measurements and repeating the experiment using Syene and Alexandria, the circumference becomes 40 074 km, very close to the accepted value of 40 075 km.

Since Eratosthenes' method for measuring the size of the Earth is relatively straight forward, it can be performed as a high school and university experiment without complicated or expensive equipment [2–7]. The aim of the experiment described in this paper was to reproduce Eratosthenes' measurement in two countries separated by an ocean (Australia and New Zealand) and see if a value of comparable or better accuracy can be achieved. Eratosthenes' method complements other methods of measuring the size of the Earth [8].

2. Method

To perform a measurement of the size of the Earth, three locations were used: Jimboomba, Queensland, Australia; Rosebud, Victoria, Australia; and Christchurch, New Zealand. Participants in each location were assigned the task of measuring the shadow and height of a vertical object of their choice at their local midday (figure 1). The participants agreed on a date: Saturday 14 September 2013. For a given location, the local midday ensures that the Sun casts shadows directly southwards, and the Sun is at the highest elevation. Eratosthenes performed his measurements at the summer solstice at the local midday. Although the actual time of local noon is not actually required to perform the measurements—local noon is simply the time the shadow of a vertical stick is at the shortest length—local noon times are mentioned here for completeness.

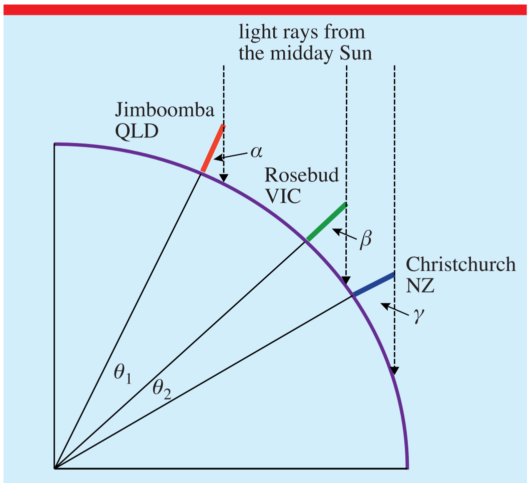

Figure 1. Schematic diagram of the experiment, showing the three measurement locations in Australia and New Zealand.

On the chosen date, Christchurch had a local midday of 12:25 pm New Zealand Standard Time (NZST, GMT + 12). The coordinates for the location were 43.536058 S, 172.635874 E. The participant chose to measure the shadow of a stick 140.9 cm in length.

Rosebud, 70 km south of Melbourne, had a local midday of 12:16 pm Australian Eastern Standard Time (AEST, GMT + 10). The coordinates for the location were 38.3560 S, 144.9180 E. The participant chose to measure a standard 30 cm rule (actual height 31.4 cm).

For Jimboomba, the local midday occurred at 11:44 am AEST, the same as Brisbane (Jimboomba is 52 km almost directly south from Brisbane). The coordinates for this location are 27.8333 S, 153.0333 E. The shadow angle was found by calculating the inverse tan of the shadow length divided by the pole height.

The north–south distance (D_{NS}) between each pair of cities was calculated from the difference between the latitudes (ΔL) divided by 360 and multiplied the mean circumference (C_m) of the Earth:

$$D_{NS} = \frac{\Delta L}{360} \times 40\,030 \text{ km}.$$

The north–south distances were Jimboomba to Christchurch 1746 km, Jimboomba to Rosebud

Table 1. Measurements and calculations specific to location, including measurement error margins.

	Jimboomba	Rosebud	Christchurch
Latitude	27.8333 S	38.3560 S	43.5360 S
Longitude	153.0333 E	144.9180 E	172.6358 E
Height of pole	285 cm	31.4 cm	140.9 cm
Length of shadow	170.8 cm	28.0 cm	148.1 cm
Error	±0.1 cm	±0.05 cm	±0.05 cm
Local time	11:48 am	12:16 am	12:25 pm
Midday	11:44 am	12:16 am	12:25 pm
Error	±30 s	±30 s	±30 s
Shadow angle at midday	30.93 ± 0.02° (α)	41.7 ± 0.1° (β)	46.43 ± 0.02° (γ)

Table 2. Results for each city pair. The first column shows the city pair (J = Jimboomba, R = Rosebud, C = Christchurch), the second column shows the difference in the latitude of the city pair, the third column the difference between the angles of the shadows cast at midday at the city pair, the fourth column the calculated circumference of the Earth, and the fifth column the percentage discrepancy.

City pair	Difference in latitude (degrees)	North–south distance (km)	Difference between angle of midday Sun (degrees)	Calculated circumference (km)	% discrepancy
J–R	10.5227	1170	10.79	39 038	2.48
J–C	15.702758	1746	15.49	40 572	−1.35
R–C	5.180058	575	4.70	44 089	−10.14

1170 km and Rosebud to Christchurch 575 km. The angle (θ) of the shadow cast by the midday Sun at each city was calculated from figure 1.

Jimboomba experienced some overcast weather, delaying the acquisition of data by 4 min, but time was kept well on time in the other two locations.

3. Results

The results are shown in tables 1 and 2. The circumference of the Earth was calculated using the average of nine shadow angles, with the propagated error. The error in the shadow angle was estimated by adding the measurement error to the height, and subtracting the measurement error from the shadow length, and inserting the values in the inverse tan expression.

4. Discussion

The results show that accurate Earth circumferences (<2.5%) can be obtained across oceans. When the Christchurch shadow length is increased from 148.1 to 150.6 cm, the discrepancy in the Rosebud–Christchurch circumference reduced from −10.14 to 0.03%, whilst the Jimboomba–Christchurch discrepancy changed from −1.35 to +1.68%. This suggests that the Christchurch shadow length is slightly out.

An interesting question in view of the techniques used in this paper is how Eratosthenes performed his calculations. He may have used an abacus or perhaps some ancient technique lost to the world when the library of Alexandria was burned to the ground.

5. Conclusion

In conclusion, the 2200 years of scientific progress and technological advancement has given ordinary students the ability to measure the Earth using a telephone, tape, and Google. Eratosthenes' accomplishment is still an amazing feat, as he obtained an accurate measure of the Earth's size using very basic 'equipment', which is a testament to the ingenuity of the ancient philosophers and mathematicians. The exercise described in this paper would be suitable as a collaborative project between schools in different countries.

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