Chapter 1

THE SAGNAC EFFECT IN THE GLOBAL POSITIONING SYSTEM

Neil Ashby

Department of Physics, UCB 390.

University of Colorado.

Boulder, CO 80309-0390 USA.

n_ashby@mobek.colorado.edu

Abstract In the Global Positioning System (GPS) the reference frame used for navigation is an earth-centered, earth-fixed rotating frame, the WGS-84 frame. The time reference is defined in an underlying earth-centered locally inertial frame, freely falling with the earth but non-rotating, with a time unit determined by atomic clocks at rest on earth's rotating geoid. Therefore GPS receivers must apply significant Sagnac or Sagnac-like corrections, depending on how information is processed by the receiver. These corrections can be described either from the point of view of the local inertial frame, in which light travels with uniform speed c in all directions, or from the point of view of an earth-centered rotating frame, in which the Sagnac effect is described by terms in the fundamental scalar invariant that couple space and time. Such corrections are very important for comparing time standards world-wide.

1. Introduction

The purpose of the Global Positioning System (GPS) is accurate navigation on or near earth's surface. GPS also provides an accurate world-wide clock synchronization and timing system. Most GPS users are interested in knowing their position on earth; the developers of GPS have therefore adopted an Earth-Centered, Earth-Fixed (ECEF) rotating reference frame as the basis for navigation. Specifically, in the WGS-84(873) frame, the model earth rotates about a fixed axis with a defined rotation rate, $\omega_E = 7.2921151247 \times 10^{-5}$ rad s^{-1}.[1],[2]

In an inertial frame, a network of self-consistently synchronized clocks can be established either by transmission of electromagnetic signals that propagate

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with the universal constant speed c (this is called *Einstein synchronization*), or by slow transport of portable atomic clocks. On the other hand it is well-known[3] that in a rotating reference frame, the Sagnac effect prevents a network of self-consistently synchronized clocks from being established by such processes. This is a significant issue in using timing signals to determine position in the GPS. The Sagnac effect can amount to hundreds of nanoseconds; a timing error of one nanosecond can lead to a navigational error of 30 cm.

To account for the Sagnac effect, a hypothetical non-rotating reference frame is introduced. Time in this so-called Earth-Centered Inertial (ECI) Frame is adopted as the basis for GPS time; this is discussed in Section 2. Of course the earth's mass encompasses the origin of the ECI frame and has significant gravitational effects. To an extremely good approximation in the GPS, however, gravitational effects can be simply added to other effects arising from special relativity. In this article gravitational effects will not be considered. Even time dilation, which is an effect of second order in the small parameter v/c, where v is the velocity of some clock, will be neglected. I shall confine this discussion to effects which are of first order (linear) in velocities. The Sagnac effect is such an effect.

A description of the GPS system, of the signal structure, and the navigation message, needed to understand how navigation calculations are performed, is given in Section 3. In comparing synchronization processes in the ECI frame with those in the ECEF frame, taking into account relativity principles, it becomes evident that the Sagnac effect is a manifestation of the relativity of simultaneity. Observers in the rotating ECEF frame using Einstein synchronization will not agree that clocks in the ECI frame are synchronized, due to the relative motion. In fact observers in the rotating frame cannot even globally synchronize their own clocks, due to the rotation. This is discussed in Section 4. Section 5 discusses Sagnac corrections that are necessary when comparing remote clocks on earth by observations of GPS satellites in common-view. Section 6 introduces the GPS navigation equations and discusses synchronization processes from the point of view of the rotating ECEF frame. Section 7 develops implications of the fact that GPS navigation messages provide satellite ephemerides in the ECEF frame.

2. Local Inertial Frames

Einstein's Principle of Equivalence allows one to discuss frames of reference which are freely falling in the gravitational fields of external bodies. Sufficiently near the origin of such a freely falling frame, the laws of physics are the same as they are in an inertial frame; in particular electromagnetic waves propagate with uniform speed c in all directions when measured with standard rods and atomic clocks. Such freely falling frames are called *locally inertial*