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Light propagation and local speed in the linear Sagnac effect

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ABSTRACT

We investigate rigorously the behaviour of light propagation in the closed contour of the linear Sagnac effect. Assuming that the local light speed is c in a section of the contour, our approach makes it possible to determine the local speed in the other sections. We show that, if standard clock synchronization is adopted, the speed c turns out to be invariant in an open section of the contour only. Our result is due to the distinctive physical feature of the 'time gap' introduced by relative simultaneity in the closed contour.

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1. Introduction

Special Relativity (SR) is often necessarily taught at a somewhat superficial level and limited to certain foundational aspects such as the Lorentz transformations and the interpretation of seminal experiments. Some of the deeper foundational issues are typically either not presented or discussed only briefly, for instance, the difficulties involved in the measurement of the one-way speed of light and the conventionality of clock synchronization in SR. Of course, these topics have been explored by many workers for more than a century, for example in the early works of Poincaré (*1*), Einstein (*2*), Reichenbach (*3*), and Grünbaum (*4*). According to Einstein's second postulate of SR, the local one-way speed of light measured in an inertial frame is constant (invariant), always equal to c in all directions, and independent of the velocity of the source. This postulate relies on the well-known approach to clock synchronization proposed by Einstein, in which a light signal is sent from a clock to a mirror at the distance L. After reflection the signal returns to the clock that measures the round-trip time, T, and the one-way speed c is the average speed $c = 2L/T$, assumed by Einstein to be the same when propagating first one-way toward the mirror and then back from it during the return trip. Our intention here is to contribute to the ongoing discussions about this key foundational aspect of SR by presenting a new approach for interpreting the classical optical test performed in 1913 by Sagnac (*5*).

1.1. Background

There is a long and rich history of attempts to understand how methods of synchronization come into play in measuring the speed of light. We only mention here that, in relation to the standard Einstein synchronization, Poincaré (*6*) objected that Einstein's procedure can represent the one-way speed of light c when performed by observers stationary relative to each other. Similarly, Reichenbach (*3*) argued that by introducing a mirror for measuring the average speed, the procedure involves circular reasoning as it leaves undetermined the one-way speed. It follows that a nonstandard synchronization convention can be adopted, where the average speed is c, but with unequal values of the speed of light in opposite directions. This argument has led to the 'conventionalist thesis', alleging that Einstein's procedure is only a convention and claiming that the one-way speed c cannot be measured in principle. Because of the conventionality of the one-way speed c, Mansouri and Sexl (*7*) showed that relativistic theories can be formulated using coordinate transformations with the same rod-contraction and clock-retardation as the Lorentz transformations (LT), while the time transformation depends on an arbitrary synchronization parameter, which can be chosen to conserve simultaneity. Depending on the synchronization procedure adopted, Einstein or absolute, we may have the following coordinate transformations between the inertial frame S and the frame S' in motion with velocity

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$\mathbf{u} = \hat{\mathbf{i}}u$ relative to S:

$$\begin{aligned} &GT \quad t' = t \quad ; x' = x - ut; y = y; z' = z \\ &LT \quad t' = \gamma(t - ux/c^2); x' = \gamma(x - ut); y = y; z' = z \\ &LTA \quad t' = t/\gamma \quad ; x' = \gamma(x - ut); y = y; z' = z, \end{aligned} \tag{1}$$

where GT stands for the Galilean transformations, LT for the standard Lorentz transformations adopting Einstein synchronization, and LTA (*8*) for the Lorentz transformations adopting absolute synchronization, which preserves simultaneity. We can see from (1) that the absolute time of Newton ($t' = t$ with the GT), after Einstein becomes relative ($t' = \gamma(t - ux/c^2)$ with the LT) because it depends on the relative velocity (through the factor $\gamma = (1 - u^2/c^2)^{-1/2}$) and the position x. Thus, with the LT, simultaneity is not conserved and is relative. If, instead of Einstein synchronization, we adopt absolute synchronization for the LTA, we obtain the relation $t' = t/\gamma$, where time is still relative because of the factor γ that provides the relativistic effect of time dilation, confirmed by experiment. However, as with the GT, simultaneity is conserved with the LTA because if two events are simultaneous in S ($\Delta t = 0$), they are simultaneous also in S' ($\Delta t' = 0$).

Since the LTA displays the same relativistic factor γ as the LT, as shown by several authors (*7*, *9*–*13*), 'preferred frame' relativistic theories based on absolute synchronization are capable of interpreting all the performed experiments supporting *standard* SR with Einstein synchronization. Hence, according to the conventionalist thesis, SR can be formulated equivalently by either adopting absolute or Einstein synchronization. From the pedagogic perspective, an example where absolute (LTA) instead of relative (LT) synchronization has been used in the context of special relativity is given by the description of the Sagnac (*5*) effect by Kassner (*14*).

However, talking about clock synchronization, we believe it convenient to cite Lundberg (*15*) who points out how the term 'synchronization' is often misused and misinterpreted in the literature:

> I have said several times that Einstein specified a procedure for coordinating clocks, but I have not said that he specified a procedure for synchronizing clocks. 'Coordinating' is a safe and unproblematic word in this context. To coordinate clocks is merely to connect the clock settings in some way or other, so that they are not independent of each other. Einstein's procedure clearly does that. 'Synchronizing' is more specific than 'coordinating ', but in a way that is not obvious.

In fact, difficulties in the application of Einstein synchronization may emerge because of the unfeasibility of performing Einstein synchronization along a closed path, as pointed out by Selleri (*12*), Gift (*16*), and mentioned by Weber (*17*), Anandan (*18*), Klauber (*19*), Field (*20*), and Landau and Lifshitz (*21*), who over 50 years ago stated:

> . . . However, synchronization of clocks along a closed contour turns out to be impossible in general. In fact, starting out along the contour and returning to the initial point, we would obtain for dx° a value different from zero . . . .

With reference to our citation to Lundberg's paper (*15*) and before going into the next section, we comment briefly on the notations used in Equation (1). The GT differ from the LT and LTA in the sense that, according to many users of GT, time is not thought of in terms of a set of clock-variable axes belonging to different coordinate systems. Thus, time is a property of the universe that transcends coordinate systems and it could be more appropriate to use different notations for the coordinate-system-independent time that appears in the GT, and the one appearing in the two coordinate systems that figure in LT and LTA. However, to introduce different notations could mislead some of the readers and, thus, we leave it unchanged but only after having pointed out the mentioned unique time conception inherent to the GT.

Another important issue concerns the preferred frame S related to the LTA of Equation (1). According to the conventionalist thesis (*7*) the preferred frame is not 'identifiable' and can be chosen arbitrarily. Because of this, conventionalists have been considering the LTA to be equivalent to the LT. Instead, as claimed by the authors of Ref. (*13*) and as assumed in the present paper, the reference frame S associated with the LTA is the unique 'identifiable' preferred rest frame where space is isotropic and the one-way speed of light is c. The natural properties of this 'identifiable' preferred frame make it possible to single it out by means of the approach described in Ref. (*13*) that invalidates the conventionalist thesis. It follows that the frame S cannot be chosen arbitrarily. In this case, since the arbitrariness has been removed, the LTA associated with the 'identifiable' preferred frame are no longer physically equivalent to the LT.

2. A simple 'Gedankenexperiment'

The relevant difference between LT and LTA is the clock synchronization adopted, Einstein ('internal') or absolute ('external') synchronization, respectively (the terms 'internal' and 'external' have been first used by Mansouri and Sexl (*7*)). Ideally, with our Gedankenexperiment we expect to determine which of the two transformations, LT or LTA, describes coherently the known experiments supporting SR, without signs of internal inconsistencies.

Consider now the following thought experiment where a light pulse, previously emitted from the origin

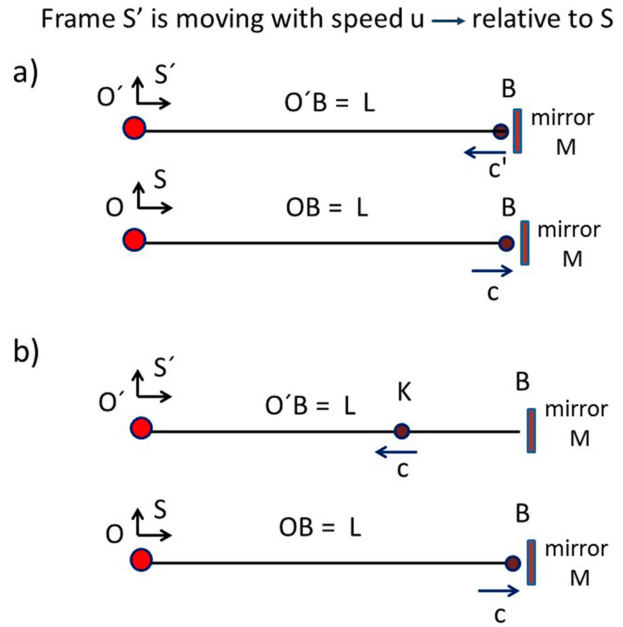

Figure 1. To explore the timing of a light pulse propagation in the new Gedankenexperiment, the arrangement in (a) will employ absolute synchronization, while that of (b) will employ relative synchronization. At $t = t' = 0$, the origins O and O' (of the overlapping inertial frames S and S' in relative motion) coincide. In both (a) and (b), a light pulse, previously emitted from the origin O of frame S, is reflected by the mirror at B and returns toward the origin. Point B is fixed to the mirror, which can be moving relative to O, but (regardless of the mirror's relative motion) we have $OB = L$ when the light pulse reaches the mirror. As seen from frame S', in the case (a) of absolute simultaneity, when O and O' coincide, the reflected pulse is at B (being $O'B = L$) from where it moves toward O'. In the case (b) of relative simultaneity, when O and O' coincide, after being reflected earlier at B, the pulse is already at K, from where it moves toward O'.

O of the inertial frame $S(\mathbf{x}, t)$ shown in Figure 1, is travelling in the x-direction towards point B. For the purpose of relating our thought experiment to the Sagnac effect, we assume that B is moving with velocity v relative to frame S, as shown in the next sections. When the pulse reaches point B at the time $t = 0$, B is located at the distance $x_B = OB = L$ from the origin O. At this moment ($t = 0$), the pulse is reflected back by the mirror shown in Figure 1(a,b). Next, we consider the physical situation described from the inertial frame $S'(\mathbf{x}', t')$, overlapping with frame $S(\mathbf{x}, t)$ and moving with velocity $u = 2v$ relative to S. If the respective origins O' and O coincide at $t' = t = 0$, the Lorentz transformations between S' and S are, $t' = \gamma(t - ux/c^2)$; $x' = \gamma(x - ut)$, while the LTA are, $t' = t/\gamma$; $x' = \gamma(x - ut)$, in agreement with (1). We denote by the symbol C a clock at the origin O of S and co-moving with it and, similarly, by the symbol C' another clock co-moving with the origin O' of S'. When, at $t = 0$, the light pulse is reflected by the mirror at B, its distance from clock $C' \equiv O'$ is given by

$$x'_B = O'B = \gamma x_B = \gamma L \simeq L \qquad (2)$$

for both the LT and LTA. For the purpose of our Gedankenexperiment we need to consider only the relevant terms, which are of first order in u/c, and thus, for simplicity, we shall neglect higher order terms (such as u^2/c^2, or higher) and approximate $\gamma = (1 - u^2/c^2)^{-1/2} \simeq 1$, for the last term of (2). Thus, the length $O'B \simeq L$ in (2) represents, at $t = 0$, the distance to be covered in S' by the reflected pulse in order to reach O'.

Since for S the light pulse is at B at the time $t = 0$, the position of the reflected pulse in frame S at the time t is $L - ct$, while that of the origin O' is ut. Then, the pulse reaches O' when $L - ct = ut$ and the time of flight from B to O' in S is

$$t_{BO'} = \frac{L}{c+u}. \qquad (3)$$

We wish to find the time displayed by clock C' when the pulse, reflected from B to $C' \equiv O'$, reaches it. Calculations can be performed in any inertial reference frame as, obviously, after transforming to frame S', the final result will be the same.

Absolute synchronization: In the case of absolute synchronization and as indicated in Figure 1(a), for an observer of frame S' the light pulse is still at B at the time $t' = t = 0$. Thus, with $t' = t/\gamma \simeq t$, we obtain immediately,

$$t'^{abs} \simeq t_{BO'} = \frac{L}{c+u}, \qquad (4)$$

valid to first order in u/c. Result (4) is in line with the composition of velocities of absolute synchronization, according to which the speed of the pulse relative to S' is $\mathrm{d}x'/\mathrm{d}t' \simeq \mathrm{d}(x - ut)/\mathrm{d}t = -(c + u)$, i.e. superluminal.

Relative synchronization: If relative (Einstein) synchronization is used, with $\gamma \simeq 1$ the time t and t' are connected by the relation $t' \simeq t - ux/c^2$ (or the inverse time relation $t \simeq t' + ux'/c^2$). With this relation, we find that, at $t = 0$ and due to non-conservation of simultaneity, in S' the light pulse is reflected by the mirror at B at the earlier time

$$t'_B \simeq 0 - \frac{uL}{c^2} = -\frac{u}{c}\frac{L}{c} = -\delta t', \qquad (5)$$

and not at $t' = 0$, as in the case of absolute simultaneity. Since the pulse is reflected at B at the earlier time $t'_B \simeq -\delta t'$ given by (5), O' has not yet reached O, which is still at the small distance $x'_O \simeq u\delta t' = u^2L/c^2$ away from O'. Thus, when O' and O coincide at the later time $t' = t'_B + \delta t' = 0$, the reflected light pulse has already moved from B to point K, having covered the sizeable distance

$BK \simeq c\delta t' = uL/c$, as shown in Figure 1(b). For the trip from K to $C' \equiv O'$, the equation of motion of the light pulse in frame S' is, $KO' - ct' \simeq (L - uL/c) - ct' = 0$, which gives, $t' = t'^{rel} \simeq L(1 - u/c)/c$. Since we are considering results valid to the lowest order in u/c, we may use the series expansion $1/(1 + u/c) \simeq 1 - u/c$ and, for simplicity, we shall replace from now on the symbol $\simeq$ by $=$. Then, we may write,

$$t'^{rel} = t'^{rel}_{KO'} = t'^{abs} = t_{BO'} = \frac{L}{c+u} = \frac{L(1-u/c)}{c}, \quad (6)$$

which is the same as (4) and, thus, synchronization-independent. Although quantitatively the two results t'^{rel} and t'^{abs} are the same, there is a difference in their interpretation. In fact, the last term of (6) indicates that, if the local speed is imposed by Einstein synchronization to be c in frame S', due to non-conservation of simultaneity, at $t' = 0$ the light pulse is at K and the distance covered by the light pulse during the time interval $t'^{rel}_{KO'}$ is $L(1 - u/c) < x'_B \simeq L$.

To sum up, if absolute synchronization is adopted, when O' and O coincide, the light pulse is at B for both frame S and S', as shown in Figure 1(a). However, if Einstein synchronization is adopted, when O' and O coincide, the light pulse is at B for an observer of frame S (as in Figure 1(a,b)) and, due to relative simultaneity, at K for an observer of frame S' (as in Figure 1(b)). For both synchronizations the path to be covered by the pulse from B to O' has length L, as measured from S'. However, there is an important aspect to consider, consequence of Einstein synchronization and crucial in the interpretation of experiments: The path section BK has been covered by the light pulse before O reaches O', so that the corresponding time delay $\delta t'$ is not included in the time interval $t'^{rel}_{KO'}$ measured by $C' \equiv O'$ in (6).

2.1. Accelerating a clock from frame S to frame S′

Let us suppose now that clock C, initially coincident with O ($C \equiv O$) at $t = 0$, is accelerated in a short period of time in such a way that, after it reaches the velocity u, is co-moving with O' ($C \equiv O'$). The exact time delay τ experienced by clock C during the process of acceleration can be calculated without approximations using the general theory of relativity and using the principle of equivalence (22). Then, the small contribution $\delta\tau$ (due to the acceleration) to the resulting time delay τ can be separated from the kinematic contribution due to the velocity (negligible in our case). Similar considerations are made in the interpretation of the clock readings in the Hafele–Keating experiment (23), where the effect due to the difference in gravitational potential between the surface of the earth and the flight altitude of airplanes, is separated from the kinematical effect of time dilation due to the velocity. As also considered in Appendix, the time delay τ (independent of L), including the effect due to acceleration, is negligible with respect to $\delta t'$ if L is such that $\tau c \ll uL/c = \delta t' c$, as we assume in the present context. Therefore, within the approximation made, clock C, originally co-moving with the inertial frame S, after the negligible time interval τ, is now co-moving with clock $C' \equiv O'$ and the inertial frame S'. In this case, events taking place after the time $t' \simeq 0$, obviously coincide for both C and C' as they are now measured by the co-moving clocks $C \equiv C'$. In the sequence of events measured by C', with Einstein synchronization we find first the light pulse at $x'_B = L$ at the earlier time $t'_B = -\delta t'$ and then the (reflected) light pulse at $x'_K = L(1 - u/c)$ at the later time $t' \simeq 0$. For clock C we have first (before accelerating), the light pulse at $x_B = L$ at the time $t = 0$. However, after the time $t' \sim \tau \simeq 0$, when C coincides with $O' \equiv C'$ after accelerating, the sequence of events (corresponding to the evolving proper time of clock C) is the same as C' and begins with the light pulse being at $x'_K = L(1 - u/c)$ at $t' \sim \tau \simeq 0$. As is well-known, the proper time of clock flows continuously regardless of its state of motion (22). However, while clock C is accelerating until finally is co-moving with $C' \equiv O'$ (and the other synchronized clocks of S'), the effect of non-conservation of simultaneity seems to require that the distant light signal has been shifting to the future, 'jumping' at the nearly infinite speed $(u/c)L/\tau \simeq \infty$ from point B (at the distance L) to point K (at the distance $L(1 - u/c)$). Such a process takes place in the negligible time interval τ recorded by clock C in accelerating from frame S to frame S'.

A similar behaviour is found if, instead of C, clock C' is decelerated until co-moving with O and frame S. In this case, it is as if clock C' 'sees' initially the light pulse at K at the time $t' = 0$ and then, when it is co-moving with O after decelerating, 'sees' the light pulse shift to the past of S', jumping back to B at the epoch $t \simeq \tau \simeq 0$.

These properties of the time transformation and their consequences, directly related to non-conservation of simultaneity, are well-known and have been considered by different authors (12, 16, 24), in different contexts. The possible existence of superluminal light signal has been discussed in the area of superluminal information transfer, time orientation, and causality structure of the theory (25–28). As long as light propagation takes place in an open path, there are no problems with relative simultaneity. However, as mentioned above, problems emerge when Einstein synchronization is applied along a closed contour. In fact, when we compare the physical reality reflected by the set of Einstein-synchronized clocks of frame S along the x-axis from B to point O, with the physical reality reflected by the set of Einstein-synchronized clocks of frame S' along the x' axis from $O' \equiv O$ to B, we are (ideally) covering the closed contour $BO + O'B$ ($O \equiv O'$) and are met with the 'time gap' $\delta t'$ pointed out in Refs. (*12*, *14*, *16*, *21*, *24*), just to mention a few authors. From a physical point of view, relative simultaneity has been questioned throughout the development of SR, but it is generally accepted by supporters of the theory because necessary, from a mathematical perspective, for sustaining the constancy of the speed of light, $c' = c$. Considering that for the case under consideration, as for many other cases, the results for absolute and relative simultaneity, (4) and (6), are the same, the majority of physicists has paid little attention to the inconsistency that emerges in applying Einstein synchronization along a closed contour. However, this inconsistency persists and plays an important role in the interpretation of experiments, such as the linear Sagnac effect, which is reconsidered below for the convenience of the reader in a didactical presentation.

3. Absolute *versus* relative simultaneity

The conventionalist scenario implies that the Lorentz transformations can be substituted by transformations based on absolute simultaneity, a substitution hardly acceptable by physicists who have been using the symmetry properties of the Lorentz group for decades in applications in several branches of modern physics, such as elementary particle physics, astrophysics, and quantum physics. Moreover, epistemologists (*29*) claim that a theory is physically meaningless unless its basic postulates can be tested. From that perspective, it is crucial for the standing of the theory that the physical equivalence (or not) between preferred frame theories and SR be satisfactorily tested and verified, at least in principle. Advances have been made by Spavieri, Rodriguez and Sanchez (*13*) who have recently shown that, in general, Einstein and absolute synchronization predict different observable results, thus suggesting that the two synchronizations are not physically equivalent. It follows that, at least in principle, Einstein's second postulate of a universally constant speed of light can be verified experimentally. In this case, standard SR with Einstein synchronization maintains its unique physical meaning and can be tested against theories that assume the existence of an identifiable preferred frame.

Nevertheless, several interpretations of the experiments supporting SR and solutions to presumed paradoxes of SR have been given (*7*, *14*, *30*) within the assumption that the conventionalist point of view is valid, while now this is being questioned (*13*) with implications for the equivalence between SR and preferred frame theories (*31*). Because of the above-mentioned difficulties in the interpretation of the Sagnac effect, there seem to be sufficient motivations for looking into the validity of Einstein's second postulate on the constancy of the speed of light c. For the convenience of the reader we review in the next sections some of the interpretations of optical experiments related to the Sagnac effect, without assuming that absolute and Einstein synchronizations are equivalent. Several authors have already pointed out that when adopting relative simultaneity, inconsistencies with Einstein's second postulate emerge (*12*, *16*, *20*, *24*). Instead, as generally acknowledged (*7*, *12*, *14*, *16*, *20*, *24*, *30*), no inconsistencies are found by adopting absolute simultaneity.

3.1. The local light speed in the Sagnac effect

In the standard (circular) Sagnac effect (Figure 2(a)), two counter-propagating light signals (photons) are emitted by the source (interferometer, or clock C) co-moving with a rotating disk along the circumference $2\pi R$. If $v = \omega R$ is the velocity of the clock (or interferometer) fixed on the rotating disk circumference relative to the laboratory frame, to the first order in v/c the time interval recorded by clock C for round-trip light counter-propagation is given by $t_+ = 2\pi R/(c + v)$, and $t_- = 2\pi R/(c - v)$ for light co-propagation. Then, the time delay between the time of arrival of the two light signals back to clock C observed by Sagnac, is $\Delta t = t_- - t_+ = 4\pi(v/c)R/c$, valid to the first order in v/c (non-relativistic approximation where the Newtonian and Einsteinian interpretations coincide). There are countless descriptions and interpretations of the Sagnac effect in the literature. The most common interpretation (*32*) is done from the laboratory frame, where the centre of the disk is stationary and the light speed c is assumed to be isotropic and constant. In this case, the result that t_+ is different from t_- is due to the fact that the two light signals are seen as traversing paths of different lengths, as measured from the lab frame, in their motion relative to clock C.

However, the problem is not in foreseeing the result of the Sagnac experiment, but in verifying the validity of Einstein's second postulate of the constancy of the local speed of light along its circular path, considering that Newton and Einstein predict different values for the speed of light relative to the observer co-moving with clock C. In the Sagnac effect, the local speed c at a point on the circumference $2\pi R$ of the rotating disk of radius R, is the local speed measured in a Lorentz inertial frame instantaneously co-moving with the rotating frame at the distance R from the centre. Thus, Sagnac (*5*) claimed that his experiment disproves standard SR because, relative to clock C, the length of the round-trip path is $2\pi R$ for both light signals and the correspondent time recorded by the

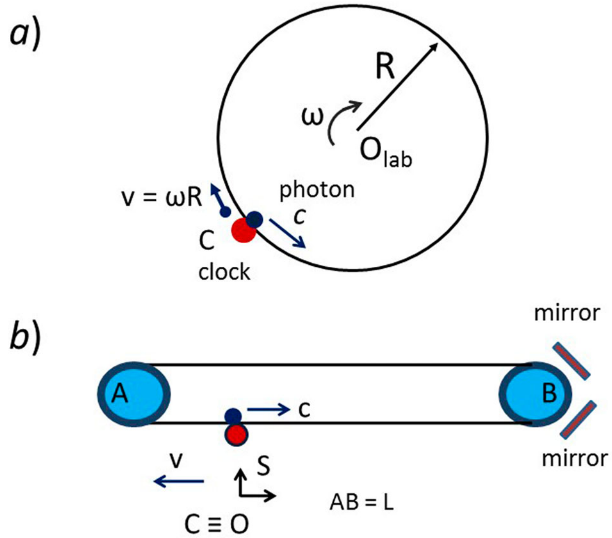

Figure 2. The experimental arrangement for the standard Sagnac effect is shown in (a), where counter-propagating light pulses are emitted from the location of clock *C* on the turntable, which is rotating with constant angular velocity. The 'conveyor belt' linear equivalent of the classical Sagnac arrangement is shown for the new Gedankenexperiment in (b), where now the clock *C* is fixed to the moving belt and co-moving with the origin of frame *S*, in motion with velocity *v* relative to the pulleys *A* and *B* driving the clockwise movement of the belt. A light pulse, emitted from clock *C*, is propagating with speed *c* relative to the clock and frame *S*.

clock should be $t_\pm = t = 2\pi R/c$. If this were the case, we have $\Delta t = t_- - t_+ = 0$, contrary to observation and thus disproving Einstein's second postulate on the constancy of the speed of light. Sagnac's contentions were later highlighted by Selleri's paradox (*12*), which indicates that, in special conditions when the circular motion of the clock is nearly rectilinear, the interpretation of the Sagnac effect requires the local one-way speed of light to be $c + v$ or $c - v$ at a point of the disk circumference moving with velocity $v = \omega R$, again invalidating Einstein's second postulate.

Concerning the interpretation of paradoxes – such as the 'twin paradox', which (as pointed out by Mansouri and Sexl (*7*)) is no longer a paradox when interpreted assuming absolute simultaneity – if absolute and Einstein synchronization are equivalent, it can be argued that the same paradox must be interpretable equally well by adopting either absolute or relative simultaneity. Similar considerations can be made in relation to any other paradox, such as the Selleri paradox (*12*) related to the Sagnac effect. The Sagnac effect and the Selleri paradox have been addressed by several authors (e.g. (*14*, *16*, *19*, *20*, *30*)) and most of them describe the problem from a single frame of reference, generally the laboratory frame (*32*), even adopting absolute synchronization (instead of relative) (*14*, *30*) adhering to the view of conventionalism that absolute and Einstein synchronization are equivalent. In effect, as it occurs for other experiments supporting SR, the outcome of the Sagnac effect is synchronization-independent and, thus, foreseen by SR adopting either Einstein or absolute synchronization. Because of this, many interpretations of the Sagnac effect are missing this crucial point: the fact that different synchronizations provide the same round-trip result in the Sagnac effect, does not tell or reveal what the local speed of light is in every section of the light path. Therefore, these descriptions are not suitable for discriminating absolute from relative simultaneity and unable to verify the validity of Einstein's second postulate on the constancy of the speed of light.

Thus, for the purpose of testing absolute *vs* relative simultaneity, we reconsider below the Sagnac effect in its linear version (*20*, *24*, *30*, *33*) and, by means of our Gedankenexperiment, check which synchronization describes it coherently.

4. Time continuity in the flow of space-time events. Connection with the linear Sagnac effect

In the context of relativistic theories in classical physics, the readings of a clock display progressively the flow of time in correspondence to the sequence of physical events that are taking place in space-time (It is not excluded that time is discrete at the quantum level, where the smallest time interval can be related to the Planck constant.). In order to better visualize and interpret the proper time intervals measured by the (synchronization-independent) clock *C* when the light pulse performs a round trip, starting from *C* and then back to *C*, we connect our thought experiment with the actual experiment that verifies the linear Sagnac effect.

As a linear version of the Sagnac effect (*20*, *24*, *30*, *33*), our Gedankenexperiment is represented by the conveyor belt system shown in Figure 2(b), equivalent to the standard circular version of Figure 2(a). The sequence of events taking place within the conveyor belt system is the following. Clock *C* is initially co-moving with the origin *O* of *S*, as shown in Figure 2(b) and 3 (upper scenario). The clock, co-moving with the belt clockwise has velocity $v = u/2$ relative to the conveyor system S_{cs} and the centres of the pulleys driving the belt. Since for the purpose of our thought experiment relativistic effects of order higher than the first in v/c (or u/c) can be neglected in the interpretation of the Sagnac experiment, it is simpler to use the transformations (1) to first order by setting the factor $\gamma \simeq 1$ from the start (for transformations from S to S_{cs} or from S to S'). When located in the lower section

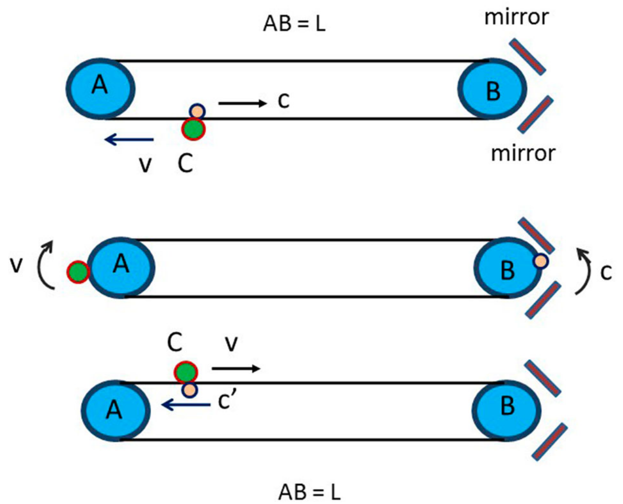

Figure 3. In the upper scenario of the figure, the clock *C* is fixed to the frame *S* instantaneously co-moving with the lower section of the belt towards pulley *A*, and it emits a light pulse toward the pulley *B*. Then, as shown in the middle scenario, after reaching the pulley *A*, clock *C* continues to co-move with the belt, but now on its upper part. Meanwhile, the light pulse is reflected from the mirror arrangement shown around pulley *B*. (An equivalent arrangement would have the pulse change direction if propagating inside an optical fibre that runs along the length of the belt.) Finally, as shown in the lower scenario, the pulse eventually reunites with clock *C* in the upper section of the belt.

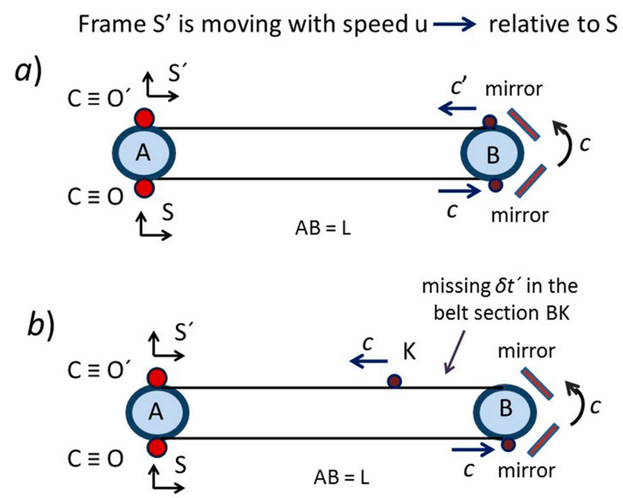

Figure 4. A complete visualization of the new linear Sagnac Gedankenexperiment is presented here in a form that shows its equivalence with the original conceptual arrangement presented in Figure 1.

of the belt, clock C emits a light pulse toward the pulley B and then, after reaching the pulley A, is co-moving with the upper part of the belt, as in Figure 3 (middle scenario). In the meantime, the light pulse reaches the pulley B from where it is reflected (or is made to change direction if propagating inside an optical fibre-belt) as in Figure 3 (middle scenario). The light signal eventually reunites with clock C in the upper section of the belt, as shown in Figure 3 (lower scenario).

Our purpose here is to check the consistency of the theory interpreting the experimental results and verify what is the value of the local light speed along the belt light path. Obviously, the local speed must correspond to the measurements of clocks stationary on the belt and co-moving with it, such as clock C.

Let us suppose that, the light pulse has been emitted by clock C (acting as a source and co-moving with the conveyor belt) at some previous time and has reached the pulley B, in motion with velocity $v = u/2$ relative to C, when it is at the distance L from C at the later time $t = 0$. This is possible if we assume in frame S (which could be coinciding with the unique 'identifiable' preferred frame where space is isotropic and the speed of light is c) the following initial conditions. Since the conveyor arm AB has length L, if B is initially at the distance $L(1 - v/c)$ from $C \equiv O$, at the time t the pulse has moved to ct and point B has moved to $L(1 - v/c) + vt$. Then, the pulse reaches B when $ct = L(1 - v/c) + vt$, i.e. after the time interval $t = T_{CB} = L/c$. Since the pulley A is initially at the distance $-(v/c)L$ from $C \equiv O$, we find that A reaches C when $-(v/c)L + vt = 0$, i.e. after the same time interval L/c. Therefore, the two events ('light pulse at B' and 'clock C at A') are simultaneous in frame S and, concerning what is seen by an observer in frame S at $t = 0$, the physical situation of Figure 4 matches the one of Figure 1.

We assume that the conveyor arm $AB = L$ is much larger than the radius r of the pulley, specifically, $(u^2/c^2)L >> r$, as derived in Appendix. In this case, the circular Sagnac effect of Figure 2(a) is essentially completely linearized by the equivalent system of Figure 2(b). At the time $t = t' = 0$, clock C, being fixed to the moving conveyor belt, moves from frame S to frame S' after being accelerated and acquiring the velocity u relative to S, in the negligibly short time $\tau \ll uL/c^2$ (as shown in Appendix). This way, starting from the time $t' \simeq 0$, clock C will be co-moving with O' of frame S'. In the meantime, the light pulse reflected at B is now moving toward $C \equiv O'$ in the upper part of the conveyor belt and the physical situation is the same as described in our Gedankenexperiment. The length of the conveyor belt (the 'ground' path) is $2L$ and the sequence of path sections, covered by the light pulse in its round trip, starts from $C \equiv O$ to reach B and then, after reflection, from B back to $C \equiv C' \equiv O'$. The round-trip proper time T_{round}, taken by the light pulse to cover the round-trip ground path of the conveyor belt and measured by the single clock C, is known and must correspond to the sum of the proper time interval measured by $C \equiv O$ when in the lower part of the conveyor belt, plus the proper time interval measured by $C \equiv C' \equiv O'$ when in the upper part. Depending on the type of synchronization adopted and its interpretation, the sequence of time intervals displayed by the measuring device (clock C), following the progression of the light pulse in its round trip, is the following.

In the case of absolute synchronization (Figure 4(a)), T_{round} may be expressed as

$$\text{abs simult}: T_{round} = T_{CB} + T'_{BC} = T_{CB} + t'^{abs}_{BO'} = \frac{CB}{c} + \frac{BC}{c+u} = \frac{L}{c} + \frac{L}{c+u} = \frac{2L}{c+u/2} = \frac{2L}{c+v}, \tag{7}$$

where $u \simeq 2v$ and v is the velocity of the belt relative to the conveyor arm AB. In (7), the term $T_{CB} = L/c$ represents the proper time interval, measured by clock $C \equiv O$ co-moving with O, taken by the pulse to reach B starting from C in the outward path. The term $T'_{BC} = t'^{abs}_{BO'} = L/(c+u)$ represents the time interval measured by $C \equiv O'$ taken by the pulse to cover the return path $BC = L$ and reach clock $C \equiv O'$. The average light speed over the ground path $2L$ is superluminal and given by $c+v$. The local light speed is c along the lower section of the belt CB, and $c+u = c+2v$ along the upper section of the belt in the return trip.

If instead Einstein synchronization is applied in S' (Figure 4(b)), the round-trip proper time T_{round} is expressed as

$$\text{rel simult}: T_{round} = T_{CB} + T'_{KC} = T_{CB} + t'^{rel}_{KO'} = \frac{CB}{c} + \frac{KC}{c} = \frac{L}{c} + \frac{L(1-u/c)}{c} = \frac{2L(1-v/c)}{c} = \frac{2L}{c+v}, \tag{8}$$

where the term $T_{CB} = L/c$ represents again the proper time interval measured by clock $C \equiv O$ before turning around the pulley A. However, because of non-conservation of simultaneity, when at $t' = 0$ clock $C \equiv O'$ starts co-moving with O', the pulse is already at point K, at the distance $L(1-u/c)$ from C. Then, if the pulse travels at the local speed c in the frame S' where C is now at rest, the term $T'_{KC} = t'^{rel}_{KO'} = L(1-u/c)/c$ in (8) measured by the clock C, represents the proper time taken by the light pulse to cover the path section KC.

We stress that in both (7) and (8) the correct interpretation of the proper time of clock C requires separating the readings displayed when C is co-moving with O in S, from those displayed when it is co-moving with O' in S'. In any case, the readings of C correspond always to the readings of an inertial frame, S in the outward trip, and S' in the return trip. We notice that, with absolute simultaneity, to the round-trip proper time T_{round} in expression (7) contribute the partial time delays taken by the light pulse to cover every section of the belt of length $2L = CB + BC$. However, with relative simultaneity, in the round-trip time T_{round} of expression (8), we have the partial time delays corresponding to the sections CB and KC, but the time delay taken by the light pulse to cover the section BK is missing! The closed contour $CB + BC$ is reduced to the open contour $CB + KC$.

It follows that, by adopting Einstein synchronization in order for the local speed of light to be c in the return trip, non-conservation of simultaneity introduces a space-time discontinuity that eliminates the section BK $= \delta t' c = (u/c)L$ and shortens the return light path L to $L(1-u/c)$. Notice that the term $\delta t'$ is related to Kassner's 'unphysical' time gap (14), as pointed out by Selleri (12), Gift (16), Spavieri, Rodriguez and Sanchez, and Spavieri and Haug (24), among many other authors. If the section BK is not eliminated and the contour is closed, result (8) becomes,

$$\text{rel simult}: T_{round} = T_{CB} + T'_{BK} + T'_{KC} = \frac{CB}{c} + \frac{BK}{c} + \frac{KC}{c} \tag{9}$$

$$= \frac{L}{c} + \frac{u}{c}\frac{L}{c} + \frac{L(1-u/c)}{c} = \frac{2L}{c}, \tag{10}$$

in agreement with Einstein's second postulate but in conflict with observation (as Sagnac claimed). However, agreement with observation is restored if we accept the possibility that the speed of light in the section BK is not c, but superluminal and nearly infinite, c_∞, so that $T'_{BK} = BK/c_\infty \sim 0$ is negligible.

4.1. Remarks on the discontinuity in the flow of time

In his Minkowsky analysis of the circular Sagnac effect, Kassner (14) is met with a discontinuity related to the speeds $c+v$ and $c-v$ of Selleri's paradox. Because of it, in order to confirm that the local speed of light is c along the disk circumference, Kassner tries to justify the discontinuity by introducing the unusual concept of a 'time gap' and states that 'the speed of light is c everywhere except at the point on the circle where we put the time gap '. Such a proposal has been objected to by Gift (16) who claims that it is a 'theoretical construct that has no basis in reality because based on an unphysical time discontinuity '. Interestingly, as another attempt to solve the paradox, Kassner (*14*) suggests what was already proposed by Selleri (*12*), i.e. that SR could, or should, adopt absolute synchronization for the purpose of interpreting the Sagnac effect. This way, a light speed $c + v$ or $c-v$ is obtained and, thus, Kassner dismisses Selleri's paradox (implying a light speed $c + v$ or $c-v$) on the basis that the Sagnac effect can be equivalently described with either Einstein or absolute synchronization. A similar position in relation to the Sagnac effect is held by other authors, such as Tartaglia (*30*) who states that '... In the observer's reference frame, light will be expected to have speed $c-v$ in one direction (forward) and $c + v$ in the other'. Obviously, a light speed different from c is in contrast with Einstein's second postulate and it is not clear why, by choosing to adopt absolute instead of Einstein synchronization (when there are problems with it), we should expect that the latter is being corroborated. In fact, if we consider possible, as indicated recently by Spavieri et al. (*13*), that Einstein and absolute synchronization are not physically equivalent, the one-way speed is either constant or not, and thus absolute and relative simultaneity cannot be arbitrarily interchanged. Moreover, a synchronization procedure is physically meaningful if it can provide a coherent interpretation of a physical effect, such as the one of Sagnac. Consequently, if there are problems or inconsistencies in interpreting the Sagnac effect with a specific synchronization, these inconsistencies need to be solved within the framework of the specific synchronization adopted and cannot disappear by interpreting the Sagnac effect by means of a different synchronization. Indeed, if a synchronization (such as absolute synchronization) coherently describes physical effects, it does rule out any other synchronization that does not provide an equivalent coherent description.

Physically, the standard perspective is that space-time events occur sequentially in correspondence with the continuos flow of time that can be measured by a convenient device (clock C, in our case). In the sequence of time intervals measured by clock C of expression (8) derived with Einstein synchronization, we have neglected the small time delay τ experienced by clock C in passing from frame S to S'. Then, according to our Gedankenexperiment applied to the linear Sagnac effect, we find that, for an observer in S the light pulse is at B when clock $C \equiv O$ displays the time $t = 0$, as indicated in Figures 1 and 4. Instead, due to non-conservation of simultaneity, for an observer in S' the pulse is at K when clock $C \equiv O'$ displays essentially the same epoch $t' = t = 0$, as shown in Figure 1(b) and Figure 4(b). It is apparent from (8) that relative simultaneity has the effect of introducing a discontinuity in the flow of time measured by clock C. Indeed, since the pulse covers the section BK at $t' < 0$, the interval $\delta t' = uL/c^2$, corresponding to the time taken by the pulse to cover the section $BK = \delta t' c = uL/c$, is not accounted for in (8). It could be argued that the theory is incapable of justifying the unusual behaviour of the light pulse in the section BK. In fact, space-time continuity requires that the time interval $\delta t'$ appears in the sequence of measurements made by clock C, as done in (9). Although unlikely, a possibility for maintaining space continuity along the closed contour is to conceive that the speed of light in the section BK is superluminal and nearly infinite, c_∞, so that $T'_{BK} = BK/c_\infty \sim 0$ becomes negligible.

When the clock (measuring apparatus) moves from frame S to S', the effect of non-conservation of simultaneity on the behaviour of the ray of light in its round trip becomes manifest, as described above. The mentioned effect is not perceivable when, during the photon's round trip, the clock keeps on the same section of the belt (e.g. the lower section), because non-conservation of simultaneity and Einstein synchronization are not involved. Then, in this case, it is not easy to verify that the local light speed be c in the other section, a task that we shall consider in future contributions.

In any case, it seems that the meaningful expression that preserves space-time continuity and coherently interprets the readings of clock C, is the one derived adopting absolute simultaneity and given by (7). If the local speed of light is c in the belt lower section CB, the expression $L/(c + 2v) = L/(c + u)$ of (7) implies the superluminal local light speed $c + 2v$ in the belt upper section BO', leading to the superluminal average light speed $c + v$, as in the circular Sagnac effect.

5. Conclusions

We have considered the behaviour of a light pulse when performing a closed path, determining the local speed of light in the different sections of the path when both absolute and Einstein synchronization are separately adopted. In the interpretation of expression (8), the approach with relative simultaneity seems unable to justify the shorter light path $2L(1 - v/c)$ covered by the photon and the unusual behaviour of light propagation along the section BK. Researchers (*12*, *14*, *16*, *21*, *24*) ascribe this difficulty to the well-known fact that Einstein synchronization along a closed contour is impossible in general. However, by adopting the LTA with absolute synchronization, we obtain a coherent interpretation of light propagation in the linear Sagnac effect, as reflected in expression (7). We do not exclude a priori that other interpretations, eventually linked to quantum mechanics, are possible. Nevertheless, in the context of classical relativistic theories, our Gedankenexperiment would favour absolute simultaneity, in line with and corroborated by the criticism to relative simultaneity pointed out by several authors in the context of the Sagnac effect (*5*, *12*, *16*, *20*, *24*). Any inconsistency inherent to Einstein's second postulate can be interpreted as an additional indication that Einstein and absolute synchronizations may not be physically equivalent, as noted independently in Ref. (*13*), where it is shown that, in principle, testing absolute versus relative simultaneity is possible.

Students and researchers are routinely taught that Einstein has the merit of having introduced the relativistic effects of time dilation and length contraction, inherent to the LT. These relativistic effects are inherent to the LTA also and, representing the core of the theory, make possible the interpretation of the experiments supporting SR (*7*, *9*, *10*, *12*, *14*, *16*, *19*, *20*). Thus, in a balance between Einstein and absolute synchronization, considering that with absolute synchronization the Sagnac effect and Selleri's paradox can be interpreted and solved without inconsistencies, the formulation of SR with absolute simultaneity appears to be an interesting approach that might stimulate new debate on an old topic. With absolute synchronization, SR reinstates conservation of simultaneity, but allows for all relativistic effects, including the relativity of time because of its dependence on the velocity, as expressed by the effect of time dilation. Therefore, it could be suggested that one possible research strategy, when faced with choosing either the LT or the LTA for developing a theory or a physical model, might consist of exploring how the theory or model is modified by adopting absolute instead of relative simultaneity. Most likely, this approach will lead to interesting new scenarios that can advance our knowledge of relativistic theories.

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Appendix. Estimating the time delay τ of clock C while turning around the pulley

Let us indicate by τ the time delay suffered by C in the process of turning around the pulley A. Generally speaking, τ may depend on the chosen radius r of the pulley and the velocity $u = 2v$, but not on the length L of the arm AB. Since τ is independent of L, we can choose L to be large enough so that $\tau \ll uL/c^2 = \delta t'$. In this case, the delay τ is small relative to the time delay $\delta t'$ due to non-conservation of simultaneity and can be neglected. Furthermore, we may relate τ to u and r as follows. Let us suppose that clock C, initially at rest in frame S, while turning around the pulley is accelerated up to the velocity u in the time interval $\Delta t \simeq \tau$. Expressing the average acceleration as $a = u/\Delta t$, we have $\Delta t \simeq \tau \simeq u/a$. For a pulley of radius r, $a = v^2/r \simeq u^2/r$ and $\Delta t \simeq \tau \simeq r/u$. With the assumption $\tau \ll uL/c^2$, we find $L >> rc^2/u^2$ for our thought experiment applied to the linearized version of the Sagnac effect.

The acceleration a has the effect to slow down the rate of time of the moving clock, as foreseen by the principle of equivalence (*22*). In this case, the time variation of the accelerated clock can be expressed as $\Delta t' = \Delta t(1 + \Delta\Phi/c^2)$, where $\Delta\Phi = ah = uh/\Delta t$ is the change of the corresponding equivalent gravitational potential Φ and h the change in position. Then, with $h \simeq u\Delta t \simeq a\Delta t^2 \simeq u^2/a$, we have $\Delta t' \simeq \Delta t(1 + ah/c^2) \simeq \Delta t(1 + u^2/c^2)$, indicating that the small contribution $\delta\tau \simeq \Delta t(u^2/c^2)$ (due to the acceleration) to the resulting time delay τ is of second order in u/c and, thus, negligible within our approximation.