The Lunar Orbit Throughout Time and Space

Adrienne Dove
Stuart Robbins
Colin Wallace

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What We See Today

To First Order

- The Moon Orbits the Earth*
- *Actually, the Moon orbits the Sun and is perturbed by the Earth. This orbit is elliptical (but the shape is changing) with a varying eccentricity, recessing nodes, and precessing apsides. The orbit is also inclined 5° with respect to the ecliptic.

The Orbit of the Moon and the Wobble of Earth

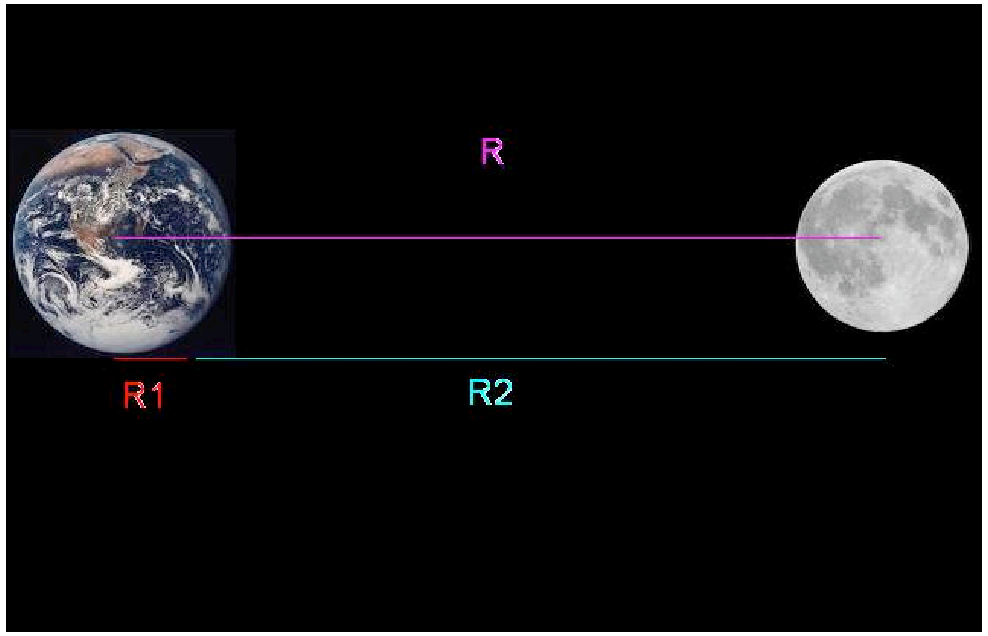
- The center of mass of the Earth-Moon system is given by:

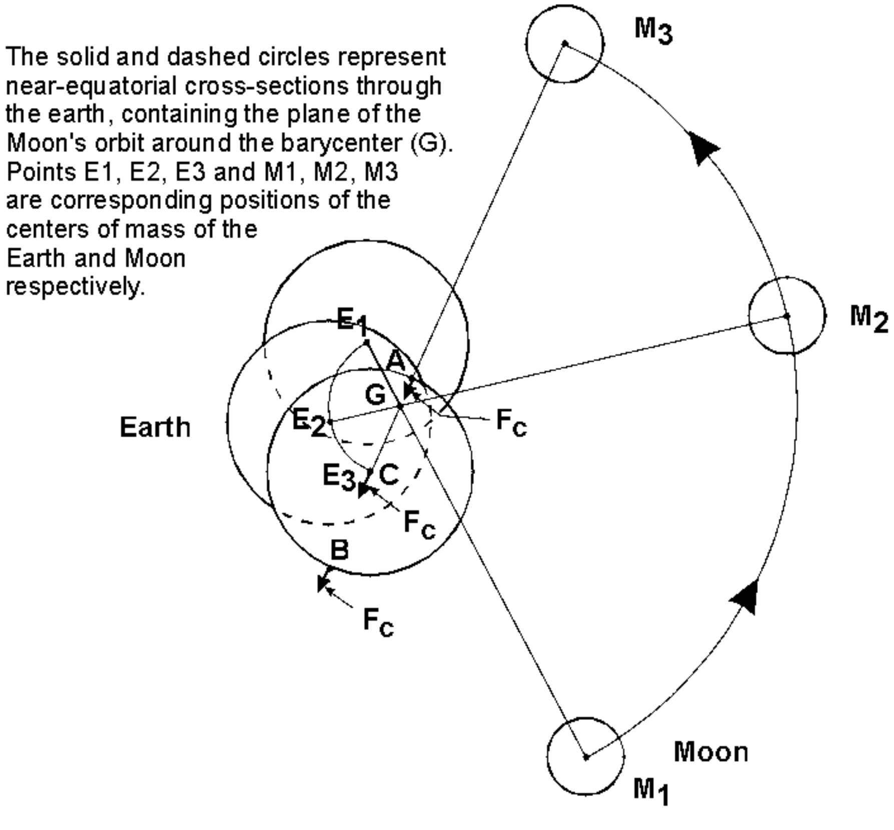
$$M_{\oplus} R_1 = M_{\text{moon}} R_2$$

$$R_1 = \frac{M_{\text{moon}}}{M_{\oplus}} R_2 = \frac{M_{\text{moon}}}{M_{\oplus}} \left(R - R_1 \right)$$

$$R_1 = R \frac{M_{\text{moon}}}{M_{\oplus} + M_{\text{moon}}}$$

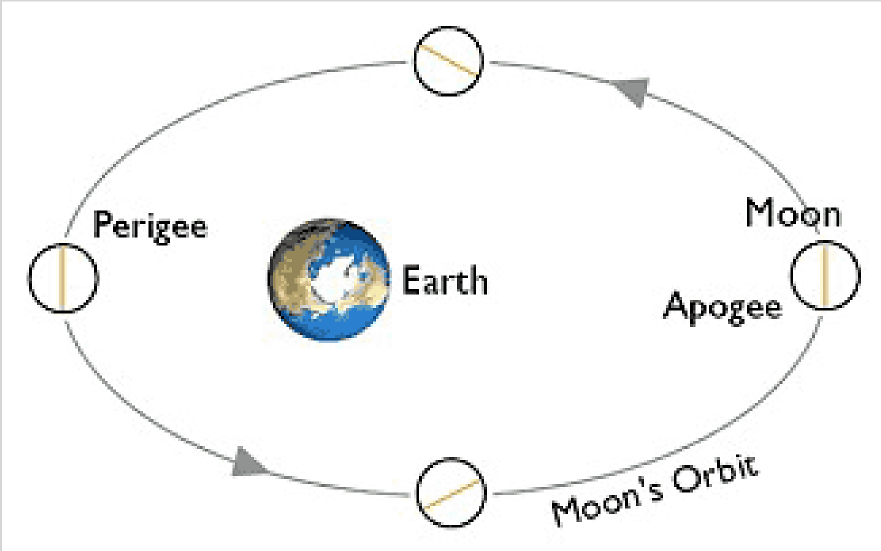
- This lies ~1800 km below Earth's surface -- thus, the Earth "wobbles."

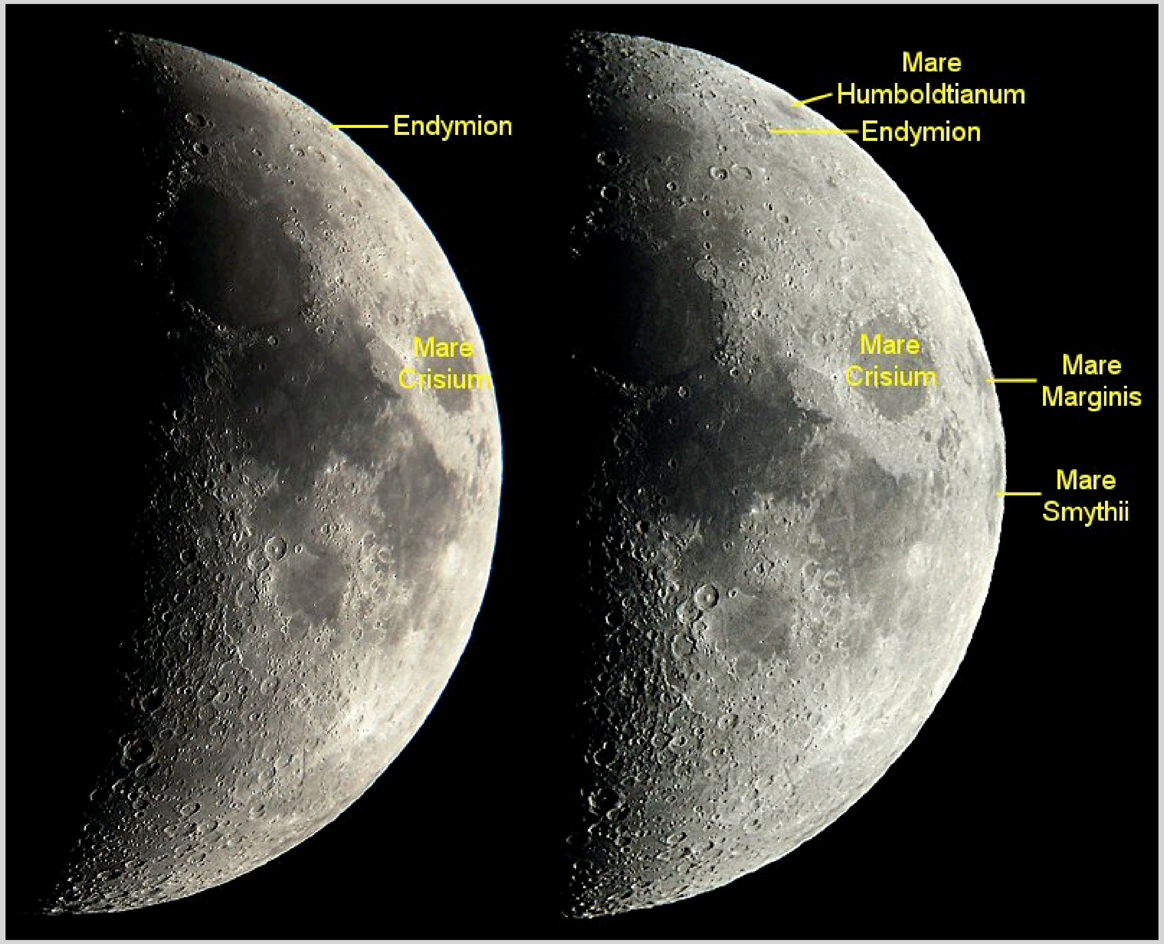




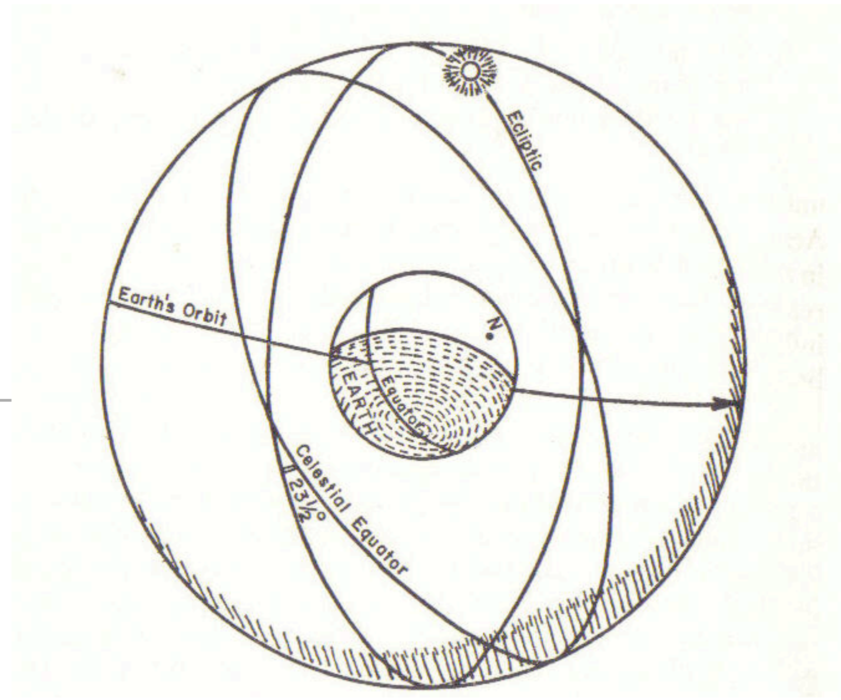
Longitudinal Librations in Action

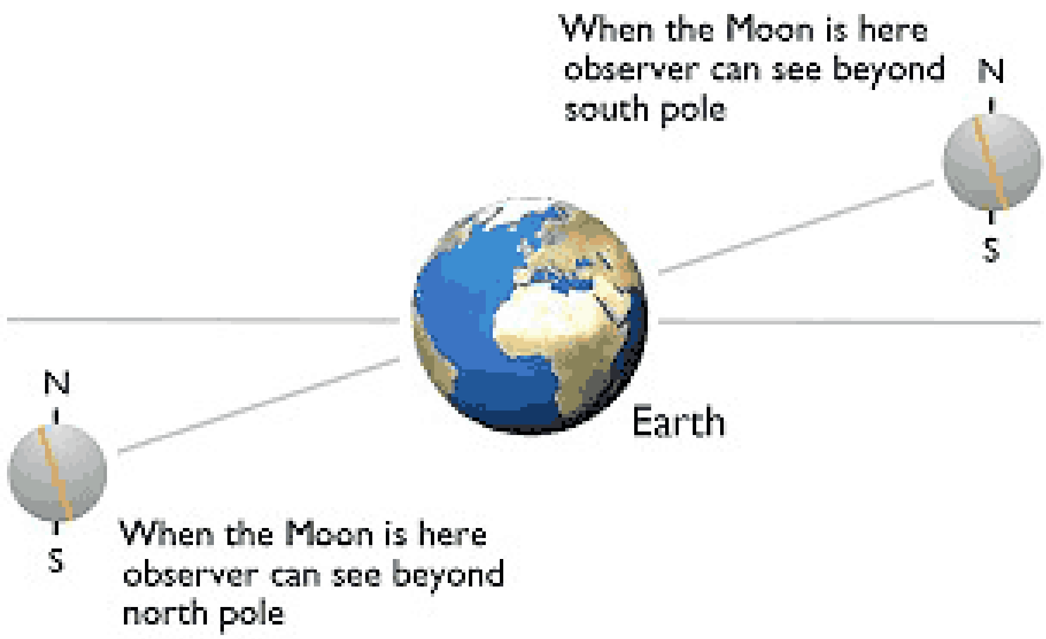
While the Moon's rotation rate may be constant, the fact that its orbit is slightly eccentric means that its orbital velocity changes. This causes longitudinal librations.





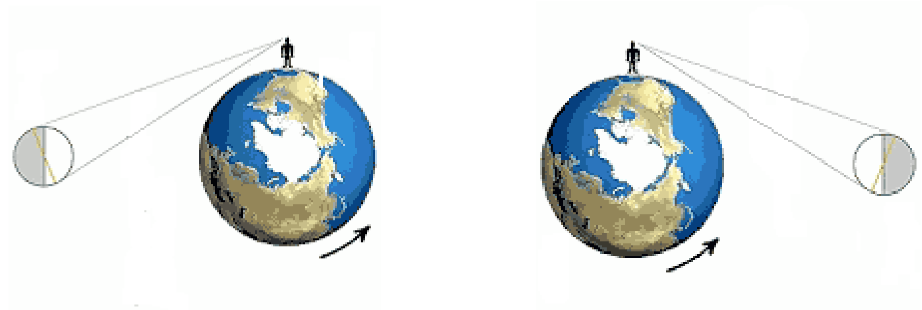
The Moon's Orbit is Inclined 5° to the Ecliptic

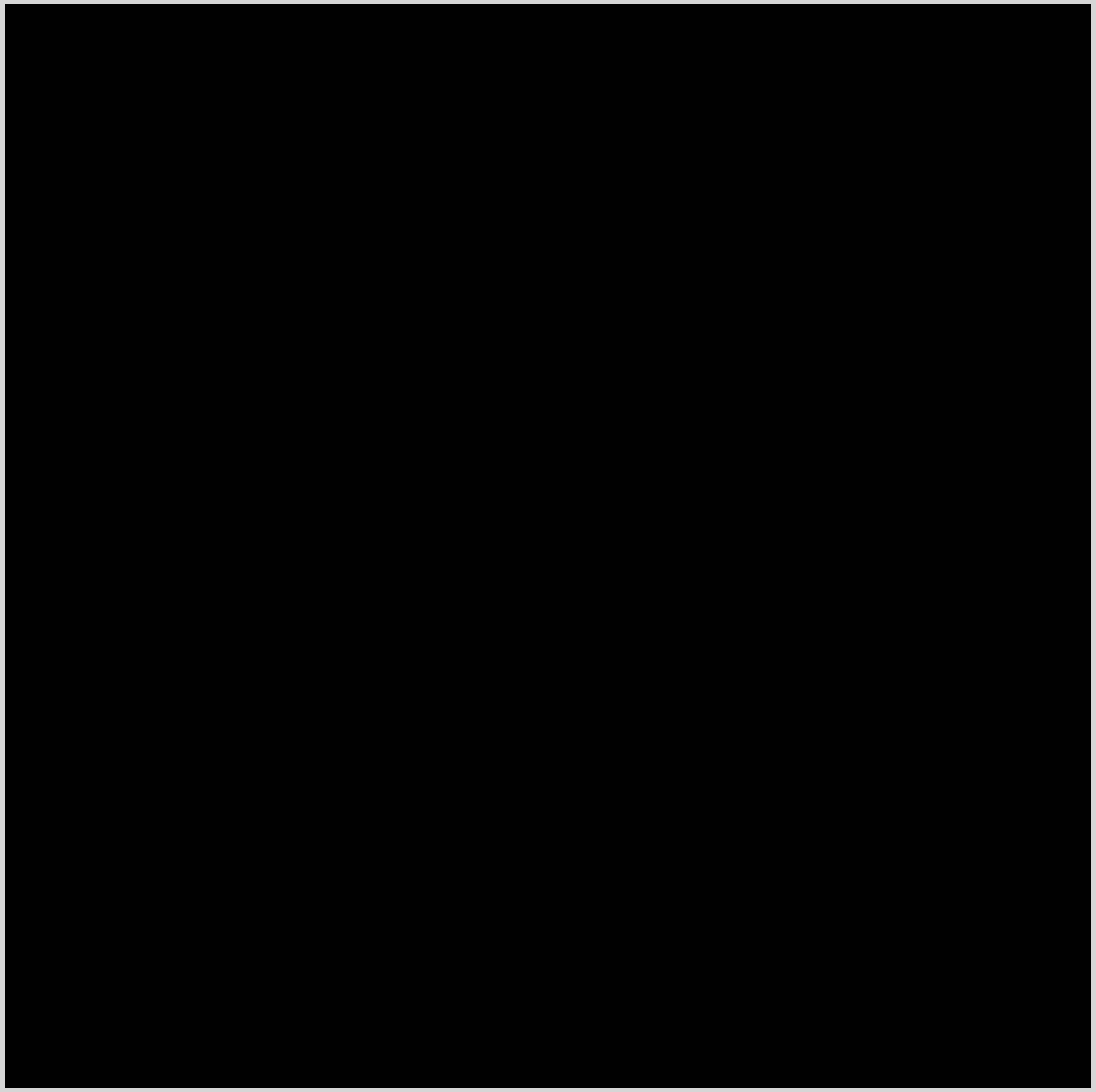




This results in latitudinal librations.

For completeness, there are also diurnal librations.

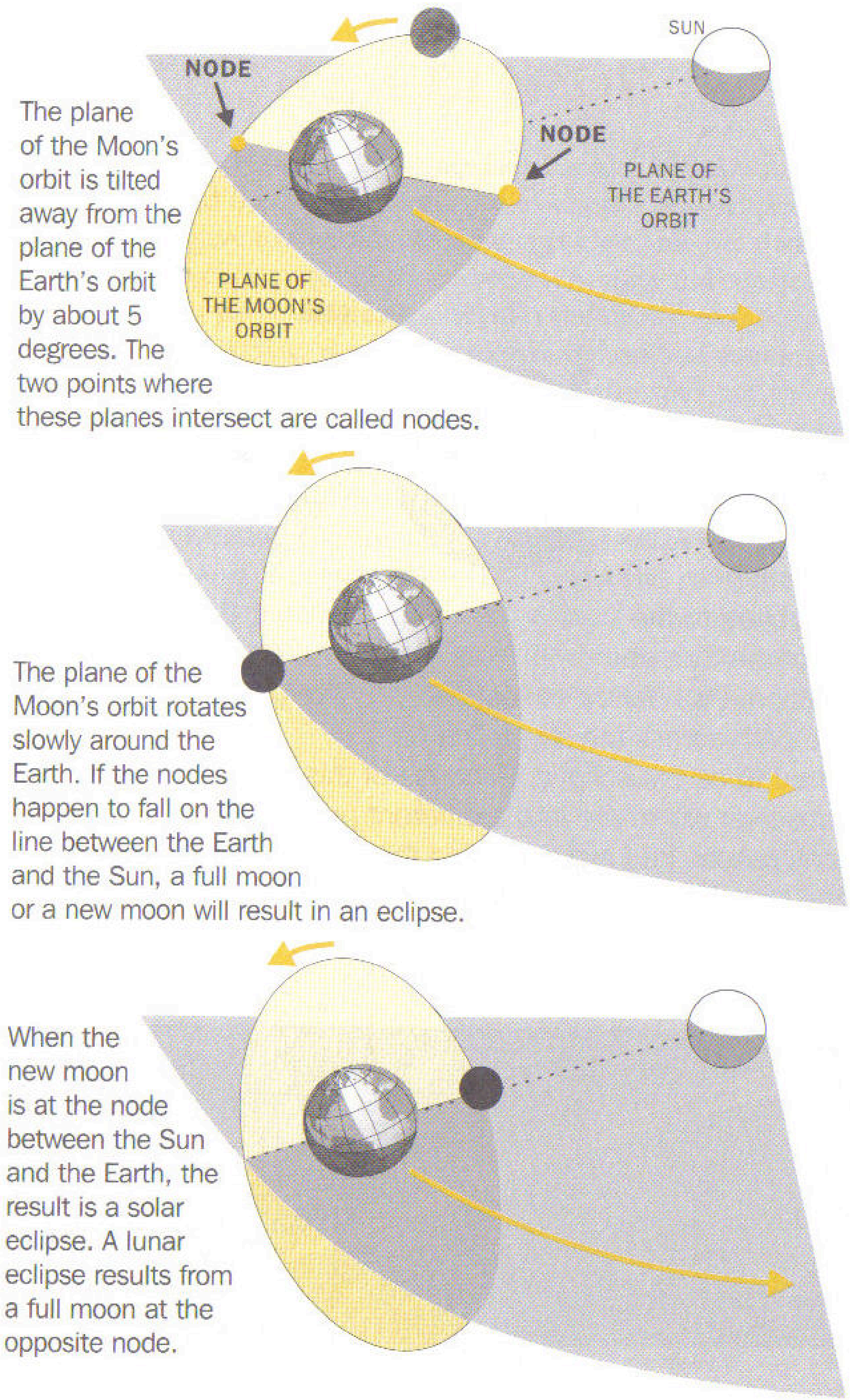


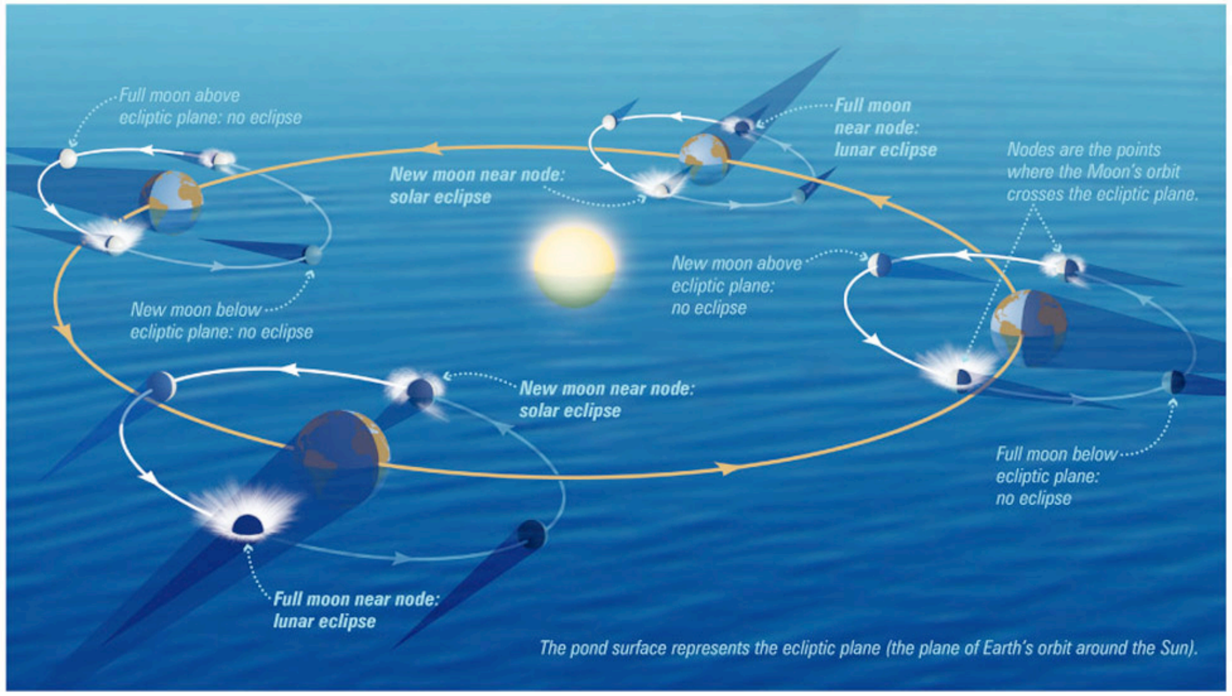


APOD: http://antwrp.gsfc.nasa.gov/apod/ap051113.html

One Lunation

Lunar Nodes

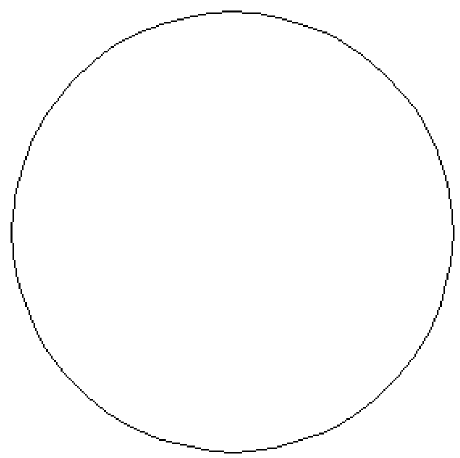




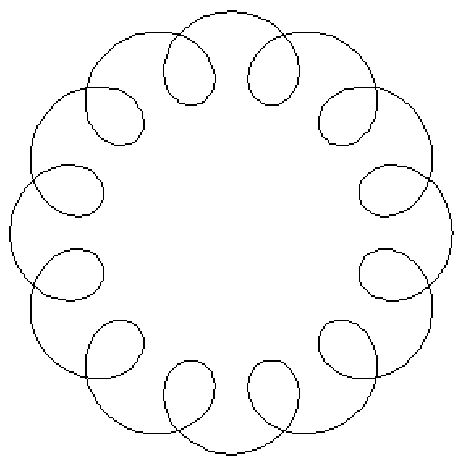
The two points where the lunar orbit intersects the ecliptic are called *nodes*. The location of a node determines whether or not an eclipse will occur.

The Moon's orbit "twists" over time and the position of the nodes changes. The nodes recess over a period of 18.6 years.

Moon's Orbital Path



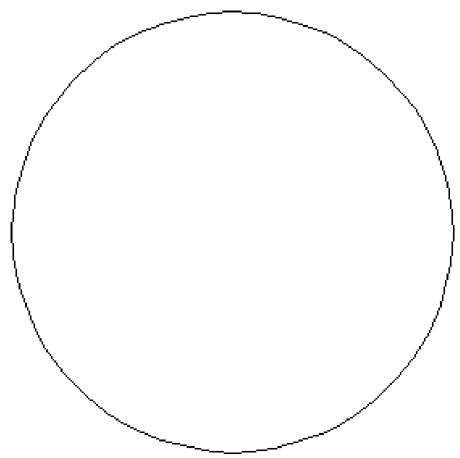
The Moon's orbit with respect to the Sun looks like this.



The Moon's orbit with respect to the Sun does NOT look like this.

Conclusion: *The Moon's orbit with respect to the Sun is convex.* But why?

Moon’s Orbital Path



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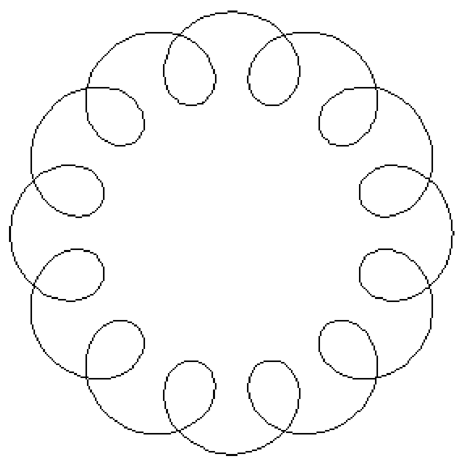

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Conclusion: *The Moon’s orbit with respect to the Sun is convex.* But why?

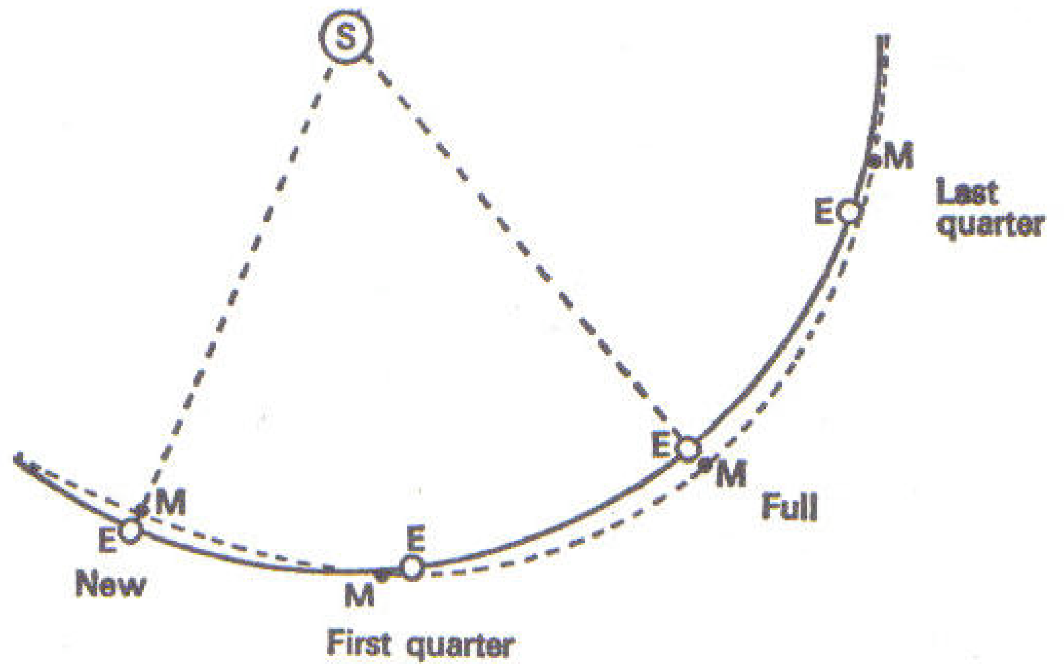
The gravitational force of the Sun on the Moon is more than twice the gravitational force of the Earth on the Moon:

$$\frac{F_{SM}}{F_{EM}} = \frac{GM_{\odot}M_{\text{moon}} \big/ R_{\oplus}^2}{GM_{\odot}M_{\text{moon}} \big/ R_{\text{moon}}^2} \approx 2.21$$

Moon’s Orbital Path

So the net force is directed toward the Sun. Force is proportional to acceleration via Newton’s second law. The acceleration is the second derivative of position, and concavity depends on the second derivative. Ergo, the Moon’s orbit is convex with respect to the Sun.

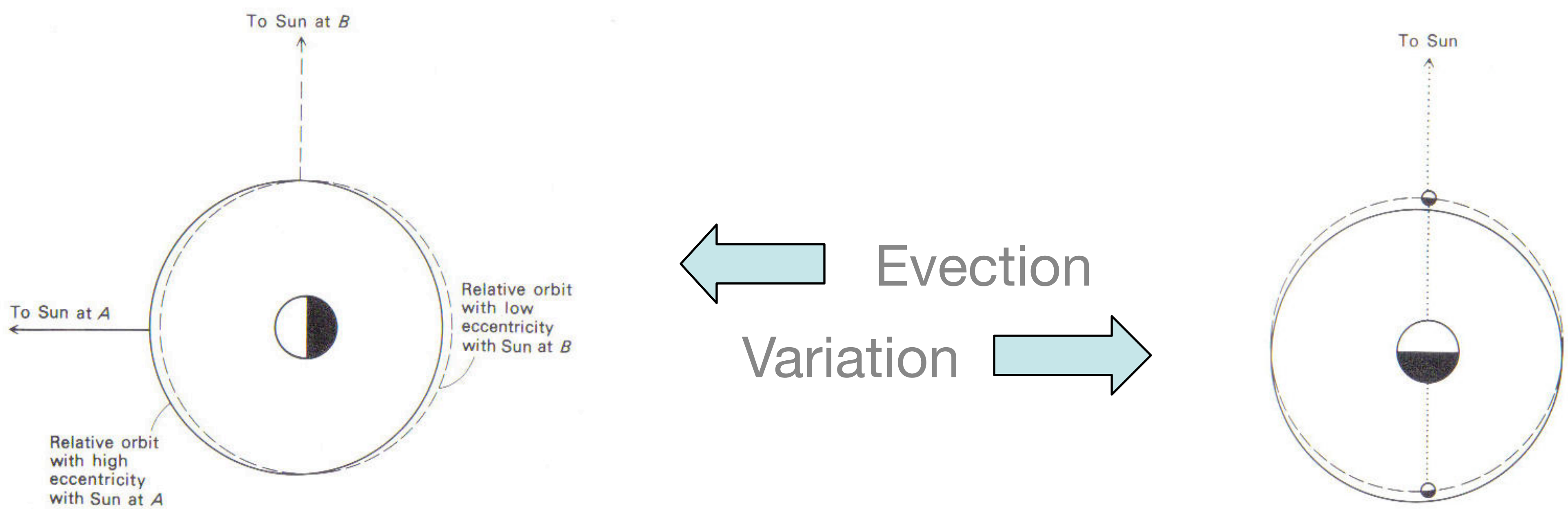
“…to an observer in space the Moon must appear as a normal planet, traveling in an elliptical orbit with the Sun in one of the foci.” (Moore 2001)



“The moon is really orbiting the sun, in a manner that is significantly perturbed by the gravitational field of the earth.” (Hodges 2002)

Apsidal Precession

The speeding up and slowing down of the Moon's circumsolar motion relative to Earth causes the line of apsides to precess with a period of 8.85 years.



Century	Max. Perigee	Max. Apogee
20th	221,451 miles	252,731 miles
21st	221,535 miles	252,728 miles

How the Moon's Orbit Evolved Through Time

Evidence for a Changing Orbit: Present-Day

- The Experiment: Apollo Lunar Laser Ranging (LLR)
- Observatories throughout the world have used these for 30+ years.
- The Results: Lunar recession rate is presently 3.82±0.07 cm per year.
- Running the results backwards, the moon is impossibly young, with its orbit only 1.5 Gyrs old.

$$\Delta t_0 = \frac{-2}{13}\frac{a^{13/2}}{f} = \frac{-2}{13}\frac{\left(3.84\cdot 10^{8}\ \mathrm{m}\right)^{13/2}}{6.29\cdot 10^{45}\ \mathrm{m}^{13/2}\cdot \mathrm{yr}^{-1}} \approx 1.5\ \mathrm{Gyr}$$

Evidence for a Changing Orbit: Historical

- Fossil Evidence for Tidal Period
- Sedimentary Evidence for Tidal Period
- Models Based Upon Ocean Resonances

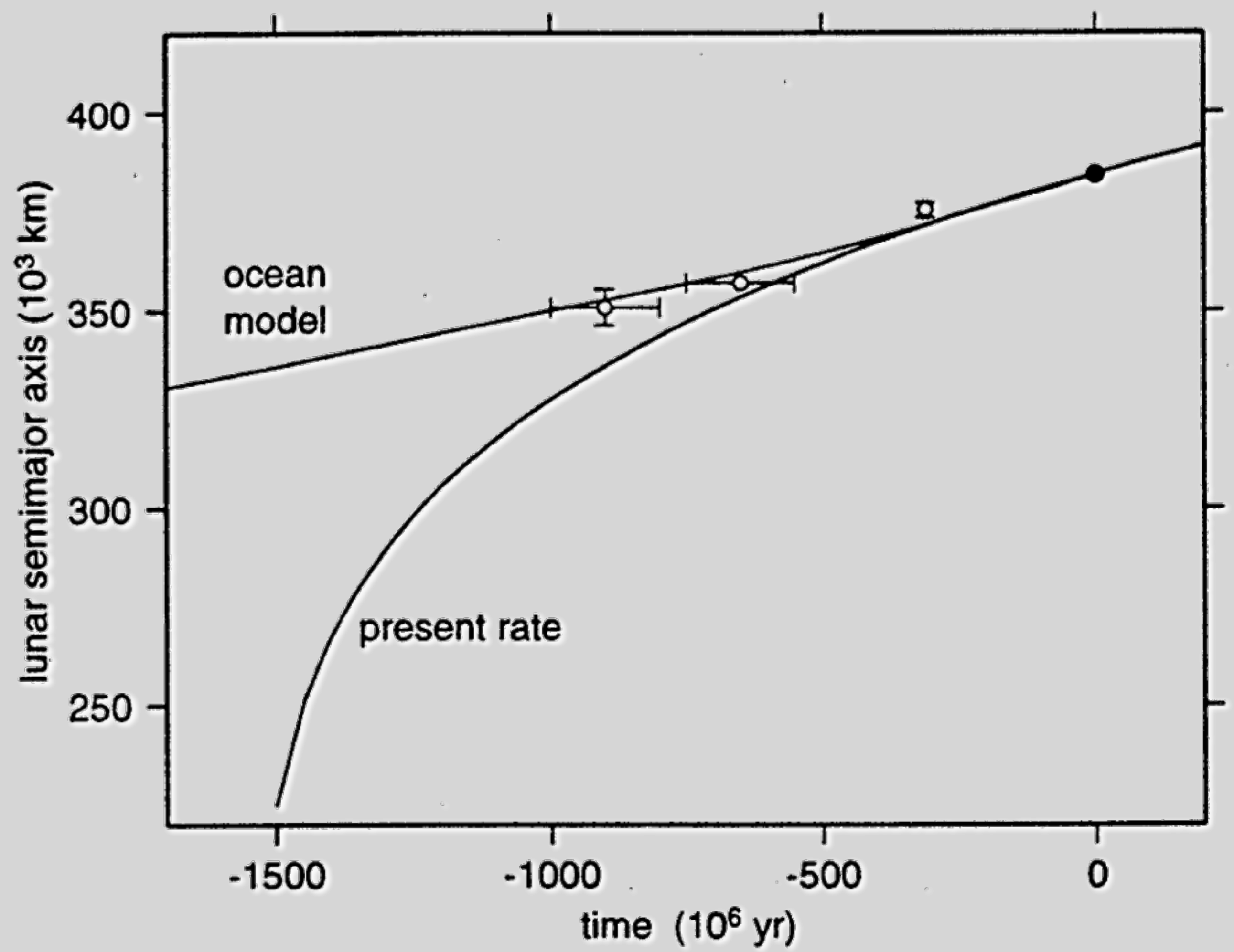
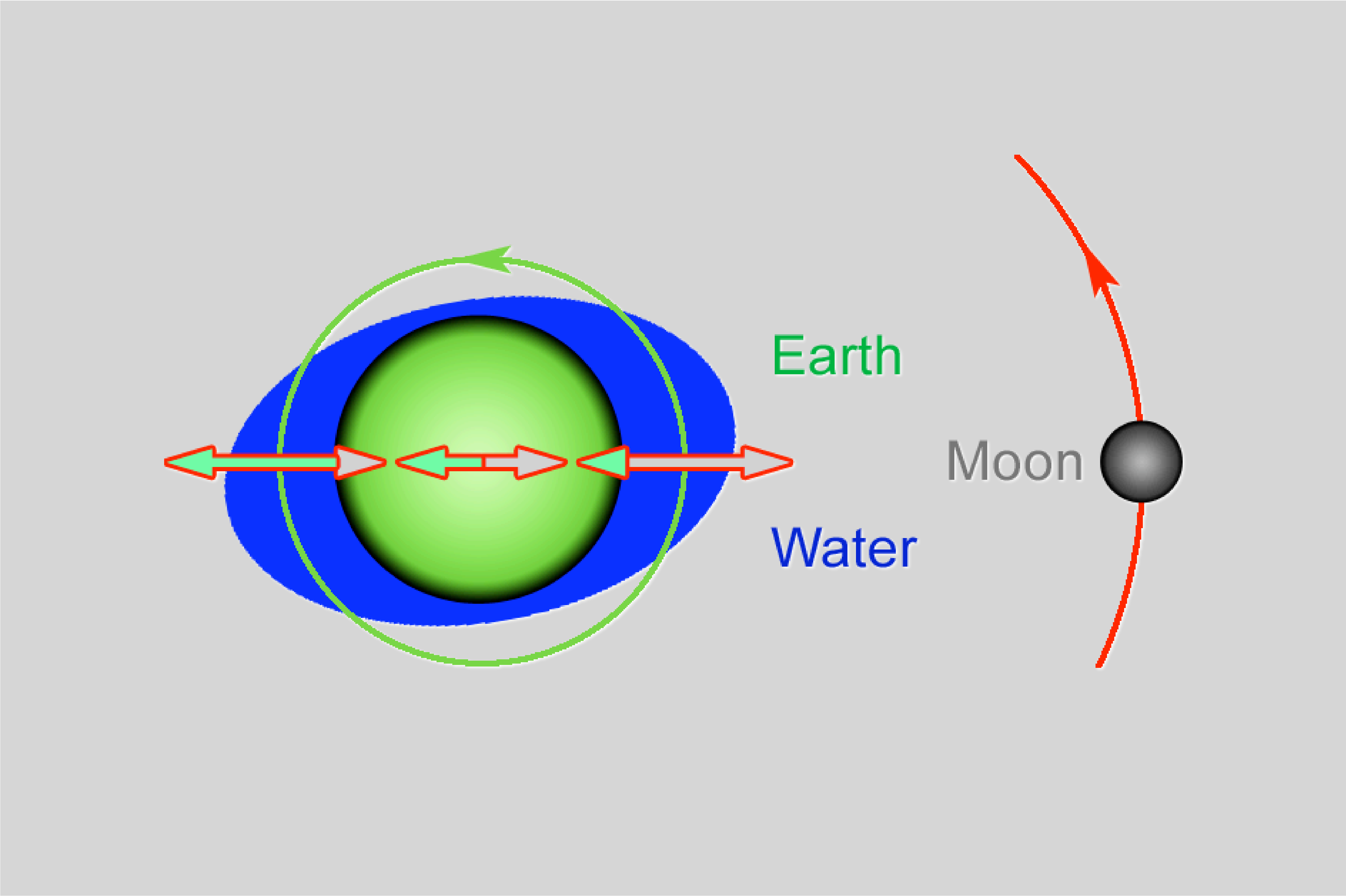


Figure 1, Bills & Ray (1999)



What Governs a Changing Orbit: The Picture

What Governs a Changing Orbit: The Equation

$$\frac{da}{dt} = f \cdot a^{-11/2} = \left(3 \frac{k}{Q} \frac{M_{\mathrm{moon}}}{M_{\oplus}} R_{\oplus}^{5} \left(G \left(M_{\mathrm{moon}} + M_{\oplus} \right) \right)^{1/2} \right) a^{-11/2}$$

- This shows how to calculate the change in the moon's semi-major axis over time.
- To get the "age of the lunar orbit" value from before, a constant *f* was assumed.
- But this is not the case ...

What Governs a Changing Orbit: The Equation

$$\frac{da}{dt} = f \cdot a^{-11/2} = \left(3 \frac{k(t)}{Q(t)} \frac{M_{\text{moon}}}{M_{\oplus}} R_{\oplus}^{5} \left(G \left(M_{\text{moon}} + M_{\oplus} \right) \right)^{1/2} \right) a^{-11/2}$$

- This shows how to calculate the change in the moon's semi-major axis over time.
- To get the "age of the lunar orbit" value from before, a constant *f* was assumed.
- But this is not the case ... because the Earth+ocean tidal Love number and the dissipation quality factor change though time.

What Governs a Changing Orbit: Graph

- Dating shows the Moon is over 4 Gyrs old.
- k/Q must have changed through time.
- We are in an anomalously high k/Q ratio today, meaning that the tidal bulge on Earth is larger, forcing a greater recession rate.
- k/Q can change based upon where land masses are.

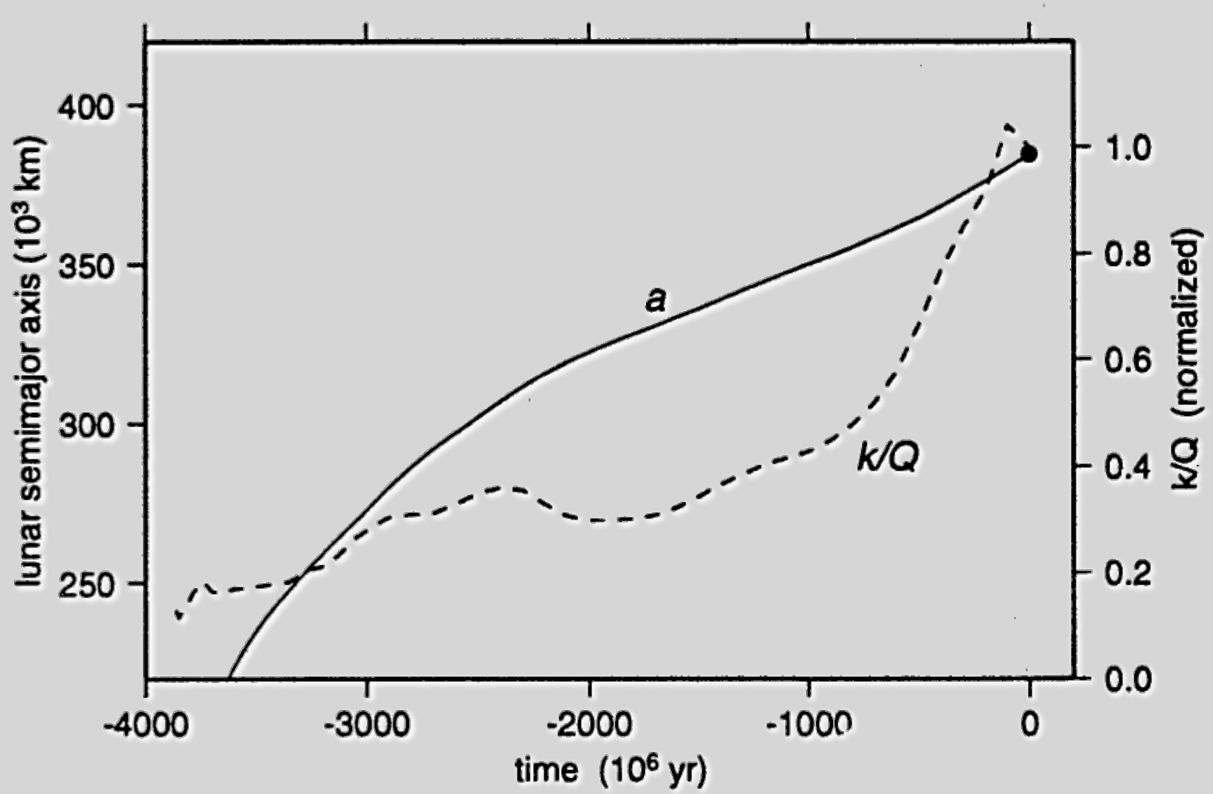


Figure 2, Bills & Ray (1999)

Various Tidal Models

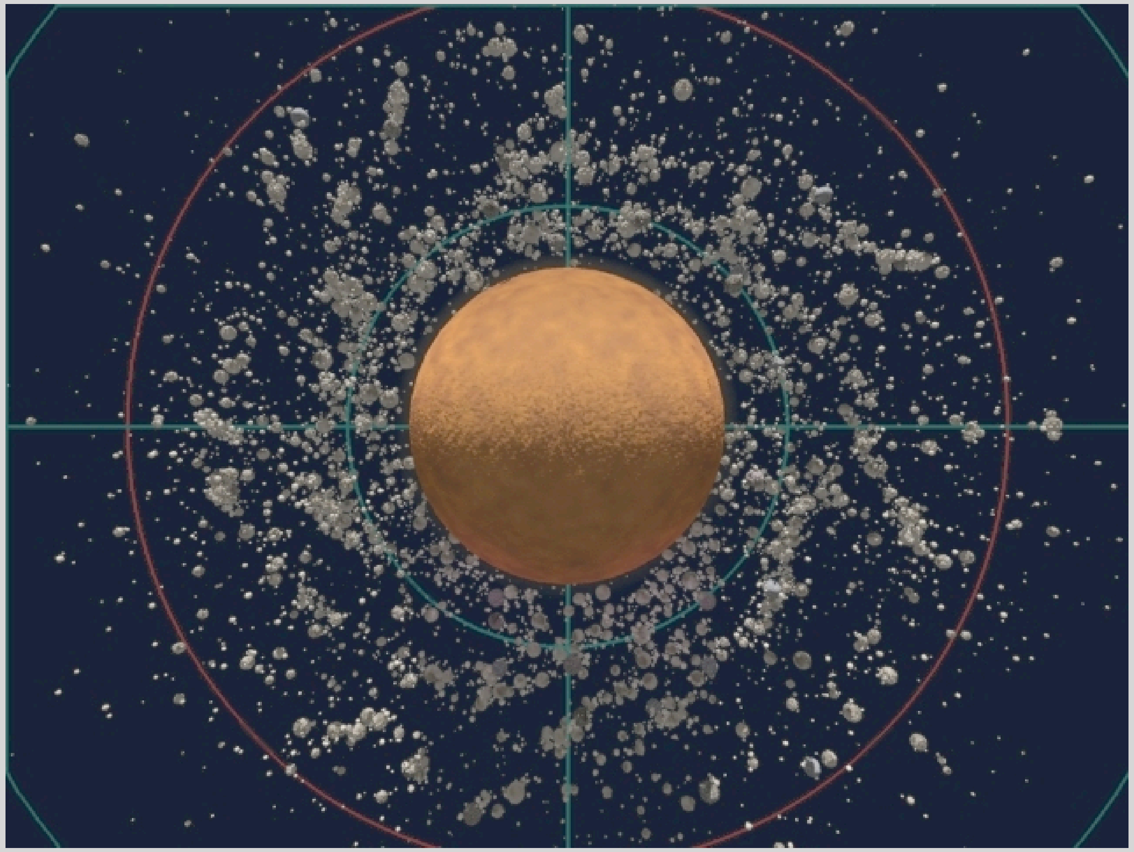
- Simplify: Assume that orbits are simple.
- Darwin Tides: Fourier expand the tide potential and introduce friction through numerous (and unconstrained) phase lags.
- MacDonald Tides: Model distortion of the body as a second harmonic distortion; adds friction as a delay mechanism for a constant (arbitrary) phase lag (doesn't make sense for an eccentric orbit).
- Mignard's Darwin Tides: Simple analytic form with a second harmonic distortion, but delayed relative to the tide potential by a constant time lag. This is the same as Darwin's if the phase factors are proportional to frequency.

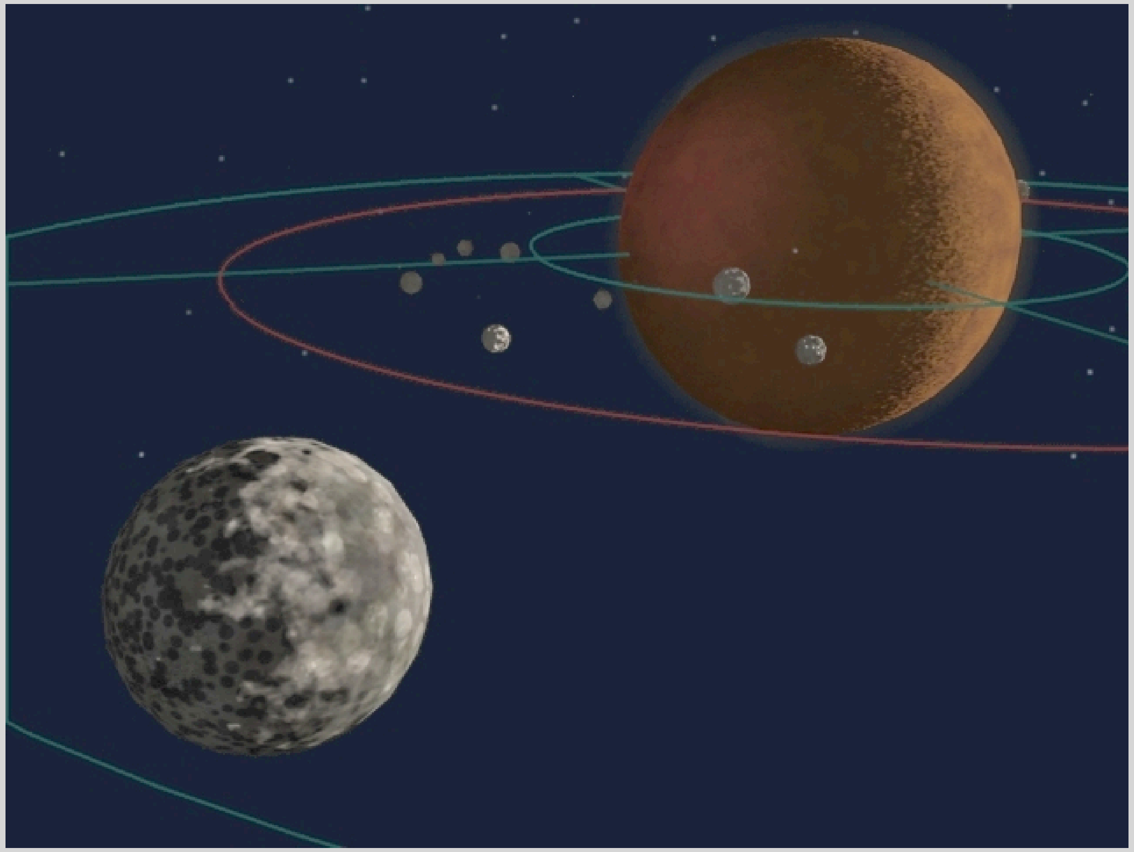
What That Means for Where the Moon Was (Maybe)

Where Was the Moon?

- Assume it formed from a ring of material orbiting the Earth after the Giant Impact

- So you would initially expect it to be orbiting in the equatorial plane of the Earth





http://th.nao.ac.jp/~kokubo/moon/kit/movie.htmL

Calculating a Previous Orbit

"Calculation of the History of the lunar orbit is fraught with difficulties."
~ Wisdom (AJ, 2006)

- Difficulties:
  - Rate of dissipation
  - Inclination
  - Eccentricity

Initial Conditions

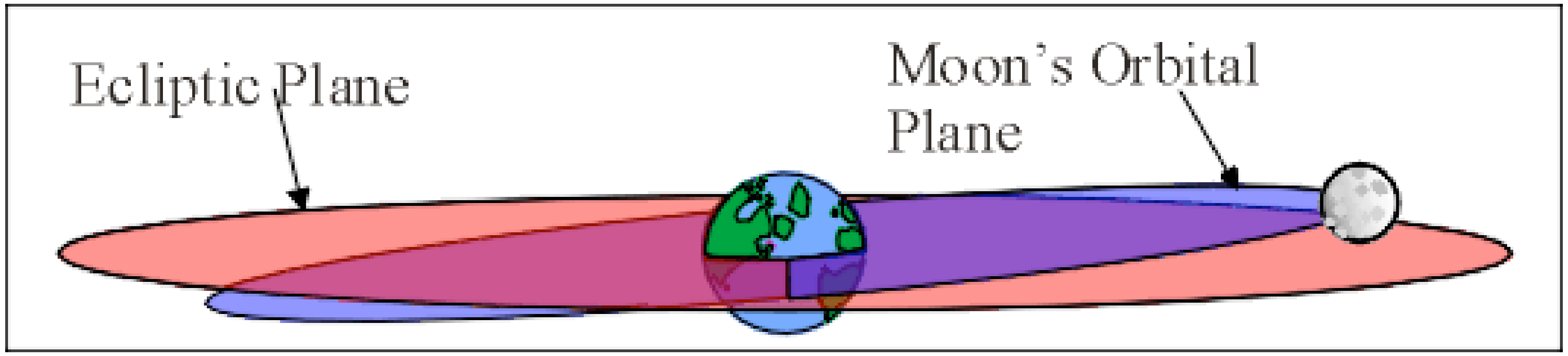
- Remember current values:
  - $a = 3.84402 \cdot 10^8$
  - $da/dt = 3.82 \pm 0.07$ cm/yr

- Records indicate that 620 Mya:
  - A day was 21.9±0.4 hrs
  - There were 13.1±0.1 synodic months per year
  - There were 400±7 solar days per year
  - $a/a_0 = 0.965 \pm 0.005$
  - The average recession rate was 2.17±0.31 cm/yr

- Records indicate that 2.45 Gya:
  - There were 14.5±0.5 synodic months per year
  - $a/a_0 = 0.906 \pm 0.029$
  - The average recession rate was 1.24±0.71 cm/yr

Taken together, this indicates that the moon wasn't close to Earth 1.5 Gya.

Inclination



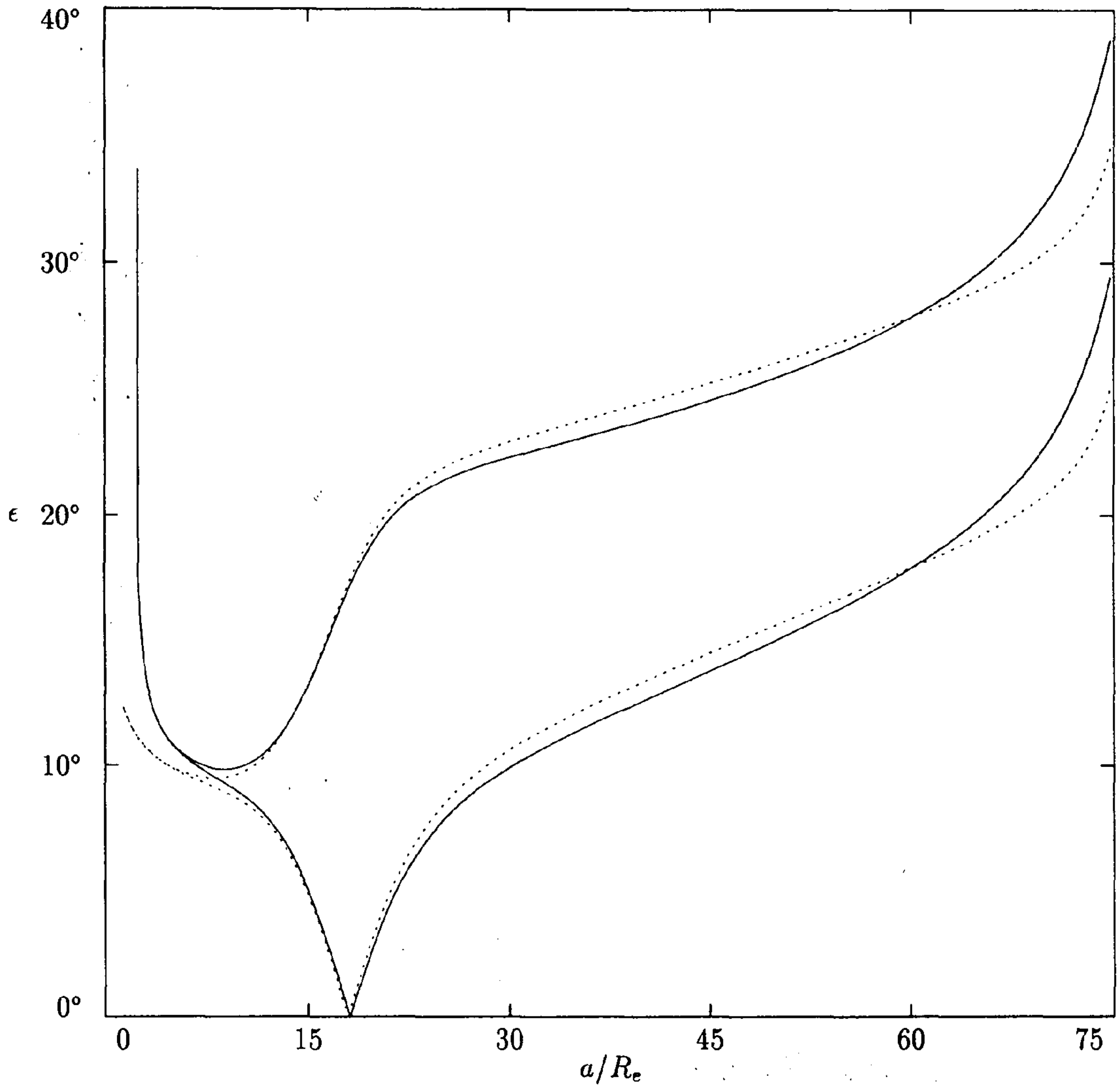
Orbital Plane is inclined ~5° to the ecliptic.

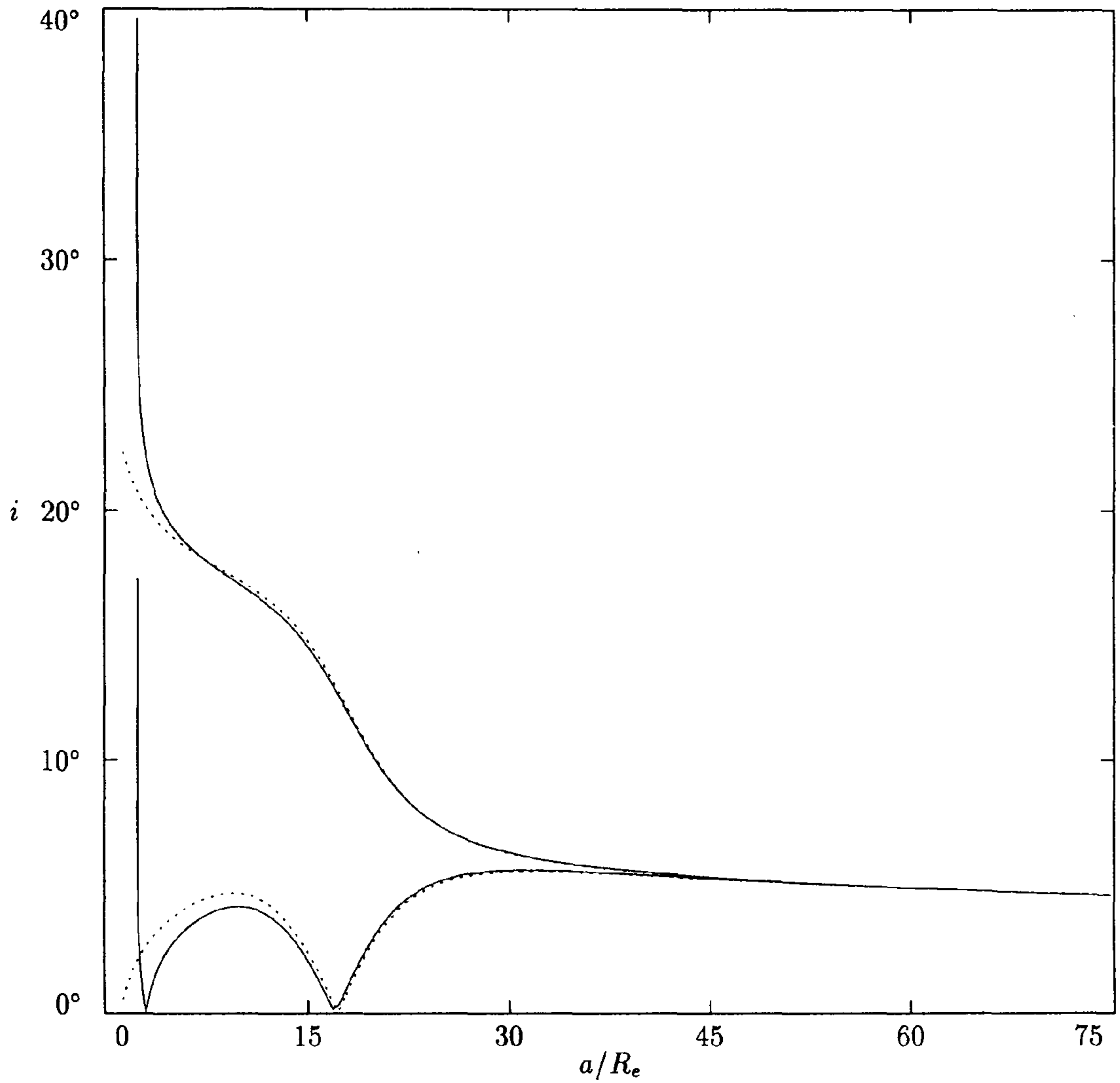
(But this doesn't match what we'd expect if it had formed in the plane of Earth's rotation)

http://ww.ap.stmarys.ca/demos/content/astronomy/lunar_eclipse/lunareclipse.html

Calculating a Previous Orbit

- Remember current values:
  - Rate of dissipation
  - Inclination
  - Eccentricity
- Many parameters, so you need to simplify.
- Most models assume:
  - Synchronously locked
  - Low eccentricity


40°
30°
ϵ 20°
10°
0°
0
15
30
45
60
75
a/R_e



40°
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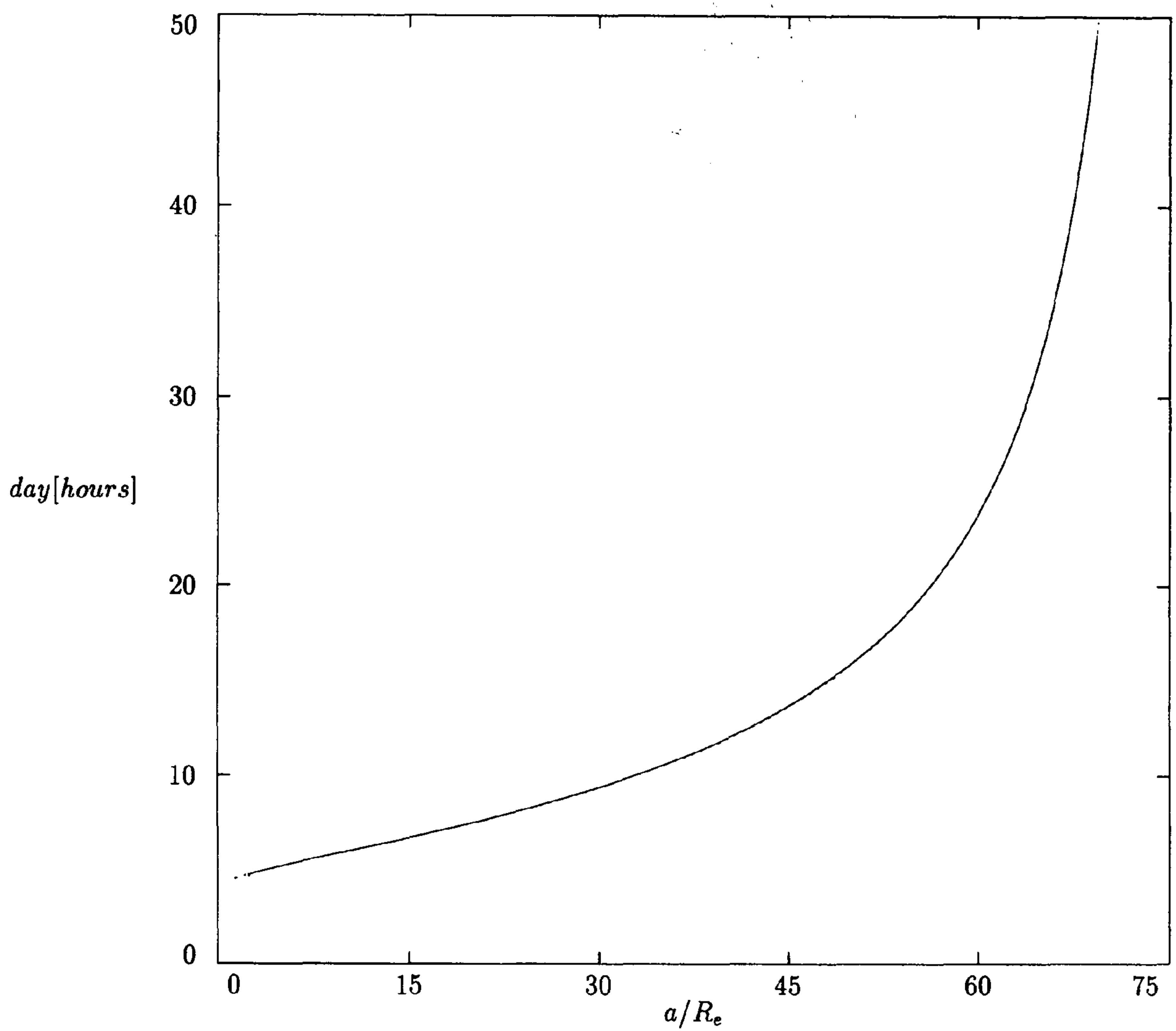
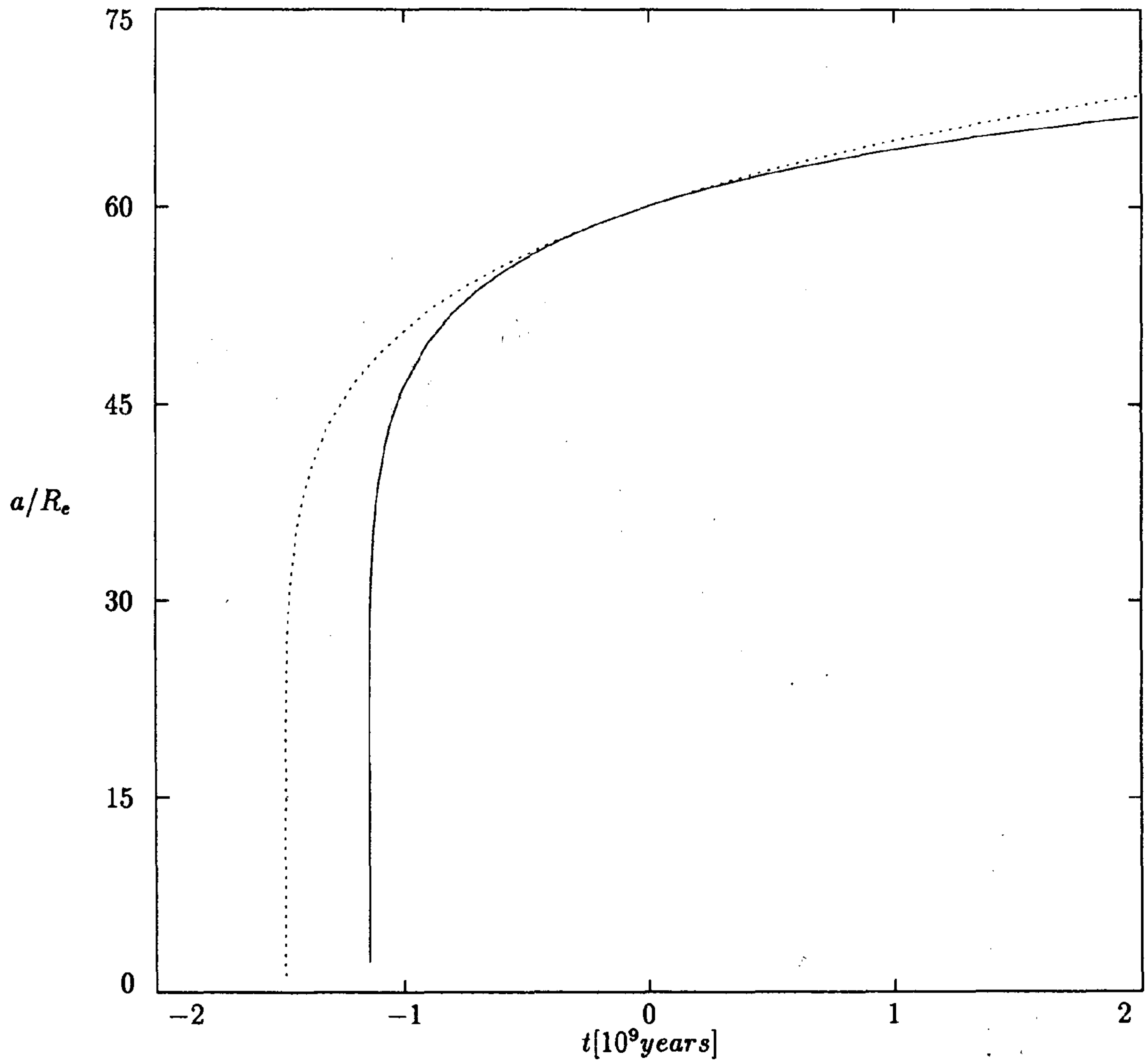
60
75
a/R_e



50
40
30
20
10
0
$day[hours]$
0
15
30
45
60
75
a/R_e


75
60
45
30
15
0
a/R_e
−2
−1
0
1
2
$t[10^9 years]$

Highly Eccentric Past?

"Evidence for a Past High-Eccentricity Lunar Orbit."
~ Garrick-Bethell et al. (Science, 2006)

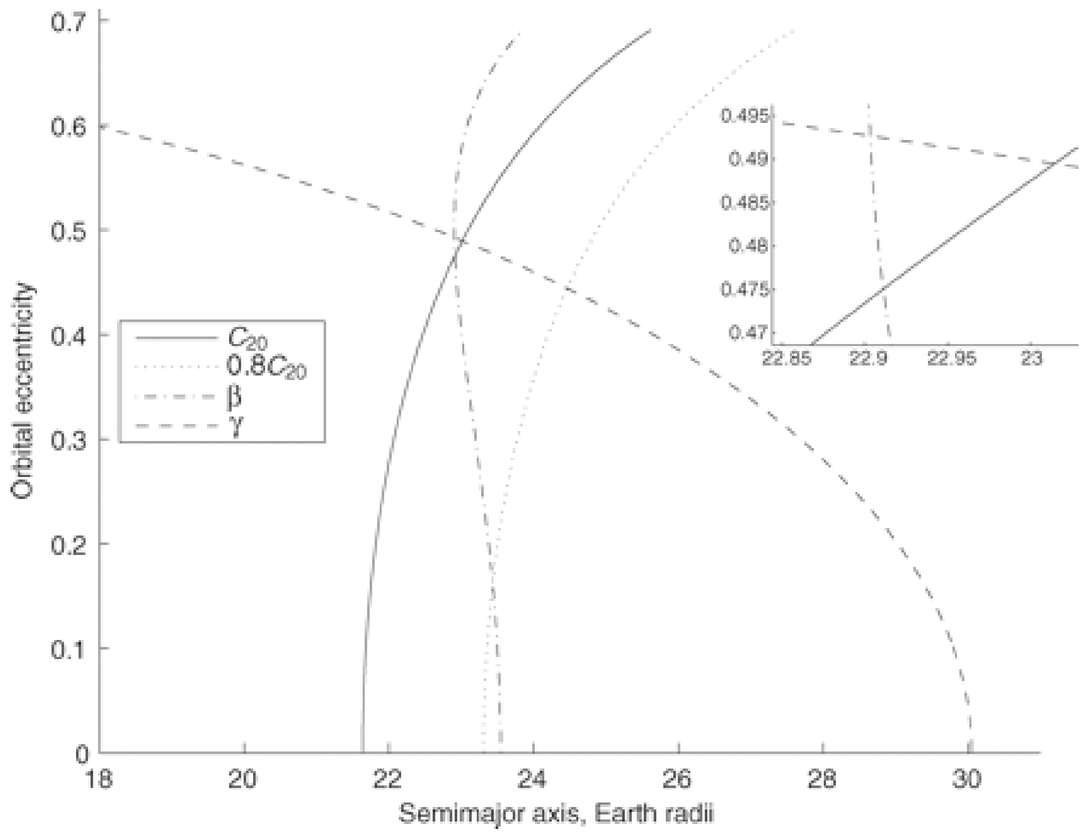
- Possible impact parameters:
    - with a giant impact, material at ~4 r_E
    - Synchronous orbital periods > 10 hrs
    - Could have period as fast as 1.8 hrs (which gives more angular momentum possibilities)

- High-*e* means system evolves faster.

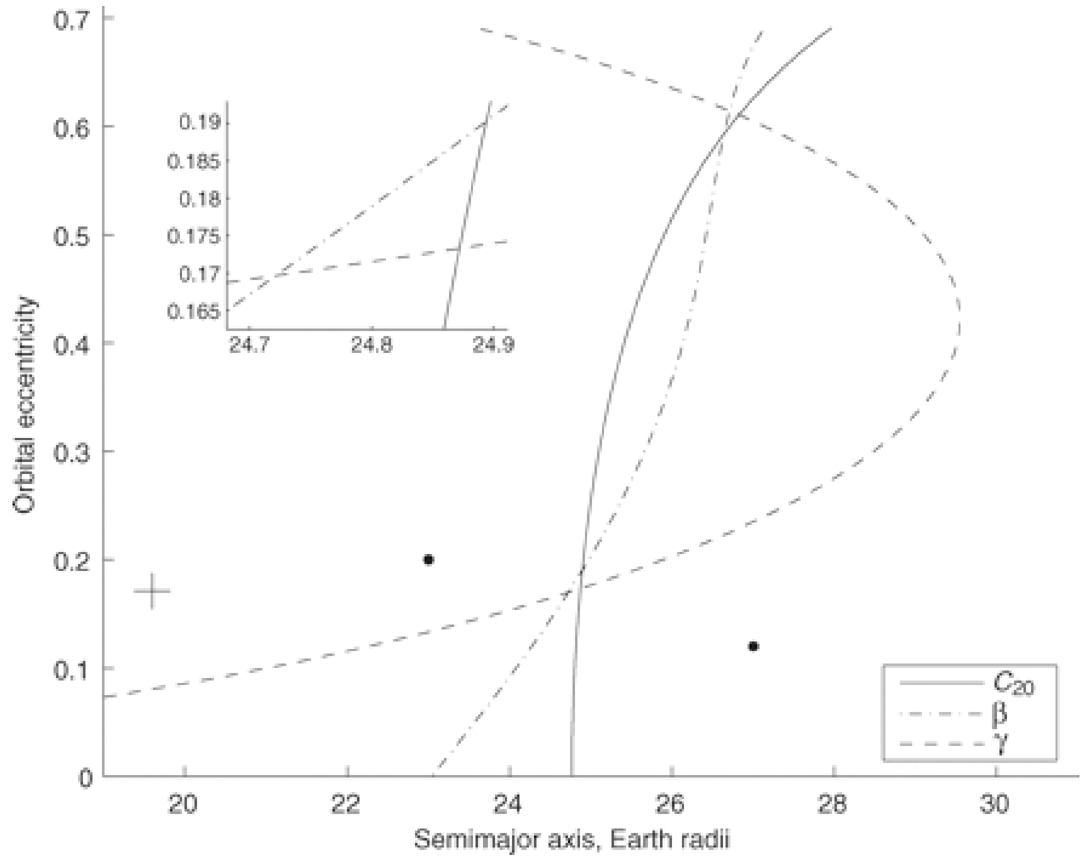
Important Features of the Models

- High-*e*
- Looked at orbits:
  - in synchronous rotation
  - in 3:2 resonance

Model Results



Synchronous Orbit



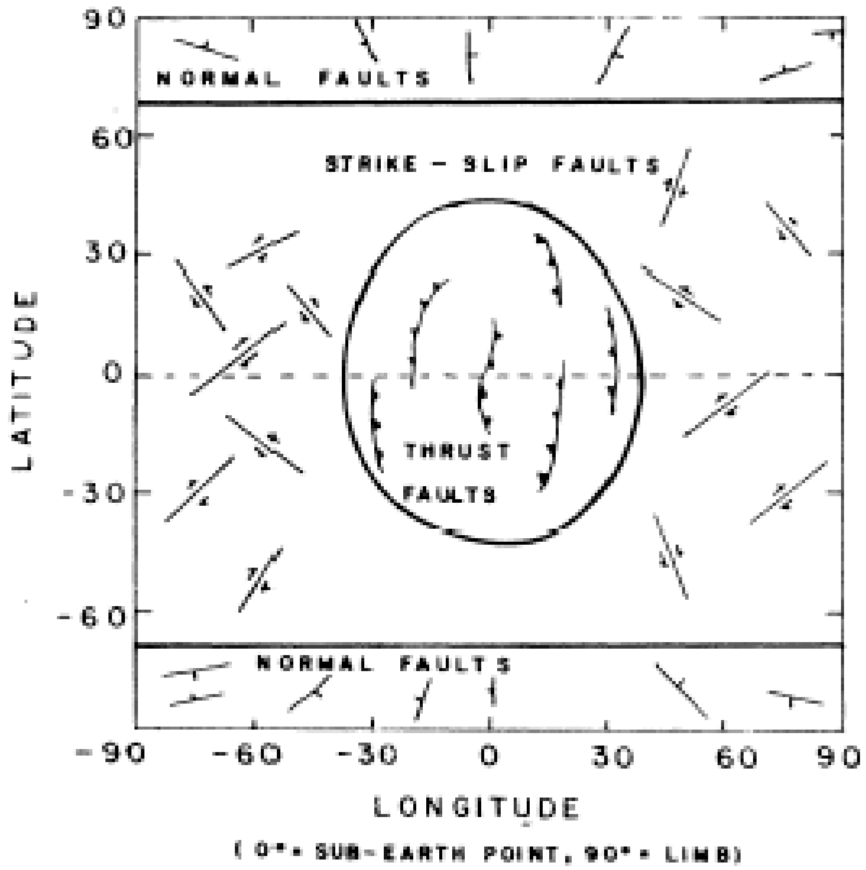
3:2 Resonance

Important Features of the Models

- High-*e*
- Looked at orbits:
    - in synchronous rotation
    - in 3:2 resonance
- Models present solution to “fossil bulge.”

The Fossil Bulge

Back in 1799, Laplace noticed that the moon was flatter than it should be given its rotation rate (27.3 days). Subsequent investigations confirmed the presence of this "fossil bulge" at the lunar equator.



If the moon had once been spinning more rapidly than at present, then as it slowed it would have left a fossil bulge and a pattern of faulting.

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