ANOTHER ETHER-DRIFT EXPERIMENT.

BY ROY J. KENNEDY.

SYNOPSIS.

Introduction.—The ether drift experiments heretofore performed seem to have proven the reality of the Lorentz-Fitzgerald contraction, but the explanation of the negative results requires nothing further. The experiment forming the subject of this paper is designed to detect ether drift or to confirm the time transformation between relatively moving systems, which is an essential part of the theory of relativity.

Theory.—Assuming Huyghen's principle and the Lorentz-Fitzgerald contraction it can be shown that the paths of light rays relative to a system are unaffected by its motion or orientation in the medium transmitting the light. The relative phases of two homogeneous beams superposed under large retardation are unaffected by the orientation but not by the velocity of the system. The effect sought in this experiment depends on the velocity change due to the alternate adding and subtracting of the earth's orbital motion and that of the solar system as a whole.

Apparatus and Method.—A delicate optical system is used in which a half-shade analyzer measures the variation with change of phase of component beams, of the position of the plane of polarization of a beam resulting from the superposition of two oppositely circularly polarized ones. The change manifests itself as an unbalancing of a uniform photometric field. The smallest effect to be expected according to the classical ether theory is shown to be several times the least detectable with the experimental arrangements, so that conclusive evidence for or against the relativity principle should be obtained.

Introduction.—The negative results of all experiments devised to detect relative motion between the earth and the ether, which gave rise to the theory of relativity, can be explained without modification of the classical electromagnetic theory if the forces on which the size and shape of rigid bodies depend are assumed to be electrical. For then the Lorentz-Fitzgerald contraction of the experimental apparatus follows necessarily, and nothing further than the contraction is required to account for the absence of any physical effect of motion with the arrangements heretofore used. For this reason the hypothesis of the relativity of time, which is essential to the theory of relativity, appears to be a gratuitous assumption. This paper is an outline of an experiment which is intended to detect ether-motion or, failing that, to establish the validity of the Lorentz-Einstein time transformation. It is being performed by the writer at the Palmer Physical Laboratory.

Apparatus.—The arrangement of apparatus is as shown in Fig. 1. A beam of homogeneous plane-parallel light polarized in a plane inclined at an angle of 45° to that of the paper falls on a Wollaston prism W which divides it into two beams of equal intensity and plane polarized in planes at right angles to one another. The one whose plane of polarization is in the plane of the paper passes to the mirror M_1, from which it is reflected to mirror M_2, and thence to a partially silvered mirror M_3. Here

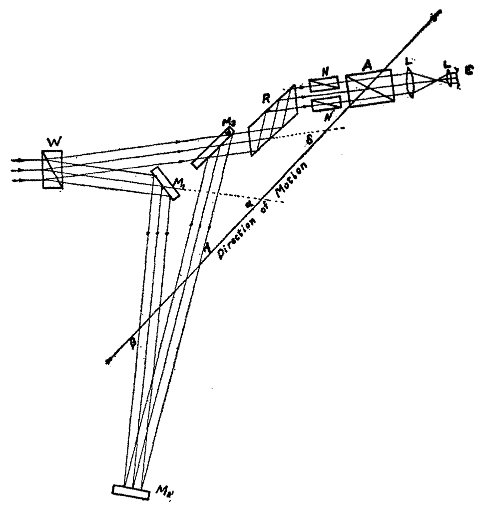

Fig. 1.

it is reflected and superposed on the other ray, which is polarized in a plane at right angles to the paper and passes through M_3. The elliptically polarized beam resulting from this superposition passes through a Fresnel rhomb R, so placed as to render it plane polarized, the inclination of the plane of polarization depending on the phase relation between the two beams combining at M_3. From R the beam passes between and through the small nicols NN, whose principal sections are parallel, through the analyzing nicol A and to the eye at E by way of the lens system LL. The nicols NN are fixed in position, while the nicol A is rotatable about an axis in the direction of the rays through it, its position being read off a graduated circle. The optical parts are mounted so that temperature changes are compensated for and do not affect the paths of the rays.

Such an arrangement provides a very sensitive measure of any change in relative phase of the two superposed beams. It can be shown that if the plane of polarization of the light emerging from the rhomb R makes an angle θ with the planes of the principal sections of the nicols NN, and ψ is the angle between the principal sections of NN and of A when the latter prism is set so that it lets through equal amounts of the light passed through and between NN, then to a small change $\delta\theta$ in θ there corresponds a change in ψ

$$\delta\psi = \frac{2\ \delta\theta}{1 + 3\cos^2\theta}.$$

Since for the greatest sensibility θ must be small (not greater than two or three degrees), $\delta\psi$ will be about half of $\delta\theta$. The light emerging from R may be regarded as the resultant of two oppositely circularly polarized beams; a little consideration will show that its plane will rotate through 180° as the relative phase of the component beams varies through 360° or one wave. With a sufficiently intense source of light the analyzer A can be set accurately to one-hundredth of a degree; that is, a change $\delta\psi = 0.01$ can be detected. Therefore a change in the phase ϕ of

$$\delta\phi = \frac{2 \times .01^\circ}{180^\circ} = \frac{1}{9000}\text{ wave}$$

can be detected. An ordinary Michelson or Fabry and Perot interferometer could be used but for their low sensibility.

Theory.—Assuming the existence of an ether with respect to which the surface of the earth and the apparatus are in motion with a velocity v, and the Lorentz-Fitzgerald contraction which reduces dimensions in the direction of motion in the ratio $\sqrt{1 - \frac{v^2}{c^2}} : 1$ (where c is the velocity of light in ether), the theory of the experiment is as follows:

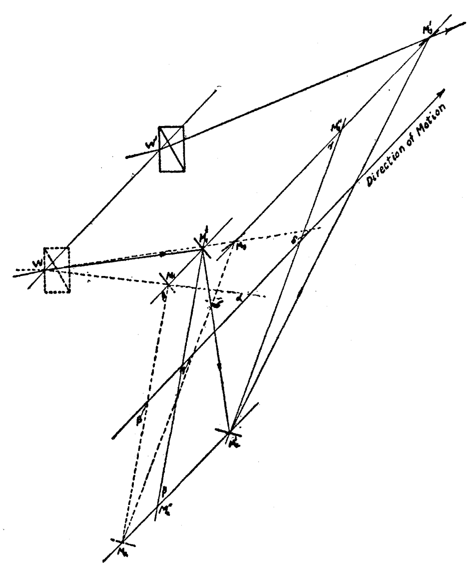

Fig. 2.

In Fig. 2 the dotted broken lines, $WM_1M_1M_3$ and WM_3 represent the paths of the two beams relative to the apparatus. The full lines $WM_1'M_2'M_3'$ and $W'M_3'$ are the paths as referred to the ether. The dotted line sketch of the Wollaston prism and the mirrors shows them in the position occupied at the instant the lower beam leaves the prism. The full line sketches show the positions of the mirrors as they are successively reached by the wave front, and of the prism at the instant of starting of the upper train which will combine at M_3' with the lower. M_2'' is position of mirror M_2 at the instant when the wave front starts from M_1' and M_3'' is that of the mirror M_3 when the wave front leaves M_2'.

Let the lengths WM_1 equal l_1, M_1M_2 equal l_2, M_2M_3 equal l_3, and WM_3 equal l_4, these being the values which would be measured by an observer stationary in the ether — the "contracted" values. Let the direction of motion makes angles α with WM_1, β with M_1M_2, γ with M_2M_3 and δ with WM_3. Consider first the ray from the prism to the mirror M_1. Due to the motion of the system, its actual path in the ether will be along WM_1' instead of WM_1, and if the time required for the light to travel this distance is t_1, we have from the geometry of the figure,

$$\begin{aligned} c^2t_1^2 &= \overline{WM_1}^2 + \overline{M_1M_1'}^2 - 2\,\overline{WM_1}\,\overline{M_1M_1'}\cos \angle WM_1M_1' \\ &= l_1^2 + v^2t_1^2 - 2\,l_1vt_1\cos(\pi - \alpha) \\ &= l_1^2 + v^2t_1^2 + 2l_1vt_1\cos\alpha. \end{aligned}$$

After reflection at M_1' the light travels to M_2' in time t_2 a distance whose square is

$$\begin{aligned} c^2t_2^2 = \overline{M_1'M_2'}^2 &= \overline{M_1'M_2''}^2 + \overline{M_2''M_2'}^2 - 2\,\overline{M_1'M_2''}\,\overline{M_2''M_2'}\cos\beta \\ &= l_2^2 + v^2t_2^2 - 2\,l_2vt_2\cos\beta. \end{aligned}$$

After reflection at M_2' the ray proceeds to M_3' in time t_3 a distance whose square is

$$\begin{aligned} c^2t_3^2 = \overline{M_2'M_3'}^2 &= \overline{M_2'M_3''}^2 + \overline{M_3''M_3'}^2 - 2\,\overline{M_2'M_3''}\,\overline{M_3''M_3'}\cos(\pi-\gamma) \\ &= l_3^2 + vt_3^2 + 2\,l_3vt_3\cos\gamma. \end{aligned}$$

At M_3' the ray combines with one directly from W' which has gone in time t_4 a distance given by

$$c^2t_4^2 = \overline{W'M_3'}^2 = l_4^2 + v^2t_4^2 + 2\,l_4vt_4\cos\delta.$$

Solving these four equations and rejecting one physically meaningless solution of each we obtain

$$t_1 = \frac{l_1}{c^2 - v^2}\left(v\cos\alpha + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\alpha}\right),$$

$$t_2 = \frac{l_2}{c^2 - v^2}\left(- v \cos \beta + c\sqrt{1 - \frac{v^2}{c^2}\sin^2 \beta}\right),$$
$$t_3 = \frac{l_3}{c^2 - v^2}\left(v \cos \gamma + c\sqrt{1 - \frac{v^2}{c^2}\sin^2 \gamma}\right),$$
$$t_4 = \frac{l_4}{c^2 - v^2}\left(v \cos \delta + c\sqrt{1 - \frac{v^2}{c^2}\sin^2 \delta}\right).$$

If we represent by primed letters the distances between pairs of points in the system when it is at rest (which are the distances actually obtained by measurement by an observer moving with the system)then any length $l = l'\sqrt{1 - (v^2/c^2)\cos^2 \omega}$ where ω is the angle between direction of the motion and the line joining the two points. For the component l_y of the length perpendicular to the motion is unaffected while the parallel component l_x is contracted to $\sqrt{1 - (v^2/c^2)}$ times its value when stationary, and

$$l = \sqrt{l_y^2 + l_x^2} = \sqrt{(l' \sin \omega)^2 + \left(l'\sqrt{1 - \frac{v^2}{c^2}}\cos \omega\right)^2}$$
$$= l'\sqrt{\sin^2 \omega + \cos^2 \omega - \frac{v^2}{c^2}\cos^2 \omega}$$
$$= l'\sqrt{1 - \frac{v^2}{c^2}\cos^2 \omega}.$$

Therefore

$$l_1 = l_1'\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha}, \qquad l_3 = l_3'\sqrt{1 - \frac{v^2}{c^2}\cos^2\gamma},$$
$$l_2 = l_2'\sqrt{1 - \frac{v^2}{c^2}\cos^2\beta}, \qquad l_4 = l_4'\sqrt{1 - \frac{v^2}{c^2}\cos^2\delta}.$$

Substituting these values in formulas (1)

$$t_1 = \frac{l_1'}{c^2 - v^2}\left(v \cos \alpha\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha}\right.$$
$$\left. + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\alpha - \frac{v^2}{c^2}\cos^2\alpha + \frac{v^4}{c^4}\sin^2\alpha \cos^2\alpha}\right)$$
$$= \frac{l_1'}{c^2 - v^2}\left(v \cos \alpha\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha} + c\sqrt{1 - \frac{v^2}{c^2}}\right),$$

neglecting the fourth order term $(u^4/c^4) \sin^2\alpha \cos^2\alpha$. Similar relations hold for t_2, t_3 and t_4 so that the retardation of the reflected ray relative to the direct ray may be written

$$T = t_1 + t_2 + t_3 - t_4$$

$$= \frac{1}{c^2 - v^2}\Bigg[v\bigg(l_1'\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha}\cos\alpha - l_2'\sqrt{1 - \frac{v^2}{c^2}\cos^2\beta}\cos\beta + l_3'\sqrt{1 - \frac{v^2}{c^2}\cos^2\gamma}\cos\gamma - l_4'\sqrt{1 - \frac{v^2}{c^2}\cos^2\delta}\cos\delta\bigg) + c\sqrt{1 - \frac{v^2}{c^2}}(l_1' + l_2' + l_3' - l_4')\Bigg].$$

The coefficient of v in this expression is zero, since it is the algebraic sum of the projections of the sides of a closed figure on a line (the direction of motion). Hence

$$T = \frac{c\sqrt{1 - \frac{v^2}{c^2}}(l_1' + l_2' + l_3' - l_4')}{c^2 - v^2},$$

$$= \frac{1}{c\sqrt{1 - \frac{v^2}{c^2}}}(l_1' + l_2' + l_3' - l_4').$$

As would be expected, this retardation is not a function of the orientation of the system; this is indicated by the absence of α, β, γ and δ from the expression for T. Its value does, however, depend on the velocity of the system. For another value v' of the velocity, the new retardation

$$T' = \frac{1}{c\sqrt{1 - \frac{v'^2}{c^2}}}(l_1' + l_2' + l_3' - l_4')$$

and

$$T - T' = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}}\right)\frac{l_1' + l_2' + l_3' - l_4'}{c}$$

$$= \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{e^4} \cdots\cdots - 1 - \frac{1}{2}\frac{v'^2}{c^2} - \frac{3}{8}\frac{v'^4}{c^4}\cdots\cdots\right)\frac{l_1' + l_2' + l_3' - l_4'}{c}$$

$$= \frac{v^2 - v'^2}{2c^2}\left(l_1' + l_2' + l_3' - l_4'\right)$$

to a very close approximation. This quantity is the difference between the time-retardations for the two velocities. The change $\delta\phi$ in the number of waves corresponding to this time is $T—T'$ multiplied by the frequency of the light employed. But the frequency equals c/λ, so that

$$\delta\phi = \frac{c}{\lambda}\left(T - T'\right) = \frac{v^2 - v'^2}{2c^2}\left(\frac{l_1' + l_2' + l_3' - l_4'}{\lambda}\right).$$

Up to the last step, nothing in the argument is contrary to the relativity hypothesis if the times T and T' be taken as determined by an observer stationary in the ether. The departure is in assuming λ to be the same for both velocities, v and v'. For according to the relativity time transformation, the indication of a clock in one system during the interval between two events, as observed from the other system, is equal to $\sqrt{1 - (v^2/c^2)}$ times the indication of a clock in the latter between the same events. To an observer stationary in the ether, then, any time indicator in the apparatus of this experiment should run slow in the above ratio. A luminous emitter is a form of clock of which the indication is expressed as a certain number of waves. If its rate decreases to $\sqrt{1 - (v^2/c^2)}$ times its rest value, the wave length corresponding to a given spectral line will increase in the inverse ratio; *i.e.*, if λ_0 be the wave length corresponding to the line when the system is at rest in the ether, the wave length for a velocity v will be $\dfrac{\lambda_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

If this value is substituted in the expression for the retardation in waves ($= T$ times the frequency) we have

$$\frac{Tc}{\lambda} = \frac{c}{\dfrac{\lambda_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} c \sqrt{1 - \dfrac{v^2}{c^2}}} (l_1' + l_2' + l_3' - l_4')$$

$$= \frac{l_1' + l_2' + l_3' - l_4'}{\lambda_0}.$$

This is quite independent of v and would predict a negative result for the experiment.

The effect to be expected if the frequency is not affected by the motion is well within the range of the apparatus. It has been found possible to secure interference with a retardation (equal to $(l_1' + l_2' + l_3' - l_4')/\lambda$) of over a million waves, so it is probable that half that retardation will be found feasible. It was shown above that a value of $\delta\phi$ equal to $\frac{1}{5000}$ wave could be detected under good conditions. As a conservative estimate let us assume that about $\frac{1}{2000}$ wave is the least value under the less favorable condition of high retardation that will obtain in the experiment. The least difference of the squares of the velocities (in miles per second) detectable will then be

$$V^2 - V'^2 = \frac{2c^2\,\delta\phi}{\left(\dfrac{l_1' + l_2' + l_3' - l_4'}{\lambda}\right)} = \frac{2 \times (186{,}000)^2 \times .0005}{500{,}000}$$

$$= 69, \text{ approximately.}$$

This is about four times the value of the diurnal difference of squares if the earth is supposed to have a velocity with respect to the ether equal to its orbital velocity merely (the least that can be ascribed to it). If the velocity is twice this at some time of the year the effect would be just observable. The yearly variation due to the motion of the solar system as a whole should be many times as great. If this latter speed is no greater than that towards the constellation Hercules — which is supposed to be the average velocity with respect to the whole sidereal system — which has a maximum component in the earth's orbital plane of slightly more than six miles per second, the maximum and minimum values of the velocity are $18 + 6$ and $18 - 6$, so that $V^2 - V'^2 = 24^2 - 12^2 = 432$. It is quite probable that the actual velocity is much greater.

If the experiment does show relative motion between earth and ether, it will be possible to determine the absolute value of it. For the difference between the first powers of the two velocities will be known from the earth's orbital and rotational velocities, and the experiment will give the difference between their squares, thus supplying the two equations necessary to determine two unknowns. If no positive effect is obtained, the relativity time transformation between a system stationary in the ether and one moving with respect to it would seem to be confirmed.

Palmer Physical Laboratory,
Princeton University,
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