A confirmation of the Allais and Jeverdan-Rusu-Antonescu effects during the solar eclipse from 22 September 2006 , and the quantization of behaviour of pendulum

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In loving memory of our former Proffesors
Gheorghe Jeverdan, Gheorghe Il.Rusu, Virgil Antonescu

Abstract

The experiments made with a paraconical pendulum at Suceava Planetarium (Romania) during annular solar eclipse from 22 September 2006 confirm once again the existence of the Allais effect (change of speed of rotation of plane of oscillation of a pendulum during an eclipse) and Jeverdan-Rusu-Antonescu effect (change of period of oscillation of a pendulum during an eclipse)

Also is take in evidence the existence of the quantization of the azimuth of plane of oscillation of a pendulum which can be treated as a quantum oscillator.

A large number of the excited states for a quantum Foucault pendulum are doubly degenerate in a similar way as the time dependence of the azimuths for a paraconical pendulum with a high sensitivity.

The quantum eigenstates for a large energy of a Foucault pendulum predict that the probability density of finding the particle is largest near the classical trajectories.

Although the annular solar eclipse from 22 September 2006 was not optical visible from Romania, a gravitational perturbation was detected with a sensitive paraconical pendulum and leds to ideia that gravitational perturbations who occur during an eclipse are similarly with tide when the Moon are at antimeridian .

Keywords: Quantum mechanics, Foucault pendulum, Eclipse, Gravitation

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1. Introduction

The Foucault effect

It is well known that in the year 1851 J.B.L. Foucault has showed that the Earth rotation around his axis can be demonstrated by the rotation of the plane of oscillation of a long pendulum. This rotation is made constant after the law $F = 15^o \sin\varphi/h$, where F is the rate at which the azimutal angle change in time, φ is the latitude of the place of observation and h is the time in hours, and is named *the Foucault effect.*

The Allais effect

During solar eclipses from 30 June 1954 and 22 October 1959 Maurice Allais (1968 Nobel Prize in Economics) utilized a short pendulum suspended on a ball (named paraconical pendulum) have discovered that during a solar eclipse, the speed of rotation of plane of oscillation of pendulum there are not constant as in the case of Foucault experiments, this is *the eclipse effect or, the Allais effect*[1].

The Jeverdan-Rusu-Antonescu effect

During solar eclipse from 15 February 1961, Gheroghe Jeverdan, Gheorghe I. Rusu and Virgil Antonescu, have discovered that during a solar eclipse the period of oscillation of a Foucault pendulum is changed. In the same time this means that also the value of the gravitational acceleration ***g*** is changed, this is *The Jeverdan-Rusu-Antonescu effect* [2].

The pioneering effect

In the years 1980 and 1989 the NASA's specialists have discovered that between the position of Pioneer 10, Pioneer 11 and Ulysses made by Doppler determinations and the positions calculated theoretical were a difference of around 400000 km. This anomaly is named now the Pioneering effect [3].

A similar gravity anomaly was measured during the line-up of Earth-Sun-Jupiter-Saturn in May 2001 [4]. During the total solar eclipse in 1977, the measurements with a high-precision gravimeter have detected a decrease in the earth's gravity and the effect occurred immediately before and after eclipse but not at its height [5].

These anomalies have determined us to do pendulum experiments in distinct mode such: solare eclipses, lunar eclipses, planetary alignments etc.
One of these results is presented in this papier.

2. The confirmation of the Allais effect during solar eclipse from 22 September 2006

In order to measure the azimuth of the plane of oscillation, the second author used a pendulum of 3.05m length and 8kg mass suspended with a ball (paraconical pendulum) with shaped like a horizontal biconvex lens and with distinctively rounded edges so as to reduce wind drag.

For measure the azimuth is utilized a vernier with 0.1° precision.

Figs.1-3 show the experimental azimuth as a function of time for 6 series (successive chained observations each of 9 minutes) and the expected exact azimuth for Foucault effect (line named F), during an annular solar eclipse of 22 September 2006. Its maximum occurred at 14h 40.2 min Romanian Summer Time at Suceava, or 11h 40.2 min UT.

From these graphs we see that in the days before and after eclipse on 21 and 23 September the curves have the tendency to have negative values whereas in the day of eclipse the tendency is to have positive values. This fact confirms the existence of the Allais effect.

Another very interesting fact is that the position of plane of oscillation of pendulum has quantized values! The series 1, 2, 3 and 4, 5, 6 from 21 September 2006 (Fig.1) can be considered together because of similarities for the time dependence of the azimuth, analog with the degenerate states of quantum levels from atomic physics. Also, the series 3 and 4 from 22 September 2006 (Fig.2) can be grouped together. This leads us to attempt to give a quantum treates of behaviour of pendulum.

3. Classical Foucault pendulum

For a pendulum small compared with the earth, the vertical motion can be neglected and at the same time the term in ω^2 where ω is the angular velocity of the earth. Using the relations between the coordinates q_1, q_2, q_3 and the length l of the pendulum

$$q_1^{\ 2} + q_2^{\ 2} + q_3^{\ 2} = l^2, q_3 = -l\sqrt{1 - \frac{q_1^{\ 2} + q_2^{\ 2}}{l^2}} \cong -l + \frac{q_1^{\ 2} + q_2^{\ 2}}{2l} + \frac{(q_1^{\ 2} + q_2^{\ 2})^2}{8l^3} \qquad (1)$$

and changing the origin in the center of the oscillating body, we have

$$q_3 = \frac{q_1^{\ 2} + q_2^{\ 2}}{2l} \qquad (2a)$$

or

$$q_3 = \frac{q_1^{\ 2} + q_2^{\ 2}}{2l} + \frac{(q_1^{\ 2} + q_2^{\ 2})^2}{8l^3} \qquad (2b)$$

where

$$\left| \frac{q_1^{\ 2} + q_2^{\ 2}}{2l} \right| < 1 \qquad (3)$$

Using the approximations (2a), the Lagrange L and Hamiltonian H functions for a Foucault pendulum are given by the relations

$$L = \frac{m(\dot{q}_1^2 + \dot{q}_2^2)}{2} + m\omega \sin\lambda (q_1 \dot{q}_2 - q_2 \dot{q}_1) - \frac{mg}{2l}(q_1^2 + q_2^2) \qquad (4)$$

$$H = \frac{p_1^2 + p_2^2}{2m} + \omega \sin\lambda (p_1 q_2 - p_2 q_1) + \frac{mg}{2l}(q_1^2 + q_2^2) \qquad (5)$$

where m is the mass of the particle, l is the length of the pendulum, g is the earth acceleration, λ is the latitude, q_1 and q_2 are the Cartesian coordinates in the horizontal plane, $\dot{q}_1$ and $\dot{q}_2$ are the velocities and p_1, p_2 are the momentum values

$$p_1 = m\dot{q}_1 - m\omega \sin \lambda q_2 \tag{6}$$

$$p_2 = m\dot{q}_2 + m\omega \sin \lambda q_1 \tag{7}$$

The corresponding motion equations are a system of coupled second ordinary differential equations that are the mathematical model for the Foucault pendulum within the approximation of little amplitude displacements

$$\ddot{q}_1 - 2\omega \sin \lambda \dot{q}_2 + \frac{g}{l} q_1 = 0 \tag{8}$$

$$\ddot{q}_2 + 2\omega \sin \lambda \dot{q}_1 + \frac{g}{l} q_2 = 0 \tag{9}$$

Multiplying the equation (9) by i and adding it to the equation (8) gives the differential equation for motion in the horizontal plane

$$\ddot{\xi} + 2i\omega \sin \lambda \dot{\xi} + \omega_0^2 \xi = 0 \tag{10}$$

where $\xi = q_1 + i\, q_2$ is a complex variable and $\omega_0^2 = g/l$ .The solution of the equation (10) is (ω^2 has been neglected in comparison with ω_0^2)

$$\xi(t) = \left[A \exp(i\omega_0 t) + B \exp(-i\omega_0 t)\right] \exp\left(-i\omega_3\, t\right) \tag{11}$$

where $\omega_3 = \omega \sin(\lambda)$. If the pendulum is released from a point of maximum amplitude a $= q_1(0)$, $q_2(0) = 0$, $\dot{q}_1(0) = 0$, $\dot{q}_2(0) = 0$ (the initial conditions), it never passes through ξ = 0 and the solutions of the motion equations becomes

$$q_1(t) = a \cos\left(\omega_3 t\right) \cos(\omega_0 t) + \frac{a\omega_3}{\omega_0} \sin\left(\omega_3 t\right) \sin(\omega_0 t) \tag{12}$$

$$q_1(t) = -a \sin\left(\omega_3 t\right) \cos(\omega_0 t) + \frac{a\omega_3}{\omega_0} \cos\left(\omega_3 t\right) \sin(\omega_0 t) \tag{13}$$

In the polar coordinates (ρ and θ) we obtain

$$\theta = \arctan\left[\frac{q_2}{q_1}\right], \dot{\theta} = -\frac{2\omega_3 (-g + l\omega_3{}^2) \sin^2(\omega_0 t)}{g + l\omega_3{}^2 + (-g + l\omega_3{}^2) \cos(2\omega_0 t)} \tag{14}$$

The laws of conservation of energy E and angular momentum L_a are satisfied with these coordinates

$$E = \frac{ma^2}{2l}(g - l\omega_3{}^2) \tag{15}$$

$$\left|\vec{L}_a\right| = \left|\vec{r} \times m\vec{v}\right| = m(q_1\dot{q}_2 - q_2\dot{q}_1) + (q_1{}^2 + q_2{}^2) m\omega_3 = ma^2\omega_3 \tag{16}$$

Changing to a new variable

$$\begin{pmatrix} q_1' \\ q_2' \end{pmatrix} = \begin{pmatrix} \cos\left(\omega_3 t\right) & -\sin\left(\omega_3 t\right) \\ \sin\left(\omega_3 t\right) & \cos\left(\omega_3 t\right) \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \tag{17}$$

we obtain

$$q_1' = a \cos(\omega_0 t) \tag{18}$$

$$q_2^{'}=\frac{a\omega_3}{\omega_0}\sin(\omega_0 t) \tag{19}$$

in a rotating system with the angular velocity $\omega\sin(\lambda)$. In this system, the Hamiltonian (5) is

$$H^{'}=\frac{p_1^{'2}+p_2^{'2}}{2m}+\frac{m}{2}(\omega_3^2+\omega_0^2)(q_1^{'2}+q_2^{'2}) \tag{20}$$

The period on elliptical path is

$$\tau=\frac{2\pi}{\sqrt{\omega_0^{\,2}+\omega_3^{\,2}}} \tag{21}$$

and the time for a complete rotation of the ellipse plan is

$$T=\frac{2\pi}{\omega_3}=\frac{2\pi}{\omega\sin(\lambda)} \tag{22}$$

The experiment on the Foucault pendulum is accompanied by the Airy precession (small ellipses appear) with an angular velocity

$$\omega_A=-\frac{3}{8}\alpha\beta\omega_0 \tag{23}$$

where α and β are the major and minor axes in radians of the elliptical trajectory of the pendulum

$$\alpha=\frac{a}{l},\ \beta=\frac{\mathrm{a}\omega_3}{l\sqrt{\omega_0^{\,2}+\omega_3^{\,2}}} \tag{24}$$

In this case in the place of $\omega_3=\omega\sin(\lambda)$ we must to put

$$\omega_3^c=\omega_3(1-\frac{3a^2}{8l^2}) \tag{25}$$

in the relations (12) and (13). Thus in place of the relations (14), (15), (16) and (21) we obtain

$$\dot{\theta}^c=\frac{(3a^2-8l^2)\omega_3\left(-64gl^3+(3a^2-8l^2)^2\omega_3^{\,2}\right)\sin^2(\omega_0 t)}{8l^2\left(64gl^3\cos^2(\omega_0 t)+(3a^2-8l^2)^2\omega_3^{\,2}\sin^2(\omega_0 t)\right)} \tag{26}$$

$$E^c=\frac{ma^2}{128l^4}\left(64gl^3-(3a^2-8l^2)^2\omega_3^{\,2}\right) \tag{27}$$

$$\left|\vec{L}_a^c\right|=ma^2\omega_3(1-\frac{3a^2}{8l^2}) \tag{28}$$

$$\tau^c=\frac{2\pi}{\sqrt{\omega_0^{\,2}+\omega_3^{\,2}(1-\frac{3a^2}{8l^2})^2}} \tag{29}$$

If we use the approximation (2b), then in place of the relation (20) we have

$$H^{'}=\frac{p_1^{'2}+p_2^{'2}}{2m}+\frac{m}{2}(\omega_3^2+\omega_0^2)(q_1^{'2}+q_2^{'2})+\frac{mg}{8l^3}(q_1^{'4}+q_2^{'4}+2q_1^{'2}q_2^{'2}) \tag{30}$$

For a classical Foucault pendulum with l = 3.05m, m = 8Kg, g = 9.81 m/s^2, we obtain E = 0.68058884j, E^c = 0.68058885j, L_a = 0.0000228075Kgm^2/s, L_a^c =

0.0000228076 Kgm^2/s, τ = 3.5034448s and τ^c = 3.50523186s. For Suceava the astronomical latitude is $\lambda = 47^o 39'$ and $\omega_3 = \omega \sin(\lambda)$ = 0.0000538929rad/s.

Fig. 4 shows a simulation of the path of the Foucault pendulum (a), of the Allais pendulum (b), the differences between their coordinates (c), the temporal derivative of the azimuth for a Foucault pendulum (d), the differences between temporal derivatives for both pendulums (e) and the elliptical trajectory (the major and minor axes of the elliptical trajectory of the pendulum are a = 0.23m and $b = 6.9\times10^{-6}$m, respectively) in the rotating reference system of the Foucault pendulum (f).

Fig. 5 shows the same graphs as in Fig.4 but for a longer pendulum (l =3.05×20m, τ = 15.66788s, a = 0.23m, $b = 3\times10^{-5}$m). $\dot{\theta}$ is zero for t = τ, 2τ, 3τ, 4τ, etc. An increase in the length of the pendulum leads to a larger of the Foucault effect in comparison with the Allais effect and the differences are visible in a perpendicular direction to that of the swing direction (Figs.4-5, (c)).

4. Quantum Foucault pendulum

The Hamiltonian operators for a quantum Foucault pendulum in a fixed system on the earth, without and with the Airy precession are, respectively

$$\hat{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{m}{2}\omega_0^2\left(x^2 + y^2\right) - a^2 m\omega_3^2 \tag{31}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{m}{2}\omega_0^2\left(x^2 + y^2\right) - a^2 m\omega_3^2 (1 - \frac{3a^2}{8l^2})^2 \tag{32}$$

In this case the corresponding Schrödinger equations take the form

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \frac{m}{2}\omega_0^2\left(x^2 + y^2\right)\psi - a^2 m\omega_3^2\psi = E\psi \tag{33}$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \frac{m}{2}\omega_0^2\left(x^2 + y^2\right)\psi - a^2 m\omega_3^2 (1 - \frac{3a^2}{8l^2})\psi = E\psi \tag{34}$$

The Hamiltonian operator and the Schrödinger equation for a quantum Foucault pendulum in a rotating system are given by

$$\hat{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{m}{2}\left(\omega_0^2 + \omega_3^2\right)\left(x^2 + y^2\right) \tag{35}$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \frac{m}{2}\left(\omega_0^2 + \omega_3^2\right)\left(x^2 + y^2\right)\psi = E\psi \tag{36}$$

If we use the approximation (2b), then in place of the relation (36), we have

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \frac{m}{2}\left(\omega_0^2 + \omega_3^2\right)\left(x^2 + y^2\right)\psi + \frac{mg}{8l^3}(x^4 + y^4 + 2x^2y^2)\psi = E\psi \tag{37}$$

The values of the energy can be obtained from the solution of the Schrödinger equations (33), (34), (36) and (37). A large number of the excited states for a quantum Foucault pendulum are pair of the doubly degenerate.

We have solved the Schrödinger equations for the given boundary conditions (the Dirichlet boundary condition at the ends of the interval where the wave function can be approximated with 0) by using the Galerkin's variant of the finite element method, with triangular grid and variable step [7].

Fig. 6 show the calculated probability density, with the Schrödinger equation (37), for the first pairs of excited doubly degenerate states of a quantum Foucault pendulum (l = 3.05m, a = 0.23m, b = 0.023m, m = 8Kg, g = 9.81 m/s^2) with the energies (in $\hbar$ = 1 units): e1 = e11 = 18.588058; e2 = e22 = 31.521995; e3 = e33= 48.414085; e4 = e44 = 63.733378; e5 = e55 = 68.378136; e6 = e66 = 84.758123; e7 = e77 = 91.269980; e8 = e88 = 112.904708; e9 = e99 = 117.030554; e10 = e1010 = 136.740011; e11 = e1111= 144.932609; e12 = e1212 = 145.616593; e13 = e1313 = 163.043895 (for two higher states e = ee = 291.502732; E = EE = 899.22807).

The quantum eigenstates for a large energy of a Foucault pendulum predict that the probability density of finding the particle is largest near the classical trajectories (compare the Figs.6 (E),(EE) with Fig.4 (a)).

A large number of the excited states for a quantum Foucault pendulum are doubly degenerate in a similar way as the time dependence of the azimuths for a paraconical Allais pendulum with a high sensitivity.

5. The variation of the gravitational acceleration - the JRA effect

First we use the method of Jeverdan-Rusu-Antonescu (applied for the first time to the eclipse from 15 February 1961, developed and utilized by Olenici beginning with solar eclipse from 11 August 1999) and we have determined the experimental value of the period of oscillation of the pendulum before (T_b = 3.5060877s) and after (T_a = 3.5061041s) the maximum of the eclipse with the average (T_m = 3.5060959s) assumed to correspond to the local (Suceava) gravitational acceleration g_s = 9.81m/s^2.

The experimental value of the period of oscillation of the pendulum during the maximum of the eclipse is T_e = 3.5060706s.

From the relations $T_m = 2\pi\sqrt{l/g_s}$ and $T_e = 2\pi\sqrt{l/g_e}$ we obtain $g_e = g_s(T_m/T_e)^2$ = 9.8101412 m/s^2. Thus, our measurements show that during the maximum of the sun eclipse from 22 September 2006, the gravitational acceleration g_e is increased. This confirme the existence of the Jeverdan-Rusu-Antonescu effect.

For measure of period of oscillation was utilized a cronometer with 0.01s precision, electrically ordered by point of pendulum, and where take in account 150 oscillation at every determination.

Also, in order to estimate the variation of the gravitational acceleration with a new method, we put in the relation (14), the experimental azimuth value (in radians) in the place of θ and solve a transcendental equation in g (we take only a value which is very close to a given g).

Tables 1 – 3 and Fig. 7 show the calculated gravitational acceleration as a function of 6 series in the day (22 September 2006) of the annular solar eclipse.

Fig. 8 shows the calculated differences between the gravitational accelerations for successive series in the day (22 September 2006) of the annular solar eclipse. The results are qualitatively similar with the measurements of Wang et al. [5] and the comments of Duif [8] (a decrease in the gravitational acceleration before and after eclipse and an increase in the time of eclipse).

6. Conclusions

Although the annular solar eclipse from 22 September 2006 was not optical visible from Romania, a gravitational perturbation was detected with a sensitive paraconical pendulum. This show us that gravitational perturbations who appear during an eclipse in a zone of antieclipse are similarly with tide produced by the Moon when are at antimeridian.

Thus we obtained a change in the velocity of the azimuth of plane of oscillation of a pendulum and a confirmation of the Allais effect for a measurement in a place where the eclipse is not optical visible.

The results obtained by using two methods show an increase of the gravitational acceleration at the maximum of the eclipse and a confirmation of the Jeverdan-Rusu-Antonescu effect (eclipses from 15 February 1961 , July 1991 , August 1999 etc).

A large number of the excited states for a quantum Foucault pendulum are doubly degenerate in a similar way as the time dependence of the azimuths for a lenticular aerodynamic paraconical pendulum with a high sensitivity in the time of the annular solar eclipse from 22 September 2006.

This result consolidate the ideia of quantization of gravity and the existence of a quantum mecanics at cosmical level.

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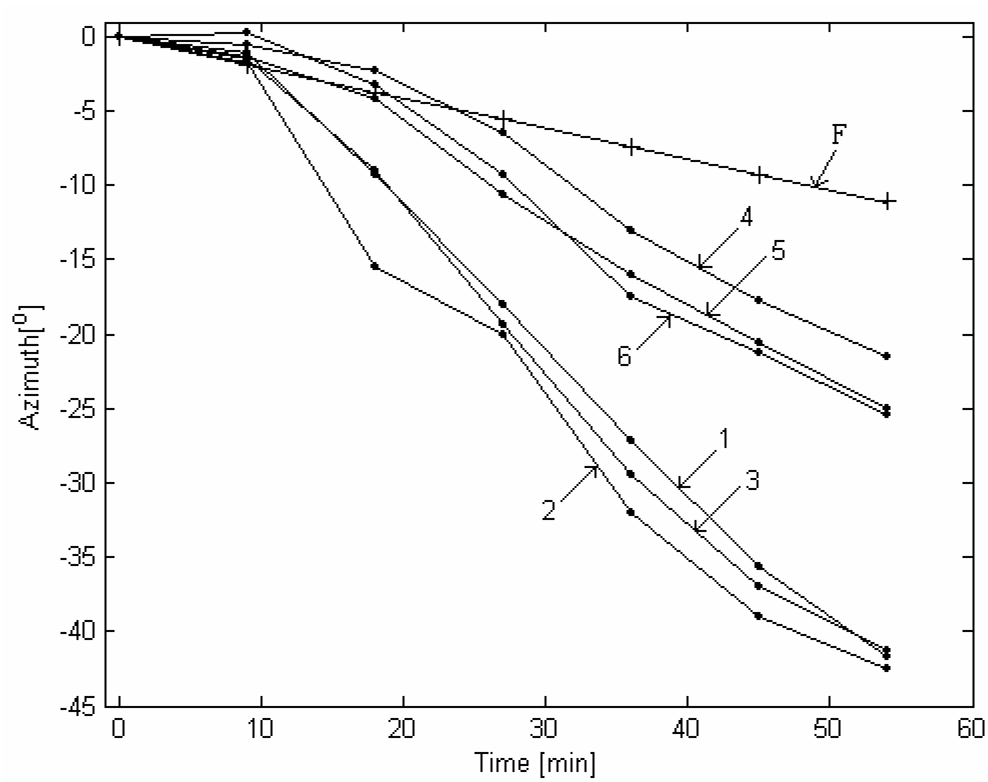


Fig. 1. The experimental azimuth as a function of time for 6 series (1 from 11h 40min to 12h 40min, 2 from 12h 40min to 13h 40min, 3 from 13h 40min to 14h 40min, 4 from 14h 40min to 15h 40min, 5 from 15h 40min to16h 40min, 6 from 16h 40min to 17h 40min) and the expected exact azimuth for Foucault effect (line F) in a day (21 September 2006) before a partial solar eclipse.

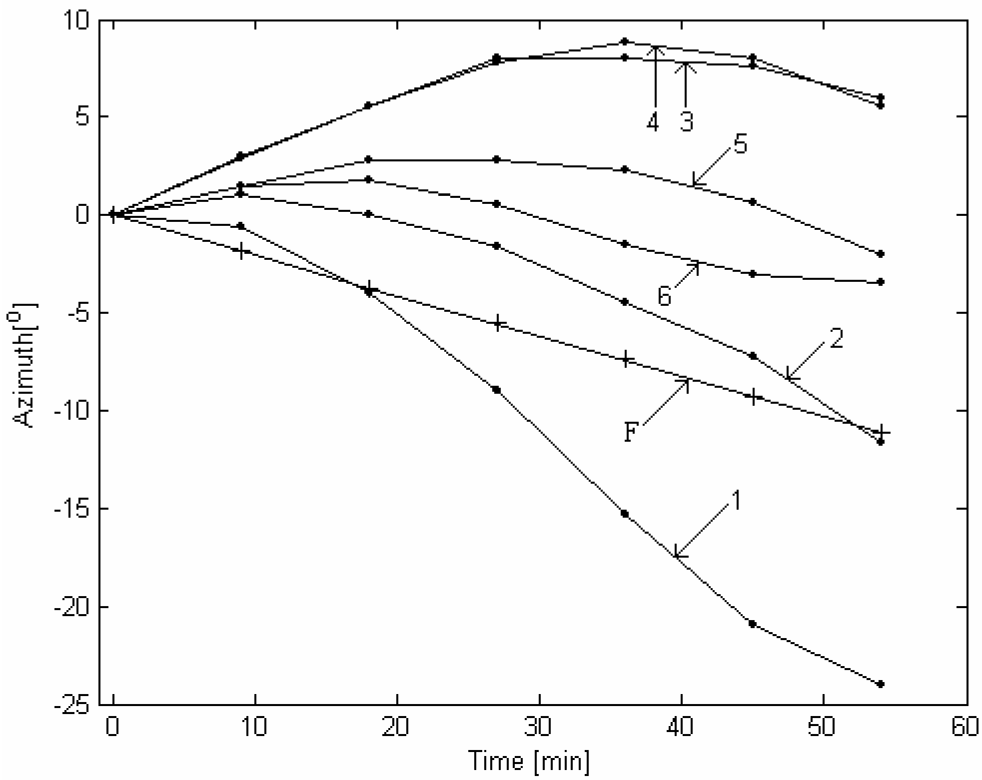


Fig. 2. The experimental azimuth as a function of time for 6 series (1 from 11h 40min to 12h 40min, 2 from 12h 40min to 13h 40min, 3 from 13h 40min to 14h 40min, 4 from 14h 40min to 15h 40min, 5 from 15h 40min to 16h 40min, 6 from 16h 40min to 17h 40min) and the expected exact azimuth for Foucault effect (line F) in the day (22 September 2006) of the annular solar eclipse (its maximum occurred at 14h 40.2min Romanian Summer Time or 11h 40.2 min UT at Suceava).

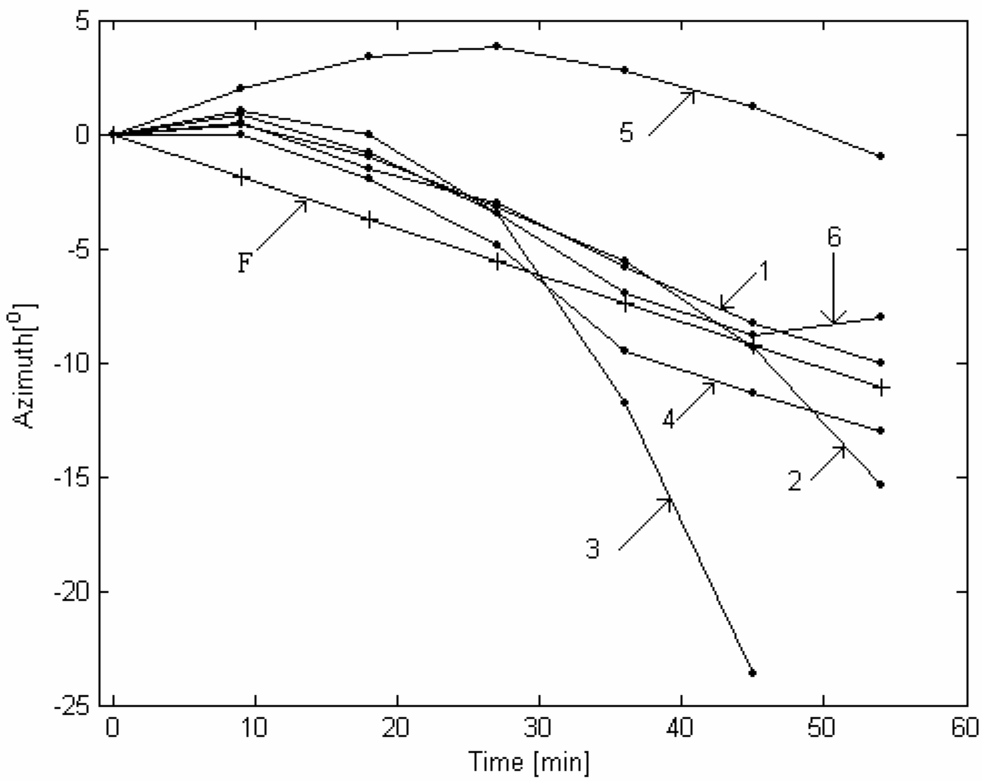


Fig. 3. The experimental azimuth as a function of time for 6 series (1 from 11h 40min to 12h 40min, 2 from 12h 40min to 13h 40min, 3 from 13h 40min to 14h 40min, 4 from 14h 40min to 15h 40min, 5 from 15h 40min to 16h 40min, 6 from 16h 40min to 17h 40min) and the expected exact azimuth for Foucault effect (line F) in a day (23 September 2006) after eclipse.

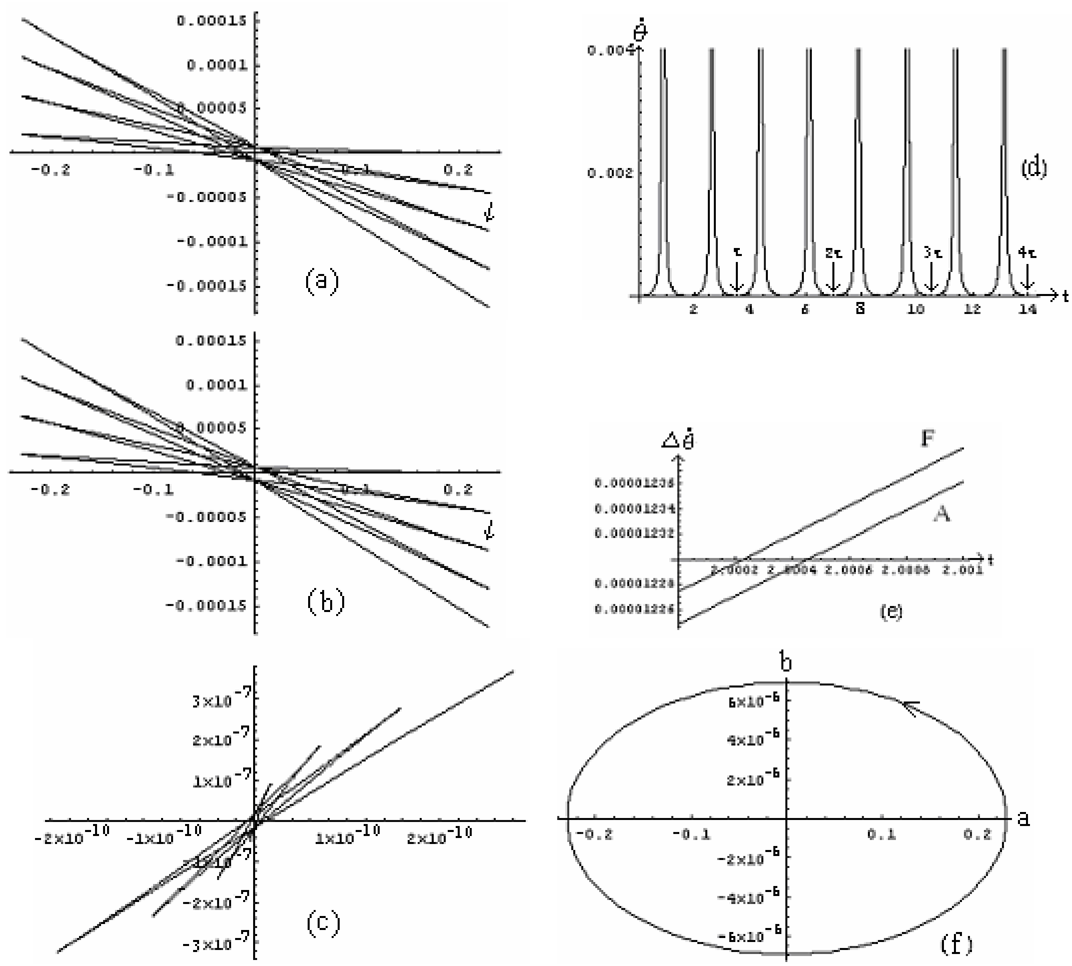


Fig. 4. Simulation of the path of the Foucault pendulum (a), of the paraconical pendulum (b), the differences between them (c), the temporal derivative of the azimuth for a Foucault pendulum (d), the differences between derivatives for both pendulums (e) and the elliptical trajectory in the rotating reference system of the Foucault pendulum (f) for $l = 3.05$m, $\tau = 3.5034448$s, $a = 0.23$m, $b = 6.9\times10^{-6}$m, $b/l = 2.3\times10^{-6}$.

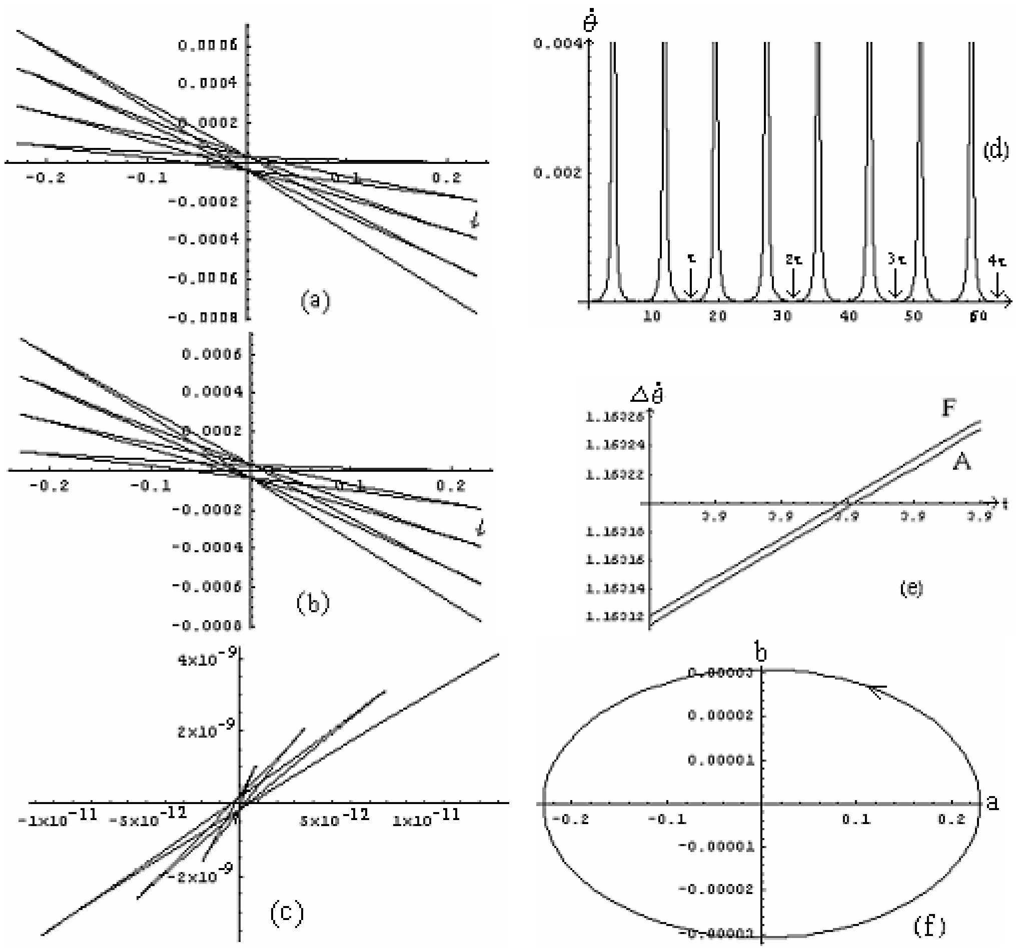
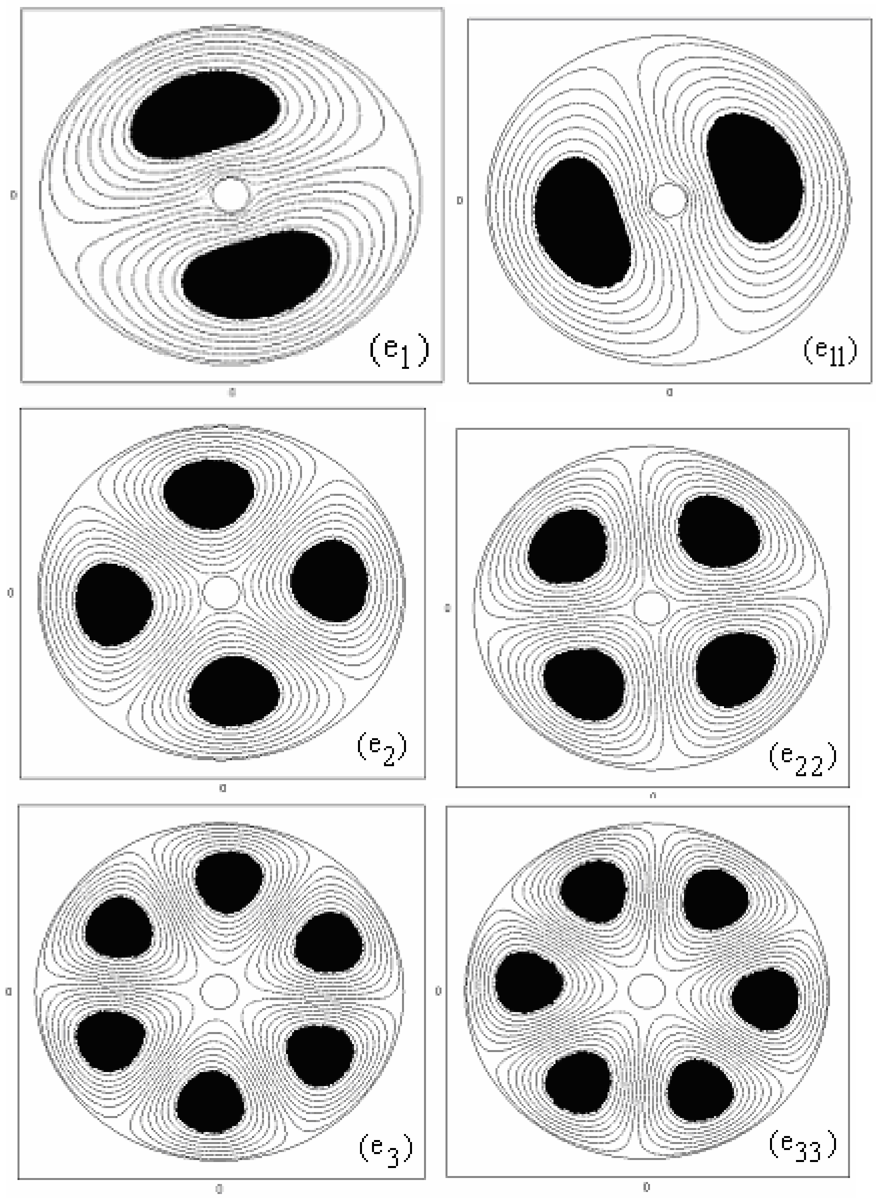
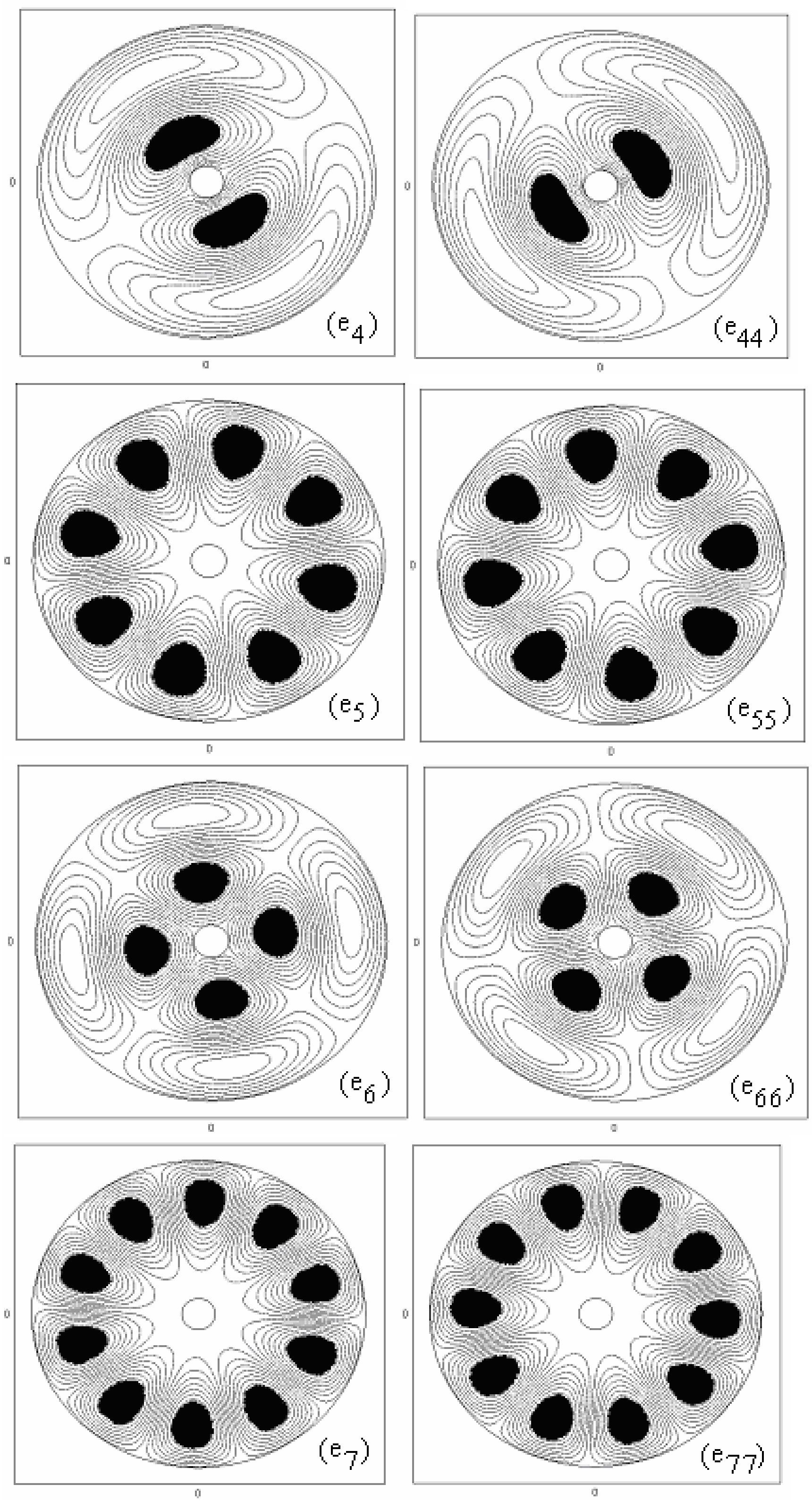


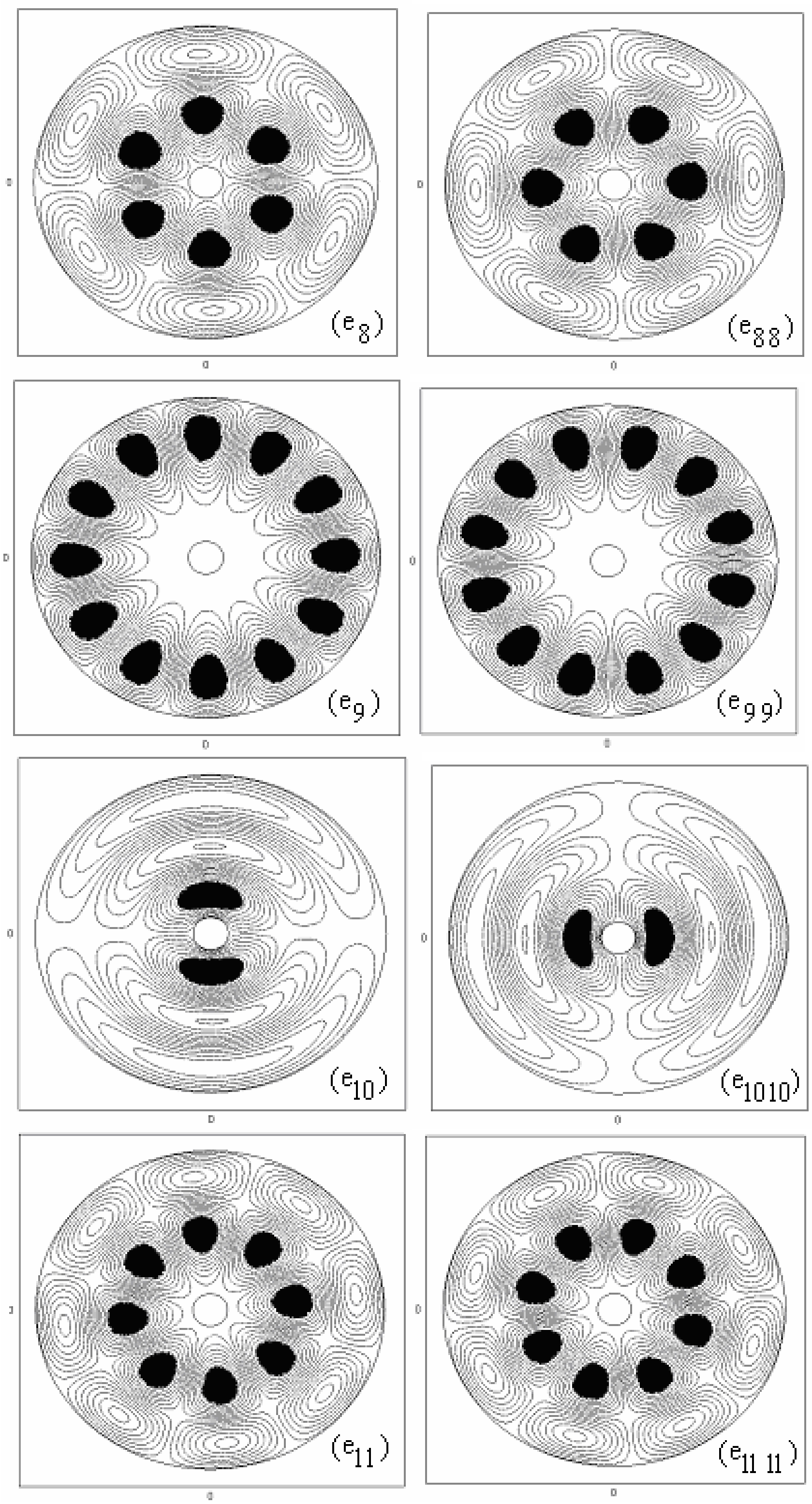
Fig. 5. Simulation of the path of the Foucault pendulum (a), of the paraconical pendulum (b), the differences between them (c), the temporal derivative of the azimuth for a Foucault pendulum (d), the differences between derivatives for both pendulums (e) with the same precision as in Fig. 1, and the elliptical trajectory in the rotating reference system of the Foucault pendulum (f) for $l = 3.05\times20$m, $\tau = 15.66788$s, $a = 0.23$m, $b = 3\times10^{-5}$m, $b/l = 5.1\times10^{-7}$.



(e_1)
(e_{11})
(e_2)
(e_{22})
(e_3)
(e_{33})



(e$_4$)
(e$_{44}$)
(e$_5$)
(e$_{55}$)
(e$_6$)
(e$_{66}$)
(e$_7$)
(e$_{77}$)


(e_8)
(e_{88})
(e_9)
(e_{99})
(e_{10})
(e_{1010})
(e_{11})
$(e_{11\,11})$

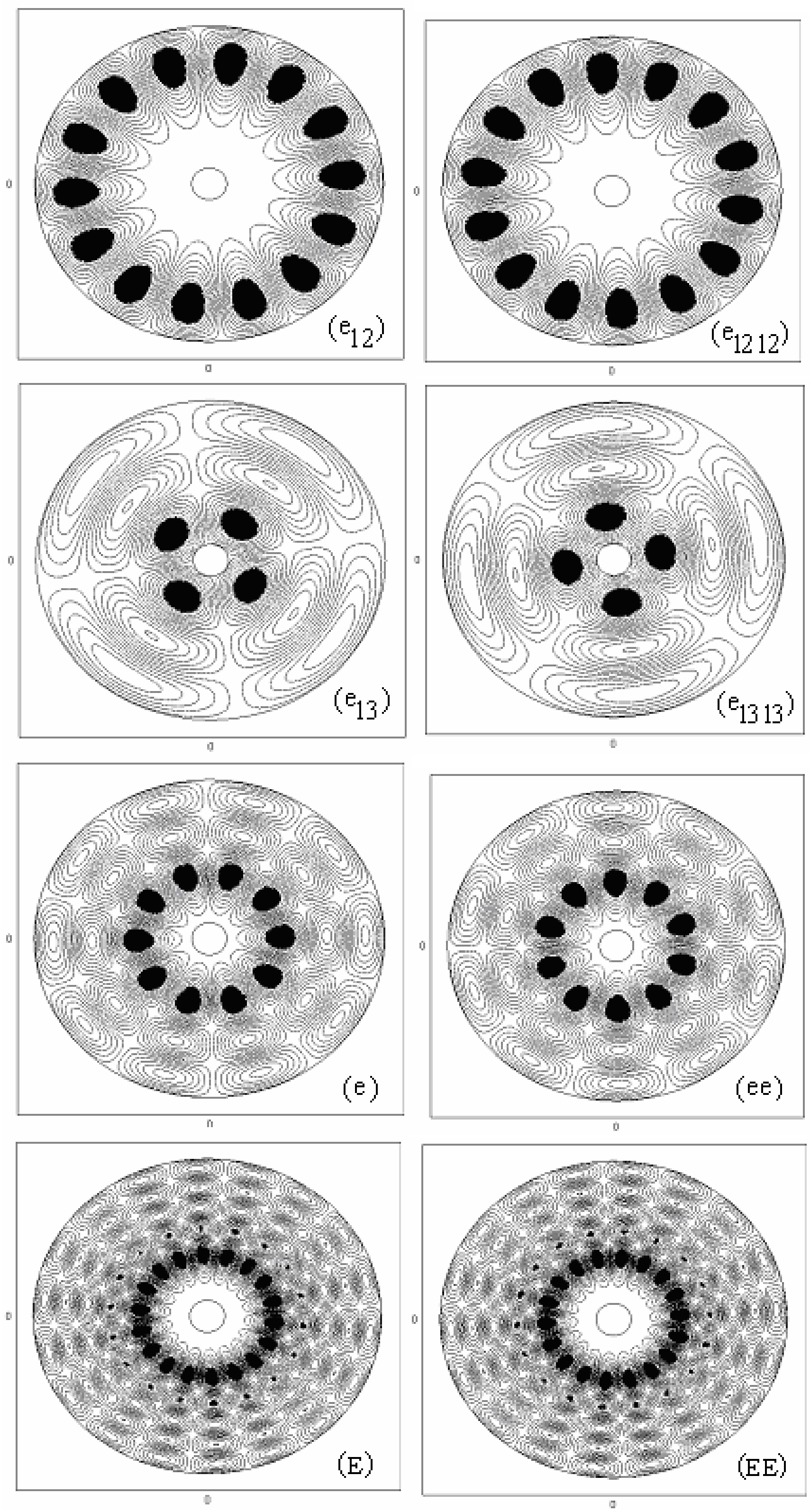
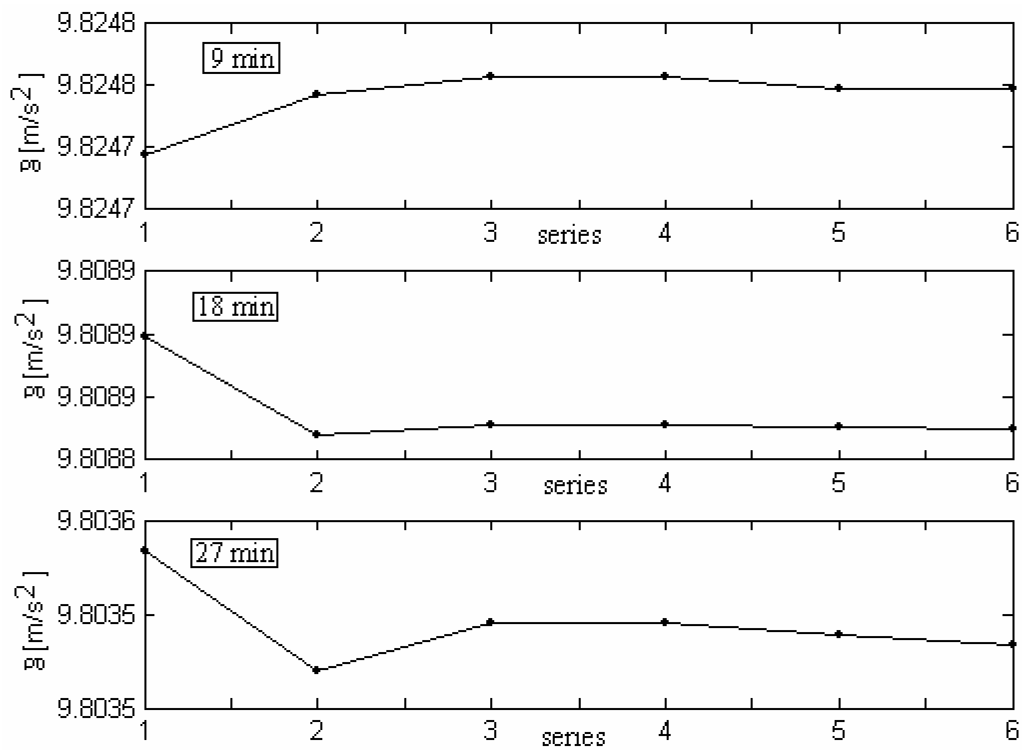


Fig. 6. The calculated probability density (with a contour plot) with the Schrödinger equation (37) in $\hbar$ = 1 units, for the first pairs of excited doubly degenerate states of a

quantum Foucault pendulum (l = 3.05m, a = 0.23m, b = 0.023m, m = 8Kg, g = 9.81 m/s^2).



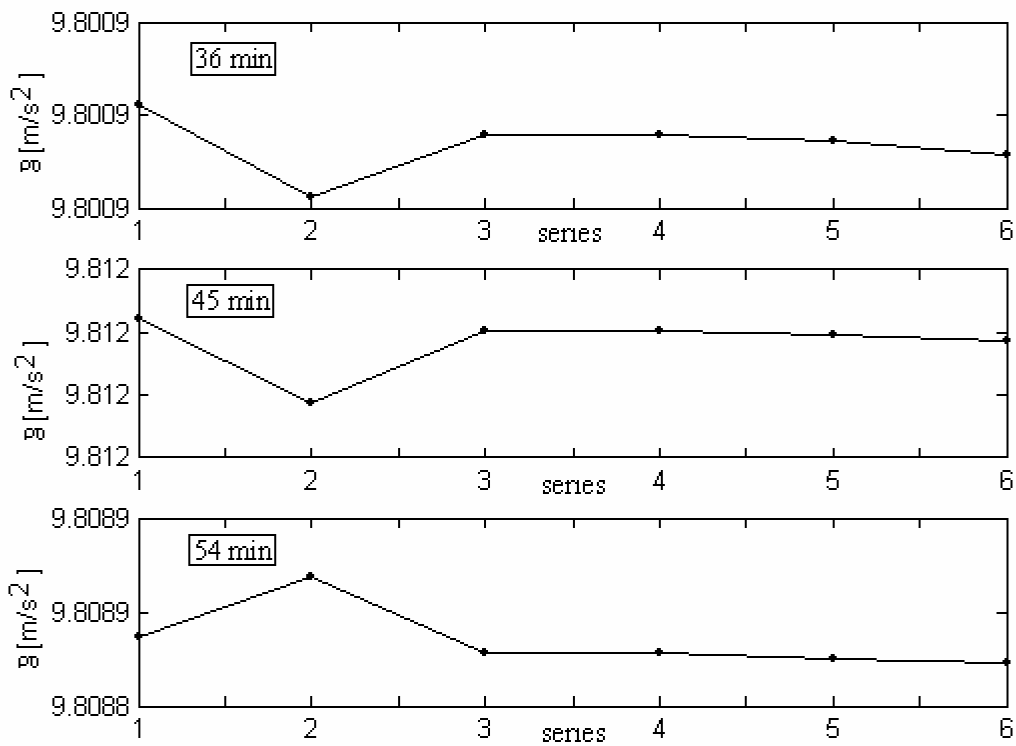


Fig. 7. The calculated gravitational acceleration as a function of 6 series in the day (22 September 2006) of a annular solar eclipse.

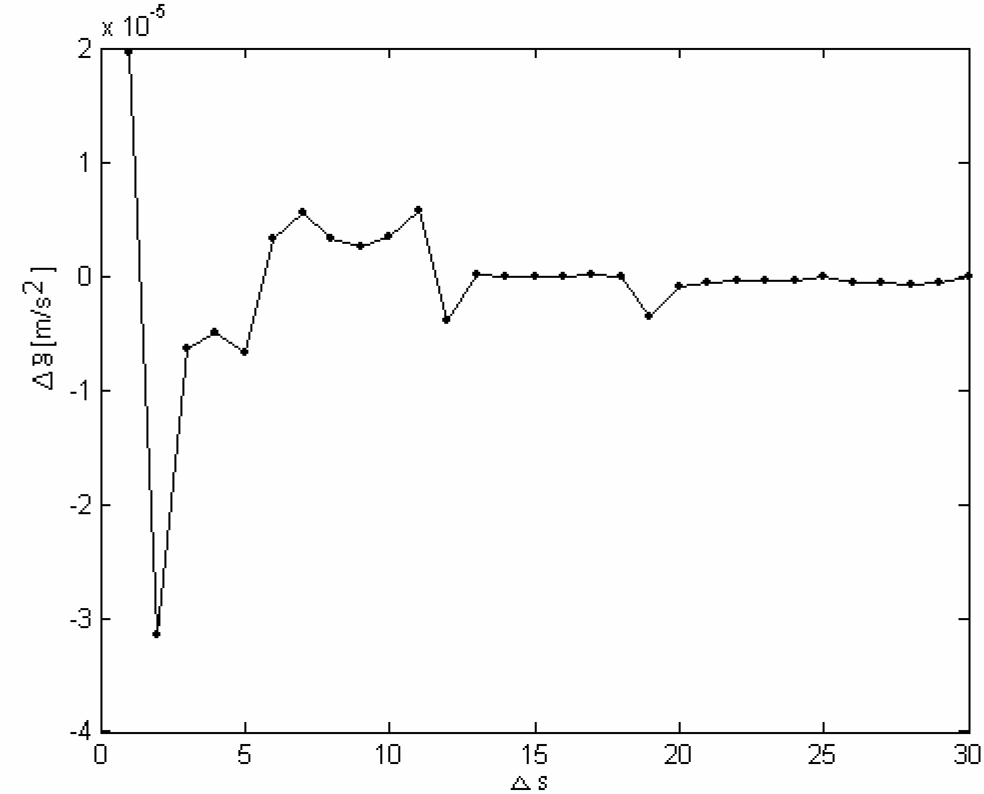


Fig. 8. The calculated differences between the gravitational accelerations for successive series (the differences between the successive columns in Tables 2-3) in the day (22 September 2006) of the annular solar eclipse.

Table 1.
The calculated gravitational acceleration as a function of 6 series in the day before (21 September 2006) and after (23 September 2006) of a partial solar eclipse. For the grouped series we have used a mean value.

t(min)	g (21 Sept.) (11^h40^m-14^h39^m)	g (21 Sept.) (14^h40^m-17^h39^m)	g (23 Sept) (11^h40^m – 17^h39^m)
9	9.8246212	9.8247403	9.8247576
18	9.8088554	9.8086101	9.8088481
27	9.8035510	9.8035535	9.8035456
36	9.8008999	9.8009006	9.8009027
45	9.8120359	9.8120358	9.8120361
54	9.8088538	9.8088537	9.8088549

Table2. The calculated gravitational acceleration as a function of 6 series in the day (22 September 2006) of the annular solar eclipse.

t(min)	g (22 Sept.) (11^h40^m-12^h39^m)	g (22 Sept.) (12^h40^m-13^h39^m)	g (22 Sept) (13^h40^m-14^h39^m)

9	9.8247375	9.8247572	9.8247626
18	9.8088794	9.8088480	9.8088513
27	9.8035533	9.8035470	9.8035495
36	9.8009006	9.8008955	9.8008990
45	9.8120361	9.8120295	9.8120352
54	9.8088536	9.8088569	9.8088529

Table3. The calculated gravitational acceleration as a function of 6 series in the day (22 September 2006) of the annular solar eclipse.

t(min)	g (22 Sept.) (14^h40^m -15^h39^m)	g (22 Sept.) (15^h40^m -16^h39^m)	g (22 Sept) (16^h40^m-17^h39^m)
9	9.8247628	9.8247592	9.8247592
18	9.8088513	9.8088504	9.8088498
27	9.8035495	9.8035489	9.8035483
36	9.8008990	9.8008986	9.8008979
45	9.8120352	9.8120348	9.8120343
54	9.8088529	9.8088525	9.8088523