

Lagrangian description of the wave equation: Global positioning system depends on Stokes' ether dragging hypothesis

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Abstract: Ether, which was considered undetectable in 1905, can be detected in the global positioning system (GPS) experiments of the 1980s. If the existence of ether is not assumed, we cannot explain the experimental data of the GPS. In this paper, we use the Lagrangian description of the wave equation, which makes wave equation Galilean invariant. Thereafter, the stellar aberration experiment with a water-filled telescope by Airy can be explained by Stokes' ether dragging hypothesis. The GPS experimental results indicate that the ether hypothesis is more suitable than both the principle of relativity and the principle of invariant light speed. © 2014 *Physics Essays Publication*. [<http://dx.doi.org/10.4006/0836-1398-27.1.68>]

Résumé: L'éther, qui était considéré comme indétectable en 1905, peut être détecté dans les expériences réalisées sur le système de localisation mondial (GPS) dans les années 1980. Les données expérimentales GPS ne peuvent être expliquées sans supposer l'existence de l'éther. Dans cet article, nous utilisons la description lagrangienne de l'équation d'onde, qui rend l'équation d'onde invariante par transformation de Galilée. L'expérience de l'aberration stellaire avec un télescope rempli d'eau réalisée par Airy peut être expliquée par l'hypothèse de Stokes sur la résistance de l'éther. Les résultats expérimentaux avec le GPS indiquent que l'hypothèse de l'éther convient mieux que le principe de relativité et le principe de l'invariance de la vitesse de la lumière.

Key words: Wave Equation; Stokes' Ether Dragging Hypothesis; Lagrangian Description; Lorentz Transformation; Galilean Invariance; Global Positioning System; Earth-Centered Locally Inertial Coordinate System.

I. INTRODUCTION

The global positioning system (GPS) causes current discussions in the foundations of physics. One is the preferred reference frame: that is, the earth-centered locally inertial (ECI) coordinate system is the preferred reference frame. The time dilation of the GPS satellites is caused by the velocity as well as the gravitational effect. The time dilation caused by the velocity of the GPS satellite only depends on the relative velocity defined in the ECI coordinate system. We will make this point clear in Section III. The other is the Newtonian time: all clocks are synchronized in the ECI coordinate system. Of course, in the GPS, the relativistic as well as gravitational effects are taken into consideration; therefore, in individual measurements we do not need to take these effects into consideration. This is because all clocks are synchronized in advance. The lengths of the position vectors are defined using the speed of light c . In the use of the GPS, we are in a three dimensional Euclidian space and one dimensional time: that is, the space and the time are independent, respectively. The idea of Minkowski's spacetime is not used.

Ether was discussed by many great scientists; however, these discussions disappeared at the end of the 1920s

because ether was not observed experimentally. Michelson,^{1,2} Morley,² Miller,³ and other great physicists continued performing experiments to detect ether because they believed in the existence of ether. Michelson^{1,2,4,5} performed the experiments for 50 years, although it was widely believed that there was no ether (or that ether was undetectable). In 1924, the Michelson–Gale–Pearson⁴ experiment measured the velocity of the ground with respect to ether (rotation speed of the earth); however, the experimental results were considered to be explained by the theory of special relativity, and the existence of ether was not accepted.

In 1725, the stellar aberration (shown in Fig. 1) was observed by Bradley. He explained the stellar aberration using Newton's particle model of light. Figure 2 illustrates the relative motion of the earth at a revolution velocity of 30 km/s and the photons from the top at the speed of light c . The light from the top of his head, angle α , is calculated as $\sin \alpha \sim \alpha = v/c$. Thus, $\alpha = 30 \div 300,000 \text{ km/s} = 10^{-4}$, which is approximately 20 arc sec.

It is pointed out that the aberration only depends on the revolution velocity of 30 km/s. Although the solar system moves in the galaxy at 220 km/s, the earth orbital motion is slipped cycloid in the galaxy; however, the stars shows the aberration angle $\alpha = 10^{-4}$; the motion of the earth looks circular around the sun. We will discuss and summarize this point in Section VII.

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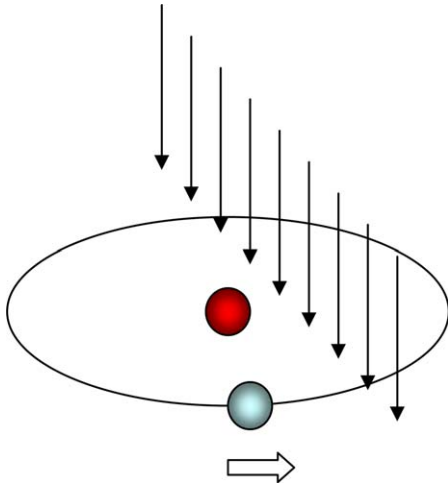


FIG. 1. (Color online) Explanation of the aberration by Bradley: The earth's revolution velocity (30 km/s) makes the stellar light from the top seem as if it comes from the front.

Two important ether dragging models have been produced: one is Fresnel's model⁶ of a partial ether drag determined by Fresnel's dragging coefficient, and the other is Stokes' model⁷ of complete ether drag. In the gravitational field of the earth, we assume the Stokes' model of complete ether drag; ether is dragged not by the mass but by the gravitational field. In the end of 1800s, there were proposed other versions of Stokes' model that ether dragging is proportional to the gravitational mass (see Section VI C).

Fresnel's model of a partial ether drag indicates that the light is dragged by the medium $((n^2 - 1)/n^2)v$ (n is the refractive index, and v is the velocity of the medium). This equation was experimentally confirmed by Fizeau^{8,9} in 1851; Michelson and Morley¹⁰ reconfirmed Fizeau's experiment in 1886; in these days, the experimental setups were reproduced for use in the undergraduate curriculum.¹¹ In water, Fresnel's coefficient is $(n^2 - 1)/n^2 = 0.434$ ($n = 1.33$); in air, the light is dragged by $(n^2 - 1)/n^2 = 5.8 \times 10^{-4}$ ($n = 1.000292$), and for optical prism, it becomes 0.677 ($n = 1.76$). In 1818, Fresnel⁶ inferred from his dragging coefficient that the aberration

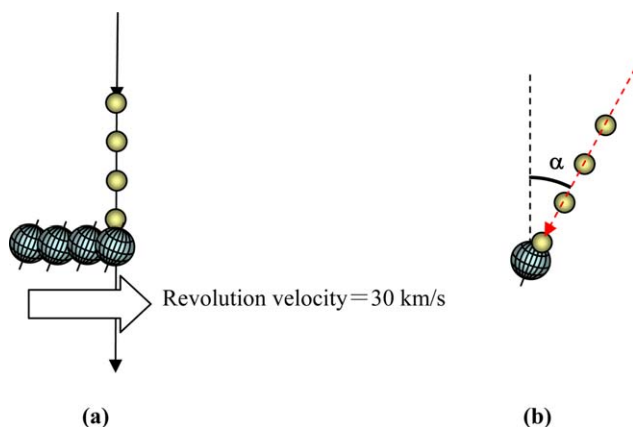


FIG. 2. (Color online) Relative motion of the earth at a revolution velocity of 30 km/s, and the photons from the top at the speed of light c seen (a) from the solar system and (b) from the earth.

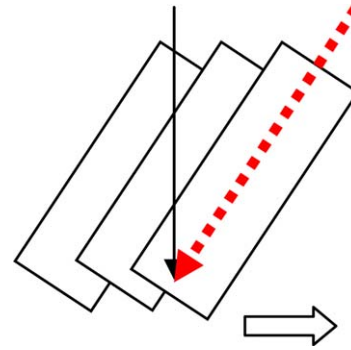


FIG. 3. (Color online) Explanation of the aberration by Bradley: The earth's revolution velocity (30 km/s) makes the stellar light from the top seem as if it comes from the front. The dotted line was considered to be the apparent direction of the light.

would be unaffected by the presence of the water (Boscovich–Airy experiments) in his letter to Arago. In 1871, Airy's experiment with a water-filled telescope demonstrated that the aberration is unaltered by water.

The problem in Fresnel's coefficient n which depends on the wavelength of light was experimentally demonstrated; the aberration does not show any dispersion. We will make another scenario using not Fresnel's model but Stokes' model⁷ in Section IV. Figure 3 provides an explanation of the aberration by Bradley: The Earth's revolution velocity (30 km/s) makes the stellar light from the top seem as if it comes from the front. Figure 4 presents the water-filled telescope experiment by Airy: The direction of the light was unchanged. To satisfy the experiments in Figs. 3 and 4, the dotted line was considered not to be the apparent but rather the true direction of the light. Explanation of the stellar aberration is shown in Section V.

In 1903, Poynting¹² was the first to realize that the sun's radiation can draw in small particles towards it: this was later named the Poynting–Robertson effect.¹³ The small particles orbit around the sun suffer the deceleration; that is, photons from the sun hit the front of the particles (aberration) as shown in Fig. 5. Thus, the particle decelerated to lose its momentum and kinetic energy to fall toward the sun. The

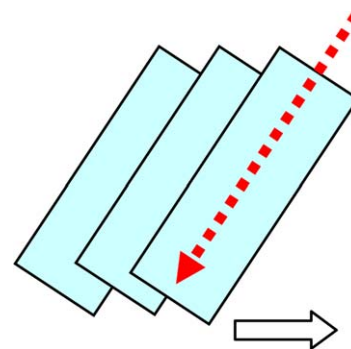


FIG. 4. (Color online) Water-filled telescope experiment by Airy: The direction of the light was not changed. The dotted line was considered to be not the apparent but the true direction of the light.

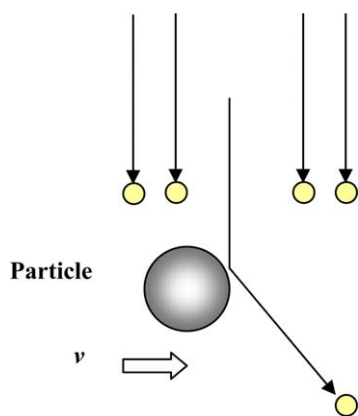


FIG. 5. (Color online) Explanation of the Poynting–Robertson effect by Compton effect: Photons from the sun hit the front of the particle, thus, the particle decelerated to lose its momentum and kinetic energy to fall toward the sun. This is because the moving particle observes the aberration. The radiation pressure moves particle outwards; particles of intermediate size will either spiral inwards or outwards depending on their size and their initial velocity vector.

Poynting–Robertson effect can be explained using Fig. 2. The Poynting–Robertson effect shows a relative motion between the particle and photon; that is, we cannot distinguish whether we are approaching the photon as shown in Fig. 2(a) or photons are traveling toward us as shown in Fig. 2(b). As far as the relative motion, there is a critical difference between the stellar aberration and the Poynting–Robertson effect; we will explain in Section V.

In 1907, Laue¹⁴ showed that the theory of special relativity calculates the Fresnel’s dragging coefficient, thus, predicts the result of the Fizeau experiment from the velocity addition law without any need for the ether. However, it cannot be applied to the Airy’s experiment; this is because the dragging direction is transverse to that of light. Therefore, to explain the Airy’s experiment, the Fresnel’s dragging coefficient or alternative explanation are required.

Eisner¹⁵ discussed the aberration of light from binary stars: “It is argued that aberration does not depend on the relative velocities of source and observer: it depends only on the change in velocity of the observer between the times when the two measurements from which the aberration is deduced are made.” The aberration was discussed from the viewpoint of relativity.¹⁶ We will discuss the aberration of binary stars in Section VII C.

Van Flandern¹⁷ noted “Airy found that aberration did not change for a water-filled telescope, and therefore did not arise in the telescope tube. That suggests it must arise elsewhere locally. Michelson–Morley expected the Earth’s velocity to affect the speed of light because it affected aberration. But it did not. If these experimenters had realized that the aether was not a single entity but changed with the local gravity field, they would not have been surprised. It might have helped their understanding to realize that Earth’s own Moon does not experience aberration as the distant stars do, but only the much smaller amount appropriate to its small speed through the Earth’s gravity field.” Thereafter, “Rather, the relative velocity between local and distant gravity fields determined aberration.” The moon-to-earth mass ratio is

0.01230 ($\approx 1/81$). Thus, the earth is almost stationary in the earth–moon system. The GPS satellites also do not show any aberration caused by the relative velocity of 4 km/s; this is because the earth is stationary in the ECI coordinate system. Only diurnal aberration caused by the earth’s spin is observed.

In 1881, Michelson¹ concluded in his paper that “The result of the hypothesis of a stationary ether is thus shown to be incorrect,” where a stationary ether implies that it is stationary in the universe. Thereafter, he added an extract from an article by Stokes that “All these results would follow immediately from the theory of aberration which I proposed in the July number of this magazine: nor have I been able to obtain any result admitting of being compared with experiment, which would be different according to which theory we adopted. This affords a curious instance of two totally different theories running parallel to each other in the explanation of phenomena. I do not suppose that many would be disposed to maintain Fresnel’s theory, when it is shown that it may be dispensed with, inasmuch as we would not be disposed to believe, without good evidence, that the ether moved quite freely through the solid mass of the earth. Still it would have been satisfactory, if it had been possible to have put the two theories to the test of some decisive experiment.” Michelson considered that as far as the revolution velocity of 30 km/s was concerned, Airy’s experiment and his own experiment were compatible not with Fresnel’s theory but with Stokes’ theory.

In 1886, Michelson and Morley¹⁰ reconfirmed Fizeau’s experiment; they concluded that “The result of this work is therefore that the result announced by Fizeau is essentially correct; and that the luminiferous ether is entirely unaffected by the motion of the matter which it permeates” (see Section IV D).

The famous Michelson–Morley² paper in 1887 noted that “On the undulatory theory, according to Fresnel, first, the ether is supposed to be at rest except in the interior of transparent media” and thereafter that “The experimental trial of the first hypothesis forms the subject of the present paper.” The experimental data indicated that “It seems fair to conclude from the figure that if there is any displacement due to the relative motion of the earth and the luminiferous ether, this cannot be much greater than 0.01 of the distance between the fringes.” Figure 6 presents the data. Michelson and Morley described that “It appears, from all that precedes, reasonably certain that if there be any relative motion between the earth and the luminiferous ether, it must be small; quite small enough entirely to refute Fresnel’s explanation of aberration. Stokes has given a theory of aberration which assumes the ether at the earth’s surface to be at rest with regard to the latter.” The conclusion was “the ether is at rest with regard to the earth’s surface.” Not only according to Airy’s experiment but also the Michelson–Morley experiment, Stokes’ ether theory was concluded to be correct.

In 1924, the Michelson–Gale–Pearson⁴ experiment was performed to determine the effect of the earth’s rotation on the velocity of light. They assumed a fixed ether (to the ECI coordinate system) and the principle of invariant light speed (the theory of special relativity). The experimental results

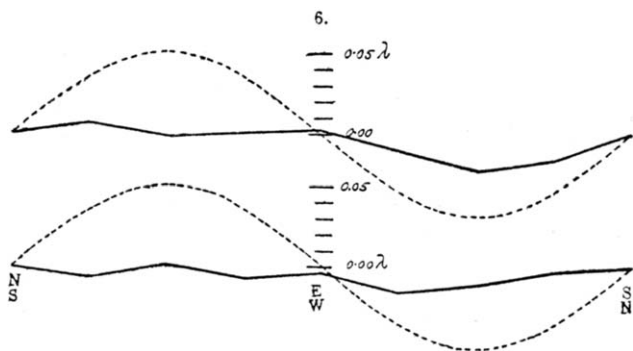


FIG. 6. Michelson–Morley² experiment in 1887: “The upper is the curve for the observations at noon, and the lower that for the evening observations. The dotted curves represent *one-eighth* of the theoretical displacements. It seems fair to conclude from the figure that if there is any displacement due to the relative motion of the earth and the luminiferous ether, this cannot be much greater than 0.01 of the distance between the fringes.”

provided the angular velocity of the earth in accordance with the theory of special relativity and experimentally demonstrated the existence of fixed ether.

In 1929, Michelson, Pease, and Pearson⁵ observed one fifteenth of the speed of the solar system in the galaxy (300 km/s); that is, 20 km/s. Although, this experimental value was considered to be equivalent to the revolution speed of the earth (30 km/s), however, they concluded one fifteenth of the speed of the solar system; this is because they consistently believed that the ether is at rest with regard to the surface of the earth. The 1886 and 1924 experiments reconfirmed the Fresnel’s stationary ether. The 1881, 1887, and 1929 experiments confirmed the Stokes’ completely dragged ether. Both the Fresnel’s stationary ether and the Stokes’ completely dragged ether are correct.

In the early of the 20th century, the difference between Newtonian mechanics and the Maxwell equations in the Galilean transformation was a serious problem; the Newtonian mechanics are Galilean invariant, whereas the Maxwell equations are not. In those days, the Maxwell equations were considered to suggest that the speed of light is independent of the reference frame; that is, the speed of light is invariant in stationary ether. This is caused because the Eulerian (partial time) derivative is used. Thus, the Maxwell equations are not Galilean invariant. Let us back to the Maxwell’s original Trieste.

In 1873, the Maxwell¹⁸ equations were represented using the total time derivatives d/dt in the original text. Darrigol¹⁹ noted that “In the most complete and concise form later given by Oliver Heaviside and Heinrich Hertz,” thereafter “our ‘Maxwell equations’ are in the case of bodies at rest”; that is, the drifting velocity is zero. At the early of the 20th century, the Maxwell equations became today’s vector notation using partial time derivatives $\partial/\partial t$. Maxwell as well as Hertz²⁰ considered that the Maxwell equations should be satisfied in drifting ether. Figure 7 illustrates the physical meaning of the original Maxwell equations. The ECI coordinate system enclosed by a capsule moves in the solar system at the drift velocity of 30 km/s. The ether surrounding the earth

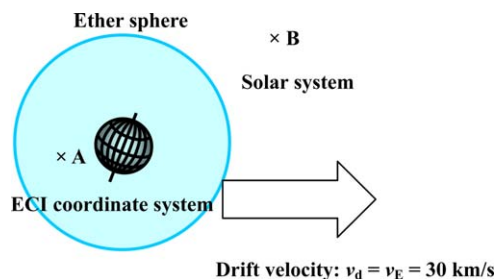


FIG. 7. (Color online) Physical meaning of the wave equation: The ECI coordinate system enclosed by an ether sphere moves in the solar system at a drift velocity of 30 km/s. From solar system, we see the wave on the medium drift.

is enclosed by an ether sphere, and the ether in the solar system is also enclosed by a large ether sphere; thus, the ether is stationary. In the ECI coordinate system (point A), as well as in the solar system (point B), the Maxwell equations are satisfied. If we determine the Maxwell equations in the ECI coordinate system from the solar system, the original Maxwell equations should be applied. These equations become Galilean invariant. We will discuss the original Maxwell equations in Section II.

Thus, in 1893, Hertz²⁰ clarified that the Maxwell equations were defined in the dragged ether; he substituted Lagrangian derivatives for Maxwell’s total time derivatives. Phipps²¹ noted that Hertz clarifies Maxwell equations Galilean invariant. Hertz²⁰ gave complete and concise representation of the Maxwell’s original equations; the drifting velocity \vec{v}_d is set zero, that is, in the case of bodies at rest. Hertz made clear that this representation can be applied on the surface of the earth; that is, the Eulerian derivatives are fixed to the ECI coordinate system. However, in the early of the 20th century, Eulerian (partial time) derivatives arose a misunderstood problem: the Newtonian mechanics are Galilean invariant, whereas the Maxwell equations are not. Of course, this is not correct: the Maxwell equations are Galilean invariant (see Section II). Hertz¹⁸ clearly showed Eulerian (partial time) derivatives are valid in the gravitational field of the earth.

Lorentz’s ether theory was based on a completely motionless ether. In 1899, Lorentz published several important papers,^{22–24} he noted that “Prof. Plank of Berlin had the kindness to call my attention to the fact that both condition might be satisfied at the same time, if the aether were compressive and subject to gravity, so that it could be condensed around the earth like a gas.”²² Thereafter, “Let the aether obey Boyle’s law and be attracted by the earth according to the law of Newton.”

Lorentz²² used description $\dot{\delta} = (\partial\delta/\partial t) - v \cdot (\partial\delta/\partial t)$ in the Maxwell equations, where δ is the dielectric displacement in the ether, dot represents the total time derivative. In 1904, Lorentz²⁵ showed that using the Lorentz transformation, the form of the Maxwell equations is not changed by the velocity of translation v (see Section III). Thereafter, Einstein²⁶ discerned the principle of relativity: the laws of physical systems are unchanged by the inertial motion relative to one another.

TABLE I. Maxwell equations and ether experiments.

Year	Equations and experiments
1725	Stellar aberration by Bradley.
1766	Boscovich: argument of an experiment of aberration with water-filled telescope.
1818	Fresnel's model ⁶ of a partial ether drag. He inferred that the aberration would be unaffected by the presence of the water.
1845	Stokes' model ⁷ of complete ether drag.
1851	Fizeau' ether dragging experiment demonstrated that the Fresnel's model ⁶ of partial ether dragging is correct.
1871	Airy's experiment with water-filled telescope: The aberration is unchanged for water. This experimentally showed that the Stokes' model ⁷ of complete ether drag is correct.
1873	Maxwell ¹⁶ equations were represented using the total time derivatives; that is, the original Maxwell ¹⁶ equations are Galilean invariant.
1881	Michelson ¹ experiment: He concluded, "The result of the hypothesis of a stationary ether is thus shown to be incorrect" and, thereafter, referred to an article by Stokes. That is, Michelson considered the Stokes' model ⁷ of complete ether drag is correct.
1886	Michelson and Morley ¹² repeated Fizeau's experiment; they concluded that "The result of this work is therefore that the result announced by Fizeau is essentially correct; and that the luminiferous ether is entirely unaffected by the motion of the matter which it permeates." They reconfirmed that the Fresnel's model ⁶ of a partial ether drag is also correct.
1887	Michelson–Morley ² experiment: They concluded, "the ether is at rest with regard to the earth's surface." They confirmed both Fresnel's and Stokes' models of ether drag are correct.
1893	Hertz ²⁰ gave complete and concise representation of the original Maxwell equations; the drifting velocity \vec{v}_d is set zero, that is, in the case of bodies at rest.
1913	Sagnac ²⁸ published a paper entitled, "The demonstration of the luminiferous aether by an interferometer in uniform rotation" (English translation).
1924	Michelson–Gale–Pearson ⁴ experiment: They experimentally observed the earth rotation velocity and concluded that the fixed ether with respect to the ECI coordinate system was valid. This experiment was considered to be equivalent to the Sagnac experiment.
1929	Michelson, Pease, and Pearson ⁵ observed one fifteenth of the speed of the solar system in the galaxy (300 km/s); that is, 20 km/s. They concluded one fifteenth of the speed of the solar system; they consistently believed that the ether is at rest with respect to the surface of the earth.

In 1909, Ehrenfest²⁷ discussed "Uniform rotation of rigid bodies and the theory of relativity," thereafter concluded that "the elements of a radius cannot show a contraction compared to the state of rest." We consider that Ehrenfest is correct; rigid bodies do not contract by rotation. However we do not have any experimental evidences.

In 1913, Sagnac^{28,29} published two important papers titled, "The demonstration of the luminiferous aether by an interferometer in uniform rotation" and "On the proof of the reality of the luminiferous aether by the experiment with a rotating interferometer" (English translation), which clearly demonstrates that he believed in the existence of ether. Today, the Sagnac effect is considered as denying the existence of ether; however, the Sagnac effect can be easily explained using the ether hypothesis.

Table I summarizes the stellar aberration, the Michelson–Morley experiments, and the Sagnac experiments from 1725 to 1929. The Maxwell equations were considered to be defined in the ether. Thus, Maxwell used total time derivatives. Maxwell believed in the existence of ether; therefore, he encouraged Michelson to perform the interferometer experiment to detect the relative velocity of the earth with respect to ether. Hertz also believed in the existence of ether.

In the 1980s, GPS began to be used. In the early period of GPS experimentation,^{30,31} interesting trials concerning the theory of special relativity were performed. For example, in 1985, an around-the-world relativistic Sagnac experiment using the stations on earth and GPS satellites was conducted. The GPS–Sagnac effect can be easily understood using an illustration.³²

Figure 8 illustrates how the signal from the GPS satellite is observed by the station on the rotating earth. During the flight time of the signal from the GPS satellite, the station on earth moves toward the satellite; thus, the station on earth receives the signal earlier than the stationary station in the ECI coordinate system.

Selleri^{33,34} fundamentally and precisely argued the speed of light in the rotating platform using the theory of special and general relativities; that is, noninvariant one-way speed

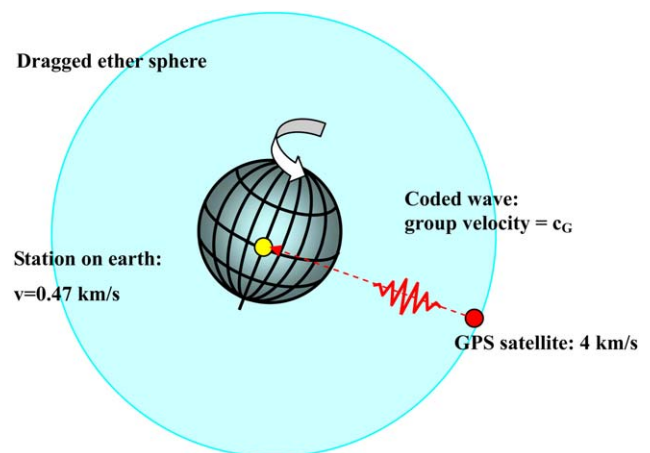


FIG. 8. (Color online) Sagnac effect in GPS³²: Station on earth moves in the ECI coordinate system at the velocity $v = 0.47$ km/s. Coded wave travels in the ECI coordinate system at the speed of light c . Station observes not only Doppler shift but also Sagnac effect. The ECI coordinate system looks a preferred coordinate system.

of light. Acceleration caused by the circular motion is carefully eliminated to make phenomena in linear motion. As was experimentally demonstrated by Wang *et al.*,^{35,36} the Sagnac effect exists in linear motion. We will follow the Selleri's argument using the GPS experiments with two observers. Selleri used the Einstein method to synchronize clocks; however, for simplicity we use the ECI coordinate system and synchronized time (see Section III A). Acceleration is also neglected.

Let us make it clear that the group velocity of the electromagnetic wave c_G is observed differently depending on the observer's velocity v .³⁷ Two observers 1 and 2 (on the equator) are connected via a rigid rod of length L , as shown in Fig. 9(a). Not only observer 1, but also observer 2, observes the Sagnac effect. That is, observer 2 moves at the velocity v during the flight time of light between observers 1 and 2. Thus, observer 2 also detects a Sagnac effect of $v/c = 1.57 \times 10^{-6}$; which indicates that the light reaches the observers earlier than in the stationary state. The Lorentz factor $\gamma(\gamma - 1 = 1.23 \times 10^{-12})$ is rather small; this is because the Sagnac effect is 10^6 times larger than the Lorentz transformation. Therefore, the Lorentz transformation cannot compensate the Sagnac effect to make the speed of light c . At this stage, we neglect the Lorentz transformation.

The group velocity of light in the moving frame is calculated as

$$c_G = \frac{L}{\Delta t_S} = \frac{L}{t_2 - t_1},$$

where t_1 is the time when the light reaches observer 1, t_2 is the time when the light reaches observer 2, and

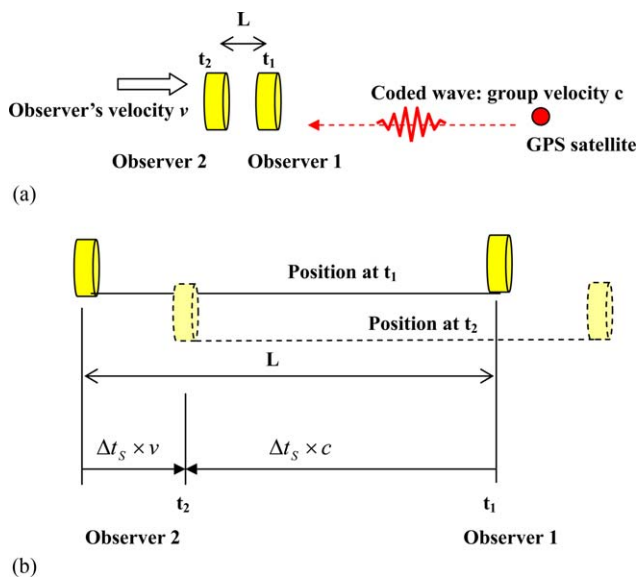


FIG. 9. (Color online) (a) Sagnac effect on the group velocity measurement using GPS: pulse coded signal is detected by observers 1 and 2. The detected times t_1 and t_2 suffer the Sagnac effect. Thus, the differential time $\Delta t_S = t_2 - t_1$ becomes smaller than Δt_0 in which observers 1 and 2 are in the stationary states. (b) Derivation of the equation of the Sagnac effect: after observer 1 detects the coded wave, observer 2 moves towards the GPS satellite. At the time t_2 , when the coded wave reaches at observer 2, which moves $\Delta t_S \times v$. Thus, we obtain $\Delta t_S \times c = L - \Delta t_S \times v$.

$\Delta t_S = t_2 - t_1$. Let the differential time of stationary observers be Δt_0 ; thus, we obtain

$$c = \frac{L}{\Delta t_0}.$$

Figure 9(a) shows the Sagnac effect between observers 1 and 2; after observer 1 detects the coded wave, observer 2 moves towards the GPS satellite. At the time t_2 , observer 2 moves $\Delta t_S \times v$. Thus, from Fig. 9(b) we obtain

$$\Delta t_S \times c = L - \Delta t_S \times v.$$

$$\therefore \Delta t_S = \frac{L}{c + v}.$$

$$c_G = \frac{L}{\Delta t_S} = c + v.$$

This equation is obtained on the condition that the group velocity defined in the ECI coordinate system is the speed of light c : that is, light in a vacuum propagates with the speed c regardless of the motion of the light source. At the same time, this discussion is supported by the experimental results by the Sagnac effect using the GPS;³⁰ Sagnac effects were experimentally observed within a 2% deviation.

The station on earth has a relative velocity in the ECI coordinate system. We do not assume invariant light speed for moving observer. The group velocity of the electromagnetic wave c_G is observed differently depending on the observer's velocity v , as $c_G = c \pm v$ (c : speed of light; “+” indicates that c and v are in the opposite direction); that is, the Galilean transformation. Not only the theory of general relativity (gravity and acceleration) but also the theory of special relativity (time dilation, phenomena of second order of $(v/c)^2$) is negligible for the Sagnac effects (first order of v/c). As was described by Sagnac, we consider that the Sagnac effect is nonrelativistic phenomena as well as the evidence of the ether.

As mentioned previously, we have not yet found any reasonable explanations for the Airy's water-filled telescope experiments using the theory of special relativity. The reason why one should replace the “relativity approach” by an “ether approach” is: (1) there are no reasonable explanations for the Airy's water-filled telescope experiments; (2) no periodic time derivations are observed in the GPS satellites' clock.³⁸

In this paper, we will take a lesson from the past. It is very important to access the original papers to understand the authors' conclusions. For example, Michelson noted that the two totally different theories (of Fresnel and Stokes) are running parallel to each other in the explanation of phenomena. Michelson and Morley concluded that “the ether is at rest with regard to the earth's surface;” however, the conclusion of the Michelson—Morley experiment was widely described as “null results.” Maxwell¹⁶ equations were represented using the total time

derivatives in the original text; that is, they are Galilean invariant.

II. LAGRANGIAN DESCRIPTION AND GALILEAN INVARIANCE

A. The theory of special relativity and the ether

Einstein²⁶ discerned two fundamental propositions: the principle of relativity and the principle of invariant light speed. The Principle of Relativity: The laws of physical systems are unchanged by the inertial motion relative to one another. The Principle of Invariant Light Speed: Light is always propagated in empty space with a definite speed, c , which is independent of the state of motion of the emitting body. In the 1905 paper, the constancy of the speed of light on the observer's velocity was also described. In later years, it was clearly stated that the speed of light is invariant with respect to the observer's velocity.³⁴ We consider this conclusion to be incorrect: as was described in Section I, the speed of light clearly depends on the observer's velocity. Only the description of "The Principle of Invariant Light Speed" in the 1905 paper is correct.

The principle of relativity and the principle of invariant light speed are represented by the original Maxwell equations.

B. Physical meaning of the principle of relativity: Lagrangian description by Hertz

Let us make our point clear for the theory of special relativity. We return to the ether hypothesis prior to 1905; two hypotheses by Einstein are summarized into the classic ether hypothesis. Ether is completely dragged by the gravitational field of the earth: that is, it is the gravitational Stokes ether, which is equivalent to the ECI coordinate system. Let us consider the wave on the medium drift.

As described by Hertz in 1893, Lagrangian derivatives $(D/Dt) = (\partial/\partial t) + \vec{v}_d \cdot \nabla$ are used to analyze the wave propagation in the drifting ether, where \vec{v}_d is the drift velocity of the ether, and $\nabla = (\partial/\partial x) + (\partial/\partial y) + (\partial/\partial z)$. The Lagrangian derivative D/Dt represents the derivative on the ether drift at velocity \vec{v}_d (see Section IV D). The Eulian derivative $\partial/\partial t$ represents the derivative on the fixed point in a space coordinate. Hertz had already known the results of the Michelson–Morley experiment (the ether is at rest with regard to the earth's surface), and thus, the solar system is considered to be in the stationary inertial frame in which the earth completely drags the ether (that is, the Stokes ether). Therefore, at the earth's surface, he set $\vec{v}_d = 0$. The application of fluid mechanics on ether is simpler than the assumption of the principle of relativity.

C. Galilean invariance of the wave equation

In this section, we discuss the Galilean invariance of the wave equation. The wave equation derived from the original Maxwell equations is represented as

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{D^2 E}{Dt^2} = 0, \quad (1)$$

where E is the electric field of an electromagnetic wave. Let us substitute total time derivatives (Lagrangian derivatives) D/Dt to analyze the waves on the medium flow, where D/Dt is rewritten using partial time (Euler) derivatives,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_d \cdot \frac{\partial}{\partial x}. \quad (2)$$

where v_d is the drift velocity of the medium. The second-order Lagrangian derivative D^2/Dt^2 is represented as

$$\begin{aligned} \frac{D^2}{Dt^2} &= \frac{D}{Dt} \left(\frac{D}{Dt} \right) = \frac{\partial}{\partial t} \left(\frac{D}{Dt} \right) + v_d \cdot \frac{\partial}{\partial x} \left(\frac{D}{Dt} \right) \\ &= \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + v_d \cdot \frac{\partial}{\partial x} \right) + v_d \cdot \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} + v_d \cdot \frac{\partial}{\partial x} \right) \\ &= \frac{\partial^2}{\partial t^2} + \frac{\partial v_d}{\partial t} \frac{\partial}{\partial x} + v_d \frac{\partial}{\partial t} \frac{\partial}{\partial x} + v_d \frac{\partial}{\partial t} \frac{\partial}{\partial x} \\ &\quad + v_d \frac{\partial v_d}{\partial x} \frac{\partial}{\partial x} + v_d^2 \frac{\partial^2}{\partial x^2}. \end{aligned} \quad (3)$$

Let us assume a uniform and stable flow, that is,

$$\frac{\partial v_d}{\partial x} = \frac{\partial v_d}{\partial t} = 0. \quad (4)$$

Thus, Eq. (3) becomes

$$\frac{D^2}{Dt^2} = \frac{\partial^2}{\partial t^2} + 2v_d \frac{\partial^2}{\partial x \partial t} + v_d^2 \frac{\partial^2}{\partial x^2}. \quad (5)$$

Therefore, Eq. (1) becomes

$$\begin{aligned} c^2 \frac{\partial^2 E}{\partial x^2} - \frac{D^2 E}{Dt^2} &= (c^2 - v_d^2) \frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial t^2} - 2v_d \frac{\partial^2 E}{\partial x \partial t} \\ &= 0. \end{aligned} \quad (6)$$

Let us take a wave with phase velocity ω/k ,

$$E = \tilde{E} \exp i(kx - \omega t). \quad (7)$$

The dispersion relation becomes

$$(c^2 - v_d^2)k^2 - \omega^2 + 2v_d k \omega = 0, \quad (8)$$

$$\left(\frac{\omega}{k} \right)^2 - 2v_d \left(\frac{\omega}{k} \right) - (c^2 - v_d^2) = 0. \quad (9)$$

The phase velocity is

$$\frac{\omega}{k} = v_d \pm c. \quad (10)$$

This result demonstrates the Galilean invariance of the wave equation.

Hertz set $\vec{v}_d = 0$ in Eq. (6), thus, today's wave equation is represented as

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0. \tag{11}$$

Although Hertz derived Eq. (11) for the surface of the earth where the ether is at rest; however, in the early of the 20th century, this equation was walking alone as the wave equation for universal condition.

D. Physical meaning of the principle of invariant light speed

We do not consider that the principle of invariant light speed (the constancy of the speed of light) indicates that the moving observer detects the invariant light speed. The light speed is defined by the wave equation; thus, the light speed is a physical substance. The moving observer obviously detects the variant light speed. The motion of the light source and that of observers are not compatible. We adopt the principle of invariant light speed in Einstein's 1905 paper.²⁶

The physical meaning of the principle of invariant light speed is that the light speed is defined with respect to ether once the light leaves the source.

III. GPS EXPERIMENTS

GPS uses the ECI coordinate system and synchronized time because this system is a preferred reference frame, and time is independently defined in space. In this section, we demonstrate that these selections are not made for convenience but due to physical requirements.

Figure 8 shows the GPS satellites' motions at the relative velocity $v_G = 4 \text{ km/s}$ in the ECI coordinate system which moves in the solar system at the relative velocity $v_E = 30 \text{ km/s}$. The GPS satellites motions in the ECI coordinate system represent the motion in the Fresnel's ether. In the solar system, the GPS satellites motions represent that in the Stokes' ether.

The time dilation is caused on the condition that the clocks travel in the ether; that is, the clocks have a relative velocity to the surrounding ether. Figure 10 shows the GPS satellites have a relative velocity to the ether of $v_G = 4 \text{ km/s}$; it is not $v_E = 30 \text{ km/s}$.

A. ECI coordinate system and synchronized time

According to the Interface Control Document^{38,39} (ICD 200 c, p. 102), the geometric range (D) from a satellite to a receiver in an ECI coordinate system is defined as $D = |\vec{r}(t_R) - \vec{R}(t_T)|$, where t_T and t_R are the GPS times of transmission and reception, respectively, and where $\vec{R}(t_T)$ is the position vector of the GPS satellite in the selected ECI coordinate system at time t_T , and $\vec{r}(t_R)$ is the position vector of the receiver in the selected ECI coordinate system at time t_R .

These definitions imply that in the GPS, times and positions are defined in the ECI coordinate system. All clocks are synchronized, and the lengths of the position vectors are defined using the speed of light c . The time and position of

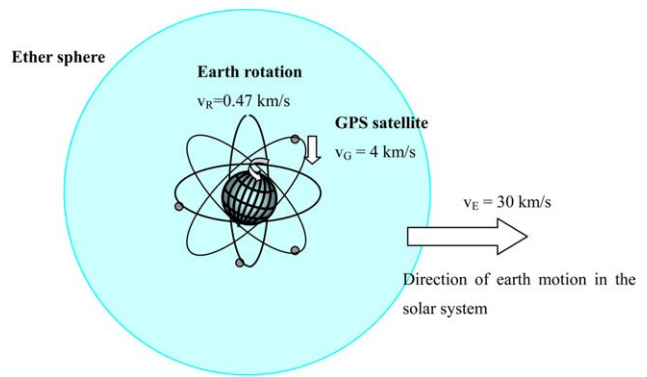


FIG. 10. (Color online) GPS experiment: GPS satellites' motions at the relative velocity $v_G = 4 \text{ km/s}$ in the ECI coordinate system which moves in the solar system at the relative velocity $v_E = 30 \text{ km/s}$. The earth rotates in the ECI coordinate system at $v_R = 0.47 \text{ km/s}$. The ECI coordinate system is equivalent to the ether sphere.

every point are defined; The ECI coordinate system is perfect for GPS calculations. We only assume time dilation by the velocity defined in the ECI coordinate system; that is, all clocks show the same time dilation. Furthermore, we do not need to assume the Lorentz contraction of the length because every point is defined using the speed of light c . In the GPS, the idea of Minkowski spacetime is not used; that is, space and time.

B. Comparison of the Fresnel's and Stokes' ethers in the GPS experiments

In this section, we explain the difference between the Lorentz and Galilean transformations: the Lorentz transformation relates to Fresnel's ether; the Galilean transformations relates to Stokes' ether. We now clarify the difference between Fresnel's model of a partial ether drag and Stokes' model of complete ether drag. The water flow (for example 1 m/s) in the ECI coordinate system agrees with Fresnel's model, as experimentally demonstrated by Fizeau, however, the water flow in the solar system at 30 km/s does not agree with Fresnel's model. For example, water on the earth does not look to drag the ether; the dragging velocity becomes $30 \text{ km/s} \times 0.434 = 13 \text{ km/s}$. If the Michelson–Morley experiments are carried out in the water bath, this velocity will not be observed. The experiments by Airy as well as Michelson–Morley demonstrate that Stokes' model is correct. Therefore, the Fresnel's and Stokes' ethers show a different aspect of the ether. If we see the ether from the GPS satellites, it is Fresnel's ether. If we see the ether on the surface of the earth from the solar system, it is Stokes' ether. The GPS satellites passing through the ether at the velocity of $v_G = 4 \text{ km/s}$ suffer time dilation. The GPS satellites also move in the solar system at the velocity of $v_E = 30 \text{ km/s}$; however, they do not suffer any time dilation; this is because the GPS satellites are in the drifting ether at the velocity of $v_E = 30 \text{ km/s}$. The ether is very similar to the atmosphere which is dragged by the gravitational field of the earth.

Table II summarizes the comparison of the Fresnel's ether seen from the GPS satellite and the Stokes' ether seen from the stationary point in the ECI coordinate system.

TABLE II. Comparison of the Fresnel's and Stokes' ethers seen from the GPS satellite.

Term	Fresnel's ether	Stokes' ether
1 Measurement point	The GPS satellite in the ECI coordinate system: $v_G = 4$ km/s.	The stationary point in the ECI coordinate system: $v_G = 0$ km/s ($v_E = 30$ km/s in the solar system).
2 Reference frame	ECI coordinate system	Solar system
3 Experiments	Fizeau (1851); Michelson and Morley (1886)	Michelson (1881) Michelson–Morley (1887) Michelson, Pease, and Pearson (1929)
4 Drift velocity of the ether in the reference frame	0	$v_E = 30$ km/s
5 Relative velocity with the ether	$v_G = 4$ km/s	0
6 Dragging coefficient	Fresnel's dragging coefficient	Complete dragging by the gravitational field of the earth
7 Time dilation	Lorentz transformation $\therefore \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma$ $V: v_G = 4$ km/s	Non
8 Doppler shift	Relativistic Doppler shift	Classic Doppler shift
9 Transformation of space	Galilean transformation for v/c	Galilean transformation

IV. LORENTZ TRANSFORMATION IN THE FRESNEL'S ETHER

Lorentz¹⁹ postulated the Lorentz contraction to resolve the problem in Fresnel's coefficient (frequency dependence) and to explain the negative result of the Michelson–Morley experiment. He used the description of time derivative $\dot{\delta} = (\partial\delta/\partial t) - v \cdot (\partial\delta/\partial x)$ in the Maxwell equations; dot represents the total time derivative, and v is the velocity of translation. Lorentz showed using the Lorentz transformation $c^2/(c^2 - v^2) = \gamma^2$, $\therefore 1/(\sqrt{1 - (v/c)^2}) = \gamma$, where c is the speed of light, and γ is the Lorentz factor; that the form of the Maxwell equations is not changed by the velocity of translation v . The time dilation by the velocity of translation v was also proposed; this is the Lorentz transformation of time.

As was described in Section I, the speed of light depends on the velocity of moving observer; this makes it difficult to derive the Lorentz contraction. However, Lorentz derived the Lorentz transformation without the postulate of invariant light speed for moving observer. Therefore, the Lorentz contraction is not excluded by noninvariant one-way speed of light. However, there is no experimental evidence of the Lorentz contraction. Let us summarize the Lorentz contraction in Table III. Lorentz and Selleri postulated that the Lorentz

contraction is true phenomena; Einstein postulated it is apparent. We assume there is no Lorentz contraction.

The contraction hypothesis proposed by Fitzgerald and Lorentz is rather attractive; however, Michelson and Morley's conclusion that "the ether is at rest with regard to the earth's surface;" is more reasonable. The Fitzgerald–Lorentz contraction is not denied; this is because it occurs in the ECI coordinate, for example, contractions of the GPS satellite itself and length of the equator. The Lorentz contraction by the earth's spin ($v = 0.47$ km/s) is small enough to be negligible; the equator is about 40,000 km = 4×10^{13} μ m long; thus, the Lorentz contraction is 49 μ m.

Time dilation is widely accepted. We derive the Lorentz transformation of time from the consideration of the physical meaning and the experimental results of the GPS.

A. Light clock and Lorentz factor

In this section, we introduce the Lorentz transformation of time using light clock model; the Lorentz contraction is not derived. A light clock shows that in a moving frame a photon travels a longer path than in a stationary frame. This causes time dilation.

Einstein³⁷ explained the Lorentz transformation of reference time using an idea of light clock. Feynman⁴⁰ used a

TABLE III. Summary of Lorentz contraction.

Term	Lorentz	Einstein	Selleri	This proposal
Condition	Maxwell equations are unchanged	Invariant light speed for a moving observer	Absolute simultaneity	Time dilation formula
Time dilation	$\Delta \rightarrow \bigcirc$	\bigcirc	\bigcirc	\bigcirc
Lorentz contraction	\bigcirc	Δ	\bigcirc	\times
Ether	\bigcirc Stationary	\times	\bigcirc Stationary	\bigcirc Atmospheric property

Note: \bigcirc : True, Δ : Apparent, \times : Non.

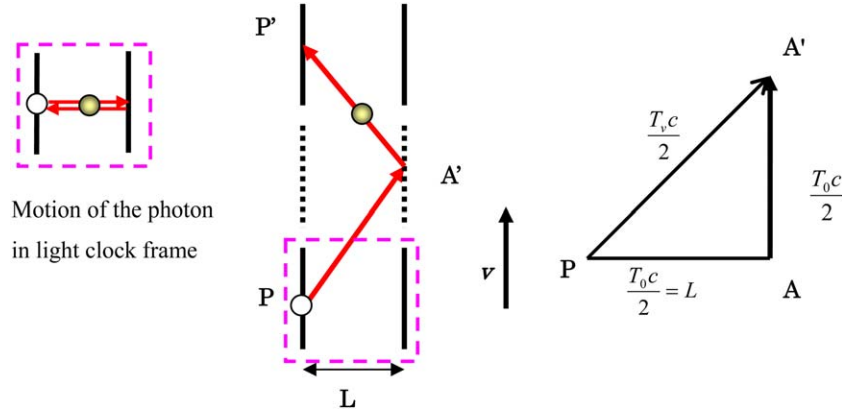


FIG. 11. (Color online) Light clock in motion: In a moving frame a photon travels a longer path than in a stationary ether frame. This causes time dilation.

light clock to visualize time dilation by motion. To obtain the physical meanings of the Lorentz transformation, let us introduce the linear light clock as shown in Fig. 11. A photon that is radiated from photon source P is reflected by the mirror and back to the photon source P. In the stationary frame, the reference time T_0 is defined as follows:

$$T_0 = \frac{2L}{c},$$

where L is the distance between photon source P and mirror, and c is the speed of light.

Figure 11 shows the light clock in motion at velocity v . In this condition, the Pythagorean theorem can be applied. The speed of light is assumed to be constant and independent on the motion of the light source. When the system moves at velocity v , point A moves to point A': thus a photon has to move the distance OA' (the traveling time of the distance OA' is represented as T_v). From the Pythagorean theorem, we obtain as follows:

$$\left(\frac{T_v c}{2}\right)^2 = \left(\frac{T_0 c}{2}\right)^2 + \left(\frac{T_v v}{2}\right)^2,$$

$$\therefore T_v^2 = \frac{(T_0 c)^2}{(c^2 - v^2)},$$

where the subscripts 0 and v represent the reference frame at rest and the moving frame at velocity v , respectively (T_0 is the reference time in the stationary state, and T_v is that of the moving frame at velocity v). The Lorentz transformation of the reference time moving at velocity v is represented as

$$T_v = \frac{T_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}.$$

Using the reference times T_v and T_0 , we can define the times as

$$t_0 = \frac{\beta}{T_0}, \quad t_v = \frac{\beta}{T_v},$$

where t_0 is the time in the stationary state, t_v is the time of moving object in the earth-centered locally inertial (ECI)

coordinate system, and β is a constant. Thus, the equation of time transformation is obtained as follows:

$$\frac{T_v}{T_0} = \frac{t_0}{t_v} = \frac{dt_0}{dt_v} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma. \quad (12)$$

Equation (12) shows Lorentz factor γ .

B. Relativistic mass in the Fresnel's ether

The inertial mass increase due to the velocity is observed in the Lorentz force in the magnetic field. The momentum P' of a moving particle at velocity $v|_v$ is represented as Eq. (13). Velocity v is defined with respect to the ECI coordinate system, $v|_v$ means v is measured in the moving frame at the velocity v .⁴¹

$$p' = mv|_v = m \frac{dx}{dt_v} = m \frac{dt_0}{dt_v} \frac{dx}{dt_0}$$

$$= \frac{m}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \frac{dx}{dt_0} = \gamma mv|_0, \quad (13)$$

m is the inertial mass in the stationary state, and $v|_0$ is the velocity measured in the stationary frame. Equation (13) shows that the relativistic mass can be explained using the reference time transformation;⁴¹ the physical meaning of relativistic mass is $\gamma mv|_0 = mv|_0 + (\gamma - 1)mv|_0$.⁴²

Introducing Lord Kelvin's vortex ring,⁴³ let us show circumstantial evidences. Figure 12 shows the motion of massive particle (the momentum is $mv|_0$) in the permittivity and

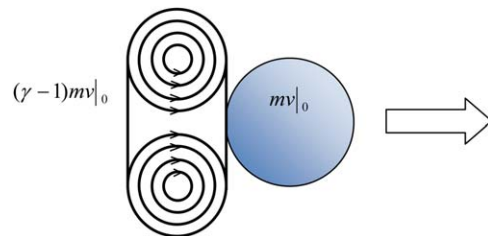


FIG. 12. (Color online) Massive particle motion in the permittivity and permeability of free space: The vortex rings in the permittivity and permeability show an adhered photon that is in accordance with the illustration of Lord Kelvin's vortex atoms⁴³.

permeability of free space. The vortex rings in the permittivity and permeability of free space are assumed to be adhered photon; the momentum is $(\gamma - 1)mv|_0$. An analogy of wave-making resistance⁴⁴ gives an illustration of relativistic mass; eddy making resistance in the ether is the relativistic mass. Bremsstrahlung and Cherenkov radiation are the experimental evidences of adhered photon release. We consider that in those days, the property of ether was widely considered to be the permittivity and permeability of free space. We suppose that Maxwell described the physical meaning of the ether. However, we have not yet found any references.

Maxwell⁴⁵ described that “Newton himself, however, endeavoured to account for gravitation by differences of pressure in an aether...; but he did not publish his theory.” Although Maxwell did not consider that the ether explains the gravitation; however, the ether is required to explain electromagnetic phenomena. Newton’s idea⁴⁶ of gravitation is discussed in Section VII B.

C. Explanation of Fresnel’s coefficient

Fresnel’s coefficient was experimentally demonstrated. Laue¹⁴ explained the Fresnel’s coefficient using the relativistic velocity addition law. Einstein³⁷ noted in his book that the relativistic velocity addition law explains the Fizeau’s experimental results.

$$W = \frac{v + w}{1 + \frac{vw}{c^2}} \approx (w + v) \left(1 - \frac{vw}{c^2}\right) \approx w + v \left(1 - \frac{1}{n^2}\right), \quad (14)$$

W = velocity of light in the moving liquid, w = velocity of light in the stationary liquid, v = velocity of the liquid in the axis of tube, c = the speed of light in vacuum, and n = refractive index of water.

Equation (14) shows the interference pattern has frequency dependence; that is, the Fizeau’s experiment shows dispersion. Lahaye *et al.*¹¹ reported that experimental results using a white-light source instead of a laser shows dispersion.

Michelson and Morley¹⁰ described that “let l be the length of the part of the liquid column which is in motion.”

$$\frac{l}{w - vx} - \frac{l}{w + vx} = \frac{2lvx}{w^2 - v^2x^2} \approx \frac{2lvx}{w^2},$$

vx = acceleration of the light, x = dragging coefficient, λ = wavelength (570 nm), and $n^2 = 1.78$.

They continued “If Δ is the double distance traveled in this time in air, ...”

$$\Delta = \frac{4lvn^2x}{\lambda c}, \quad x = \frac{\lambda c \Delta}{4ln^2v}.$$

In their experiments, the final weighted value of Δ for all observations is $\Delta = 0.1840$. Thus, x was 0.434 with possible error ± 0.02 . They calculated Fresnel’s coefficient was $(n^2 - 1)/n^2 = 0.437$. The experiment was also tried with air moving with a velocity of 25 m/s. The displacement was

about 0.01 of a fringe; a quantity smaller than the probable error of observation. The value calculated from $(n^2 - 1)/n^2$ would be 0.0036. Lahaye *et al.*¹¹ reconfirmed the experimental results with air by Fizeau and Michelson and Morley.

D. Physical meanings of Lagrangian, Eulerian, and Lorentz descriptions

Figure 13 shows the Lagrangian and Eulerian descriptions. If we see drifting vortex from drifting boat, it is Lagrangian description (D/Dt); if we see drifting vortex from bridge, it is Eulerian description ($\partial/\partial t$). Lagrangian description as well as Eulerian description is not affected by the drifting ether: this is because Lagrangian description sees the phenomena from the coordinate fixed to the drifting ether (i.e., from the drifting boat); Eulerian description sees the phenomena from the fixed coordinate (i.e., from the bridge).

If we see drifting vortex from a fixed point to the pier in the river, it is Lorentz description $\dot{\delta} = (\partial\delta/\partial t) - v \cdot (\partial\delta/\partial t)$, which represents the effect of translation velocity v : that is, the Lorentz transformation of time. Lorentz discussed the relative motion with respect to the ether. The observer as well as the source is affected by the drifting ether. The time dilation is true physical phenomena; however, the Lorentz contraction is not true but apparent phenomena. Lorentz description ($\dot{\delta}$) is affected by the drifting ether: this is because the observer is in the drifting ether (i.e., fixed to the pier). The direction of the velocity of translation v is opposite (“-” sign in front of v) to that of Lagrangian description, which causes physical difference. The difference between “+” sign and “-” sign in front of v is; “+” sign indicates to see phenomena on the moving frame at the velocity v (the relative velocity between the observer and the medium is 0), “-” sign indicates that the relative velocity between the observer and the medium is v . Only Lorentz description derives the time dilation. The Lagrangian description as well as Lorentz description assumes the drifting or moving ether; although the Eulerian description assumes the stationary

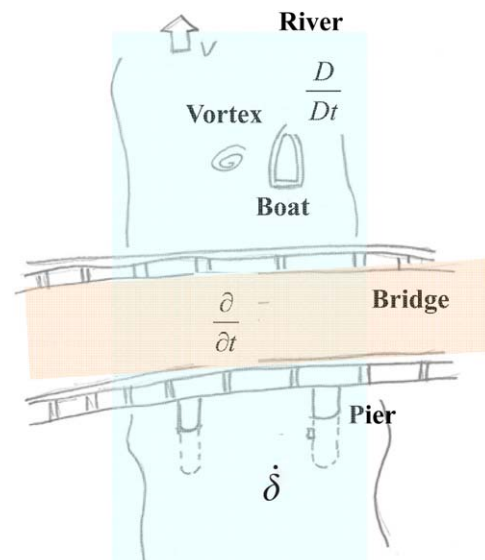


FIG. 13. (Color online) Lagrangian and Eulerian descriptions: Lagrangian description (D/Dt), Eulerian description ($\partial/\partial t$), and Lorentz description ($\dot{\delta}$).

ether (or widely believed to be no ether). Lagrangian description (D/Dt), Eulerian description ($\partial/\partial t$), and Lorentz description (δ) are equivalent on the condition $v=0$; that is, $(\partial\delta/\partial t) = \dot{\delta} = (D\delta/Dt)$.

V. ABERRATION AND DOPPLER SHIFT IN THE GRAVITATIONAL FIELD

A. Explanation of the stellar aberration: dragged ether sphere

The result of Airy’s experiment with a water-filled telescope showed the experimental evidence of the existence of the dragged ether around the earth. The light is discussed from the viewpoints of the wave and particles. In this paper, we provide an explanation using not only the particle property but also the wave property.

We consider that the Stokes’ ether is completely compatible with the aberration; it is rather difficult to find the counterargument against the Stokes’ hypotheses. From Wikipedia,³⁸ we found that Lorentz argued that if the ether has the same normal component of velocity as the earth, it would not have the same tangential component of velocity. At this stage, this is an argument against the Stokes’ hypotheses. However, this problem will disappear using the idea of dragged ether sphere. Let us make this point clear using an analogy of raindrops. If we are in a moving car, vertically falling raindrops strike a moving car at an angle; they hit front window. If we open the window, raindrops come into the car nearly at that angle.

Figure 14(a) explains the aberration using the Stokes’ ether model in the distance scale of the earth and the moon (the radius of the ether sphere is more than 380,000 km). Both the particle model and the wave model, at the surface of the dragged ether sphere, the particle and the wave refract according to $\sin \alpha \sim \alpha = v/c$. The photons hit the front of the ether sphere; thus, we see the photons at an angle α according to Huygens’ principle which shows the front surface becomes a new source of light. The aberration is caused by the refraction by moving ether. The wave front changes its direction to enclose the dragged ether sphere. The height of the dragged ether sphere from the ground is more than 380,000 km, which is the distance from the earth to the moon. The minimum distance of 380,000 km is estimated from the experimental evidence that there is no aberration of the moon light.¹⁵ We consider the explanation of the aberration by the Stokes’ ether dragging hypothesis is simple. As shown in Fig. 14(a), the photons represent the true direction of the light with respect to the ECI coordinate system.

Figure 14(b) shows the aberration seen from the solar system. Although photons look to come from opposite direction (they appear to be coming from the rear), in the ECI coordinate system, the aberration becomes $\sin \alpha \sim \alpha = v/c$ independently on the refractive index n . The angle of the telescope is unaffected by the water. This is because α is the true angle of photon; therefore the velocity of photon is slowed down in the water, however, the angle α does not depend on the refractive index n of the medium in the telescope. As was argued by Lorentz, if the ether has the same normal component of velocity as the earth, it would not have

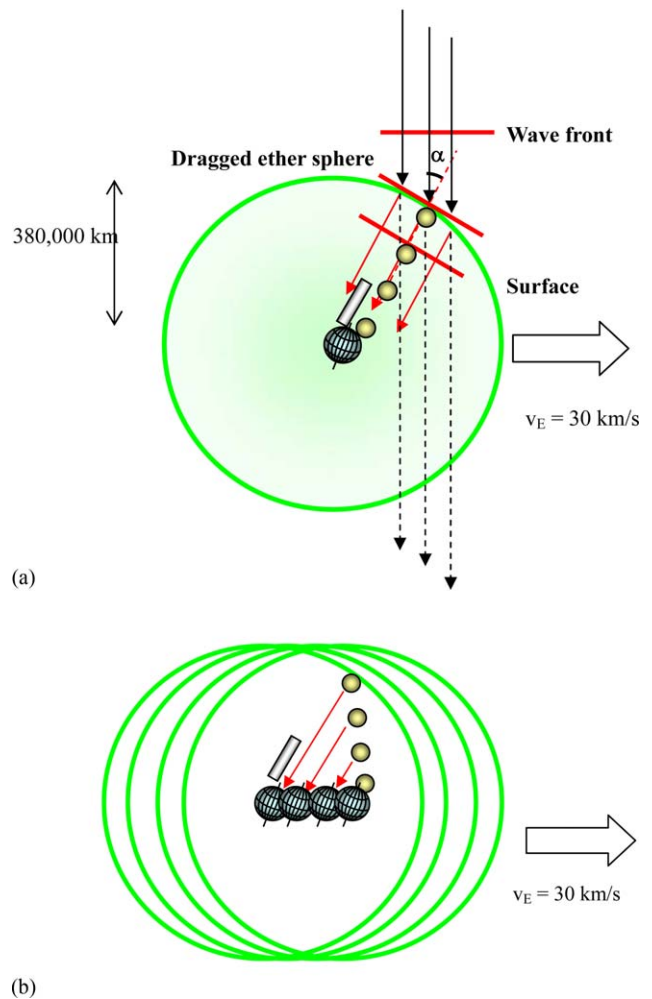


FIG. 14. (Color online) (a) (seen in the ECI coordinate system) Explanation of the stellar aberration by Stokes’ ether model: Using the particle model as well as the wave model, at the surface of the dragged ether sphere, the particle as well as the wave refract according to $\sin \alpha \sim \alpha = v/c$. The aberration is caused by the refraction by the moving ether sphere. The wave front changes its direction to enclose the dragged ether sphere. The light refracts at a point more than 380,000 km above the ground. This is explained not by the theory of special relativity but by an ether approach. (b) (seen from the solar system) The aberration becomes $\sin \alpha \sim \alpha = v/c$ independently on the refractive index n . From the solar system, the photons appear to be coming from the opposite direction. However, in the ECI coordinate system, the aberration angle becomes α .

the same tangential component of velocity; however, if the photons come into the sphere with an angle as shown in Fig. 14(a), the angle will be unchanged. Therefore, the aberration is observed.

B. Doppler shift in the Gravitational field

The idea of aberration by Van Flandern¹⁷ can be applied to the Doppler shift. The Doppler shift is considered to occur at the boundary of the dragged ether sphere. In the gravitational field of the earth (i.e., the ECI coordinate system), photon travels at the speed of light c with respect to the ECI coordinate system.

Figure 15 shows the explanation of the Doppler shift, which is explained not by the theory of special relativity but by an ether approach. The relative velocity between the ECI coordinate system and the solar system is $v_E = 30 \text{ km/s}$. The

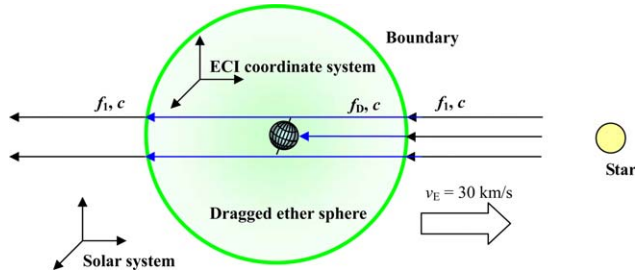


FIG. 15. (Color online) Explanation of the Doppler shift by an ether approach.

Doppler shift occurs at a boundary more than 380,000 km above the ground. Both in the ECI coordinate system and the solar system, the speed of light is c . In the solar system the frequency is f_1 ; in the ECI coordinate system, the frequency becomes the Doppler shifted frequency f_D . Therefore, if the detector on the earth observes the Doppler shifted photons, they already have suffered the Doppler shift before detection. Without detection after passing through the ECI coordinate system, the frequency of photons recovers to f_1 at the boundary.

C. Ω - k diagram of photon and the constancy of the speed of light

Figure 16 shows the ω - k diagram of photon in the solar system (dotted line) and ECI coordinate system (solid line): Photon (ω_1, k_1) in the solar system is Doppler (blue) shifted to $[(1 + (v_E/c))\omega_1, (1 + (v_E/c))k_1]$ in the ECI coordinate system, where v_E is the relative velocity. Both in the solar system and the ECI coordinate system, the phase velocity ω/k as well as the group velocity $\partial\omega/\partial k$ become the speed of light c ; that is, $(\omega/k) = (\partial\omega/\partial k) = c$; the constancy of the speed of light is satisfied. This shows that the ether should have nondispersive property. The Maxwell equations can be represented as the same form both in the solar system and the ECI coordinate system; the theory of relativity. The transformation between the solar system and the ECI coordinate system is that of Galilean; the compatibility with the Newtonian mechanics.

In sum, the theory of special relativity does not assume the ether; however, without the ether, the boundary of the coordinate system cannot be defined. Therefore, it is difficult to define the Maxwell equations for the solar system and the ECI coordinate system, respectively. The merit of an ether

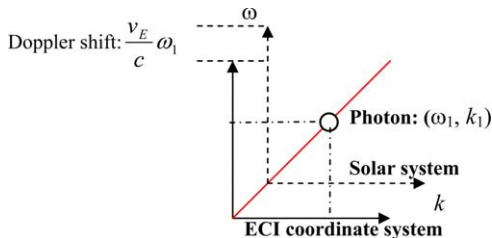


FIG. 16. (Color online) Ω - k diagram of photon in the solar system and ECI coordinate system: Photon (ω_1, k_1) in the solar system is Doppler shifted to $[(1 + (v_E/c))\omega_1, (1 + (v_E/c))k_1]$ in the ECI coordinate system. The phase velocity ω/k as well as the group velocity $\partial\omega/\partial k$ become the speed of light c ; that is, $(\omega/k) = (\partial\omega/\partial k) = c$.

approach is that the Eulerian descriptions $(\partial/\partial t)$ are defined in the respective coordinate systems.

VI. DISCUSSION

The authors thank the reviewers for their two important references and instructions. One is: Fresnel's letter ^{7,47} to Arago in 1818; Fresnel correctly predicted the results of Boscovich–Airy experiments. The other is: Hertz's equations are not compatible with the experimental results by Eichenwald. As was pointed out by the reviewer, Hertz's equations lead to quite a different story (of geomagnetism). Hertz considered Stokes' ether, thus, a dielectric rotating in the laboratory produces some magnetic field. However, Eichenwald showed that magnetic effects by a rotation of a dielectric are experimentally undetectable. Rotating a large gravitational scale dielectric, for example, the sun may show a magnetic effect. Hertz's equations can be applied for gravitational scale not for laboratory scale. In this paper, we focus on the wave equation; this is because the Hertz's equations are beyond the theme of this paper.

The Fresnel's ether was proven by the Fizeau's experiment; at the same time, the Stokes's ether was supported by the conclusion of the Michelson–Morley experiment that “the ether is at rest with regard to the earth's surface.” Although the prediction by Fresnel for Boscovich–Airy experiment is correct; however, it does not deny the Stokes's ether, which can simply explain the Boscovich–Airy experiment. Both the Fresnel's ether and the Stokes's ether should be admitted; this is because the Fresnel's ether as well as the Stokes's ether shows the physical aspects of the ether, respectively.

The authors were pointed out by the reviewer that Einstein described in 1905 paper that the speed of light is invariant with respect to the observer's velocity; as required by the principle of the constancy of the velocity of light, in combination with the principle of relativity. However, we cannot find any reasonable explanation that the speed of light is invariant with respect to the observer's velocity. Lorentz assumed the Lorentz contraction to explain the Michelson–Morley experiments; how the speed of light was observed to be independent of the reference frame. The time dilation formula has a physical explanation by Einstein's light clock (see Section IV A) as well as experimental evidence of the GPS experiments.^{30,31}

A. Stokes' ether and Minkowski spacetime

Stokes's ether is considered to be dragged by objects; however, a dielectric rotating in the laboratory does not produce any magnetic field. Ether dragging effects by a rotation of a dielectric are experimentally undetectable. In those days, this was considered to be serious problem for Hertz's equations. Let us make this problem clear via comparison between the Fresnel's ether and the Stokes's ether. Although the revolution of the earth drags the ether, the spin of the earth does not. That is, the ECI coordinate system looks to be dragged ether, in which the earth rotates. Stokes assumed that the ether is completely irrotational. We consider the Fresnel's ether smoothly connects to the Stokes's ether. In

the Stokes’s ether, the velocity v is the ether velocity (or drift velocity); in the Fresnel’s ether, it is the field detector velocity or translation velocity.

In 1908, Minkowski⁴⁸ noted that “At the present time, different opinions are being held about the fundamental equations of Electrodynamics for moving bodies. The Hertzian forms must be given up, for it has appeared that they are contrary to many experimental results.” (English translation) In those days, the ether hypothesis was considered to face many problems. The spacetime resolved the problems; however, the GPS uses three-dimensional space and time as shown in Section III A. Einstein’s time synchronization is not used in the GPS; this makes the GPS simple. Stokes’ and Fresnel’s ethers can separate space and time.

B. Gravitational ether drag: Stokes–Planck hypothesis

From Wikipedia:⁴⁹ “Another version of Stokes’ model was proposed by Wien. The gravitational ether drag is proportional to the gravitational mass. The ether is completely dragged by the earth, and partially dragged by smaller objects on earth. And to save Stokes’s explanation of aberration, Max Planck (1899) argued in a letter to Lorentz, that the aether might not be incompressible, but condensed by gravitation in the vicinity of earth, and this would give the conditions needed for the theory of Stokes (“Stokes–Planck theory”).” We were surprised to read Lorentz papers,^{22,23} this is because, Lorentz as well as Plank already discussed the gravitational ether drag. They also discussed that “the aether might not be incompressible, but condensed by gravitation.” That is, if the ether is condensed like the atmosphere, the speed of light becomes small on the surface of the earth; this causes the time dilation by the gravity.³²

Thereafter, also from Wikipedia:⁴⁹ “However, this theory was directly refuted by the Michelson–Gale–Pearson experiment (1925). The great difference of this experiment against the usual Sagnac experiments is the fact that the rotation of earth itself was measured. If the aether is completely dragged by the Earth’s gravitational field, a negative result has to be expected—but the result was positive.” This notation is not correct. As was discussed in Section VI A, Stokes assumed that the ether is completely irrotational, the Michelson–Gale–Pearson experiment shows that the earth drags the ether, in which the earth rotates. The Stokes–Planck hypothesis is very attractive.

C. Meaning of the velocity v_d in the Lagrangian description

The authors were suggested by one of the reviewers that the velocity parameter appearing in the total time derivative form of field theory as field detector velocity. Hertz considered the velocity v_d the drift velocity of the ether; Lorentz described the velocity of translation. Both Hertz and Lorentz considered that v_d is defined with respect to the ether. As was pointed out by the reviewer, the interpretation of the velocity leads to quite a different story; that is, the Lorenz transformation. However, at this stage, we consider that v_d is the drift velocity of the ether. Let us make this point clear using Fig. 10, we consider that v_d corresponds to the velocity

$v_E = 30$ km/s in the solar system, the velocity of translation or field detector velocity corresponds to $v_G = 4$ km/s in the ECI coordinate system. Again, Hertz’s equations are beyond the theme of this paper.

VII. SUMMARY

A. Stokes’ ether dragging model

Figure 17 illustrates the proposed ether dragging model. The ECI coordinate system (ether sphere of the earth), the solar system (ether sphere of the sun) and the galaxy are, respectively, in their local stationary ethers. Each gravitational field drags the ether around its gravitational field. Many gravitational fields exist, and thus, many local stationary states exist. If we leave the ECI coordinate system, we will be in the local stationary state of the solar system. The galaxy moves in the CMB at 600 km/s, the solar system moves in the galaxy at 220 km/s, and the ECI coordinate system moves in the solar system at 30 km/s. The GPS satellite in the ECI coordinate system registers 4 km/s; however, it does not detect the relative velocity in other coordinates.

B. Time dilation by the velocity and gravity

The time dilation of the GPS satellites is caused by not only the velocity but also the gravitational effect. The authors thank the reviewer for introducing the paper by Hill.⁵⁰ Using the data by Taylor, Hill⁵⁰ noted that the arrival time residuals for PSR 1937+21 with small monthly term shows periodic variations (around $\pm 1.5 \mu\text{s}$) with respect to the phase of the moon; that is, the clock on earth suffers periodic variations. According to the time variations on the equator, Hill⁵⁰ reported $2.1 \mu\text{s}$ slow at 6 a.m. and fast at 6 p.m. local time, thereafter noted that “the effect can be significantly larger for atomic clocks aboard satellites.” He explained these time variations by the spin velocity (0.47 km/s) on the equator. However, we do not consider that these time variations are not caused by the velocity; they are caused by the gravitational effect of the sun.

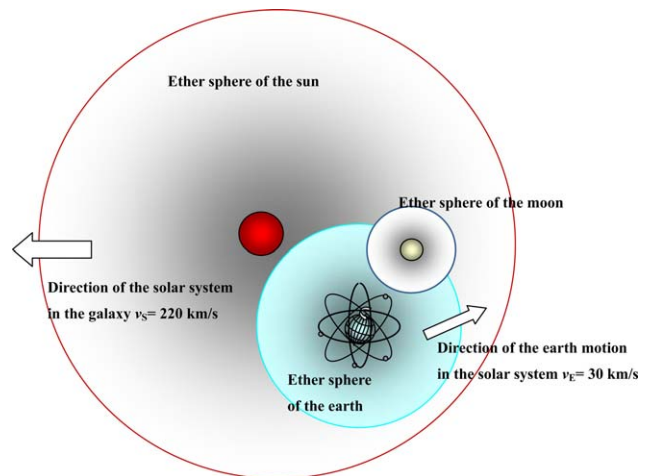


FIG. 17. (Color online) The motion of the GPS satellite in the galaxy: Although the GPS satellite is affected by the gravity of the sun as well as the moon, as far as the velocity is concerned, the time dilation mainly depends on the velocity 4 km/s.

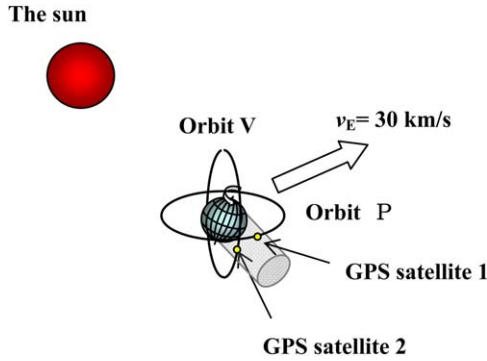


FIG. 18. (Color online) Gravity eclipse by the earth (shadowed area): Both GPS satellite 1 on orbit P and GPS satellite 2 on orbit V are eclipsed by the earth. The clocks on the GPS satellites show periodic variations. Not the velocity but the eclipse by the earth affects the reference times of the GPS satellites.

The gravity of the sun causes the deviation of GPS satellites' clocks. Figure 18 shows that both GPS satellite 1 on orbit P and GPS satellite 2 on orbit V are eclipsed by the earth. Orbit P is parallel to the velocity v_E , and orbit V is vertical to the velocity v_E . Thus, GPS satellite 1 suffers periodic velocity modification (slipped cycloid motion). The motion of GPS satellite 2 becomes helical. The eclipse by the earth affects the reference time of the GPS satellite. The density

profile of the ether sphere of the earth is modified by the gravity of the sun (shadowed area in Fig. 18), which makes the speed of light slow in the eclipse by the earth. In the shadowed area, the density of the ether becomes large, which makes the speed of light slow. The reference time depends on the speed of light; therefore, the time dilation occurs in the shadowed area.

We explain the time variations of $2.1 \mu\text{s}$. Let us introduce the classic idea of gravitation and ether.^{45,46} Figure 19 shows that the modified ether density profile by the gravity of the sun varies the speed of light; at noon it is c_h , and c_l at night ($c_h > c_l$). The curve shows the ether density profile. On the ground, the density of the ether differs at noon and night; it becomes high at night. This causes the time variations of $2.1 \mu\text{s}$. This is because the reference time relates to the inverse of the speed of light c ; that is, $T_0 = 2L/c$. Figure 19 shows the Stokes–Planck theory: the ether might not be incompressible, but condensed by gravitation in the vicinity of earth. The idea by Newton that the gradient of the ether density is the gravitation: $f \propto -(\partial\rho_E/\partial x)$ is also illustrated. Table IV summarizes the relations between physical term and the property of ether.

Senior *et al.*⁵¹ reported periodic variations (around 12 h) in the GPS satellite clocks. They noted periodic variations of 3 ns at the eclipse seasons of the satellite by the earth. The

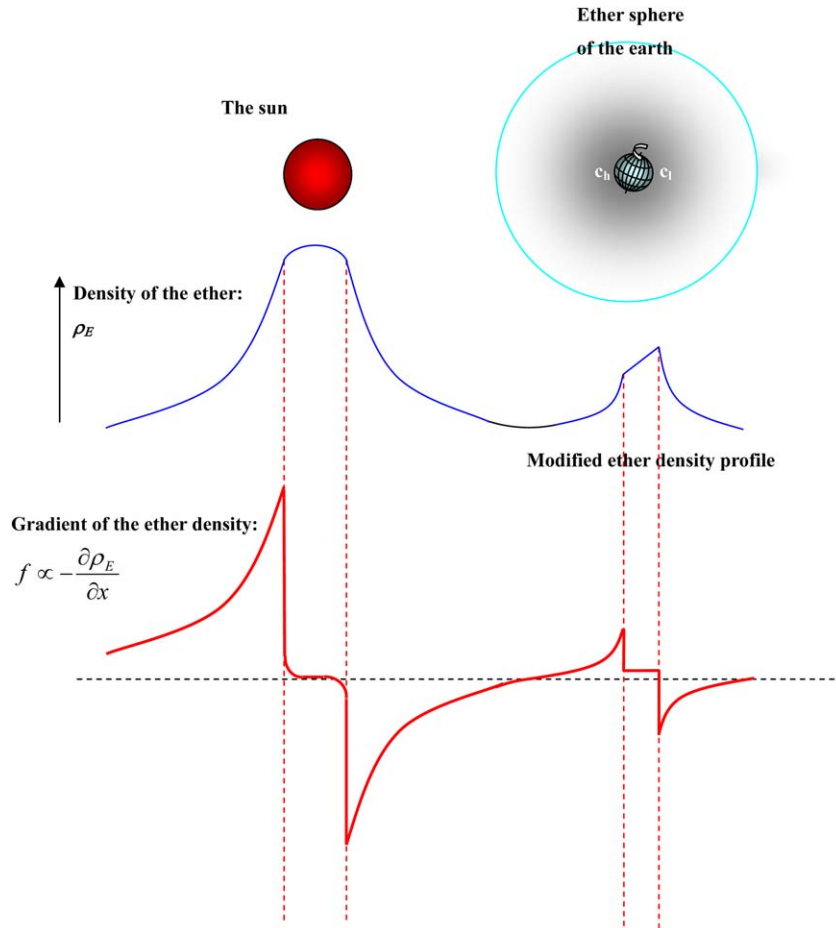


FIG. 19. (Color online) Modified ether density profile by the gravity of the sun varies the speed of light; at noon it is c_h , and c_l at night ($c_h > c_l$). This causes the time variations of $2.1 \mu\text{s}$. On the ground the density of the ether differs. At the orbits of the GPS satellites, the ether densities at noon and night become the same. The curves show the ether density profile (upper) and the gradient of the ether density (lower).

TABLE IV. Relations between physical term and the property of ether.

Physical term	Property of ether
1 Speed of light	Density of the ether: ρ_E
2 Time dilation	Density of the ether: ρ_E
3 Refraction of light	Gradient of the ether density: $\frac{\partial \rho_E}{\partial x}$
4 Gravitation	Gradient of the ether density: $f \propto -\frac{\partial \rho_E}{\partial x}$

ether sphere shields the ether wind; however, the gravities of the sun and the moon modify the ether sphere of the earth as well as the ether density profile. Thus, the GPS clocks are affected by the gravities of the sun and the moon.

From the experimental data on the ground and GPS satellites, it is considered that on the ground the ether density differs at day and night; at the orbits of the GPS satellites, the ether density becomes equal.

C. Ether spheres and aberration

In this section, we explain the reason why the aberration depends only on the earth’s revolution velocity of 30 km/s. The aberration shows sinusoidal variation of the angle $\alpha = 10^{-4}$ depending on the revolution velocity of 30 km/s. Although the solar system moves in the galaxy at around 220 km/s, that is, the earth orbital motion is slipped cycloid in the galaxy as shown in Fig. 20. This indicates that the aberration angle α does not show annual sinusoidal change.

The earth’s revolution velocity (30 km/s) as well as the velocity of the solar system (220 km/s) causes the aberration. If the solar system is the ether sphere, it moves almost linearly at 220 km/s; thus, the secular aberration angle $\alpha (= 220 \text{ km/s} \div 300,000 \text{ km/s} = 7.3 \times 10^{-4})$ becomes constant (permanent). Therefore, we cannot observe the aberration by the velocity of the solar system. The stellar aberration looks to depend only on the revolution velocity of 30 km/s. Figure 21 shows the schematic diagram of the ether spheres and the aberration: the ECI coordinate system orbits in the solar system. This is the reason that the aberration depends only on the revolution velocity of 30 km/s. It is concluded that the Stokes’ model⁷ of complete ether drag is needed to explain the aberration. The discussion of the binary star by Eisner¹³ is explained by the ether sphere model; only the revolution velocity (30 km/s) affects the aberration.

There are three types of aberrations. Annual aberration is caused by the motion of an observer on the earth revolving around the sun. The sun and solar system are revolving around the center of the galaxy, which causes secular aberration. Diurnal aberration is caused by the velocity of the observer on the surface of the rotating earth; it is only $0''.32$ in the case of an observer at the equator. Secular and annual



FIG. 20. The earth orbital motion is slipped cycloid in the galaxy; however, the aberration angle differs not cycloid but sine curve, annually.

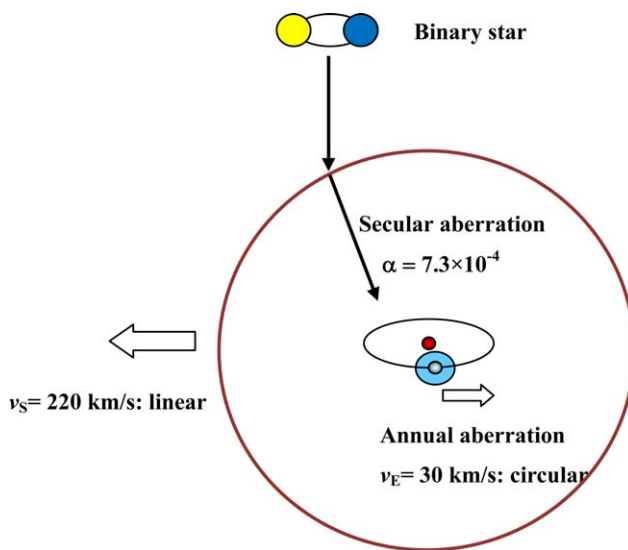


FIG. 21. (Color online) Explanation of the stellar aberration: The velocity of the solar system (220 km/s) causes the secular aberration. The ECI coordinate system (ether sphere of the earth) moves in the solar system (ether sphere of the sun). The annular aberration becomes sine curve.

aberrations are unaffected by the Airy experiment; however, diurnal aberration may be affected by the Airy experiment. This is because earth’s revolution drags the ether; however, earth’s spin does not.

Until now, no experimental data have invalidated the ether hypothesis. All of the experiments in Table I support the ether hypothesis. In particular, Airy’s aberration experiment and the Michelson–Morley experiments strongly support the ether hypothesis. In addition, the GPS experiments further demonstrate the existence of the ether.

VIII. CONCLUSION

If the Stokes’ ether completely dragged by the gravitational field of the earth is assumed, the principle of relativity does not need to be proposed. As shown by Hertz, the Maxwell equations are Galilean invariant and thus compatible with Newtonian mechanics. Thus, Airy’s aberration experiment and the Michelson–Morley experiments are easily explained. The aberration and the Maxwell equation strongly support the ether hypothesis. It is important to take a lesson from the past using the ether hypothesis; this is because the past works clearly relate to current discussions in the foundations of physics.

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