Tim Good · Follow
Optical Engineer at Honeywell (company) · Updated Nov 6

I've been in a "discussion" with Michael Brenner recently on the topic of projectile trajectories and how that relates to orbital mechanics. He denies the accuracy of the description of an orbiting body as being in free fall, where the body is sometimes described as constantly falling but missing the planet.

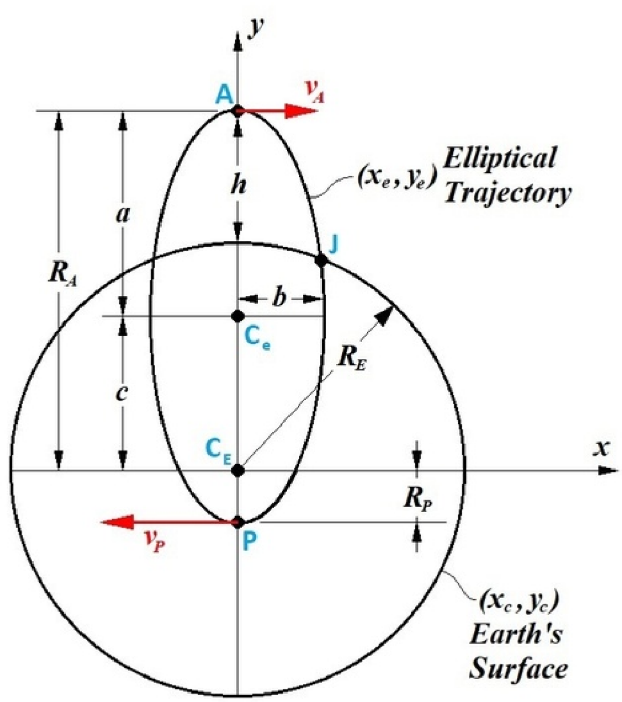
Honestly, I can understand how that description falls somewhat flat (no pun intended), but I provided a clear example to him of how the trajectory of an object moving at orbital velocity can be calculated using either a circular path or parabolic path over a short distance, and the two methods provide the exact same answer (within seven significant digits over 1 km). He refused to accept this but also refused to explain where I made a mistake. Instead he simply kept saying "what about an object 1 km high launched horizontally at 1000 m/s?" I told him the elliptical path which that would produce would result in significantly more difficult math to solve, and why can't he just tell me what's wrong with my simple circular orbit example? But, he thought he could just take my claim that the math is hard as some kind of victory.

Well, I decided I needed to challenge myself. After all, I love math. So, this can't be that hard, right? But part of me said "he doesn't deserve the dignity conferred on him by my answering his question when he refused to answer mine." But, it kept eating at my curiosity. So, no, he doesn't deserve it, and he likely won't appreciate it either, but I just had to prove (once again) to myself that the world continues to makes sense, and I can look at the work I did and feel a deep sense of satisfaction, regardless of what he thinks.

So, here it is. I use a few basic principles:

1. Newton's Law of Gravity
2. Kepler's second law
3. Conservation of energy

Initial conditions are nothing more than a given altitude and horizontal velocity.



Calculation of an elliptical projectile trajectory over a spherical Earth given initial height and horizontal velocity:

Elliptical trajectory: $\frac{(y_e-c)^2}{a^2}+\frac{x_e^2}{b^2}=1$ Eq. (1) Surface of the Earth: $y_c^2+x_c^2=R_E^2$ Eq. (2)

Distance from ellipse center to ellipse focal point: $c=\sqrt{a^2-b^2}$ Eq. (3)

Distance to apogee and perigee: $R_A=a+c$ Eq. (4) $R_P=a-c$ Eq. (5)

Application of Kepler's 2nd Law: $\frac{v_P}{v_A}=\frac{R_A}{R_P}$ Eq. (6)

Change in kinetic energy from apogee to perigee: $\Delta E_v=\frac{1}{2}mv_P^2-\frac{1}{2}mv_A^2$ Eq. (7)

Change in potential energy from apogee to perigee:

$$\Delta E_G=\int_{R_P}^{R_A}\frac{GM_Em}{r^2}dr=-\frac{GM_Em}{r}\Big|_{R_P}^{R_A}=GM_Em\left(\frac{1}{R_P}-\frac{1}{R_A}\right)\quad \text{Eq. (8)}$$

Conservation of energy: $\Delta E_v=\Delta E_G\rightarrow\frac{m}{2}(v_P^2-v_A^2)=GM_Em\left(\frac{1}{R_P}-\frac{1}{R_A}\right)$ Eq. (9)

Solve Eq. (9) for the velocity at perigee: $v_P=\sqrt{v_A^2+2GM_E\left(\frac{1}{R_P}-\frac{1}{R_A}\right)}$ Eq. (10)

Substitute Eq. (10) into Eq. (6): $\sqrt{v_A^2+2GM_E\left(\frac{1}{R_P}-\frac{1}{R_A}\right)}=v_A\frac{R_A}{R_P}$ Eq. (11)

Rearrange Eq. (11) into quadratic form:

$$R_P^2\left(1-\frac{2GM_E}{R_Av_A^2}\right)+R_P\left(\frac{2GM_E}{v_A^2}\right)+(-R_A^2)=0=AR_P^2+BR_P+C\quad \text{Eq. (12)}$$

Solve Eq. (12) for perigee using the quadratic formula: $R_P=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$ Eq. (13)

Calculate the major semi-axis of the ellipse: $a=\frac{R_A+R_P}{2}$ Eq. (14)

Calculate the minor semi-axis using Eqs. (3) & (5): $b=\sqrt{a^2-(a-R_P)^2}$ Eq. (15)

Rearrange Eq. (1): $-b^2=-x_e^2-\frac{b^2}{a^2}(y_e-a+R_P)^2$ Eq. (16)

Find intersection of circle and ellipse at point J, where $x_J=x_c=x_e$ and $y_J=y_c=y_e$ by adding Eq. (2) to Eq. (16): $R_E^2-b^2=(x_J^2-x_J^2)+y_J^2-\frac{b^2}{a^2}(y_J-a+R_P)^2$ Eq. (17)

Rearrange Eq. (17) into quadratic form:

$$y_J^2\left(\frac{a^2}{b^2}-1\right)+y_J\big(2(a-R_P)\big)+\left(a^2-\left(\frac{aR_E}{b}\right)^2-(a-R_P)^2\right)=0=Ay_J^2+By_J+C\quad \text{Eq. (18)}$$

Solve Eq. (18) for y_J using the quadratic formula: $y_J=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$ Eq. (19)

Solve for the projectile range, x_J, by rearranging Eq. (2): $x_J=\sqrt{R_E^2-y_J^2}$ Eq. (20)

Calculation of a parabolic projectile trajectory over a spherical Earth given initial height and horizontal velocity:

A parabolic trajectory occurs under the assumption that the acceleration due to gravity is constant with altitude, h, above sea level. The gravitational acceleration, g, will be approximated by the average of the acceleration at the initial altitude and at sea level:

$$g=GM_E\left(\frac{1}{(R_E+h)^2}+\frac{1}{R_E^2}\right)/2\quad \text{Eq. (21)}$$

Coordinates at time t_J when the projectile reaches point J:

$$x_J=v_At_J\quad \text{Eq. (22)}\qquad y_J=R_E+h-\frac{1}{2}gt_J^2\quad \text{Eq. (23)}$$

Solve Eq. (22) for t_J and substitute into Eq. (23): $y_J=R_E+h-\frac{1}{2}g\left(\frac{x_J}{v_A}\right)^2$ Eq. (24)

Solve Eq. (24) for x_J^2: $x_J^2=2v_A^2(R_E+h-y_J)/g$ Eq. (25)

Substitute Eq. (25) into Eq. (2): $2v_A^2(R_E+h-y_J)/g+y_J^2=R_E^2$ Eq. (26)

Rearrange Eq. (26) into quadratic form:

$$y_J^2(1)+y_J\left(-\frac{2v_A^2}{g}\right)+\left(\frac{2v_A^2}{g}(R_E+h)-R_E^2\right)=0=Ay_J^2+By_J+C\quad \text{Eq. (27)}$$

Solve Eq. (27) for y_J as before using the quadratic formula: $y_J=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$ Eq. (19)

Solve for the projectile range, x_J, again using Eq. (20).

Note that in all the foregoing equations the following identity applies relating the elliptical and parabolic cases: $R_A=R_E+h$ Eq. (28)

	A	B	C	D	E	F	G	H
1								
2		Input Parameters:				Elliptical trajectory over spherical earth:		
3		Gravitational constant, G:	6.6743E-11	Nm^2/kg^2		Apogee, R_A:	6372000	m
4		Earth mass, M_E:	5.9722E+24	kg		A_1:	-1.2411E+02	
5		Earth radius, R_E:	6371000	m		B_1:	7.9720E+08	
6		Altitude, h:	1000	m		C_1:	-4.06024E+13	
7		Initial velocity, v_A:	1000	m/s		Perigee, R_P:	51341.6	m
8		Gravity at sea level, g_0:	9.8202	m/s^2		Major semi-axis, a:	3211670.8	m
9		Gravity at altitude, g_h:	9.8172	m/s^2		Minor semi-axis, b:	571968.9	m
10						A_2:	30.52951033	
11		Parabolic trajectory over flat earth:				B_2:	6320658.442	
12		Fall time, t:	14.272	s		C_2:	-1.27944E+15	
13		Range, x_i:	14272.1	m		y_i:	6370983.8	m
14						Range, x_i:	14387.551	m
15		Parabolic trajectory over spherical earth:						
16		A_3:	1					
17		B_3:	-203692.774					
18		C_3:	-3.92917E+13					
19		y_i:	6370983.8	m				
20		Range, x_i:	14387.560	m				

The only math Michael knows how to do is captured in the Excel sheet in cells C12 and C13 which assumes constant gravity and a flat earth. That, of course, is wrong, because with a projectile range of 14 km the ground has started to drop, as can be seen, by about 16 m. But when an elliptical trajectory is calculated and compared to a parabolic trajectory, both over a sphere, the difference in the two results is only 9 mm over those 14 km, so once again (coincidentally) accurate to seven significant digits.

To anyone who happens to cross paths with Michael, if you see him going on about Newton's cannonball thought experiment, or ballistic trajectories, or "hammer-throw physics" (???), or saw-tooth paths, any any related nonsense, please direct him back to this derivation of the conclusive answer to his arrogant question as a reminder that math and physics doesn't care what he thinks.

Edit 11/5/23:

As anticipated, Michael claimed victory in the face of defeat but for a far more compelling reason than typical flerfer excuses. He has a formula he uses that he claims calculates the trajectory of an object using "orbital mechanics." At one point earlier in our discussion I said his formula was bogus. However, after my post to DFE, Michael showed me that his formula actually produced the same result for the [1000 m/s, 1 km] example as my derivation and to the same degree of accuracy. Admittedly, this came as a surprise, as I had not analyzed his formula in detail. So, now I have. To begin, here's his formula, which I will say is very clean and simple:

$$Range = v_A \sqrt{\frac{2h}{g_{eff} - \frac{v_A^2}{R_A}}}$$

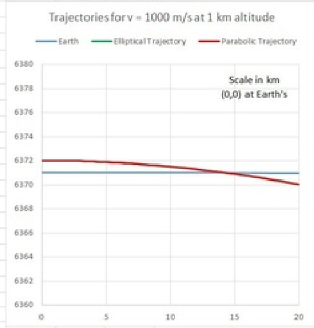
As Michael sees it, this equation combines orbital mechanics and simple ballistics by subtracting the centrifugal acceleration from the gravitational acceleration. As he sees it, when the two are equal, gravity is supposedly cancelled and the object no longer falls, so this should represent orbit. And, it is true that at the speed where the denominator goes to zero, the velocity is, in fact, at the traditional value for a circular orbit. On the other hand, if velocity is zero, the result is the simple parabolic range over a flat earth.

Michael believes that if orbital mechanics are real, then at any intermediate value of velocity the range from this equation would be the same as what you find using the simple parabolic equation over a flat plane. That does not hold, so he thinks he has proven his point. His misconception supports his notion that terrestrial ballistics and orbital mechanics are not a seamless expression of a single theory of physics.

The problem, of course, is that his understanding of this formula is flawed. He does not know how to derive it properly. His notion of subtracting centrifugal acceleration from gravity suffers from the simple fact that centrifugal force is not an actual force. Michael will disagree, of course, and he pushed back by showing me that his formula calculates the same result as mine, as shown below.

20	Square size	
0	X Primary Min: 0	
6380	Y Primary Max: 6380	
20	X Primary Max: 20	
6360	Y Primary Min: 6360	

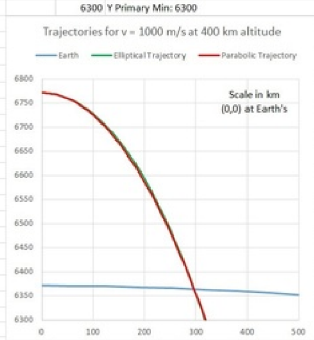
Input Parameters:			Elliptical trajectory over spherical earth:		
Gravitational constant, G:	6.6743E-11	Nm^2/kg^2	Apogee, R_A:	6372000	m
Earth mass, M_E:	5.9722E+24	kg	A_1:	-1.2411E+02	
Earth radius, R_E:	6371000	m	B_1:	7.9720E+08	
Altitude, h:	1000	m	C_1:	-4.06024E+13	
Initial velocity, v_A:	1000	m/s	Perigee, R_P:	51341.6	m
Gravity at sea level, g_0:	9.8202	m/s^2	Major semi-axis, a:	3211670.8	m
Gravity at altitude, g_A:	9.8172	m/s^2	Minor semi-axis, b:	571968.9	m
			A_2:	30.52951033	
Parabolic trajectory over flat earth:			B_2:	6320658.442	
Fall time, t:	14.272	s	C_2:	-1.27944E+15	
Range, x_I:	14272.1	m	y_J:	6370983.8	m
			x_J:	14387.551	m
			Range angle, θ_J:	0.12939	deg
Parabolic trajectory over spherical earth:			Range (arc length), s_J:	14387.563	m
A_3:	1				
B_3:	-203692.774				
C_3:	-3.92917E+13		MB Equation for Range:	14387.560	m
y_J:	6370983.8	m			
Range, x_J:	14387.560	m			
Range angle, θ_J:	0.12939	deg			
Range (arc length), s_J:	14387.573	m			



But, before we let him celebrate too much, let’s look at the proper derivation of his formula. At first, I thought it might be the full symbolic solution of one of my previous derivations, but at faster velocities or higher altitudes, all three formulas diverge, as seen below.

500	Square size	
0	X Primary Min: 0	
6800	Y Primary Max: 6800	
500	X Primary Max: 500	
6300	Y Primary Min: 6300	

Input Parameters:			Elliptical trajectory over spherical earth:		
Gravitational constant, G:	6.6743E-11	Nm^2/kg^2	Apogee, R_A:	6771000	m
Earth mass, M_E:	5.9722E+24	kg	A_1:	-1.1674E+02	
Earth radius, R_E:	6371000	m	B_1:	7.9720E+08	
Altitude, h:	400000	m	C_1:	-4.58464E+13	
Initial velocity, v_A:	1000	m/s	Perigee, R_P:	58001.9	m
Gravity at sea level, g_0:	9.8202	m/s^2	Major semi-axis, a:	3414501.0	m
Gravity at altitude, g_A:	8.6942	m/s^2	Minor semi-axis, b:	626682.5	m
			A_2:	28.68652413	
Parabolic trajectory over flat earth:			B_2:	6712998.088	
Fall time, t:	293.971	s	C_2:	-1.20457E+15	
Range, x_I:	293970.7	m	y_J:	6364087.8	m
			x_J:	296694.066	m
			Range angle, θ_J:	2.66920	deg
Parabolic trajectory over spherical earth:			Range (arc length), s_J:	296801.411	m
A_3:	1				
B_3:	-216046.8887				
C_3:	-3.91268E+13		MB Equation for Range:	296495.027	m
y_J:	6364097.0	m			
Range, x_J:	296496.413	m			
Range angle, θ_J:	2.66742	deg			
Range (arc length), s_J:	296603.544	m			



So, that’s not the answer. As it turns out, the formula is achieved when we make another parabolic assumption, this time for the shape of the earth. With both the trajectory and the surface assumed to have parabolic shape, the solution has a simple closed form.

Calculation of a parabolic projectile trajectory over a parabolic approximation of Earth’s surface given initial height and horizontal velocity:

Solve Eq. (2) for y_c and then approximate using the first term of the Taylor series expansion. This provides a parabolic approximation for Earth’s surface:

$$y_c = \sqrt{R_E^2 - x_c^2} = R_E\sqrt{1 - \frac{x_c^2}{R_E^2}} \cong R_E\left(1 - \tfrac{1}{2}\frac{x_c^2}{R_E^2}\right) = R_E - \frac{x_c^2}{2R_E} \quad \text{Eq. (29)}$$

To find the intersection of the parabolic trajectory with the parabolic Earth, set Eq. (24) equal to Eq. (29) at point J:

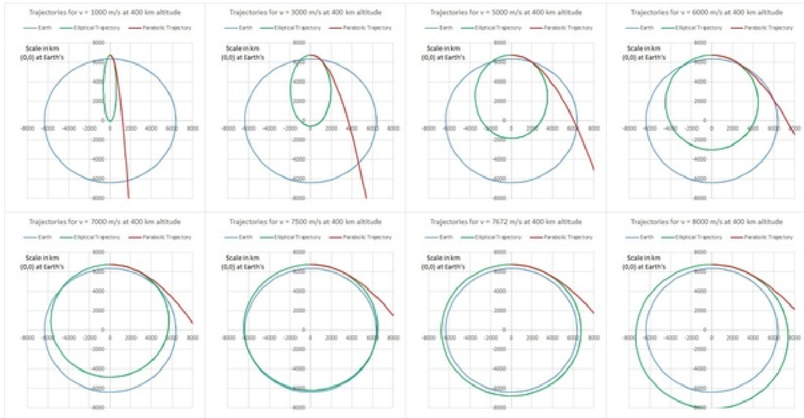
$$y_J = R_E + h - \tfrac{1}{2}g\left(\frac{x_J}{v_A}\right)^2 = R_E - \frac{x_J^2}{2R_E} \quad \text{Eq. (30)}$$

Solve Eq. (29) for x_J: $\quad x_J = v_A\sqrt{2h/\left(g - \frac{v_A^2}{R_E}\right)} \quad$ Eq. (31)

So, his formula is the exact, non-approximated solution for a parabola intersecting a parabola, having nothing to do with centrifugal forces.

Except there’s just one problem. If you’ve been paying close attention, you’ll have noticed that there is a slight difference between Eq. (31) above and “Michael’s formula,” that being that the former uses R_E and the latter uses R_A. I’m not certain if Michael even realized he betrayed his position by making that switch. Perhaps he did it knowingly, but I’ll allow that he did it by mistake.

Lastly, I’ll add one last analysis showing the proper progression from terrestrial to orbital, using the elliptical equations.



When Michael revealed his finding to me, I gave him my word that I would investigate it and update my post accordingly. I have now done that. I expect that I will hear from him again. We'll see if he has a similar commitment to integrity in how he responds.

Adrian Fagg · Oct 29
That's nice work, Tim. Michael kept wanting me to 'do the maths' in my part of the same argument, but I was too lazy and it would have taken me quite a while. But I also felt that I didn't want to waste time arguing on his terms, especially since (more)
9 Reply

Gert Van Der Walt · Oct 30
Brenner believes that, if the ISS really existed, the occupants would be stuck to the roof similar to a bucket of water swung through 360 degrees. Only this reveals his inability to understand basic physics.
8 Reply

Richard Adams · Oct 31
And yet that's precisely what a Sun and Moon whizzing around a flat disk would require; the complete denial of centrifugal forces acting on them. Flatties don't think and that's the problem right there. It's like their ice wall holding in t (more)
1 Reply

Chris Harrington · Oct 29
Wow. That's commitment. Of course, the flerf will probably just throw up his hands and say, "math isn't reality!" ...and then crap all over the chess board, over turn the table, and flap off. But I applaud you for sticking yo your guns and giving the (more)
14 Reply

Adrian Fagg · Oct 30
I don't know if you have seen Michael's maths but he has some very strange ideas. He thinks elliptical orbits are impossible because things must orbit around a centre that is normal to the orbital trajectory. Gravitational force (more)
6 Reply

Chris Harrington But… if he gets parabolic trajectories, why can't he get elliptical orbits……

J HAM · Oct 31
Bruh, are you wearing a graduation gown in your picture? I'm just curious that's all.
Reply

Chris Harrington · Oct 29
This is SO GREAT! Now all Michael has to do is actually *fire a cannon ball* from a height of 1km and then measure how far it goes! Then he'll see the Earth is a sphere! 🤣🤣🤣
5 Reply

Tim Good · Oct 29

I know, right? When I finished working it out and saw that the example conditions he chose only provide an accurate answer over a sphere even when using a parabola, I thought "great. Now he'll just say something like 'see, (more)

4 Reply

Chris Harrington Yep. Their biggest problem with math is that they just don't understand it.…

Neil Davies · Oct 29

Brenner is just an arrogant, ignorant Dunning-Kruger poster boy. He consistently gets his sorry arse handed to him by Torsten Hehl, and now you've really exploded his tiny mind. Way to go!

10 Reply

Martin Dennett · Oct 30

"He consistently gets his sorry arse handed to him by Torsten Hehl,"

Got, my dear friend. *Got*. Since he had his arse handed to him one too many times, Brenner took the only option left available to him…

3 Reply

Neil Davies You're right, of course. The dreaded "Block (insert name here)" option.

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About the Author

Tim Good

Optical Engineer at Honeywell (company)

MS in Mechanical Engineers, Lehigh University Graduated 1995

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