DRACO: A FAILURE OF THE TIDAL MODEL

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ABSTRACT

We test whether the structural properties of the nearby dwarf spheroidal (dSph) galaxy Draco, a wellstudied Milky Way companion, can be reconciled with the suggestion that dSphs are unbound tidal remnants with a large depth extent along the line of sight. In order to apply the observational test of this hypothesis, suggested by Klessen & Zhao, we use public photometric data from the Sloan Digital Sky Survey (SDSS) to explore the width of Draco's blue horizontal branch over a range of areas covering 0.06–6.25 deg² centered on Draco. The SDSS database is the only currently existing database with sufficient depth and area coverage to permit a stringent test of the tidal models. Blue horizontal branch stars were chosen as tracers of Draco's spatial extent and depth because of their low contamination by Galactic foreground stars and since they have a spatially more extended distribution than the more centrally concentrated red horizontal branch stars. Indeed, we show that blue horizontal branch stars extend beyond the previously inferred limiting radii of Draco, consistent with the observed absence of a truncated stellar surface density profile of this dSph (Odenkirchen et al.). Following the method of Klessen & Kroupa, we calculate new models for a galaxy without dark matter, using Draco's morphological properties as constraints. The resulting models are unable to reproduce the narrow observed horizontal branch width of Draco, which stays roughly constant regardless of the field of view. We conclude that Draco cannot be the remnant of a tidally disrupted satellite but is probably strongly dark matter-dominated, as suggested independently by the structural analysis conducted by Odenkirchen et al. and by the kinematic analysis of Kleyna et al.

Subject headings: galaxies: dwarf — galaxies: evolution — galaxies: kinematics and dynamics — galaxies: structure — Local Group — stellar dynamics

1. INTRODUCTION

Dwarf spheroidal (dSph) galaxies are very unusual astrophysical objects. Their stellar masses are comparable to those of massive globular clusters (Harris 1996), but unlike globular clusters, which are dense and compact, dSphs are very extended and diffuse assemblies of stars. Their limiting radii exceed those of globular clusters by factors of several hundreds (Irwin & Hatzidimitriou 1995) and reach kiloparsec scales, while the velocity dispersion of their stars again is comparable to what is measured in globular clusters. This makes them unusual in the sense that if dSphs were bound objects in virial equilibrium, then their gravitational mass as implied by the velocity dispersion must exceed the luminous mass observed in stars by 1-2 orders of magnitude. Dwarf spheroidal galaxies therefore are often claimed to be completely dominated by invisible dark matter (see, e.g., the review on that subject by Mateo 1997).

The assumption of virial equilibrium, however, may be challenged since many dSphs orbit massive L_* -type galaxies like our Milky Way and are therefore subject to strong tidal torques. This influences their internal dynamics and may bring them on the brink of disruption, as is claimed for the nearby dSph Ursa Minor (Martínez-Delgado et al. 2001) or even complete dissolution as observed in the case of the Sagittarius dwarf galaxy (Ibata, Gilmore, & Irwin 1994). It is therefore a valid approach to hypothesize that many, if

not all, dSph companions of the Milky Way may be the remnants of tidally disrupted satellites without appreciable amounts of dark matter.

The "tidal scenario" has received considerable attention. In early studies, Kuhn & Miller (1989) concentrated on the effects of resonant coupling between orbital and internal motions, while Kuhn (1993) investigated diffusion in phase and configuration space of assemblies of stars orbiting various rigid potentials. Both studies concluded that indeed tidal effects could mimic the observed large mass-to-light ratios. The opposite conclusion was derived by Oh, Lin, & Aarseth (1995), who included the effects of the satellites' self-gravity in their studies, and by Piatek & Pryor (1995), who focused on tidal effects during the satellites' perigalactic passage. Further discussion on tidal mass loss can be found in Helmi & White (1999), Johnston, Sigurdsson, & Hernquist (1999), and Bekki, Couch, & Drinkwater (2001).

The most detailed models of the long-term evolution of low-mass satellite galaxies in the tidal field of the Milky Way have been discussed by Kroupa (1997) and Klessen & Kroupa (1998), who demonstrated that a remnant containing a few percent of the initial mass prevails as a long-lived and distinguishable entity for a period of several billion years after the initial satellite dissolves. They proposed that what appears to be a bound dSph galaxy to a terrestrial observer may in fact be the unbound remnant of a tidally disrupted satellite galaxy on an eccentric orbit with two tidal arms extending along a small angle from the line of sight of a terrestrial observer. This model successfully demonstrates that high velocity dispersions in dSphs may be obtained without internal dark matter. It also accounts for the distorted morphology of, e.g., the double-peaked dSph Ursa Minor (e.g., Kleyna et al. 1998). The tidal model furthermore predicts an appreciable spread in stellar distance

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moduli as the tidal remnant will have considerable depth along the observer's line of sight, which is an observationally testable prediction. Another model prediction is a radial velocity gradient during most of the evolution after tidal disintegration. These effects have been studied in detail by Klessen & Zhao (2002), who suggested using the width of the horizontal branch (HB) in the color-magnitude diagram (CMD) as a possible test for the tidal origin of the Galactic dSphs (see also Klessen & Kroupa 1998).

In our current investigation we apply the HB test to the nearby dSph galaxy in the constellation of Draco. Our goal is to construct a tidal model for Draco that complies with all observational properties of the galaxy known to date. We will see that this attempt fails, and we thus claim that Draco cannot be of tidal origin. This implies either that Draco is strongly dark matter-dominated or, alternatively, that modifications to the law of gravity are necessary (Milgrom 1983, 1995; see Sanders & McGaugh 2002 for a review). We focus on Draco, because it is particularly well suited for testing the tidal model. Draco is the only dSph to date for which high-quality, deep, multicolor CCD imaging exists covering both Draco and a 7 times larger region than its tidal radius. Draco is a close satellite galaxy of our Milky Way. Its compact and seemingly undistorted morphological appearance (Odenkirchen et al. 2001a) as well as its thin HB in the CMD can be used to define stringent constraints on tidal models.

Our line of reasoning is as follows. First, in § 2 we discuss morphological and kinematic properties of Draco. We use public data from the Sloan Digital Sky Survey (SDSS) (see York et al. 2000; Gunn et al. 1998; and Stoughton et al. 2002 for more information) to determine the width of the HB of Draco. In § 3 we attempt to construct an appropriate model for the tidal origin of Draco. Our finding is that this is not possible. Finally, in § 4 we summarize and discuss our results.

2. MORPHOLOGY AND HORIZONTAL-BRANCH THICKNESS OF THE DRACO DWARF SPHEROIDAL GALAXY

If dSphs are unbound tidal remnants, then nearby dSphs provide the best test objects since here we can study the then expected depth effects with the greatest precision. Draco is a nearby Milky Way satellite at a distance of only \sim 80 kpc. With an absolute magnitude of $M_V = -9.4$ mag, a central surface brightness of $\mu_V = 24.4 \pm 0.5$ mag arcsec⁻², and lack of detectable gas ($M_{\rm H\,{\scriptscriptstyle I}} < 8000~M_{\odot}$), it is a typical lowluminosity Local Group dSph (see Grebel 2000 and Grebel, Gallagher, & Harbeck 2003 for a recent compilation of Local Group dSph properties). Draco is dominated by metal-poor stellar populations older than 10 Gyr and possesses a wellpopulated HB (e.g., Stetson, McClure, & VandenBerg 1985; Grillmair et al. 1998; Aparicio, Carrera, & Martínez-Delgado 2001). The metallicity spread observed in Draco (e.g., Carney & Seitzer 1986; Shetrone, Côté, & Sargent 2001) shows that its early star formation history was extended and complex, unlike that of a globular cluster.

2.1. Morphological Properties of Draco

Draco and its surroundings have been mapped in their entirety with deep, homogeneous five-color photometry by the SDSS. The SDSS obtains CCD imaging in the passbands u, g, r, i, and z of a contiguous region of less than 10^4 deg² centered on the north Galactic cap. The public data presented here are part of the SDSS Early Data Release (EDR; Stoughton et al. 2002). The SDSS multicolor photometric database permits one to employ optimized filtering techniques to enhance the signal of the stellar population associated with the target of interest versus contaminants such as foreground stars and to remove background galaxies based on shape parameters (see Odenkirchen et al. 2001a, 2001b for details). The SDSS also provides corrections for Galactic foreground extinction using Schlegel, Finkbeiner, & Davis's (1998) maps. Odenkirchen et al. (2001a) carried out a morphological study of Draco based on these SDSS data covering more than 27 deg² of the galaxy itself and its surroundings, extending earlier photographic work by Irwin & Hatzidimitriou (1995). Odenkirchen et al. found Draco to exhibit a flattened stellar surface density distribution with constant ellipticity $(e = 0.29 \pm 0.02)$, position angle $= 88^{\circ} \pm 3^{\circ}$). The limiting radius of Draco is 40".1 along the major axis from an empirical King profile (King 1962) and 49".5 from a theoretical King profile (King 1966), which is 41%–75% more extended than found in the earlier photographic study of Irwin & Hatzidimitriou (1995). Overdensities consistent with the presence of extratidal stars were not detected down to a level of 10^{-3} of Draco's central surface density or to a surface brightness of \sim 33.5 mag arcsec⁻². Odenkirchen et al. point out that Draco's density profile may be equally well fitted by a Sérsic profile with an exponent of n = 1.2, since there is no detectable break in the profile, which may indicate an even more extended distribution below the current detection limit. The isopleths of Draco's stellar surface density are highly regular, nested ellipses that retain the same position angle with increasing distance from the dSph's center. These properties led Odenkirchen et al. to infer that Draco is bound and strongly dark matter-dominated.

Aparicio et al. (2001) used B and R CCD photometry covering 1 deg², including parts of Draco and some of its surroundings. They also found a large tidal radius (41') and no evidence for extratidal stars. Another structural study of mostly noncontiguous CCD imaging in Draco and surroundings covering a total area of $\sim 1.36 \text{ deg}^2$ in V and R was carried out by Piatek et al. (2002) and again confirmed the lack of extratidal stars. A recent CCD study of the central part of Draco (0.17 deg⁻²; Bellazzini et al. 2002) using the V and I filters also found the smooth elliptical isopleths presented by Odenkirchen et al. (2001a). Aparicio et al. (2001) compared Draco's CMD to one of the models proposed by Kroupa (1997), RS1-4, according to which Draco is extended along the line of sight. Using the width of the HB of Draco, particularly of the red HB, they found an upper limit of ~ 14 kpc for Draco's depth extent, which is at most one-third of the extent predicted by Kroupa's model. Aparicio et al. conclude that their upper limit precludes the explanation of Draco's velocity dispersion as caused entirely by projection effects and that dark matter is a more likely culprit.

Finally, Kleyna et al. (2001, 2002) studied the radial velocity dispersion of Draco's red giants as a function of projected distance out to a distance of $\sim 25'$ from Draco's center. From this inner region they infer that Draco has a gradually rising rotation curve and is embedded in an extended dark matter halo rather than a halo in which mass follows light. Kleyna et al.'s (2001) calculations suggest that

Draco is close to its perigalacticon and that its tidal radius exceeds 1°.

2.2. The Horizontal Branch of Draco

Taking advantage of the large area coverage afforded by the SDSS data, we investigate the distribution of HB stars in Draco and surroundings. As compared to conventional CCD imaging data, the SDSS data allow us to study the entire projected area within which models of the kind calculated by Kroupa (1997), Klessen & Kroupa (1998), and Klessen & Zhao (2002) and in the current paper predict stars originating from a disrupted dSph galaxy without dark matter. Furthermore, the SDSS database allows us to exclude objects identified as galaxies or as quasars based on their shape parameters or location in multicolor space (see, e.g., Richards et al. 2001; Stoughton et al. 2002).

The red HB of Draco is heavily contaminated by Galactic foreground stars (Fig. 1), particularly by the metal-weak Galactic halo main sequence and, to a lesser extent, by main-sequence stars from the thick disk (Chen et al. 2001). While we can easily perform statistical field star subtractions in the color-luminosity range of Draco's HB, this contamination hampers our ability to determine the *width* of the red HB. This problem becomes particularly severe at larger distances from the center of Draco, where the stellar density of Draco drops rapidly. We therefore decided not to include red HB stars in our analysis.

Apart from a well-populated red HB, Draco contains a sizeable number of RR Lyrae stars, which overlap with the red HB locus and become, on average, increasingly more luminous toward bluer colors (e.g., Bellazzini et al. 2002; their Fig. 6). To avoid uncertainties in luminosity (and hence width) introduced by RR Lyrae variables, we excluded their loci from our analysis as well.

This leaves us with the blue HB. While Draco's blue HB is sparsely populated in comparison with the red HB, blue stars generally have the advantage of being easily recognizable even in regions where Galactic field stars dominate otherwise, since Galactic field star contamination is very low at these blue colors. The field star contribution in the area of the red HB amounts to 62.2 ± 5.3 objects per square degree, while for the blue HB it is only $1.8 \pm 0.9 \text{ deg}^{-2}$. Some of the latter contaminants may be unidentified quasars at redshifts between ~2 and ~3 (e.g., Fan 1999; Richards et al. 2001).

Many dwarf galaxies show population gradients in the sense that younger populations are more centrally concentrated than older ones (see Grebel 1997, 1999, 2001a for reviews). Gradients may even exist in the old populations of dwarf galaxies: Harbeck et al. (2001) showed that in dwarf galaxies blue HB stars often have a spatially more extended distribution than red HB stars. This "second-parameter" variation may be caused by age or metallicity, or both, or additional factors (e.g., Hurley-Keller, Mateo, & Grebel 1999; Harbeck et al. 2001). In Draco, our analysis of the SDSS data reveals that Draco's HB stars follow the same trend. Beyond galactocentric distances greater than 0°5 (Fig. 2), the contribution of red HB stars decreases: the fraction of blue HB stars (measured as the number of blue HB stars [BHB] over the sum of BHB and the number of red HB stars [RHB]) increases from less than 20% in the inner region of Draco to $\sim 90\%$ in its outskirts (Fig. 2). This makes blue HB stars useful tracers of the spatial extent of Draco.

We choose the SDSS color $(g^* - \bar{t}^*)$ to ensure a large color baseline in two of the most sensitive SDSS filters. (Note that the superscripted asterisk is conventionally used to indicate that the photometric calibration of the SDSS EDR data is not yet final and may be further refined in later data releases; see also Stoughton et al. 2002.) The SDSS g filter lies between Johnson B and V and largely overlaps with these two filters,



FIG. 1.—Color-magnitude diagrams of Draco and surroundings. The magnitudes were dereddened using the extinction maps of Schlegel et al. (1998). The left-hand panel shows all stars within a region of 0.5×0.5 centered on Draco, while the right-hand panel contains all stars within a 1.5×1.5 area. In the left-hand panel, Draco's red giant branch, prominent red horizontal branch, and diagonal RR Lyrae strip stand out. Blue horizontal branch candidates fulfilling our selection criteria (§ 2.2) are marked by the larger dots. In the right-hand panel, the increasing contamination by Galactic foreground stars, which overlap in particular with Draco's red horizontal branch stars, is obvious. In comparison, the blue horizontal branch locus suffers little contamination.



FIG. 2.—Cumulative radial distribution of blue horizontal branch (BHB; solid line) stars. The label and numbers of the left-hand side refer to the number of stars within the radial distance R from the galaxy center, $N(\leq R)$, normalized by the total number of stars, N(total). For comparison, the dotted line indicates the distribution of Galactic thin-disk stars as a homogeneously distributed, control field population unrelated to Draco (see Chen et al. 2001 for details). Open circles indicate the fraction of BHB stars with respect to the total number of horizontal branch stars [BHB/ (BHB + RHB)]. The fraction of BHB stars increases with increasing projected angular distance from the center of Draco. The inner three open circles sample the BHB fraction within 1, 2, and 3 core radii, whereas the outermost bin combines three bins of this size for improved number statistics. The bin width is indicated by the horizontal bars. The vertical bars show the uncertainty of [BHB/(BHB + RHB)]. The BHB stars can be traced out to at least the limiting radius of Draco inferred from using King (1966) models (Odenkirchen et al. 2001a). Draco's BHB stars show a more extended distribution than the more centrally concentrated red horizontal branch stars. This kind of population gradient among old stars is a fairly common feature in dwarf spheroidal galaxies (see Harbeck et al. 2001) and underlines the suitability of blue horizontal branch stars as tracers of the spatial extent of dwarf galaxies.

while the SDSS *i* filter is slightly more narrow and a bit bluer than the Kron-Cousins *I* filter (see Fig. 1 in Grebel 2001b). Transformation equations between the SDSS filter system and the conventional Johnson-Cousins system are given in Fukugita et al. (1996); for the purposes of our study we will consider SDSS g^* as equivalent to Johnson *V*. All SDSS magnitudes were corrected for Galactic foreground extinction using the appropriate values of Schlegel et al. (1998), which are provided by the SDSS database. We show the distribution of Galactic reddening in the region of Draco and surroundings in Figure 1 of Odenkirchen et al. (2001a). We assume that there is no internal extinction in Draco (see, e.g., Gallagher et al. 2003 and references therein). The resulting dereddened SDSS magnitudes are denoted g_0^* and i_0^* . To select blue HB stars, we considered only objects flagged as stars in the database, which in addition were required to fulfill the criteria

19.8 mag
$$< g_0^* < 20.3$$
 mag

and
$$-0.35 \text{ mag} < (g_0^+ - i_0^+) < -0.10 \text{ mag}$$

19.8 mag $< g_0^* < 20.7$ mag

and
$$-0.50 \text{ mag} < (g_0^* - i_0^*) < -0.35 \text{ mag}$$

for the "red" part of the blue HB and for the extended blue HB, respectively (Fig. 1, *larger dots*). We note that while extinction properties vary with stellar temperature, these effects are anticipated to be small within the range of colors considered here and correspond to an uncertainty of at most 0.01 mag in g_0^* [Grebel & Roberts 1995, their Fig. 6, *top left-hand panel*; assuming that $(M - T_2) \sim (g_0^* - i_0^*)$ and $g_0^* \sim M$].

 $g_0^* \sim M$]. We then applied the same procedure as used in Klessen HB: we calculated the average over the standard deviations $\sigma(g_0^*)$ of stars in $(g_0^* - i_0^*)$ color bins of a width of $\Delta(g_0^* - i_0^*) = 0.05$ mag for all blue HB star candidates within certain distances from the center of Draco. The results are given in Table 1. Column (1) shows the selection area centered on Draco, column (2) the shows the average of the standard deviations in g_0^* , column (3) specifies the number of $(g_0^* - i_0^*)$ color bins, and column (4) gives the number of blue HB candidates that entered into the calculations. For comparison, column (5) gives the HB thickness, $\sigma(V)$, expected from the best-fit tidal model (see § 3.5). Table 1 demonstrates that the blue HB exhibits only a small width in the center of Draco and that it remains narrow also when larger areas are considered, while tidal models predict a significant HB widening. The fractional increase in blue HB star candidates in the outermost annulus (see col. [4]) is consistent with the numbers expected from field contamination quoted above.

TABLE 1 Blue Horizontal Branch Width of Draco						
0.25 × 0.25	0.12	5	17	0.125 ± 0.023	48.0	
0.50×0.50	0.13	8	35	0.138 ± 0.023	53.8	
$1.00 \times 1.00 \dots$	0.13	8	48	0.162 ± 0.020	81.8	
1.50×1.50	0.14	8	56	0.185 ± 0.018	91.5	
2.00×2.00	0.13	8	60	0.207 ± 0.017	99.3	
2.50×2.50	0.14	8	63	0.225 ± 0.016	99.5	

^a Predicted average HB width $\langle \Delta V \rangle_{\text{HB}}$ for the best-fit tidal model 4 obtained from bootstrapping. ^b Percentage of bootstrapping attempts that delivered $\langle \Delta V \rangle_{\text{HB}}$ in excess of the observed value in col. (2).

3. A TIDAL MODEL FOR THE DRACO?

With regard to its morphology and structure, Draco is one of the best-studied dSph satellite galaxies of the Milky Way. Its compact and seemingly undistorted morphological appearance, together with a well-defined, thin HB in the CMDs, sets severe constraints on tidal models. In this section we investigate whether it is possible to construct a tidal model for Draco that is able to reproduce *all* observed morphological, kinematic, and photometric properties of this satellite galaxy.

3.1. The Numerical Method

In order to construct and constrain a tidal model for the Draco dSph galaxy, we follow the evolution and disintegration of various satellite dwarf galaxies in the tidal field of our Galaxy until well after they have completely dissolved. The considered model galaxies vary in mass and size, but all follow an initial Plummer-type equilibrium density profile with parameters as summarized in Table 2. The satellites are placed on nearly radial orbits with eccentricity $\epsilon = 0.9$ and

 TABLE 2

 Initial Properties of the Model

 Satellite Galaxies

Model	r _c (kpc)	r_t (kpc)	$M \ (M_{\odot})$
1	0.3	1.5	107
2	0.1	0.5	10^{6}
3	0.05	0.5	10^{6}
4	0.02	0.2	10^{6}

peri- and apogalactic distances 100 and 5 kpc, respectively (see Fig. 3). The model galaxies have a fixed mass-to-light ratio M/L = 3, and there is *no* dark matter component. The Milky Way, however, does contain dark matter. It dominates the gravitational potential at large scales and is described as an isothermal sphere with circular velocity of 220 km s⁻¹ and a core radius of 5 kpc.

Each satellite galaxy consists of 131,072 particles, and their time evolution in the Galactic tidal field is followed



Fig. 3.—Effect of projection on morphological appearance. The left-hand side shows the distribution of stars in Model 1 in the tidal field of the Milky Way seen at t = 4.5 Gyr, i.e., about 2 Gyr after the complete disruption of the satellite galaxy. The galaxy has dissolved into extended tidal tails on each side of the maximum of the stellar density tracing the location of the former core of the satellite (*dotted circle*). The trajectory of the satellite is indicated by the dotted line where the arrows indicate its location in intervals of $\Delta t = 0.5$ Gyr. The orbital eccentricity is $\epsilon = 0.9$, and the satellite starts at t = 0 Gyr at the right at a galactocentric distance $d_{gal} = 100$ kpc with velocity v = 25 km s⁻¹ in azimuthal direction. The axes indicate a galactocentric coordinate system with units in kpc. The location of the hypothetical terrestrial observer at $d_{gal} = 8.5$ kpc is indicated by the black dot on the vertical axis. The viewing angle θ under with the tidal satellite is seen from this position is almost zero. This leads to very compact morphological appearance of the system when projected on the observers plane of the sky and a large depth along the line of sight leading to considerable widening of the distribution of stars on the horizontal branch in the color-magnitude diagram. Three other positions for observing the satellite are indicated by open dots along the vertical axis, corresponding to $\theta \approx 15^{\circ}$, 30° , and 45° . The right-hand side of the figure illustrates how the tidal debris appears on the sky of the hypothetical observer for the four projection angles. The first column depicts the surface density distribution of particles. The contour lines are linearly spaced in intervals of 10% of the central value. The dashed contour indicates the 1% level. The second column shows the azimuthally averaged radial number density profile Σ . The arrow gives the distance at which Σ has dropped by 2 orders of magnitude. The third column depicts the resulting distrib

numerically solving the equations of motion using a standard TREECODE scheme (Barnes & Hut 1986). The Galactic tidal field hereby is treated as rigid external potential. Further details of the computational scheme are given in Klessen & Kroupa (1998).

3.2. The Effect of Projection on Morphology

The first constraints on tidal models for Draco can be derived from the very compact observed morphological appearance of the dwarf galaxy and the lack of "extratidal" stars. The radial profile of stellar density of Draco is well described by King models (King 1962, 1966) or generalized exponentials (e.g., Sérsic 1968) with core radii of $r_c \approx 8'$ and limiting radii r_t in the range 40'-50' (Odenkirchen et al. 2001a; Aparicio et al. 2001). Adopting a distance to Draco of d = 80 kpc, this corresponds to a total radius of the galaxy of about 1 kpc. Note that the available observational data do not imply that a clear outer "edge" in the galaxy has been detected (§ 2.1). The only firm statement that can be made is that the stellar density drops to less than 1% of its central value within $r_t \leq 50'$, where the galaxy blends into the distribution of foreground stars.

The angular size of Draco is very small compared to typical values reported from numerical simulations. The tidal models published to date predict limiting radii considerably in excess of 1° when the satellite galaxy is projected onto the plane of the sky of an hypothetical observer on Earth after its complete dissolution, i.e., in the phase when it clearly bears morphological and kinematic resemblance to Galactic dSphs (e.g., Fig. 2 of Klessen & Zhao 2002; see also Kroupa 1997; Klessen & Kroupa 1998). Furthermore, these model dSphs often appear quite elongated along the orbital trajectory and have no clear truncation radius, i.e., exhibit a large number of extratidal stars. The fact that Draco is compact (i.e., the surface density decreases by 2 orders of magnitude within $r_t \leq 50'$) and not very elongated (with a ratio between minor and major axis of ~ 0.7) therefore strongly restricts the overall parameter space for tidal models.

Let us first consider the effect of the orbital parameters of tidal satellite galaxies on their morphological appearance for observers on Earth. As a dwarf galaxy orbits around the Milky Way, it loses stars, which subsequently populate two extended tidal tails. After complete disruption the location of the former satellite is still discernible for 1-2 Gyr as an enhancement of stellar density, moving along the orbital trajectory in between the leading and trailing tidal arm. This remnant slowly expands and decreases in density as its stars drift apart and populate the tidal arms. If the satellite is on a relatively circular orbit, a terrestrial observer (being much closer to the Galactic center) would see the resulting elongated structure of the tidal debris from the "side" and would recognize a low-density feature that stretches over many degrees on the plane of the sky along the orbital path, bearing little resemblance to real dSph galaxies. In addition, it would be difficult to see and distinguish this feature at all, as it is likely to blend into the distribution of Galactic foreground stars. In this case only an extensive search for kinematic signatures of tidal streamers can reveal its true nature (see, e.g., Majewski et al. 2000; Helmi & Tim de Zeeuw 2000; Dohm-Palmer et al. 2001; Helmi, White, & Springel 2002; Kundu et al. 2000; Newberg et al. 2002). On the other hand, if the satellite was originally on a very eccentric orbit (as the model galaxies discussed here), it is likely that the viewing angle between the orbital trajectory and the line of sight of an observer on Earth is very small. The tidal debris is then seen along the major axis and subsequently covers a much smaller projected area on the observer's plane of the sky, with morphological and kinematic features now similar to observed dSph galaxies (Kroupa 1997; Klessen & Kroupa 1998). This "piling up" of stars along the line of sight makes the tidal remnant easier to detect against the Galactic foreground stars and leads to a large dispersion of stellar distance moduli, which in turn results in a widening of the distribution of stars in the CMD along the luminosity axis. This widening can be particularly well measured using HB stars and has been suggested as a possible observational test for the tidal models (Klessen & Zhao 2002).

Figure 3 illustrates the effect of the orbital eccentricity ϵ on the projected morphological appearance. Instead of observing satellite galaxies on different orbits (from the same position at $d_{gal} = 8.5$ kpc), we equivalently select a model with $\epsilon = 0.9$ and vary the location of the observer, such that the line of sight covers a range of viewing angles $0^{\circ} \leq \theta \leq 45^{\circ}$ with the orbital trajectory (and hence the major axis of the tidal debris). The right-hand side of Figure 3 shows that tidal models are able to reproduce a compact and roundish morphological appearance with azimuthally averaged radial stellar density profiles only for eccentricities that allow for $\theta \leq 5^{\circ}$. These profiles can be fitted by King models or generalized exponential models and exhibit a lack of significant numbers of extratidal stars. This limits the parameter space for tidal models of Draco to high orbital eccentricities. If Draco currently were near its perigalacticon, as indeed suggested by Kleyna et al. (2001), then tidal models could immediately be excluded. Note that small viewing angles favor projections where the major axis is perpendicular to the orbital trajectory, leading to line-of-sight velocity gradients that run primarily along the minor axis (although connected to very large dispersions; see \S 3.4). This is in contrast to angles $\theta > 10^\circ$, where the elongation always lies along the orbital path (similar to the line-of-sight velocity gradient).

3.3. Small Angular Size and the Timescale for Tidal Disintegration

We selected model 1 for illustration in Figure 3, because it is the largest satellite in our suite of models and it is the one most easily disrupted. Note that in the initial bound state, it is already too extended to describe Draco at a distance of 80 kpc. It is therefore necessary to study the tidal evolution and observational appearance of satellite galaxies of smaller sizes, to see whether it is possible to find a model that better resembles the observational data. Galaxies 2-4 are less massive than model 1 by a factor of 10 (10⁶ instead of 10⁷ M_{\odot}) and are increasingly more compact ranging in their initial stage in size from $r_t = 0.5$ to $r_t = 0.2$ kpc with core radii from $r_c = 0.1$ down to $r_c = 0.02$ kpc, respectively (see Table 2). Model 4 is therefore quite comparable to a massive globular cluster (Harris 1996). The time evolution of the Lagrangian radii encompassing 10%, 20%, etc., of the mass is shown in Figure 4 as the satellite galaxies orbit around the Milky Way. Models 1 and 2 dissolve after $t \approx 2.8$ Gyr during their third perigalactic passage. Model 3 needs five $(t \approx 5.2 \text{ Gyr})$ and model 4 seven passages $(t \approx 7.5 \text{ Gyr})$ to completely dissolve in the Galactic tidal field. Satellite galaxies of the same mass that are initially more compact



FIG. 4.—Lagrangian radii as function of time. The figure shows the radii (*left axis*) containing 10%, 20%, ..., 90% of the mass as the satellite galaxy dissolves in the tidal field of the Milky Way. The corresponding galactocentric distances are indicated by dotted lines (*right axis*), and the perigalactic passage that leads to complete dissolution is indicated by triangles.

would not fully dissolve over the lifetime of the Galaxy on the orbit considered here. Similar characteristics hold for satellites that are more massive but have a size comparable to model 4. This limits the available parameter space of initial satellite properties (M, r_c , and r_t) and motivates our choices in Table 2.

Within the allowed parameter range smaller initial satellites lead to more compact morphological appearance of the tidal remnant after complete dissolution. In our set of four satellite galaxies, only model 4 is able to reproduce the fact that the stellar surface density of Draco falls off to less than 1% within $\leq 1^{\circ}$. Figure 5 illustrates this point by showing the image of the satellites as seen on Earth after complete disruption at the time when they first reach distances to the observer of $d \approx 80$ kpc. The upper panel shows the projected number density Σ of N-body particles per square degree on the sky; the lower panel shows the resulting azimuthally averaged radial profile. The conversion of Σ into the corresponding *stellar* number density Σ_* depends on the number of particles used in the simulation, the mass of the galaxy, and assumptions about the initial mass function (IMF). A prediction of the observed surface density of stars (e.g., for comparing with star count results) furthermore depends on the sensitivity limit of the instrument used and the crowding (number density) of foreground stars, which

gives a constant offset. For instance, the number density of particles in the central region of model 4 is $\Sigma \approx 1.7 \times 10^5$ deg⁻². As the number of particles is 131,072 and the total mass is 10⁶ M_{\odot} , each particle carries ~7.6 M_{\odot} . We assume a standard IMF as proposed by Kroupa (2002) with average stellar mass $\langle M \rangle = 0.38 \ M_{\odot}$ and where stars with $M \ge 1 \ M_{\odot}$ contribute roughly 50% of the mass but only 6% in number. If we furthermore assume a detection limit of 1 M_{\odot} , then each N-body particle represents about 1.2 stars lying above the detection threshold, $\Sigma_* = 1.2\Sigma$. For the very inner region of Draco, model 4 thus predicts star counts of order 10⁵ deg⁻². This is consistent with the observed values (Irwin & Hatzidimitriou 1995; Odenkirchen et al. 2001a; Aparicio et al. 2001).

3.4. Rapid Expansion after Complete Tidal Dissolution

The results of the previous section confirm that it is possible to construct tidal models that resemble the morphology of the Draco dSph galaxy at least immediately after their complete disruption. However, how long does this period of time last? This question is addressed in Figure 6, which shows the projected surface density of model 4 during various phases of its dynamical evolution under the condition that the galaxy lies at a distance of roughly 80 kpc to an



FIG. 5.—Images of the model galaxies shortly after complete disruption. The upper panel shows the surface density of the model galaxies shortly after their perigalactic passage that fully disrupts them, at a time when they reach again a distance to the observer of $d \approx 80$ kpc. Continuous contours are overlaid onto the projected distribution of *N*-body particles (*gray dots*). The contour spacing is linear in steps of 10% of the central value. The dashed line indicates the 1% level. The lower panel depicts the corresponding azimuthally averaged radial profile. Σ is defined as the number density of *N*-body particles per square degree. The conversion into stellar surface density is discussed in the text.

observer on Earth. The figure depicts all such instances in the time interval t > 5 Gyr until the end of the simulation at t = 11.5 Gyr. Earlier times exhibit no substantial evolution in the projected stellar density distribution and only two snapshots are shown. Note that the satellite completely dissolves during the seventh perigalactic passage at $t \approx 7.5$ Gyr. The figure illustrates how the tightly bound satellite galaxy evolves into unbound debris in the tidal field of the Galaxy and how this tidal stripping process influences the observable structure. Note the variation of morphological parameters (e.g., in the axis ratios, the position of the major axis, or the "smoothness" of the particle distribution) among the six snapshots with t > 7.5 Gyr. Note also that only the first two snapshots after complete disintegration are compact enough to describe Draco. At later times the surface density does not drop below 1% of its central value within radii of $\sim 2^{\circ}.5$, largely exceeding the limiting radius of Draco $r_t \leq 50'$. Figure 6 therefore indicates that the time interval during which tidal models are able to reproduce properties of Draco is quite limited (to about 1 Gyr). This implies that we would observe the dSph galaxy at a very special instance in time (namely, right after its tidal disruption) if the tidal models were correct. This result is further supported by the time evolution of the line-of-sight velocity dispersion σ_{LOS} , as indicated in Figure 7. For model 4, σ_{LOS} lies within the observed margin 8 km s⁻¹ < $\sigma_{\rm LOS} \lesssim 10$ km s⁻¹ only for a period of about 1 Gyr following the complete disruption. At earlier times σ_{LOS} is smaller, and at later times it is larger. Similar timescales hold for the other models.

3.5. Horizontal Branch Thickness

As discussed above, the compactness of Draco at a distance of 80 kpc, the shape of its radial surface density distribution and the measured value of its line-of-sight velocity dispersion severely limit the available parameter space for

tidal models and make their applicability to Draco very unlikely. To fully exclude a tidal model for Draco, one needs to consider its HB. As discussed in § 2.2, the HB of stars in the CMD is remarkably thin with $\langle \Delta g_0^* \rangle_{\rm HB} \approx 0.13$ mag, and it shows very little variation with extent of the region considered. Values range from $\langle \Delta g_0^* \rangle_{\rm HB} \approx 0.12$ mag for stars within a radius of 0°25 around the center of Draco to $\langle \Delta g_0^* \rangle_{\rm HB} \lesssim 0.14$ mag when considering all HB stars within $2^{\circ}.5 \times 2^{\circ}.5$. The best-fit tidal model exhibits comparable values only in the innermost region. When considering larger areas, it shows a HB thickness significantly larger than observed. It is a general property of tidal models to predict a strong correlation between the HB thickness and the angular size of the considered field. This is in sharp contrast to the observational data of Draco, which reveal no significant variation of HB thickness with area. As illustration, Figure 8 shows the CMD obtained for fields of different sizes from model 4 at time t = 7.6 Gyr when it reaches a distance of 80 kpc for the first time after complete disruption. The corresponding stellar density distribution can be seen in the last column of Figure 5. The synthetic CMDs are obtained using the method described by Klessen & Zhao (2002) with the globular cluster M3 as template, and with $\langle \Delta V \rangle_{\rm HB}$ computed as in § 2.2. M3 has metallicities similar to Draco and has a very thin HB with $\langle \Delta V \rangle_{\rm HB} \lesssim 0.08$ mag. Hence, any significant HB widening apparent in Figure 8 stems from stellar distance moduli variations in the projected tidal debris. The larger the considered area, the larger is the derived HB thickness. Stars that are farther away from the projected center of density are likely to be farther downstream or upstream along the tidal arms. The inclusion of those stars into the analysis leads to a larger range of distance moduli and subsequently to a widening of the luminosity distribution in the CMD. This is summarized in Figure 9, which shows the time evolution of the HB thickness for all four models for four different field sizes each.



FIG. 6.—Morphological appearance of model 4 as function of time. The figure describes the projected particle distribution (*upper panels*) and radial surface density profile (*lower panels*) at times when the distance between observer and the density center of the satellite galaxy reaches $d \approx 80$ kpc. All such instances are shown after the satellite completely dissolves in the perigalactic passage at $t \approx 7.5$ Gyr. At earlier times, the satellite exhibits no substantial signs of evolution in projection and we show only two snapshots, one immediately after the start of the calculation, t = 0.3, and the other before complete disruption, t = 7.0.

To test the statistical significance of our result we need to address two questions. First, how does the HB thickness of the best-fit tidal model vary within the evolutionary window allowed from morphological and kinematic considerations? Second, what is the effect of the small sampling size in the observations—the HB thickness for the innermost region of Draco is obtained from only 17 BHB stars? In the previous section we have demonstrated that the line-of-sight velocity dispersion in the best-fit tidal model lies within the observed margins only for a period of about 1 Gyr after complete disruption. Furthermore, Draco is at a distance of ~80 kpc. Inspection of the satellite trajectories in Figure 4 reveals that model 4 complies with both requirements only twice, at t = 7.6 Gyr and at t = 8.2 Gyr. When we allow for a distance uncertainty of Draco of 5%, which corresponds in the models to a time span of roughly 50 Myr, and compare the HB morphology within and between these two intervals, we see that the variations are very small. This enables us to derive a well-defined and meaningful average HB thickness. The second question can be addressed by bootstrapping. From the large number of HB stars in the model we randomly select a small subset corresponding in number to the observed sample (e.g., 17 for the innermost 0°.25 × 0°.25) and compute the resulting HB width. We repeat the procedure 10,000 times for each field size. The resulting HB thickness $\langle \Delta V \rangle_{\rm HB}$, together with its standard deviation averaged over all bootstrapping attempts and averaged over all allowed snapshots of model 4, is listed in column (5) of Table 1 for direct



FIG. 7.—Time evolution of the line-of-sight velocity dispersion σ_{LOS} . For all four model galaxies, σ_{LOS} vs. *t* is calculated for three different observational field sizes (*gray area, units on the left axis*). Before complete tidal disruption, the measured velocity dispersion is lowest when considering only a small central field, as σ_{LOS} is dominated by the stars in remaining bound core. With increasing field size also unbound stars in the tidal tails begin to contribute and σ_{LOS} increases. At about 1 Gyr after disruption as the system becomes sufficiently dispersed, the influence of the central remnant vanishes and the variation of σ_{LOS} obtained from different field sizes becomes negligible. To see the relation between the observable σ_{LOS} and the galactocentric distance, d_{gal} is indicated by the dotted lines (*right axis*). The perigalactic passage that leads to complete dissolution is indicated by triangles.

comparison with the observations. The statistical error decreases slightly with increasing field size because of the increasing number of stars considered in the subsamples (col. [4]). Column (6) in Table 1 indicates in addition the percentage of the bootstrapping attempts that delivered $\langle \Delta V \rangle_{\rm HB}$ values in excess of the observations. This number

increases from $\sim 50\%$ in the inner 0.25×0.25 to about 100% in the full field and gives a clear indication that we must consider wide-field data to arrive at statistically sound statements about the nature of dSph galaxies. It should be noted that the results still hold, even if no assumption about the intrinsic HB morphology are made,



FIG. 8.—Influence of the field size on the horizontal branch thickness. The figure shows the color-magnitude diagram of model 4 at t = 7.6 Gyr using the globular cluster M3 as template for field sizes ranging from 0.25×0.25 to 2.5×2.5 centered on the maximum of the surface density of stars. The horizontal branch thickness $\langle \Delta V \rangle_{\text{HB}}$ is defined as the average value of the standard deviation of V for horizontal branch stars determined from bins of width $\Delta(B-V) = 0.05$ mag in the range $-0.15 \text{ mag} \leq (B-V) \leq 0.5$ mag (see § 2.2).



FIG. 9.—Horizontal branch thickness as function of time. For models 1–4 the figure shows $\langle \Delta V \rangle_{\text{HB}}$ vs. *t* calculated for four different field sizes (*gray area, left axis*). The galactocentric distance d_{gal} at the corresponding times is indicated by the dotted lines (*right axis*). As already indicated in Fig. 8, larger field sizes lead to larger horizontal branch thickness. The perigalactic passage that leads to complete dissolution is indicated by triangles.

i.e., even if we assume an intrinsic HB width of zero, $\langle \Delta V \rangle_{\rm HB}$ in the outer regions would be significantly in excess of the observed values.

In summary, Draco has a remarkably thin blue HB that shows no signs of variation with radius. Tidal models, however, predict a strong increase of HB thickness with considered field size. While in the central regions, the HB width in the best-fit tidal model still only marginally exceeds the values observed in Draco, this excess is highly significant at larger radii. The large area coverage offered by SDSS is needed to derive this result, and taking all together, we feel safe in concluding that even best-fit tidal models that are able to reproduce the morphological parameters of Draco cannot correctly predict its HB properties.

4. SUMMARY: THE FAILURE OF THE TIDAL MODEL FOR DRACO

The dSph galaxies in the vicinity of the Milky Way are either strongly dark matter–dominated or the remnants of tidally disrupted satellite galaxies that are seen today by terrestrial observers at certain highly favorable viewing angles. This is inferred from the observed stellar velocity dispersions, which are large with respect to the luminous mass of the galaxy. A possible third alternative is a modification to Newton's law of gravity (Sanders & McGaugh 2002). It was our attempt in this paper to construct an appropriate tidal model for the Draco dSph galaxy. Draco is particularly useful for testing tidal models, because it is one of the best-studied satellite galaxies of the Milky Way, and its compact and seemingly undistorted morphological appearance, as well as its thin HB in the CMD, sets tight constraints on tidal models. Our effort to find a model for Draco that is able to reproduce *all* properties of the galaxy known to date failed.

Our line of reasoning is the following. First, we consider the apparent lack of extratidal stars in the vicinity of Draco to constrain the orbital trajectory of the progenitor satellite galaxy. Indeed, tidal models without a noticeable number of extratidal stars are possible if the viewing angle to the galaxy is close to a tangent to the orbital trajectory (see Fig. 3), which requires that the satellites follow very eccentric orbits. Second, the observed compactness of Draco can be reproduced if the satellite galaxy itself initially is small and compact. However, this galaxy cannot be too compact, otherwise it will not be disrupted in the tidal field of the Milky Way over a Hubble time. Third, rapid expansion of the tidal debris after complete disruption limits the period during which the tidal models resemble Draco to about 1 Gyr. A similar time span can be derived from looking at the line-of-sight velocity dispersion. Together, these morphological and kinematic constraints severely limit the available

parameter space for tidal models, but they do not render them completely implausible. Finally, large field-of-view photometric data from the SDSS can be used to fully exclude tidal models for Draco on the basis of its HB thickness. The observed small variations in the position of stars in the CMD of Draco are inconsistent with the predicted wide distribution in the tidal model due to distance modulus variations along the line of sight. These models furthermore exhibit a strong increase of HB thickness with increasing field size, while Draco exhibits no such variation.

In summary, we conclude that Draco *cannot* be the remnant a tidally disrupted satellite galaxy. Instead, it most likely is strongly dark matter-dominated, as predicted by standard cosmological scenarios and, increasingly, by observational evidence such as structural studies (e.g., Odenkirchen et al. 2001a) and the radial dependence of the velocity dispersion (see Kleyna et al. 2001, 2002).

We thank J. S. Gallagher, P. Kroupa, M. Odenkirchen, R. Scholz, and H. Zhao for stimulating discussions. We also thank the referee for many useful comments and suggestions. R. S. K. acknowledges support by the Emmy Noether

- Aparicio, A., Carrera, R., & Martínez-Delgado, D. 2001, AJ, 122, 2524
- Barnes, J. E., & Hut, P. 1986, Nature, 324, 446 Bekki, K., Couch, W. J., & Drinkwater, M. J. 2001, ApJ, 552, L105
- Bellazzini, M., Ferraro, F. R., Origlia, L., Pancino, E., Monaco, L., & Oliva, E. 2002, AJ, 124, 3222
 Carney, B. W., & Seitzer, P. 1986, AJ, 92, 23

- Chen, B., et al. 2001, ApJ, 553, 184 Dohm-Palmer, R. C., Helmi, A., Morrison, H., Mateo, M., Olszewski, E. W., Harding, P., Freeman, K. C., Norris, J., & Shectman, S. A. 2001, ApJ, 555, L37
- Fan, X. 1999, AJ, 117, 2528
- Fukugita, M., Ichikawa, T., Gunn, J. E., Doi, M., Shimasaku, K., & Schneider, D. P. 1996, AJ, 111, 1748
- Gallagher, J. S., Madsen, G., Reynolds, R., Grebel, E. K., & Smecker-Hane, T. 2003, ApJ, 588, 326
- Grebel, E. K. 1997, Rev. Mod. Astron., 10, 29
- 1999, in IAU Symp. 192, The Stellar Content of the Local Group, ed. P. Whitelock & R. Cannon (San Francisco: ASP), 17
- 2000, in Star Formation from the Small to the Large Scale, ed.
 - 2001b, Rev. Mod. Astron., 14, 223
- Grebel, E. K., Gallagher, J. S., & Harbeck, D. 2003, AJ, 125, 1926 Grebel, E. K., & Roberts, W. J. 1995, A&AS, 109, 293 Grillmair, C. J., et al. 1998, AJ, 115, 144 Gunn, J. E., et al. 1998, AJ, 116, 3040 Harbeck, D., et al. 2001, AJ, 122, 3092

- Harbeck, D., et al. 2001, AJ, 122, 3092 Harris, W. E. 1996, AJ, 112, 1487 Helmi, A., & Tim de Zeeuw, P. 2000, MNRAS, 319, 657 Helmi, A., & White, S. D. M. 1999, MNRAS, 307, 495 Helmi, A., White, S. D., & Springel, V. 2002, Phys. Rev. D, 66, 63502 Hurley-Keller, D., Mateo, M., & Grebel, E. K. 1999, ApJ, 523, L25 Ibata, R. A., Gilmore, G., & Irwin, M. J. 1994, Nature, 370, 194
- Irwin, M., & Hatzidimitriou, D. 1995, MNRAS, 277, 1354
- Johnston, K. V., Sigurdsson, S., & Hernquist, L. 1999, MNRAS, 302, 771 King, I. R. 1962, AJ, 67, 471
- . 1966, AJ, 71, 64

Program of the Deutsche Forschungsgemeinschaft (DFG, grant KL1358/1) and subsidies from a NASA Astrophysics Theory Program supporting the joint Center for Star Formation Studies at NASA Ames Research Center, University of California, Berkeley, and University of California, Santa Cruz. Funding for the creation and distribution of the SDSS Archive has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Aeronautics and Space Administration, the National Science Foundation, the US Department of Energy, the Japanese Monbukagakusho, and the Max Planck Society. The SDSS Web site is http://www.sdss.org. The SDSS is managed by the Astrophysical Research Consortium (ARC) for the Participating Institutions. The Participating Institutions are the University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, the Johns Hopkins University, Los Alamos National Laboratory, the Max Planck Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, University of Pittsburgh, Princeton University, the United States Naval Observatory, and the University of Washington. This research has made extensive use of NASA's Astrophysics Data System.

REFERENCES

- Klessen, R. S., & Kroupa, P. 1998, ApJ, 498, 143 Klessen, R. S., & Zhao, H. S. 2002, ApJ, 566, 838 Kleyna, J. T., Geller, M., Kenyon, S., Kurtz, M., & Thorstensen J. 1998,
- Kleyna, J. T., Wilkinson, M. I., Evans, N. W., & Gilmore, G. 2001, ApJ, 563, L115

- Kuhn, J. R. 1993, ApJ, 409, L13

- Kuhn, J. R., & Miller, R. H. 1989, ApJ, 341, L41 Kundu, A., et al. 2002, ApJ, 576, L125 Majewski, S. R., Ostheimer, J. C., Kunkel, W. E., & Patterson, R. J. 2000, ÅJ, 120, 2550
- Martínez-Delgado, D., Alonso-García, J., Aparicio, A., & Gómez-Flechoso, M. A. 2001, ApJ, 549, L63 Mateo, M. 1997, in ASP Conf. Ser. 116, ed. M. Arnaboldi, G. S. Da Costa,
- & P. Saha (San Francisco: ASP), 259
- Milgrom, M. 1983, ApJ, 270, 371 ——. 1995, ApJ, 455, 439
- Newberg, H. J., et al. 2002, ApJ, 569, 245
- Odenkirchen, M., et al. 2001a, AJ, 122, 2538

- Collin, M., et al. 2001a, AJ, 122, 2336
 . 2001b, ApJ, 548, L165
 Oh, K. S., Lin, D. N. C., & Aarseth, S. J. 1995, ApJ, 442, 142
 Piatek, S., & Pryor, C. 1995, AJ, 109, 1071
 Piatek, S., Pryor, C., Armandroff, T. E., & Olszewski, E. W. 2002, AJ, 123, 2001 2511

- Richards, G. T., et al. 2001, AJ, 121, 2308 Sanders, R. H., & McGaugh, S. S. 2002, ARA&A, 40, 263 Schlegel, D. J., Finkbeiner, D. P., & Davis, M. 1998, ApJ, 500, 525
- Sérsic, J. L. 1968, Atlas de Galaxias Australes (Cordoba: Obs. Astron.) Shetrone, M. D., Côté, P., & Sargent, W. L. W. 2001, ApJ, 548, 592
- Stetson, P. B., McClure, R. D., & Vandenberg, D. A. 1985, PASP, 97, 908
- Stoughton, C., et al. 2002, AJ, 123, 485
- York, D. G., et al. 2000, AJ, 120, 1579