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On a Reason for the Appearance of Negative Parallaxes in the Determination of the Distances of Stars

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Everyone who has made any considerable number of determinations of stellar parallaxes by trigonometric methods has occasionally found that the usual procedure gave him a negative parallax for the star under investigation. So long as its value does not exceed $-0''.003$ to $-0''.006$ he may reasonably be content, for the parallaxes traditionally assumed for the comparison stars are of about this amount, so that the final parallax, reduced to infinity, is a small positive quantity or zero. Now and then a negative parallax turns up which is two or three times the size of the internal probable error of the determination. Sometimes this value may be modified by making another solution with measures on another set of comparison stars. The mean of the two determinations is usually given or both values may be exhibited in publishing.

Essentially, there is no mystery about negative parallaxes. At least for first order quantities they must occur for one or more of the three following reasons:

First. In case the real parallax of a given star is small, the determined value of the parallax may have an error which by chance falls on the negative side of zero, so that the parallax comes out negative. This case has been exhaustively treated by Sir Frank Dyson in *Monthly Notices 86*, p. 686. In this paper the Astronomer Royal develops the specific formulae, in the theory of probability applicable to the case, and, from the internal mean errors in an extensive series of Greenwich parallaxes of stars having proper motions greater than 20″ per century, he derives corrections to all these observed parallaxes and applies them.

Second. When parallaxes of close visual double stars are sought photographically an occasional negative parallax may be due to uncertainty regarding the relative energy of close stars in building up the composite image under varying conditions of seeing. This is, of course, especially true for double stars which change in position angle and distance during the time interval spanned by the plates.

Third. When a distant star, whether bright or faint, is observed through a scattered cluster, or perhaps rather a layer, of relatively near and faint stars, a negative parallax must be expected. Obviously, trigonometric conditions are just as valid for a distant star with comparison stars that are near to us as they are in the usual inverse situation. A large negative parallax may be just as real as an equal positive parallax. It must merely be recognized that it is the positive parallax of the comparison stars with respect to the distant star which has been gotten. In fact, a certain amount of direct information about the distances of fainter groups of stars, such as are used for comparison purposes, is available in such cases.

A promising means exists of testing whether or not the distribution of ordinary negative parallaxes bears any relation to the parallaxes of the comparison stars.

In Fig. 1, the lower curve is obtained by collecting all the negative parallaxes between declinations $+30°$ and $+60°$ in Schlesinger's *Catalogue* of parallaxes, in which all parallaxes produced up to January, 1924, have received uniform treatment. After having given each its proper weight corresponding to the probable error in the catalogue, averages were taken by three-hour groups in right ascension. The number of stars in each group is given at the bottom of the graph, a total of 78, and no star was omitted. The rather wide belt in declination, 15° on each side of declination +45°, was taken in order to get a representative number of stars.

The upper curve gives, in similar three-hour groups of right ascension, the average parallax of all stars between photographic magnitudes 9.50 and 11.00 in the *+45° Zone of Selected Areas.** Every star in this list within the limits of magnitude specified is included. The total number is 107 stars. The unit at the left is 0".001 of parallax.

There is a considerable range in the magnitudes of comparison stars used by parallax observers. Thus the average for the Yale heliometer series is about 7.8; Leander McCormick reports 9.9 visual for 700 fields; Yerkes 9.9 to 10.3 for various series. Van Maanen uses stars from 11.2 to 13.7 photographic magnitude. On the whole, the limits 9.50 to 11.00 taken above for the stars in the Selected Areas seem most comparable with the great majority of the comparison fields used for ordinary parallaxes. If stars up to 11.50 are included the number is increased to 206, but the only change of consequence is to make the upper curve more smooth and the vertical range somewhat smaller. The positions of maxima and minima, however, remain essentially the same. For reasons given above it did not seem justifiable to include this category of fainter stars. Also, it is obvious from the curves that another mode of taking means by hours of right ascension would make no appreciable difference in the shapes of the curves, provided, of course, the groups be not made too small to give significant averages or so large as to do violence to the argument.

In Fig. 2 the solid curve gives the weighted mean of the negative parallaxes in Schlesinger's *Catalogue* which occur between 0° and +30° of declination. The number of stars entering into each three-hour group in right ascension is printed below the graph. The broken curve in Fig. 2 is similar for the catalogue stars lying between +60° of declination and the north pole. The number of stars in each group is given above the graph. There are too few stars in the latter zone to give the curve much significance.

The solid curves in Figs. 1 and 2 are not similar. Considering the high individual accuracy of the parallaxes entering into them, the means in each cannot be considered to have accidental distribution. Obviously, seasonal errors in astrometry cannot be invoked to account for the distribution of negative parallaxes as shown by these curves, for, with all parallax observing confined to small hour angles, the plates of fields in all declinations are taken in the same season and should be affected by the same seasonal errors if such exist.

Lists of parallaxes published since January, 1924, have been made and averages in three-hour groups, similar to the others, have been taken in two ways. First, the probable errors, as published, were used as a basis for weighting. Then, all parallaxes were given equal weight. While in general there was corroboration of the main thesis of this paper in the resulting curves, it did not seem advisable to include these data with the material from Schlesinger's *Catalogue* until after they have had the same critical treatment.

Whatever reasons of chance may exist for the appearance of negative parallaxes, as they must exist, it seems inevitable to conclude that their distribution of size in right ascension is related to the parallaxes of the comparison stars. While the numbers of stars represented in the groups of the lower curve of Fig. 1 are small, it is probable, *a priori*, that the effects of negative chance errors, discussed under the first case above, are practically eliminated in the averages for this curve just as they probably are in the 107 stars which give the upper curve, where all positive as well as negative parallaxes were used.

But for the small available number of directly determined parallaxes of stars between magnitudes 9.50 and 11.00, one is tempted to say that all negative parallaxes, taken in groups, arising in ordinary parallax work are due to varying but adequate positive parallaxes of comparison stars.

*Zone +45° of Kapteyn's Selected Areas: Parallaxes and Proper Motions of 1041 Stars. Oliver Justin Lee. Part IV, Volume IV, Publications of the Yerkes Observatory, 1926.

It is becoming evident that when we photograph or count fainter and fainter stars we are not only exploring farther into space but we are also finding hordes of intrinsically fainter neighbors nearer home.

The curves were drawn by Miss Beatrice Rieke, secretary and computer at Dearborn Observatory.

Dearborn Observatory
June 9, 1930

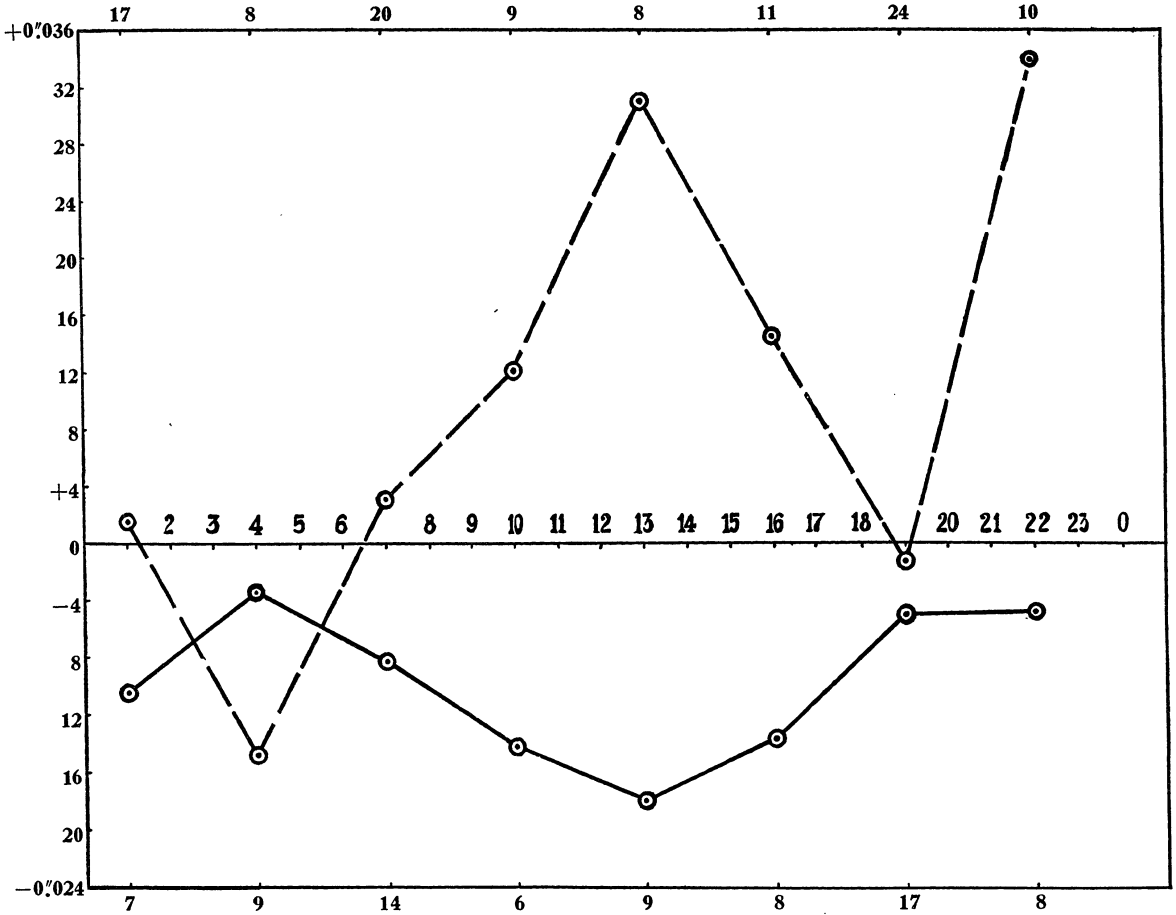

FIG. 1. Solid curve, negative parallaxes between declinations +30° and +60° in Schlesinger's *Catalogue*. Broken curve, parallaxes of all stars between magnitudes 9.50 and 11.00 from Lee's work on *Selected Areas*.

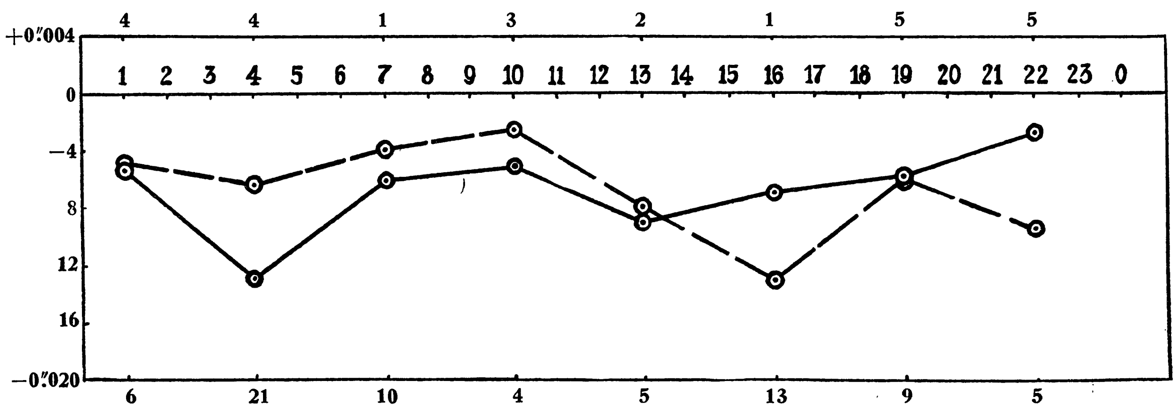

FIG. 2. Negative parallaxes in Schlesinger's *Catalogue:* Solid curve, between declinations 0° and +30°; broken curve, between declinations +60° and +90°.