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Spinning Earth and its Coriolis effect on the circuital light beams: Verification of the special relativity theory

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Abstract. Bilger *et al* (1995), Anderson *et al* (1994) and Michelson–Gale assisted by Pearson (1925) measure/mention Sagnac effect on the circuital light/laser beams on the spinning Earth. But from the consideration of classical electrodynamics, the effect measured/mentioned by those experimenters is the Coriolis effect, not the Sagnac effect. A simple experiment is suggested here that can easily settle the problem.

Keywords. Spinning Earth; circuital light beams; Michelson–Gale experiment; experiment of Bilger *et al*; Coriolis effect; Sagnac effect.

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1. Introduction

In Bilger *et al* [1], Anderson *et al* [2], and in Michelson and Gale assisted by Pearson [3], Sagnac effect on the circuital laser/light beams on the spinning Earth has been studied. The formula for Sagnac effect on the spinning Earth for circuital opposing beams of light first calculated by Silberstein [4] and used in the explanation of the Michelson–Gale experiment was as follows:

$$\text{Fringe shift} = \frac{4\Omega A}{c\lambda}\sin\alpha = \frac{4\Omega A}{c\lambda}\cos(90^\circ - \alpha) = \frac{4\Omega \cdot A}{c\lambda}, \quad (1)$$

where A is the area of the circuit, Ω is the angular velocity of the spinning Earth, α is the latitude of the place where the effect is being measured, λ is the wavelength of light used and c is the speed of light in free space.

In the calculation, Silberstein has assumed a relative spinning motion between ether and Earth at and near its surface and has reached the well-known formula of Sagnac effect for the circuital opposing light beams on the surface of the spinning Earth as given above.

But unfortunately, the same formula arises in the case of electromagnetic fields (originating from a radiating source) at and near the Earth's surface spins (and translates) with Earth and thereby Coriolis force acts on the propagation of light. Therefore, the question arises as to which effect is responsible for the fringe shifts measured/mentioned by those experimenters?

2. Calculation of fringe shift from Coriolis effect

The measure of fringe shift for two opposing light beams travelling in a circuital path on the spinning Earth is given below by considering the Coriolis force acting on the beams as calculated by Hajra [5].

Earth, just like all other physical objects, carries electromagnetic fields along with it at the vicinity of its surface. Therefore, Coriolis force due to the spinning of Earth must act on the propagation of light on the surface of Earth. This will explain the Michelson–Gale experiment assisted by Pearson and the experiment of Bilger *et al* cited in the Introduction.

Let us choose a point O (figure 1) on the surface of Earth with the latitude α° North and construct a tangential plane at this point. Now let us fix a Cartesian coordinate system in the plane such that OY represents the North and OX represents the East. Now suppose that Earth is not spinning, and an element of light beam is arranged to move from a point P in the OY axis at the instant $t = 0$ in a small circular motion in the clockwise direction such that at the time t it touches the point Q

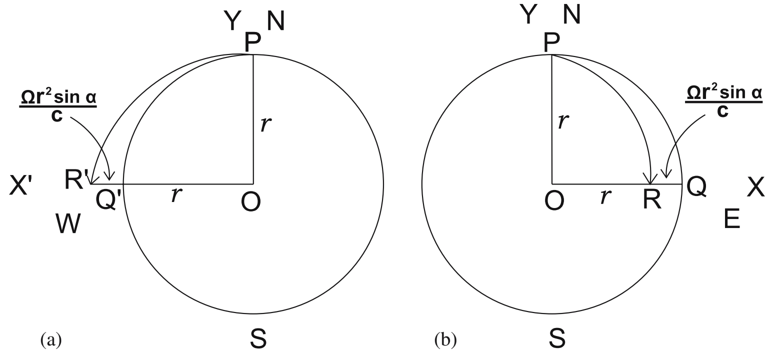

Figure 1. Paths of the opposing circuital light beams on the spinning Earth in the northern hemisphere.

in the OX axis and let OP = OQ = r. That is, when $t = 0$, then $x = 0$, $y = r$ and when $t = t$, then $x = r$, $y = 0$. Now suppose that Earth spins in its usual direction with an angular velocity Ω. Then the Coriolis force due to the spinning of Earth should deflect the beam shown mainly westward and the beam will not touch the point Q. Instead, it will touch a point R in the OX axis drawn towards the centre of the circle. Now for a rough calculation of the distance OR, let us consider the motion of the beam on the OY axis with a velocity c from the point P to the point O. In this case, we may write, for the Coriolis force acting on the beam

$$F_x = -2m\Omega \times v = -2m\Omega c \sin\alpha, \tag{2}$$

where m is the mass of a moving photon (rest mass of a photon is 0, but the mass of a moving photon is E/c^2, E being the energy of the moving photon).

Therefore, from eq. (2), we have Coriolis acceleration

$$a_x = -2\Omega c \sin\alpha, \tag{3}$$

$$\frac{d^2x}{dt^2} = -2\Omega c \sin\alpha, \tag{4}$$

$$\frac{dx}{dt} = -2\Omega c(\sin\alpha)t + C_1, \tag{5}$$

$$x = -\Omega c(\sin\alpha)t^2 + C_1 t + C_2. \tag{6}$$

Remembering the initial condition and taking into account

$$t = \frac{r}{c}, \tag{7}$$

we have

$$x = -\frac{\Omega r^2}{c}\sin\alpha, \tag{8}$$

which is the inflection of the beam towards the centre of the circle. Therefore, in that case, for the clockwise beam, the radius of the circuit (figure 1b),

$$\text{OR} = r - \frac{\Omega r^2}{c}\sin\alpha. \tag{9}$$

For the beam moving in the anticlockwise direction, the radius of the circuit (figure 1a) will be

$$r + \frac{\Omega r^2}{c}\sin\alpha. \tag{10}$$

The path difference between the anticlockwise and the clockwise beams after one complete rotation is

$$2\pi\left(r + \frac{\Omega r^2}{c}\sin\alpha\right) - 2\pi\left(r - \frac{\Omega r^2}{c}\sin\alpha\right) = \frac{4\Omega A}{c}\sin\alpha, \tag{11}$$

where A is the area of the circle.

From the last two equations we have, for one complete rotation, the time lag

$$\Delta t = \frac{4\Omega A}{c^2}\sin\alpha, \tag{12}$$

$$\text{Fringe shift} = \frac{4\Omega A}{c\lambda}\sin\alpha = \frac{4\Omega A}{c\lambda}\cos(90^\circ - \alpha) = \frac{4\Omega \cdot A}{c\lambda}. \tag{13}$$

The above formula is the same as that of Silberstein for Sagnac effect as given in eq. (1).

It is clear from eq. (13) that

(i) In any place on Earth, a laser beam traversing in a ring laser cavity in a plane perpendicular to the equatorial plane of Earth should be very stable, as in that case, no Coriolis force acts on the beam. But, the ring laser beams traversing in the planes perpendicular to the axis of Earth's rotation should be unstable as the Coriolis force acting on the beams should be maximum at these planes.

(ii) Ring laser beams moving in a plane tangential to Earth at the point O (as shown in figure 1) should suffer from the action of Coriolis force.

(iii) Coriolis force acting on the ring laser beams travelling in the tangential plane of the equator is 0.

(iv) Two laser beams travelling in opposite directions along the equator will sense Earth's rotation when they meet at the same point after travelling the circumference of Earth.

(v) The experiments of Bilger *et al* and others [1–3] clearly demonstrate that if a laser beam moves in a circuital path in the tangential plane at any point on Earth in the northern hemisphere/southern hemisphere (but not in the tangential plane at any point in the Equator), it must be affected by the Coriolis force. This implies that if the beam moves in the N–S direction, Coriolis force should act and, therefore, could sense the Earth's rotation.

(vi) Consider a ring laser gyro placed in such a way that the laser beams travelling inside the ring cavity in a plane perpendicular to the equatorial plane of Earth will give a zero output and it could not sense the Earth's rotation.

It has been shown in [6] that limiting angular momentum P of the gyroscopes is responsible for the de Sitter precession and the same momentum along with Coriolis effect is responsible for Lense–Thirring effect as has been measured in the GPB experiment. We may outline in the following section the gist of the arguments for the Conclusion.

3. Orbital (de Sitter) and equatorial (Lense–Thirring) precessions of the gyroscopes in the GPB experiment

It is well known that matter contains charges, and charges being material entities, should be subject to gravitation as well as Coriolis force when in motion. In that case, it could be easily shown from the consideration of classical physics that a material body should have the limiting momentum [6]

$$\mathbf{P} = \gamma^3 m u, \tag{14}$$

where m is the mass of the body measured at rest, u is the velocity of the body in free space and

$$\gamma = 1/k, \quad \text{where } k = (1 - u^2/c^2)^{1/2}$$

(c being the velocity of light in free space).

The gyroscopes in the GPB experiment have two motions, the significant one in the orbital plane and the insignificant second one in the tangential plane.

When a spherical gyroscope moves in a circular polar orbit around Earth, in the orbital plane of the gyroscope, each infinitesimal mass element on the body of the gyroscope, at each instant will tend to move in a direction different from the constrained direction of movement of the centre of the gyroscope due to the new momentum eq. (14). This will cause a torque on the gyroscope. If the angular momentum of the gyroscope changes only in direction, but not in magnitude, as per classical physics, the gyroscope should precess with the angular velocity in the orbital plane as observed from the centre of Earth [6]

$$(\Omega_{\mathrm{pr}})_{\mathrm{o}} = -3u^2 w_{\mathrm{g}}/(2c^2), \tag{15}$$

where w_{g} is the angular velocity of the gyroscope in its orbit. When the gyroscope rotates in the anticlockwise direction, the precession will be in the clockwise direction. In case of free fall of the gyroscope, this result should be read from Gravity Probe B type experiments which is said to have been observed in the experiment.

Apart from the orbital precession, there will be another insignificant classical precession of the gyroscope in the tangential plane because in the tangential plane there is a minute movement of the axis of the gyroscope due to the action of the Coriolis force originating from the spinning of Earth.

This precession, when averaged over the orbit, will be an equatorial precession [6]

$$(\Omega_{\mathrm{pr}})_{\mathrm{Eq}} = -3u^2 w_{\mathrm{E}}/(4c^2), \tag{16}$$

when observed from the centre of Earth, w_{E} being the angular velocity of the spinning Earth.

But the magnitude of Coriolis force must decrease by some factor K due to environmental condition and experimental restriction that the gyroscope faces around the orbit. Therefore, the above formula should be [6]

$$(\Omega_{\mathrm{pr}})_{\mathrm{Eq}} = -K3u^2 w_{\mathrm{E}}/(4c^2). \tag{17}$$

The Gravity Probe-B experiment does in no way contradict this result.

4. Sagnac effect? No, it is Coriolis effect

Michelson *et al*, Anderson *et al* and Bilger *et al* assume that ether is at relative spinning motion with respect to the Earth's surface which could explain their experiments using Sagnac effect. In that case the magnetic field due to a system of charges moving with a velocity u in the Earth's equator will be markedly different from the magnetic field of the same system of similarly moving charges at other latitudes of this planet.

As for an example, the magnetic field due to a system of charges slowly moving at a point on the Earth's equator with a velocity u with respect to Earth's surface eastward would be almost double the magnetic field of the same system of charges moving similarly at 60° latitude when $w \gg u$, w being the spinning velocity of that point of Earth with respect to ether. This is impossible and so is the assumption of Silberstein, Michelson *et al*, Anderson *et al* and Bilger *et al*.

The only alternative is: Coriolis effect (not the Sagnac effect) is responsible for the non-null result of the Michelson–Gale experiment assisted by Pearson and the experiment of Bilger *et al*. In that case the magnetic field due to the same relative motion of the charges with respect to the surface of Earth will cause the same magnetic field on any place on the surface of the spinning Earth which we always find. The effect of Coriolis force on moving charges is so small that it will not appreciably affect the magnetic field due to a system of charges moving with respect to the surface of Earth.

Electromagnetic fields carry momentum and energy which can be experienced with our sense organs. Therefore, electromagneticians have concluded that those fields are real physical entities (objects) to the same extent as charged bodies are real physical entities [7]. Electromagnetic fields are independent entities in their own right, every bit as 'real' as atoms or baseballs [8]. All physical objects are subject to gravitation; and at the vicinity of the Earth's surface they are carried with Earth. Therefore, electromagnetic fields should similarly be subject to gravitation and at the vicinity of the Earth's surface these fields should similarly be carried with this planet [6]. In that case, Coriolis force should act on the electromagnetic bodies and light beams moving near the vicinity of the Earth's surface, just like this force acts on the material bodies moving at its near vicinity, which has been pointed out earlier.

Let us now design an experiment to settle whether electromagnetic field at the vicinity of the Earth's surface is carried with Earth (and if carried, we could conclude that the effect is the Coriolis effect). The suggested experiment may also simultaneously demonstrate whether relativity principle of special relativity is correct or not.

A cylindrical box of a few meters of height is taken. A plane circular mirror (M_1) with the arrangement of tilting is placed at the centre of the topside of the box. Let the centre of this mirror be marked with O_1. A second plane circular mirror (M_2) with semitransparent metal coating is placed at the centre of the bottom side of the box. A circular photographic plate with a small hole at the centre is inserted beneath this bottom side mirror in such a way that the hole of the photographic plate is fixed at the centre O_2 of the mirror (figure 2a).

The first observation

Let the box be placed on board of an aeroplane at rest on the north pole of Earth where there is no Coriolis effect on light beams propagating vertically.

Now let a pencil laser beam at right angle to the mirror plane be passed through the minute central hole of the photographic plate to touch the point O_1 of the topside mirror and be reflected from that point. The beam must come back to the hole O_2 of the bottom side mirror (figure 2a).

The second observation

Now, suppose that the aeroplane flies up and moves eastward with a velocity u with respect to the surface of Earth. As per the relativity principle of special relativity, the observer in the moving aeroplane should find the phenomenon similar to as before, i.e., the observer on the steadily moving aeroplane must see that the light beam has come to the hole O_2.

But as per the present analysis, Earth carries electromagnetic fields along with it, but the aeroplane cannot carry the fields. Therefore, the observer on the steadily moving aeroplane finds that he is moving past the vibrating fields. In that case, he will see that the

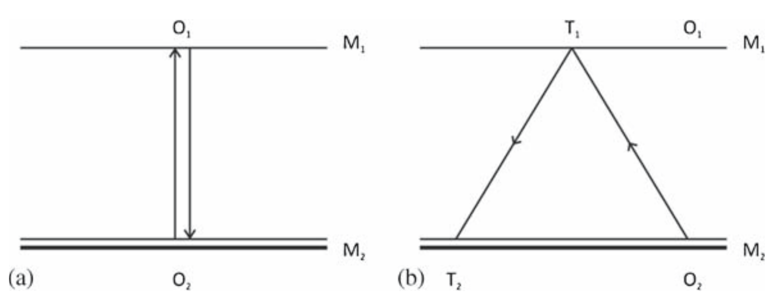

Figure 2. (**a**) The paths of the overhead and the reflected light beams in an aeroplane stationary in the north pole and (**b**) those when the aeroplane is steadily flying eastward in the north pole.

beam will not touch the point O_1 of the top side mirror. Instead, he will find that the beam touches a point T_1 adjacent to O_1 and westward to it such that $O_1T_1 = lu/c$. After reflection, it will not come back to the hole O_2. Instead, it will touch a point T_2 of the bottom side mirror adjacent to O_2 (of the bottom side mirror) and westward to it such that $O_2T_2 = 2lu/c$ and affect the photographic plate beneath T_2 where l is the distance between the centres of the topside and downside mirrors and u and c are velocities of the aeroplane and the light beam on the surface of Earth (figure 2b).

The third observation

The aeroplane now moves eastward with a velocity v. As per relativity principle of special relativity, the observer will find that the beam, after reflection, has come to the hole O_2 just like earlier. But as per the present analysis, the observer will find that the beam, after reflection, has come to a point T_4 in the bottom side mirror adjacent to O_2 and westward to it such that $O_2T_4 = 2lv/c$ and affect the photographic plate beneath T_4.

5. Measurement and conclusion

(a) If the relativity principle is correct and the experiment is carried out ideally, there will be a minute blackening of the photographic plate centring the point O_2.

(b) If our present analysis is correct, there will be blackening of the photographic plate from the point T_2 (of the bottom side mirror) westward in the second observation and there will be blackening of the photographic plate from the point T_4 (of the bottom side mirror) westward in the third observation. All the distances viz., O_2T_2 and O_2T_4 could be measured and shown to be $2lu/c$ and $2lv/c$ respectively.

The experiment could be done in any high latitude places.

The experiment will settle whether the relativity principle is correct or the electric and the magnetic fields are carried with Earth from which we could easily decide as to which effect is responsible for the non-null result of the Michelson–Gale assisted by Pearson type experiment.

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