Synchronisation of Clock-Stations and the Sagnac Effect

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It is shown that the Sagnac correction, as applied to time comparisons upon the Earth, does not derive from the normal Relativistic corrections. It is proposed that the reason given for the application of the Sagnac correction, and the circumstances appropriate to its application, require amendment.
Key words:: Clock synchronisation; Sagnac effect; Relativistic corrections.

Standards for the synchronisation of clock-stations upon the Earth are to be found in the 1990 publication of the CCIR (International Radio Consultative Committee: International Telecommunication Union) [1]. Similar rules are in the 1980 publication of the CCDS (*Comité Consultatif Pour la Définition de la Seconde: Bureau International des Poids et Mesures*) [2]. Two methods are used to synchronise clocks at different clock stations. The first method is physically to transport a clock from one site to the other, and thereby to compare the times recorded at the two clock stations. The second method is to send an electromagnetic signal, from one site to the other.

Three corrections to be applied, as listed in the above publications, are as follows:-

(a) to take account of the Special Relativistic velocity effect, caused by carrying a portable clock at speed aboard an aeroplane, from one site to the other.

(b) under General Relativity, to allow for height above sea level.

(c) a correction described as being for the *rotation of the earth*.

Correction (a) is quantified as $v^2/2c^2$. This is the slowing of time as calculated under the Special Theory of Relativity. A clock transported from one site to another will have such a correction applied, because of the ground speed v of the aeroplane; c is the velocity of light. Correction (b) is quantified as $g(\phi)h/c^2$ where g is the total acceleration at sea level (gravitational cum centrifugal) at a latitude of ϕ, and h is the height over sea level.Correction (c) is

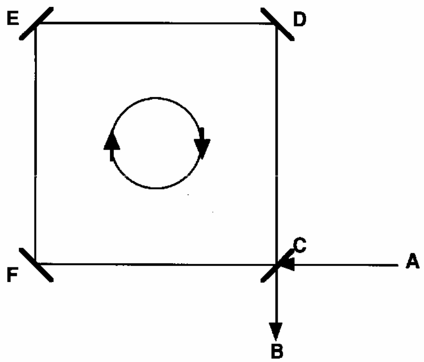

Figure 1 - Sagnac Test

quantified as $2A_E\,\omega/c^2$, where A_E is the equatorial projection of the area enclosed by the path of travel of the clock being transported from one site to another (or of the electromagnetic signal) and the lines connecting the two clock-sites to the centre of the Earth; ω is the angular velocity of the Earth. As the area AE is swept, it is taken as positive when the projection of the path of the clock (or signal), on to the equatorial plane, is Eastward.

Both reports include all three terms under the umbrella description of being *"of the first order of general relativity."* The first two corrections are clearly the result of the Special Theory and the General Theory of Relativity respectively. But, what is the third? This paper examines the precise meaning and derivation of the third correction.

To understand the meaning of the third term, we must study the Sagnac effect. Sagnac (1914) showed that light took different times to traverse a path, in opposite directions, upon a spinning disc [3]. Figure 1 shows a schematic representation of the test that was done by Sagnac. A source at A sends light to a half-silvered mirror at C. Some of the light goes from C to D, E, F and C and is reflected to a photographic plate at B. Some of the light goes the other way around. The whole apparatus (including A and B), can turn with an angular velocity of ω.When the apparatus is set spinning, a fringe shift occurs at an interferometer, indicating a difference (d*t*) in the time taken by the light to traverse the path in opposite directions. For this difference in time Sagnac derived the formula

$$\delta t = \frac{4A\omega}{c^2} \tag{1}$$

where A is the area enclosed by the path of the light signals, and ω is the angular velocity of spin in R/s.

Sagnac also showed that the centre of rotation can be away from the geometric centre of the apparatus, without affecting the results, and that the shape of the circuit was immaterial. He also proved that the tilting of the mirrors, as they spin, caused an insignificant alteration in the overall effect.

In order to derive the Sagnac equation, consider the theoretical circular model shown in Figure 2. Light is emitted at S; a portion of the signal goes clockwise (denoted by the inner line), and some goes anti clockwise, around a circular disc of radius *r*. The light source at S and the photographic recorder, also situated at S, rotate with the disc. The disc is rotating with an angular velocity ω in a clockwise direction. The anti clockwise beam is going against the rotation of the equipment, and will return to Point S when it has moved to S′. The second beam, travelling clockwise, will return when S has moved to S.″ As viewed by an observer on the spinning platform, the light signals return to the same point, but at different times.

Taking t_o as the time observed when the disc is stationary, *i.e.* the path length divided by the speed of light

$$t_o = \frac{2\pi r}{c} \tag{2}$$

Let $\delta s'$ be the distance SS′ and $\delta s''$ be the distance SS.″ Let t' be the time for the light to go from S to S′ in the anti clockwise direction.

$$t' = \frac{2\pi r - \delta s'}{c} \tag{3}$$

But, t' is also the time taken for the disc to move the distance $\delta s'$ in the clockwise direction. Therefore $t' = \delta s'/v$, and $\delta s' = t'v$; $\delta s' = (2\pi r - \delta s')v/c$; $\delta s'/v = 2\pi r/(c+v)$, and

$$t' = \frac{2\pi r}{(c+v)} \tag{4}$$

Similar calculations give the time (t'') for the light to go from S to S″ in a clockwise direction,

$$t'' = \frac{2\pi r}{(c-v)} \tag{5}$$

Subtracting equation (4) from (5), the difference (δt) between the times for the light to go clockwise (t'') and anti clockwise (t') is

$$\delta t = 2\pi r\left[\frac{1}{(c-v)} - \frac{1}{(c+v)}\right] = \frac{4\pi r v}{(c^2 - v^2)} \tag{6}$$

This is the same as equation (1), because v^2 is negligible.

From the point of view of the observer in the fixed laboratory the disc moves a distance $\delta s'$ while the light completes a distance of $2\pi r - \delta s'$ around in the other direction from S to S′. Equation (3) describes the time interval, as it would be discerned by the observer in the laboratory. From the point of view of the moving observer, upon the spinning disc, the light has, relative to that observer, completed one revolution of the disc ($2\pi r$) at velocities of $c \pm v$ in the two opposing directions. Equations (4) and (5) describe this.

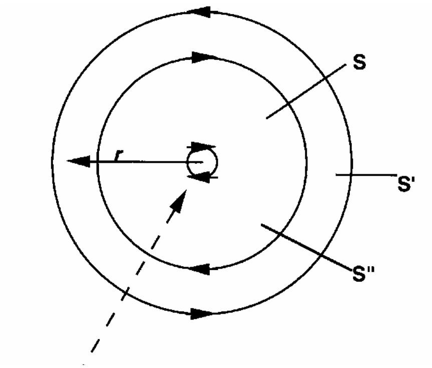

Figure 2 - Circular Sagnac Test: Whole apparatus turning at ω clockwise

In the above calculation, the light is assumed to travel at a constant velocity of c in relation to the fixed laboratory. But, the fringe shift measured solely aboard the spinning disc, and which is a record of the time difference for the light beams to complete a circuit in opposing directions, corresponds exactly to the time difference in equation (1). How can this be? The only possible explanation is that the time in the fixed laboratory, and that upon the spinning disc are precisely the same. This fact is at the core of the postulates being put forward in this paper.

The Sagnac effect shows that the velocity of the light is not affected by the movement of the source of that light (Point S); this accords with Special Relativity theory. It also shows that the light travels at the velocity c solely relative to the laboratory. Assuming that the light travels at the velocity c, relative to the laboratory, gives the correct result. The light does not adapt to the movement of the disc.

To get a fringe shift of one fringe, the velocity of Point S in Figure 2, relative to the laboratory, has to be about 13 m/s per meter of radius. This is a very low velocity. Fringe shift is got from time difference by multiplying by c/λ. Where, for example, $\lambda = 5500 \times 10^{-10}$ m, this gives $v = 13$ m/s, per meter of radius, from $1 = (4A\omega)/(c\lambda) = (4\pi rv)/(3 \times 108 \times 5{,}500 \times 10^{-10})$. In equation (4), as v approaches c, t' becomes $t_0/2$, and the speed relative to the observer is now 2c. In equation (5), as the speed v approaches c, t'' becomes infinite, because the light and the Point S are travelling in the same direction, and the time for the light signal to gain one complete circuit on the Point S is infinite; the speed of the light, relative to the observer, becomes zero.

Dufour & Prunier (1942) repeated the Sagnac test, and got the same result [4]. They then did a variation of that test. A practical example of a case where the signal is not solely in the plane of the disc is their test, in which the path of the light was partly on the spinning disc, and partly in the fixed laboratory. The light signal was introduced (Figure 3) from C out

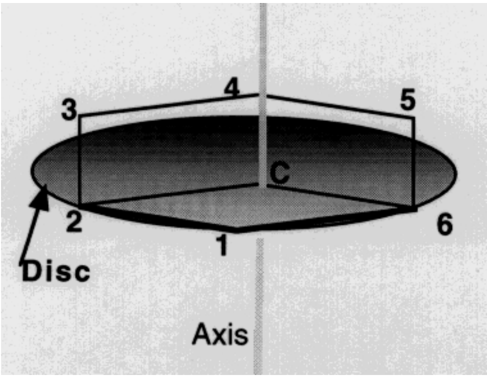

Figure 3 - Dufour & Prunier Test

to Point 1, and sent from there in opposite directions.

As shown schematically, the light went firstly on a path on the spinning disc (Point 1 to Point 2), then went vertically up to a mirror fixed to the laboratory overhead the disc (Point 3). It then traversed linear paths 3 to 4 to 5 in the fixed laboratory, and came vertically back down to the disc at Point 6, whereupon it finished the trajectory on the disc back to the starting point at Point 1. The reverse beam went the other way. The plane of the path, of the portion that was fixed in the laboratory, was parallel to the plane of the disc. Lines 3-4 and 4-5 are directly overhead 2-C and C-6. The two short connections, 2-3 and 5-6 (shown exaggerated here for clarity) were 10 cm each. The mirrors at 2 & 6 rotated with the disc. The fringe shifts were the same as in their repeat of a test with the light path solely upon the spinning disc (on the circuit 1-2-C-6-1). This test by Dufour & Prunier confirms that the light does not adapt to the movement of the disc, and that it is travelling relative to the fixed laboratory.

A young German student Harress (1911) had done a test on the refraction of light [5]. This test was later shown by von Laue (1920) to have produced the Sagnac effect, but Harress was not aware of this [6]. Harress had both the photographic equipment and the light source fixed in the laboratory, whereas Sagnac had both on the spinning disc. This shows that the photographic record of the fringe shift and/or the origin of the light may be made on or off the disc, without affecting the result; this is because it is the behaviour of the light relative to the spinning disc that is being measured. Dufour & Prunier also did tests with the light source fixed in the laboratory and with the photographic plate fixed in the laboratory; the results were the same as in a traditional Sagnac test. The fringe shift occurs, whether there is any observer (camera) present on the disc, or in the fixed laboratory. There is a slight Doppler effect in the case where the photographic equipment is in the fixed laboratory, because the disc is moving past the viewing lens. Post (1967) discusses the magnitude of the distortion introduced, and correctly dismisses the effect as too small to have any observable effect, being *"v/c times smaller than the effect one wants to observe."* [7].

Michelson & Gale (1925) showed that electromagnetic signals sent around the Earth did not travel at the same speed in the East-West direction [8]. They constructed a large rectangular piping system fixed to the Earth, and sent light signals in opposite directions around the circuit. The signals did not arrive back at the same time, as evidenced by the resulting fringe shift. That test was a Sagnac test on a disc of radius equal to that of the Earth at the Latitude concerned, and rotating at the angular velocity of the Earth. The results were within 3% of the forecast and were also in the correct direction (signal retarded in the direction of the spin of the Earth). Tests by Bilger *et al.* (1995) using a ring-laser, confirmed the Sagnac effect to better than one part in 10^{20}. This was a Michelson & Gale type test with the ring laser fixed to the Earth; the retardation of the signal was also in the direction of the spin of the

Earth (as this was done in the Southern hemisphere, the retardation was in the opposite sense to the Michelson & Gale test)[9].

Saburi *et al.* (1976) transported a clock from Washington (USA) to Tokyo (Japan), and compared the difference in the time displayed by the two clocks on the arrival of the transported clock, with the time relayed from one station to the other, *via* an electromagnetic signal.[10] The two sites were almost at the same latitude. They calculated from the Sagnac effect that there should be a difference of +0.333 ms (Japan ahead of Washington, DC, because of the direction of rotation of the Earth). The Sagnac correction, on its own, applied solely to the electromagnetic signal (and not to the time displayed by the clock that was physically transported from one site to the other), bridged the gap to a very close agreement with the test results (to –0.02 μs). The Relativistic effects applied solely to the portable clock, which was physically transported from one site to the other, amounted to +0.08 μs. The uncertainty of the reading being recorded by the portable clock was ±0.2 μs. This test could nowadays be repeated to greater accuracy.

Special Relativity has no role in trying to explain the Sagnac effect. Post (1967) states that the Sagnac effect and the Special Relativity effect are of very different orders of magnitude. He says that the alteration to be applied to the Sagnac effect under Special Relativity is a v^2/c^2 effect which is "*indistinguishable with presently available equipment*" and "*is still one order smaller than the Doppler correction, which occurs when observing fringe shifts*." Post derives the Sagnac formula as given above in equation (1) and then applies the Special Relativity γ factor to that formula; in this he distinguishes clearly between the two. Post says that "*for all practical purposes we may accept as adequate for the time interval in the stationary as well as in the rotating frame, the formula*" as in equation (1). This confirms that the difference in the time recorded in a Sagnac test is the same in the laboratory and upon the spinning disc. Post also says that "*the time interval between the consecutive positions of the beam splitter is observed in the stationary frame and is therefore dilated by a factor γ* ." Here again Post distinguishes between the Sagnac effect and the Relativistic time dilation.

The basis of timekeeping by the CCIR is time at the non-rotating centre of the Earth. It defines that the "*TAI is a coordinate time scale defined at a geocentric datum line*." The unit of time is defined as "*one SI second as obtained on the geoid in rotation*." The time scale and the unit of time are not measured at the same place; the unit of time is based upon the spinning Earth, which has motion in relation to the geocentre where the time scale is measured. The CCIR report recommends that "*for terrestrial use a topocentric frame be chosen*." It continues "*when a clock B is synchronised with a clock A (both clocks being stationary on the Earth) by a radio signal travelling from A to B, these two clocks differ in coordinate time by*" the Sagnac effect. These statements make it clear that the time upon the rotating Earth is viewed as differing from that at the geocentre. This assumption is in contradiction of the analysis in this paper, and of the conclusions of Post [7].

The CCIR report states that "*the time of a clock carried eastward around the earth at infinitely low speed at h = 0 at the equator will differ from a clock remaining at rest by –207.4 ns*." That amount is the Sagnac one-way effect. The significance of the $h = 0$ is that there would be no effect under the General Theory of Relativity. The infinitely low speed eliminates any effect from the Theory of Special Relativity. The CCIR report here assumes that when a clock is physically transported around the globe, a Sagnac-type correction has to be applied. Because the area is taken as "*positive if the path is traversed in a clockwise sense as viewed from the South Pole*," a clock transported around the Earth in a Westward direction would gain time by +207.4 ns, relative to the stationary clock. Consider two clocks that are sent, in opposite directions, around the globe at the equator at the same time; when they have completed one revolution each, there would be a supposed time difference of 414.8 ns between them, and they would each differ from a clock that remained at the starting place by 207.4 ns. They have had no effect from Special Relativity (velocity infinitely slow) or from General Relativity (at sea level). We then would have the strange situation where we have three clocks at the same spot on the Earth recording different times; we could repeat the circumnavigation as often as we wish and get clocks, at the same spot, which have had zero corrections under normal Relativity theory, recording times which are different from each other by larger and larger amounts. All times here are coordinate times as earlier defined.

Both the CCIR and CCDS reports make it clear that considering time upon the Earth, from the point of view of "a geocentric non-rotating local inertial frame," requires no Sagnac correction. But, when considering time upon the rotating Earth, they apply a Sagnac correction.. Langevin (1937) proposed that, to explain the Sagnac effect, one had to assume that either (a) the velocity of the signal was $c \pm v$ in the two directions or, (b) the time aboard the spinning disc was altered by $2A\omega/c^2$ [11]. The CCIR and CCDS reports assume that (b) is true. As we saw above, it is (a) that is the correct explanation.

Special Relativistic time dilation does nor contribute very much towards the Sagnac effect. Taking an example, where the surface velocity of the Earth at a particular latitude is $v = 300$ m/s and a portable clock is transported at, say, $x = 10$ m/s (the CCIR defines the transportation as"slowly"). In this case the difference between the $v^2/2c^2$ and $(v+x)^2/2c^2$, which is $(2vx+x^2)/2c^2$, gives a difference of 4×10^{-14} s/s. An electromagnetic signal circumnavigates the Earth in about 0.1 s. The Sagnac one-way difference $2A\omega/c^2$ for a light signal to circumnavigate the Earth is about 2×10^{-7}s/s, as calculated in the CCIR Report. The ratio of the two is thus 10^7. Thus, the two effects are not at all of the same magnitude. This agrees with the analysis by Post [7]. Another basic difference between the Relativistic and Sagnac effects, as calculated for movements measured upon the spinning Earth, is that the former is non-directional, whereas the latter is ± depending upon the direction of sending the signal West or East respectively, and zero in a North-South direction.

In the CCIR analysis, the starting point is time at the "*local non-rotating geocentric reference frame*." This is done "*to account for relativistic effects in a self-consistent manner*." If we assume that the speed of light upon the rotating Earth must be the constant value c, then perforce we must vary the time upon the Earth, by the Sagnac formula, as compared with time measured from the geocentric reference frame. Special Relativity theory is designed specifically to alter the time upon the moving object in direct accord with the requirement that the speed of light must be the constant value c.

The application of the γ factor correction, under Special Relativity, to time on the spinning Earth, as compared to time at the centre of the Earth, ensures that

a) the speed of light is c as calculated in all directions upon the spinning Earth
b) time upon the Earth is consequently calculated to vary by precisely the amount necessary to agree with the constant value for the speed of light.

If this were not so, the velocity of the electromagnetic signals upon the Earth would remain unchanged as $c \pm v$ in the opposing directions. This method arrives at a solution that conforms with Relativity theory. Is this method justifiable?

It is convenient to start with time at a geocentric datum line. This conforms with the fact that electromagnetic signals do not adapt to the spin of the Earth; this datum corresponds to the 'laboratory' in a Sagnac bench test. The speed of the signals can confidently be taken as c as measured in that frame of reference. The orbital movement of the Earth around the Sun. and its other movements in the Universe, can be ignored, and assumed to have no effect upon the results being calculated.

Allan *et al.*, compare the Sagnac correction as applied to (a) slowly moving portable clocks upon the Earth and (b) electromagnetic signals, used for clock synchronisation [12]. They state that "*the Sagnac effect has the same form and magnitude whether slowly moving portable clocks or electromagnetic signals are used to complete the circuit*." They say that the Sagnac correction applies in both cases, and that it has the same magnitude. In case (a) they define the Sagnac effect as "*being due to a difference between the second-order Doppler shift (time dilation) of the portable clock and that of the master clock whose motion is due to the Earth*'s *motion*" as "*viewed from a local nonrotating geocentric frame*." Petit & Wolf also state that the correction $2A\omega/c^2$ is applied equally "*if the two clocks are compared by using portable clocks or electromagnetic signals in the rotating frame of the Earth*." If we take the time measured at the geocentric datum line as t_o, and the time upon the spinning Earth as t', Special Relativity Theory requires that $t_o = t'\gamma$. Applying a correction of $v^2/2c^2$ to the time taken for two clocks, which move at speeds of v relative to the ground, to circumnavigate the Earth in opposing directions, as viewed from the geocentre, gives the following result.

The moving clocks have the speeds of $\omega r + v$ and $\omega r - v$ in the opposing directions, relative to the geocentric time frame; r is the radius of the Earth, and ω its angular velocity. The time dilations of the two clocks are $\int \frac{1}{2}\left[(\omega r + v)/c\right]^2 \mathrm{d}t$, and $\int \frac{1}{2}\left[(\omega r - v)/c\right]^2 \mathrm{d}t$ respectively. The difference between these two time dilations is therefore $\int \left[(2\omega r v)/c^2\right] \mathrm{d}t$.

When the two clocks have gone right around the equator (a distance of $2\pi r$) the $\int c\,\mathrm{d}t = 2\pi r$, and the difference between the time dilations is $(4\pi r 2\omega)/(c^2)$, which is the same as equation (1) (Burt, 1973) [13]. The result is independent of v, so the speed of transportation of the clocks will not affect this result. A similar analysis using electromagnetic signals to circumnavigate the globe Eastward and Westward (that is substituting c for v in the above equations) also gives the same result. In this way this analysis gives the Sagnac formula as a supposed correction for the difference in the time taken by two electromagnetic signals sent in opposing directions around the globe, or for the time correction to be applied to clocks that are physically transported around the globe in an East-West direction. This is the correction published in the CCIR and CCDS reports. It also shows that the application of the γ factor to time as measured upon a moving object agrees with the speed of light being measured upon that object as c in all directions. All of this scheme is consistent.

There is one problem. The Sagnac tests, done with ever increased accuracy down the years, show a difference in the time taken by electromagnetic signals to circumnavigate any spinning disc (including a cross-section of the Earth) and consequently a difference in the speed of the signal in the opposing directions. No difference in time for activities upon the spinning disc is required, when viewed from the stationary laboratory. This difference in the speed of the signal contradicts a basic assumption of the scheme of synchronisation that is used.

It is assumed by the CCIR that the time upon the spinning Earth is altered by the γ factor of Special Relativity in all calculations carried out on time durations upon the Earth, from the viewpoint of the geocentric non-rotating system. If no difference in time was measured in a Sagnac test, then the speed of the signal would have been measured as c in the opposing directions, upon the spinning disc. It can be argued that the rotating disc is not an Inertial Frame, and that therefore the matter is not relevant. As larger and larger discs are considered, we approach the situation where the movement is tantamount to that in a straight line at constant velocity. In this case the matter applies to an Inertial frame. There does not seem to be any plausible solution which shows the fringe shift measured aboard the spinning disc to be caused by other than a difference in the speed of light relative to that disc.

The application of the CCIR correction to time upon the spinning Earth gives a correct answer, whenever electromagnetic signals are used to compare the time being recorded at two clock stations upon the Earth. However, where the physical transportation of a clock around the globe is concerned, it introduces an error.. The CCIR report works out an example where the three specified corrections are applied to the frequency of a clock that is physically transported from one site to another. However, as discussed earlier, Saburi *et al.* showed that the physical transportation of a clock does not require the application of any Sagnac correction to the time being recorded upon that travelling clock [10]. They also confirmed that it is the electromagnetic signal speed Eastward and Westward that varies, and that requires a Sagnac correction to its speed of transmission.

Allan *et al.* (1985) did a Sagnac-type test between standard time-keeping stations in USA, Germany and Japan [12]. These tests confirm the Sagnac effect, as applied to electromagnetic signals, sent right around the Earth in opposing directions, to an accuracy of 1% over a period of 3 months. There were no further corrections made to the results (got by sending electromagnetic signals between the clock-stations) on the basis of Special Relativity or General Relativity; in this case, where electromagnetic signals are used to synchronise the clock stations, no measurable effect under Special Relativity or General Relativity are to be expected. Saburi *et al.* state that "*in a comparison experiment via a satellite, it is considered that the effect of gravitational potential on the light path is small and cancelled out by the two-way method, and that other relativistic effects are negligibly small.*"

In the CCIR report, the corrections to be applied are listed as three *viz.* "*the corrections for difference in gravitational potential and velocity and for the rotation of the Earth.*" In describing these corrections the report names them as "*corrections of the first order of general relativity.*" We now see that the third one is the Sagnac effect ($2A\omega/c^2$). By naming the Sagnac correction as a separate item from the other two factors, the CCIR report tacitly accepts that it is not a Special Relativity or a General Relativity effect. This paper shows that no such Sagnac correction should be applied to the case where a clock is physically transported from one site to another. However, in all cases of synchronising clocks by electromagnetic signal comparison, the Sagnac correction is properly quantified in the CCIR report, and thus the timekeeping authorities are applying it correctly, even if they assume that it is derived from the Theory of General Relativity. The Sagnac correction is nowadays automatically applied to all electromagnetic signals used in the synchronisation of clock stations. The CCIR report gives an incorrect value for the angular velocity of the Earth (7.992 R/s instead of 7.292 R/s); this error was not carried forward into the calculations given in examples in the report.

Winkler (1991), in a paper on the subject of the synchronisation of clocks around the world, ascribed the Sagnac effect to the General Theory of Relativity [14]. He explained the effect by saying that "*accelerations have an effect on timekeeping and on the propagation of light.*" He also stated that "*on a* *rotating system, the velocity of light must be added to (or subtracted from) the speed due to rotation, an effect that produces a time difference for two rays that travel in opposite directions around a closed path."* Here he has accepted that the velocity of the signal is different in the opposing directions, and that the signals take different times to complete the circuit, relative to an observer upon the rotating Earth.

Other publications also purport to show that the Sagnac effect is part of the General Relativity Theory. An example is the paper by Petit & Wolf (1994), which begins by assuming that the light travels relative to the stationary frame (in their case the "*geocentric 'non-rotating frame'*") [15]. They assume that the light velocity relative to the spinning object is not c. They take it as "*$c + s$ where s represents the time taken for the signal to travel the extra path due to the motion of b in the non-rotating frame during transmission*": "*b*" is the clock moving on a rotating disc. This is the same as the analysis of the Sagnac effect, given earlier in this paper, where the extra distance travelled by the Point S in Figure 2, while the signal is travelling around the circuit, yields a speed of the light of $c - v$ in one direction. But, they then assume that the time aboard the spinning Earth alters by the equivalent of the Sagnac effect; this is, as seen above, not sustainable.

Two clocks upon the Earth at the same Latitude have no relative motion in respect to each other, as considered in a geocentric Earth-fixed system. It is only when we attempt to compare the time being kept by the two clocks that we have to employ either an electromagnetic signal or a physical transportation of a comparison clock. The time keeping of those two clocks does not alter because of the measuring process. The Sagnac correction has to be applied to the time taken by the electromagnetic signal to get from one clock site to the other. No corrections apply to the time being kept by the clocks in relation to each other. By shifting the time base to the geocentre, the CCIR introduce a supposed Sagnac effect alteration to the time difference measured between the two clocks when transporting a portable clock or sending an electromagnetic signal between the two sites.

There are various reasons that can be advanced to answer the apparent contradictions between Relativistic theory and the Sagnac effect. One could say that it is correct to state that the Sagnac effect is not relativistic; but it comes out naturally if one writes the equations of time transfer, from the geocentric frame to the spinning Earth, in the context of general relativity, with some very small additional terms that are genuinely relativistic. It can be claimed that Newtonian Mechanics are not relativistic, but that General Relativity includes all terms of Newtonian theories of motion plus additional corrections. So we could claim that it is not wrong to say that the Sagnac effect is also relativistic in the sense that it also appears in the solution in a general relativity theory. Such an argument would agree that the Sagnac effect is a first order effect that cannot have any explanation purely by Relativistic theory.

It could be debated that we have to (a) adopt a relativistic model, because the classical treatment leads to contradictions with experiment, and (b) have a convention for the meaning of clock comparison. As a model, we use Einstein's General Relativity because this theory is the simplest which, up to now, agrees with all observed facts. The convention for clock comparison is based on the convention of coordinate simultaneity; the readings of the clocks take place at the same value of some specified coordinate time (geocentric in metrology on the Earth). The question, it could be claimed, is not to distinguish in the theory of clock comparison some classical terms, some terms due to Special Relativity, and some gravitational terms. General Relativity, it can be said, is a self-contained theory and provides all the terms we need, as a consequence of its basic postulates. The separation of the various terms is a consequence of the choice of coordinates we have made and of the low level of approximation which is accepted.

General Relativity theory is required to make corrections to the time keeping of the clocks. It includes the corrections for height over sea level, and also the corrections under Special Relativity (velocity effects). The setting of the atomic clocks that are to be placed aboard a satellite are made, in advance of launching the satellite, to allow for both of those corrections. These corrections anticipate the increased reading that will emerge as a result of height over sea level, and also the decreased reading that will emerge because of the higher velocity of the satellite as compared with the velocity of the surface of the Earth. The clocks are set before launch, and will then be correct in keeping time the same as upon the surface of the Earth, when they are in orbit. These alterations are appreciable, and are a precise confirmation of these two corrections. Without making these corrections, the clock on the satellite would not keep an unaltered time, as compared with a clock upon the surface of the Earth.

However, there is another correction to be made and that is the Sagnac correction, whenever one has to compare the time upon such a satellite with the time being recorded by a clock on another satellite or upon the Earth. It is this quite separate correction that is the dichotomic problem being addressed here.

Some publications try to avoid the problem of the Sagnac effect by declaring that the Theory of Special Relativity is not applicable to a rotating Frame of Reference. But, some precise explanation of the effect is required. It is not sufficient to say that the Sagnac effect is not explained by Special Relativity theory, and to leave the matter at that. Einstein (1905) seems to have accepted, in his first paper on Relativity Theory, that movement on a circular path had the same result as movement in a straight line, when considering the question of measurement of distance or time.[16]. Having derived his formula for straight line movements, he said *"it is at once apparent that this result still holds good if the clock moves from A to B in any polygonal line"* and *"if we assume that the result proved for a polygonal line is also valid for a continuously curved line, we arrive at this result: If one of two synchronous clocks at*

A is moved in a closed curve with constant velocity until it returns to A, the journey lasting t seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be ½tv^2/c^2 second slow." An observer riding upon the moving clock B will not be measuring time in an Inertial Frame with respect to clock A, but in a Rotating Frame of Reference. The argument that Special Relativity Theory is not applicable to movement in a circuit, such as that of circumnavigation of the Earth, is thus open to different interpretations. Even though the effect Einstein described is much smaller than the Sagnac effect (as shown above), it is the application, from a straight path to a curved path, that is of interest here.

The Sagnac corrections applied by the CCIR and the CCDS is not a Relativistic correction. It is not a continuing correction, such as are the Relativistic corrections. It is necessary when comparing the time being recorded at different clock stations, because the velocity of electromagnetic signals, travelling in an East-West direction, as measured upon the Earth, is not constant, but $c \pm v$, where v is the spin velocity of the surface of the Earth at the particular Latitude. The outstanding problem is to devise a theory that will fit both the Relativistic corrections, that are vindicated in everyday use, and the Sagnac correction.

The Sagnac effect is proof that light travels at a constant velocity, in relation to the fixed laboratory, and does not adapt to the movement of a spinning disc. This requires that time aboard a spinning disc is the same as time in the fixed laboratory. The Sagnac correction is being correctly applied to the sending of electromagnetic signals between standard clock stations on the Earth; the reason given (relativistic correction) is incorrect. It is proposed that the Sagnac correction should not be applied to the physical transportation of clocks between sites, as is presently done in the CCIR and CCDS rules; it is solely the Relativistic corrections, due to velocity of travel and height over sea level, that should be applied in such a case.

The CCIR report concludes by saying that "*additional definitions and conventions are under consideration.*" These are awaited with interest. An amendment to relativistic theory to accommodate the true application of the Sagnac correction would give a more precise solution to the problem of clock synchronisation.

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