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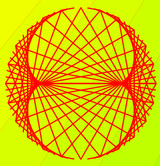

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Detection of the Relativistic Corrections to the Gravitational Potential using a Sagnac Interferometer

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General Relativity predicts the existence of relativistic corrections to the static Newtonian potential which can be calculated and verified experimentally. The idea leading to quantum corrections at large distances is that of the interactions of massless particles which only involve their coupling energies at low energies. In this short paper we attempt to propose the Sagnac intrerferometric technique as a way of detecting the relativistic correction suggested for the Newtonian potential, and thus obtaining an estimate for phase difference using a satellite orbiting at an altitude of 250 km above the surface of the Earth.

1 Introduction

The potential acting between to masses M and m that separated from their centers by a distance r is:

$$V(r) = -\frac{GMm}{r}\,, \tag{1}$$

where s the Newton's constant of gravitation. This potential is of course only approximately valid [1]. For large masses and or large velocities the theory of General Relativity predicts that there exist relativistic corrections which can be calculated and also verified experimentally [2]. In the microscopic distance domain, we could expect that quantum mechanics, would predict a modification in the gravitational potential in the same way that the radiative corrections of quantum electrodynamics leads to a similar modification of the Coulombic interaction [3].

Even though the theory of General Relativity constitutes a very well defined classical theory, it is still not possible to combine it with quantum mechanics in order to create a satisfied theory of quantum gravity. One of the basic obstacles that prevent this from happening is that General Relativity does not actually fit the present paradigm for a fundamental theory that of a renormalizable quantum field theory. Gravitational fields can be successfully quantized on smooth-enough space-times [4], but the form of gravitational interactions is such that they induce unwanted divergences which can not be absorbed by the renormalization of the parameters of the minimal General Relativity [5]. Somebody can introduce new coupling constants and absorb the divergences then, one is unfortunately led to an infinite number of free parameters. In spite the difficulty above quantum gravity calculations can predict long distance quantum corrections.

The main idea leading to quantum corrections at large distances is due to the interactions of massless particles which only involve their coupling energies at low energies, something that it is known from the theory of General Relativity, even though at short distances the theory of quantum gravity differs resulting to finite correction of the order, $O\left(\frac{G\hbar}{c^3 r^3}\right)$. The existence of a universal long distance quantum correction to the Newtonian potential should be relevant for a wide class of gravity theories. It is well known that the ultraviolet behaviour of Einstein's pure gravity can be improved, if higher derivative contributions to the action are added, which in four dimensions take the form:

$$\alpha R^{\kappa\lambda} R_{\kappa\lambda} + \beta R^2, \tag{2}$$

where α and β are dimensionless coupling constants. What makes the difference is that the resulting classical and quantum corrections to gravity are expected to significantly alter the gravitational potential at short distances comparable to that of Planck length $\ell_P = \sqrt{\frac{G\hbar}{c^3}} = 10^{-35}$ m, but it should not really affect its behaviour at long distances. At long distances is the structure of the Einstein-Hilbert action that actually determines that. At this point we should mentioned that some of the calculation to the corrections of the Newtonian gravitational potential result in the absence of a cosmological constant Λ which usually complicates the perturbative treatment to a significant degree due to the need to expand about a non-flat background.

In one loop amplitude computation one needs to calculate all first order corrections in G, which will include both the relativistic $O\left(\frac{G^2 m^2}{c^2}\right)$ and the quantum mechanical $O\left(\frac{G\hbar}{c^3}\right)$ corrections to the classical Newtonian potential [6].

2 The corrections to the potential

Our goal is not to present the details of the one loop treatment that leads to the corrections of the Newtonian gravitational potential but rather sate the result and then use it in our calculations. Valid in order of G^2 we have that the corrected potential now becomes [6]:

$$V(r) = -\frac{GMm}{r}\left[1 - \frac{G(M+m)}{2c^2 r} - \frac{122\,G\hbar}{15\pi c^3 r^2}\right]. \tag{3}$$

Observing (3) we see that in the correction of the static Newtonian potential two different length scales are involved. First, the Planck length $\ell_P = \sqrt{\frac{G\hbar}{c^3}} = 10^{-35}$ m and second the Schwarzschild radii of the heavy sources $r_{sch} = \frac{2GM_n}{c^2}$. Furthermore there are two independent dimensionless parameters which appear in the correction term, and involve the ratio of these two scales wit respect to the distance r. Presumably for meaningful results the two length scales are much smaller than r.

3 Perturbations due to oblateness J_2

Because the Earth's gravitational potential is not that of a perfect spherical body, we can approximate its potential as a spherical harmonic expansion of the following form:

$$V(r,\phi) = -\frac{GMm}{r}\left[1 - \sum_{n=2}^{\infty} J_n\left(\frac{R_e}{r}\right)^n P_n(\sin\phi)\right] = \\ = \frac{GMm}{r}\left[V_o + V_{J_2} + V_{J_3} + \ldots\right], \tag{4}$$

where:

r = geocentric distance,

ϕ = geocentric latitude.

R_e = means equatorial radius of the Earth,

P_n = Legendre polynomial of degree n and order zero,

$J_n = J_{n_0}$ jonal harmonics of order zero, that depend on the latitude ϕ only,

and the first term GMm/r now describes the potential of a homogeneous sphere and thus refers to Keplerian motion, the remaining part represents the Earth's oblateness via the zonal harmonic coefficients and [7]

$$\left.\begin{aligned} V_0 &= -1 \\ V_{J_2} &= \frac{J_2}{2}\left(\frac{R_e}{r}\right)^2\left(3\sin^2\phi - 1\right) \\ V_{J_3} &= \frac{J_3}{2}\left(\frac{R_e}{r}\right)^3\left(5\sin^3\phi - 3\sin\phi\right) \end{aligned}\right\} \tag{5}$$

similarly [8]

$$\left.\begin{aligned} J_2 &= 1{,}082.6\times10^{-6} \\ J_3 &= -2.53\times10^{-6} \end{aligned}\right\}. \tag{6}$$

Therefore equation (4) can be further written:

$$V(r,\phi) = -\frac{GMm}{r}\times \\ \times\left[1 - \sum_{n=2}^{\infty} J_n\left(\frac{R_e}{r}\right)^n P_n(\sin\phi) - \frac{G(M+m)}{2rc^2}\right] = \tag{6a} \\ = \frac{GMm}{r}\left[V_o + V_{J_2} + V_{J_3} + \ldots\; - V_{Relativistic}\right].$$

Since J_2 is 400 larger that any other J_n coefficients, we can disregard them and write the following expression for the Earth's potential function including only the relativistic correction and omitting the quantum corrections as being very small we have:

$$V(r,\phi) = -\frac{GM_e m}{r} + \frac{GM_e m R_e^2 J_2}{r^3}\left(\frac{3}{2}\sin^2\phi - \frac{1}{2}\right) + \\ + \frac{G^2 M_e m (M_e + m)}{c^2 r^2}. \tag{7}$$

Since we propose a satellite in orbit that carries the Sagnac instrument it will of a help to express equation (7) for the potential in terms of the orbital elements. We know that $\sin\phi = \sin i\, \sin(f+\omega)$ where i is the inclination of the orbit, f is the true anomaly and ω is the argument of the perigee. Ignoring long and short periodic terms (those containing ω and f) we write (7) in terms of the inclination as follows:

$$V(r,\phi) = -\frac{GM_e m}{r} + \frac{3GM_e m R_e^2 J_2}{2r^3}\left(\frac{\sin^2 i}{2} - \frac{1}{3}\right) + \\ + \frac{G^2 M_e m (M_e + m)}{c^2 r^2} \tag{8}$$

therefore the corresponding total acceleration that a mass m at $r > R_e$ has becomes:

$$g_{tot} = -\frac{1}{m}\frac{\partial}{\partial r}\left[-\frac{GM_e m}{r} + \frac{3GM_e m R_e^2 J_2}{2r^3}\times \right. \\ \left. \times\left(\frac{\sin^2 i}{2} - \frac{1}{3}\right) + \frac{G^2 M_e m (M+m)}{r^2 c^2}\right] \tag{9}$$

so that:

$$g_{tot} = -\frac{GM_e}{r^2} + \frac{9GM_e R_e^2 J_2}{2r^4}\left(\frac{\sin^2 i}{2} - \frac{1}{3}\right) + \\ + \frac{G^2 M_e (M_e + m)}{c^2 r^3}. \tag{10}$$

4 Basic Sagnac interferometric theory

The Sagnac interferometer is based on the *Sagnac effect*, reported by G. Sagnac in 1913 [8]. Two beams are sent in opposite directions around the interferometer until they meet

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N\Omega_s\nu\left(R_s\Omega_s + \pi^{-2}v_{orb}\right)}{\left(c^2 - R_s^2\Omega_s^2\right)\left[1 - \frac{R_s}{c^2}\left(-\frac{GM_e}{r^2} + \frac{9GM_eR_e^2J_2}{2r^4}\left(\frac{\sin^2 i}{2} - \frac{1}{3}\right) + \frac{G^2M_e^2}{r^3c^2}\right)\left[1 - \cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1 + \frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{15}$$

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N\Omega_s\nu\left(R_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{(R_e+z_{orb})}}\right)}{\left(c^2 - R_s^2\Omega_s^2\right)\left[1 + \frac{R_s}{c^2}\left(\frac{GM_e}{r^2} - \frac{3GM_eR_e^2J_2}{4r^4} - \frac{G^2M_e^2}{r^3c^2}\right)\left[1 - \cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1 + \frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{16}$$

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N\Omega_s\nu\left(1 + \frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{R_e}\left(1 - \frac{z_{orb}}{R_e}\right)}\right)}{c^2\left[1 + R_s\left(\frac{GM_e}{c^2R_e^2}\left(1 - \frac{2z_{orb}}{R_e}\right) - \frac{3GM_eR_e^2J_2}{4c^2R_e^4}\left(1 - \frac{4z_{orb}}{R_e}\right) - \frac{G^2M_e^2}{R_e^3c^4}\left(1 - \frac{3z_{orb}}{R_e}\right)\right)\left[1 - \cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1 + \frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{17}$$

again to create a phase pattern. By rotating the interferometer in the direction of either the clockwise (CW) or counter-clockwise (CCW) beam, a phase difference results between the two beams that its given by:

$$\Delta\Phi_{rs} = \frac{8\pi^2 R_{sag}^2 N\Omega\nu}{\left(c^2 - a^2\Omega^2\right)}, \tag{11}$$

where Ω is the angular velocity of the interferometer, R_{sag} is the radius of the interferometer, N is the number of turns of fiber around the radius and ν is the frequency of light in the fiber.

Let us now assume that the Sagnac interferometer and its light laser beams are in the region of space around the Earth where the gravitational potential is given by equation (3) and let us further assume that the quantum correction to the potential is really negligible. If the Sagnac light loop area has a unit vector that is perpendicular to the acceleration of gravity vector, then the motion of the interferometer will exhibit a red-shift that will be given by:

$$f_{rs} = \frac{f}{1 - \frac{\Delta V}{c^2}} = \frac{f}{1 - \frac{g_{cor}z}{c^2}}, \tag{12}$$

where ΔV is the difference in the potential between to different points P_1 and P_2, and g_{cor} is the corrected or total acceleration of gravity and z is the difference in vertical distance between the two beams as the interferometer coil rotates. This distance z that the laser beams see is given by:

$$z = R_{sag}\left\{1 - \cos\left[\frac{2\pi\Omega R_{sag}}{c\left(1 + \frac{R_{sag}\Omega}{c}\right)}\right]\right\}. \tag{13}$$

This Sagnac effect can also be amplified by an interferometer that is in orbit, where the orbital velocity of the interferometer with respect to the Earth's surface produces an increased phase shift. Both terms involved in the acceleration of gravity in the first one:

$$\Delta\Phi_{rs} = \frac{8\pi^2 R_{sag}^2 N\Omega\nu\left(R_{sag}\Omega + \frac{v_{orb}}{\pi^2}\right)}{\left(c^2 - R_{sag}^2\Omega^2\right)\left[1 - \frac{g_{tot}z}{c^2}\right]} \tag{14}$$

using (14) and taking into account that $M \gg m$ we further obtain (15), where M is the source of the gravitational field = the mass of the Earth in our case M_e, and R is the radius of the massive body $= R_e$, and $r = R_e + z_{orb}$ it's orbital height plus Earth radius for an Earth-based satellite.

This Sagnac effect can also be amplified by an interferometer that is in orbit, where the orbital velocity of the interferometer with respect to the Earth's surface produces an increased phase shift. Both terms involved in the acceleration of gravity in the first one:

5 Sagnac in circular orbit of known inclination

Let now a Sagnac interferometer be aboard a satellite in a circular polar orbit of inclination $i = 90$ degrees. If the inclination is 90 degrees the term $\sin^2\frac{i}{2} - \frac{1}{3} = \frac{1}{6}$ and the orbital velocity at some height z above the surface of the Earth is $v_{orb} = \sqrt{\frac{GM_e}{(R_e+Z_{orb})}}$ and (6) takes the form (16) can be finally written as (17).

6 Sagnac in elliptical orbit of known inclination

If now a satellite is carrying a Sagnac device is in an elliptical orbit of eccentricity e and semi-major axis a we have that the radial orbital vector and the orbital velocity are given by:

$$r(f) = \frac{a\left(1 - e^2\right)}{1 + e\cos f}, \tag{18}$$

$$v^2 = GM_e\left(\frac{2}{r} - \frac{1}{a}\right) = \frac{GM_e}{a}\left[\frac{2\left(1 + e\cos f\right)}{\left(1 - e^2\right)} - 1\right], \tag{19}$$

where f is the true anomaly of the orbit. Substituting now in (8) we obtain (20).

If we use the fact that $GM_e = n^2a^3$ where n is the mean motion of the satellite, equation (20) can be further written as (21).

When the satellite approaches perigee its orbital velocity will increase, so we will expect to see a higher phase difference than any other point of the orbit, and similarly the effect

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N\Omega_s \nu \left(1+\frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{a}\left(\frac{1+e^2+2e\cos f}{1-e^2}\right)}\right)}{c^2\left[1+R_s\left(\frac{GM_e(1+e\cos f)^2}{c^2a^2(1-e^2)^2} - \frac{3GM_eR_e^2J_2(1+e\cos f)^4}{4c^2a^4(1-e^2)^4} - \frac{G^2M_e^2(1+e\cos f)^3}{c^4a^3(1-e^2)^3}\right)\left[1-\cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1+\frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{20}$$

$$\Delta\phi_{rs} = \frac{8\pi^2 R_s^2 N\Omega_s \nu \left(1+\frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}na\sqrt{\left(\frac{1+e^2+2e\cos f}{1-e^2}\right)}\right)}{c^2\left[1+R_s\left(\frac{n^2a(1+e\cos f)^2}{c^2(1-e^2)^2} - \frac{3n^2R_e^2J_2(1+e\cos f)^4}{4c^2a(1-e^2)^4} - \frac{n^4a^3(1+e\cos f)^3}{c^4(1-e^2)^3}\right)\left[1-\cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1+\frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{21}$$

$$\Delta\phi_{rs}\,(perigee) = \frac{8\pi^2 R_s^2 N\Omega_s \nu \left(1+\frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{a}\left(\frac{1+e}{1-e}\right)}\right)}{c^2\left[1+R_s\left(\frac{GM_e}{c^2a^2(1-e)^2} - \frac{3GM_eR_e^2J_2}{4c^2a^4(1-e)^4} - \frac{G^2M_e^2}{c^4a^3(1-e)^3}\right)\left[1-\cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1+\frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{24}$$

$$\Delta\phi_{rs}\,(perigee) = \frac{8\pi^2 R_s^2 N\Omega_s \nu \left(1+\frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}na\sqrt{\frac{1+e}{1-e}}\right)}{c^2\left[1+R_s\left(\frac{n^2a}{c^2(1-e)^2} - \frac{3n^2R_e^2J_2}{4c^2a(1-e)^4} - \frac{n^4a^3}{c^4(1-e)^3}\right)\left[1-\cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1+\frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{25}$$

$$\Delta\phi_{rs}\,(apogee) = \frac{8\pi^2 R_s^2 N\Omega_s \nu \left(1+\frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{a}\left(\frac{1-e}{1+e}\right)}\right)}{c^2\left[1+R_s\left(\frac{GM_e}{c^2a^2(1+e)^2} - \frac{3GM_eR_e^2J_2}{4c^2a^4(1+e)^4} - \frac{G^2M_e^2}{c^4a^3(1+e)^3}\right)\left[1-\cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1+\frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{26}$$

$$\Delta\phi_{rs}\,(apogee) = \frac{8\pi^2 R_s^2 N\Omega_s \nu \left(1+\frac{R_s^2\Omega_s^2}{c^2}\right)\left(R_s\Omega_s + \pi^{-2}na\sqrt{\frac{1+e}{1-e}}\right)}{c^2\left[1+R_s\left(\frac{n^2a}{c^2(1+e)^2} - \frac{3n^2R_e^2J_2}{4c^2a(1+e)^4} - \frac{n^4a^3}{c^4(1+e)^3}\right)\left[1-\cos\left[\frac{2\pi R_s\Omega_s}{c}\left(1+\frac{R_s\Omega_s}{c}\right)^{-1}\right]\right]\right]} \tag{27}$$

will be minimum at the point of apogee because the satellite's velocity is minimal. The distance at perigee and apogee are given by the equations below:

$$\left.\begin{aligned} r_{pg} &= a\,(1-e) \\ r_{apg} &= a\,(1+e) \end{aligned}\right\} \tag{22}$$

also the corresponding velocities are:

$$\left.\begin{aligned} v_{pg}^2 &= \frac{GM}{a}\left(\frac{1+e}{1-e}\right) \\ v_{apg}^2 &= \frac{GM_e}{a}\left(\frac{1-e}{1+e}\right) \end{aligned}\right\}, \tag{23}$$

therefore the phase difference detected by the Sagnac due to the contribution of the Earth's oblateness plus relativistic correction to the potential at perigee and apogee can be written as (24) or again (25).

Similarly the phase difference at apogee can be written as (26) or again (27).

For this last case of the elliptical orbit in (25) and (26) where the Sagnac interferometer is on the satellite and we assume $R_s = 1$ m, $\nu = 2\times10^{14}$ Hz, $N = 10^6$, $\Omega_s = 400$ rad/sec, $a = 8\times10^6$ m, $e = 0.2$, $R_e = 6.378\times10^6$ meters we arrive at the following values for $\Delta\phi$:

$$\Delta\phi\,(perigee) = 3.57\times10^{-16} \text{ radians},$$

$$\Delta\phi\,(apogee) = 2.44\times10^{-16} \text{ radians}.$$

These values are based on the dominant potential correction in (11) of section 3 which is the first term in (11) or the Newtonian correction:

$$\text{Newtonian correction} = 2.17\times10^{-16} \text{ radians}.$$

In comparison, the second and third terms in (11) are the oblateness and relativistic corrections respectively and they produce the following values based on the given parameters:

$$\text{Oblateness correction} = 8.52\times10^{-20},$$

$$\text{Relativistic correction} = 7.91\times10^{-26}.$$

So by comparison of the values above, the Newtonian correction is much easier to measure.

$$\Delta\phi_{rs} = \frac{8\pi^2 a_s^2 N\Omega_s\nu\left(a_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{(R_e+z_{orb})}}\right)}{(c^2 - a_s^2\Omega_s^2)\left[1 + \frac{a_s\left[1+(e^2+e-1)\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right]}{\left(1+e\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right)}\left(\frac{GM_e}{r^2c^2} - \frac{3GM_eR_e^2J_2}{4r^4c^2} - \frac{G^2M_e^2}{r^3c^4}\right)\right]} \tag{30}$$

$$\Delta\phi_{rs} = \frac{8\pi^2 a_s^2 N\Omega_s\nu\left(1+\frac{a_s^2\Omega^2}{c^2}\right)\left(a_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{R_e}\left(1-\frac{z_{orb}}{R_e}\right)}\right)}{c^2\left[1+\frac{a_s\left[1+(e^2+e-1)\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right]}{\left(1+e\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right)}\left(\frac{GM_e}{R_e^2c^2}\left(1-\frac{2z_{orb}}{R_e}\right)-\frac{3GM_eR_e^2J_2}{4R_e^4c^2}\left(1-\frac{4z_{orb}}{R_e}\right)-\frac{G^2M_e^2}{R_e^3c^4}\left(1-\frac{3z_{orb}}{R_e}\right)\right)\right]} \tag{31}$$

$$\Delta\phi_{rs}= \frac{8\pi^2 a_s^2 N\Omega_s\nu\left(1+\frac{a_s^2\Omega_s^2}{c^2}\right)\left(a_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{a}\left(\frac{1+e^2+2e\cos f}{1-e^2}\right)}\right)}{c^2\left[1+\frac{a_s\left[1+(e^2+e-1)\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right]}{\left(1+e\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right)}\left(\frac{GM_e(1+e\cos f)^2}{c^2a^2(1-e^2)^2}-\frac{3GM_eR_e^2J_2(1+e\cos f)^4}{4c^2a^4(1-e^2)^4}-\frac{G^2M_e^2(1+e\cos f)^3}{c^4a^3(1-e^2)^3}\right)\right]} \tag{32}$$

$$\Delta\phi_{rs}\,(perigee)= \frac{8\pi^2 a_s^2 N\Omega_s\nu\left(1+\frac{a_s^2\Omega_s^2}{c^2}\right)\left(a_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{a}\left(\frac{1+e}{1-e}\right)}\right)}{c^2\left[1+\frac{a_a\left[1+(e^2+e-1)\cos\left[\frac{2\pi a_s\Omega s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right]}{\left(1+e\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right)}\left(\frac{GM_e}{c^2a^2(1-e)^2}-\frac{3GM_eR_e^2J_2}{4c^2a^4(1-e)^4}-\frac{G^2M_e^2}{c^4a^3(1-e)^3}\right)\right]} \tag{33}$$

$$\Delta\phi_{rs}\,(apogee)= \frac{8\pi^2 a_s^2 N\Omega_s\nu\left(1+\frac{a_s^2\Omega_s^2}{c^2}\right)\left(a_s\Omega_s + \pi^{-2}\sqrt{\frac{GM_e}{a}\left(\frac{1-e}{1+e}\right)}\right)}{c^2\left[1+\frac{a_s\left[1+(e^2+e-1)\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right]}{\left(1+e\cos\left[\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right]\right)}\left(\frac{GM_e}{c^2a^2(1+e)^2}-\frac{3GM_eR_e^2J_2}{4c^2a^4(1+e)^4}-\frac{G^2M_e^2}{c^4a^3(1+e)^3}\right)\right]} \tag{34}$$

The $\Delta\phi$ values given above may be more easily measured using a QPSK-modulator inserted in the CCW or CW beam path to improve phase resolution. Also, the use of higher wavelengths (factor of 10 higher in frequency) will increase resolution.

7 We suggest a Sagnac with an elliptic fiber loop

To attempt increasing the resolution of the phase difference of the Sagnac interferometer let us now propose a Sagnac loop, that has the shape of an ellipse that rotates with an angular velocity Ω. In this case it can be shown that the height difference between two points on the ellipse can be given by:

$$z = a\left[\frac{1+\left(e^2+e-1\right)\cos\theta}{1+e\cos\theta}\right]. \tag{28}$$

To check the validity of the formula we derived we can set e=0 which is the case of a circular Sagnac fiber optical path we can see that the (13) in now retrieved since

$R_{sag} = a_{loop(sag)} = a_s$ is the semi major axis of the elliptical fiber loop. When the ellipse spins with angular velocity Ω that would force it to trace out a circle whose radius r, will be that of the semi-major axis a of the ellipse, and therefore we can finally write for (13):

$$z = \frac{a_s\left[1+(e^2+e-1)\cos\left\{\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right\}\right]}{\left(1+e\cos\left\{\frac{2\pi a_s\Omega_s}{c}\left(1+\frac{a_s\Omega_s}{c}\right)^{-1}\right\}\right)}. \tag{29}$$

8 Circular orbit formula for the phase difference of the Sagnac

Let now as before have a Sagnac interferometer be aboard a satellite in a circular polar orbit of inclination $i = 90$ degrees. If the inclination is 90 degrees the term $\sin^2\frac{i}{2} - \frac{1}{3} = \frac{1}{6}$ and the orbital velocity at some height z above the surface of the Earth is $v_{orb(circ)} = \sqrt{\frac{GM_e}{(R_e+z_{orb})}}$ and (6) takes the form (30) that can be finally written as (31).

9 Sagnac in elliptical orbit of known inclination

If now a satellite is carrying a Sagnac device is in an elliptical orbit of eccentricity e and semi-major axis a we have that the radial orbital vector and the orbital velocity are given by (32).

At perigee the equation (32) becomes (33) and also (34).

For (33) and (34) above the following values are computed assuming $e = 0.2$, $\nu = 2\times10^{14}$ Hz, $a = 8\times10^{6}$ meters, $N = 1$ (because the orbit is the Sagnac loop), $R_{sag} = R_{perigee}$ or R_{apogee} as determined by (22), $\Omega_{perigee} = 0.001$ rad/sec, and $\Omega_{apogee} = 6\times10^{-4}$ rad/sec we find,

$\Delta\phi\,(perigee) = 6.05\times10^{10}$ radians,

$\Delta\phi\,(apogee) = 2.36\times10^{10}$ radians.

These values are for measuring the dominant Newtonian contribution as described in Section 6. To detect relativistic contribution which is 3.64×10^{-10} smaller than the Newtonian contribution the corresponding phase-shifts from (33) and (34) are:

$\Delta\phi\,(perigee) = 22$ radians,

$\Delta\phi\,(apogee) = 8.59$ radians.

Thus, the relativistic contribution in (11) of Section 3 is easily measurable using a Sagnac interferometer where the satellites in orbit are the Sagnac loop. In this scenario, the light path can be implemented by transmitting laser beams from one satellite to the next satellite in orbit ahead of it. Also, by using the maximum spacing possible between satellites in orbit this will allow line of site transmission while reducing the number of satellites required for the Sagnac loop. With the potential to measure such small relativistic corrections, the merit of using satellites to implement a large Sagnac loop of radius $R_s = R_{ap}$ or R_{per} is well worth considering.

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Resolving Spacecraft Earth-Flyby Anomalies with Measured Light Speed Anisotropy

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Doppler shift observations of spacecraft, such as Galileo, NEAR, Cassini, Rosetta and MESSENGER in earth flybys, have all revealed unexplained speed "anomalies" — that the Doppler-shift determined speeds are inconsistent with expected speeds. Here it is shown that these speed anomalies are not real and are actually the result of using an incorrect relationship between the observed Doppler shift and the speed of the spacecraft — a relationship based on the assumption that the speed of light is isotropic in all frames, *viz* invariant. Taking account of the repeatedly measured light-speed anisotropy the anomalies are resolved *ab initio*. The Pioneer 10/11 anomalies are discussed, but not resolved. The spacecraft observations demonstrate again that the speed of light is not invariant, and is isotropic only with respect to a dynamical 3-space. The existing Doppler shift data also offers a resource to characterise a new form of gravitational waves, the dynamical 3-space turbulence, that has also been detected by other techniques. The Einstein spacetime formalism uses a special definition of space and time coordinates that mandates light speed invariance for all observers, but which is easily misunderstood and misapplied.

1 Introduction

Planetary probe spacecraft (SC) have their speeds increased, in the heliocentric frame of reference, by a close flyby of the Earth, and other planets. However in the Earth frame of reference there should be no change in the asymptotic speeds after an earth flyby, assuming the validity of Newtonian gravity, at least in these circumstances. However Doppler shift observations of spacecraft, such as Galileo, NEAR, Cassini, Rosetta and MESSENGER in earth flybys, have all revealed unexplained speed "anomalies" — that the Doppler-shift determined speeds are inconsistent with expected speeds [1–6]. Here it is shown that these speed anomalies are not real and are actually the result of using an incorrect relationship between the observed Doppler shift and the speed of the spacecraft — a relationship based on the assumption that the speed of light is isotropic in all frames, *viz* invariant. Taking account of the repeatedly measured light-speed anisotropy the anomalies are resolved *ab initio*.

The speed of light anisotropy has been detected in at least 11 experiments [7–17], beginning with the Michelson-Morley 1887 experiment [7]. The interferometer observations and experimental techniques were first understood in 2002 when the Special Relativity effects and the presence of gas were used to calibrate the Michelson interferometer in gas-mode; in vacuum mode the Michelson interferometer cannot respond to light speed anisotropy [18, 19], as confirmed in vacuum resonant cavity experiments, a modern version of the vacuum-mode Michelson interferometer [20]. So far three different experimental techniques have given consistent results: gas-mode Michelson interferometers [7–11, 16], coaxial cable RF speed measurements [12–14], and optical-fiber Michelson interferometers [15, 17]. This light speed anisotropy reveals the existence of a dynamical 3-space, with the speed of light being invariant only with respect to that 3-space, and anisotropic according to observers in motion relative to that ontologically real frame of reference — such a motion being conventionally known as "absolute motion", a notion thought to have been rendered inappropriate by the early experiments, particularly the Michelson- Morley experiment. However that experiment was never null — they reported a speed of at least 8km/s [7] using Newtonian physics for the calibration. A proper calibration of the Michelson-Morley apparatus gives a light speed anisotropy of at least 300km/s. The spacecraft Doppler shift anomalies are shown herein to give another technique that may be used to measure the anisotropy of the speed of light, and give results consistent with previous detections.

The numerous light speed anisotropy experiments have also revealed turbulence in the velocity of the 3-space relative to the Earth. This turbulence amounts to the detection of sub-mHz gravitational waves — which are present in the Michelson and Morley 1887 data, as discussed in [21], and also present in the Miller data [8, 22] also using a gas-mode Michelson interferometer, and by Torr and Kolen [12], DeWitte [13] and Cahill [14] measuring RF speeds in coaxial cables, and by Cahill [15] and Cahill and Stokes [17] using an optical-fiber interferometer. The existing Doppler shift data also offers a resource to characterise this new form of gravitational waves.

There has been a long debate over whether the Lorentz 3-space *and* time interpretation or the Einstein spacetime inter-

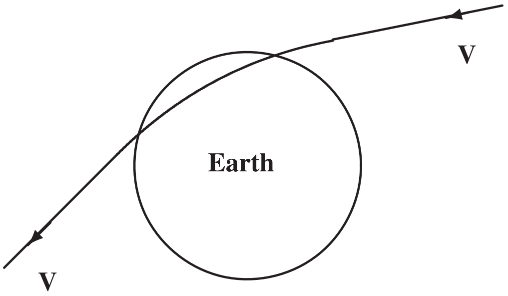

Fig. 1: Spacecraft (SC) earth flyby trajectory, with initial and final asymptotic velocity $\mathbf{V}$, differing only by direction. The Doppler shift is determined from Fig. 2 and (1). Assuming, as conventionally done, that the speed of light is invariant in converting measured Doppler shifts to deduced speeds, leads to the so-called flyby anomaly, namely that the incoming and outgoing asymptotic speeds appear to be differ, by ΔV_∞. However this effect is yet another way to observe the 3-space velocity vector, as well as 3-space wave effects, with the speed of light being c and isotropic only with respect to this structured and dynamical 3-space. The flyby anomalies demonstrate, yet again, that the invariance of the speed of light is merely a definitional aspect of the Einstein spacetime formalism, and is not based upon observations. A *neo*-Lorentzian 3-space *and* time formalism is more physically appropriate.

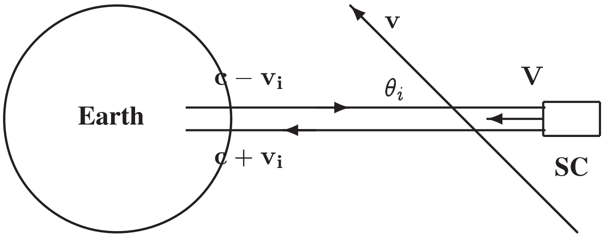

Fig. 2: Asymptotic flyby configuration in Earth frame-of-reference, with spacecraft (SC) approaching Earth with velocity $\mathbf{V}$. The departing asymptotic velocity will have a different direction but the same speed, as no force other than conventional Newtonian gravity is assumed to be acting upon the SC. The Dynamical 3-space velocity is $\mathbf{v}(\mathbf{r}, t)$, which causes the outward EM beam to have speed $c - v_i$, and inward speed $c + v_i$, where $v_i = v\cos(\theta_i)$, with θ_i the angle between $\mathbf{v}$ and $\mathbf{V}$.

pretation of observed SR effects is preferable or indeed even experimentally distinguishable. What has been discovered in recent years is that a dynamical structured 3-space exists, so confirming the Lorentz interpretation of SR [22, 24, 25], and with fundamental implications for physics. This dynamical 3-space provides an explanation for the success of the SR Einstein formalism. Indeed there is a mapping from the physical Lorentzian space and time coordinates to the non-physical spacetime coordinates of the Einstein formalism — but it is a singular map in that it removes the 3-space velocity with respect to an observer. The Einstein formalism transfers dynamical effects, such as length contractions and clock slowing effects, to the metric structure of the spacetime manifold, where these effects then appear to be merely perspective effects for different observers. For this reason the Einstein formalism has been very confusing. Developing the Lorentzian interpretation has lead to a new account of gravity, which turns out to be a quantum effect [23], and of cosmology [21, 22, 26, 27], doing away with the need for dark matter and dark energy. So the discovery of the flyby anomaly links this effect to various phenomena in the emerging new physics.

2 Absolute motion and flyby Doppler shifts

The motion of spacecraft relative to the Earth are measured by observing the direction and Doppler shift of the transponded RF transmissions. As shown herein this data gives another technique to determine the speed and direction of the dynamical 3-space, manifested as a light speed anisotropy. Up to now the repeated detection of the anisotropy of the speed of light has been ignored in analysing the Doppler shift data, causing the long-standing anomalies in the analysis [1–6].

In the Earth frame of reference, see Fig. 2, let the transmitted signal from earth have frequency f, then the corresponding outgoing wavelength is $\lambda_0 = (c - v_i)/f$, where $v_i = v\cos(\theta_i)$. This signal is received by the SC to have period $T_c = \lambda_0/(c - v_i + V)$ or frequency $f_c = (c - v_i + V)/\lambda_0$. The signal is re-transmitted with the same frequency, and so has wavelength $\lambda_i = (c + v_i - V)/f_c$, and is detected at earth with frequency $f_i = (c + v_i)/\lambda_i$. Then overall we obtain[*]

$$f_i = \frac{c + v_i}{c + v_i - V} \cdot \frac{c - v_i + V}{c - v_i}\, f\,. \tag{1}$$

Ignoring the projected 3-space velocity v_i, that is, assuming that the speed of light is invariant as per the usual literal interpretation of the Einstein 1905 light speed postulate, we obtain instead

$$f_i = \frac{c + V}{c - V}\, f\,. \tag{2}$$

The use of (2) instead of (1) is the origin of the putative anomalies. The Doppler shift data is usually presented in the form of speed anomalies. Expanding (2) we obtain

$$\frac{\Delta f_i}{f} = \frac{f_i - f}{f} = \frac{2V}{c} + \ldots \tag{3}$$

From the observed Doppler shift data acquired during a flyby, and then best fitting the trajectory, the asymptotic hyperbolic speeds $V_{i\infty}$ and $V_{f\infty}$ are inferred, but incorrectly so, as in [1]. These inferred asymptotic speeds may be related to an inferred asymptotic Doppler shift:

$$\frac{\Delta f_i}{f} = \frac{f_i - f}{f} = \frac{2V_{i\infty}}{c} + \ldots \tag{4}$$

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[*] In practice the analysis is more complex as is the doppler shift technology. The analysis herein is sufficient to isolate and quantify the light-speed anisotropy effect.

Parameter	GLL-I	GLL-II	NEAR	Cassini	Rosetta	M'GER
Date	Dec 8, 1990	Dec 8, 1992	Jan 23, 1998	Aug 18, 1999	Mar 4, 2005	Aug 2, 2005
V_∞ km/s	8.949	8.877	6.851	16.010	3.863	4.056
α_i deg	266.76	219.35	261.17	334.31	346.12	292.61
δ_i deg	−12.52	−34.26	−20.76	−12.92	−2.81	31.44
α_f deg	219.97	174.35	183.49	352.54	246.51	227.17
δ_f deg	−34.15	−4.87	−71.96	−20.7	−34.29	−31.92
α_v deg(hrs)	108.8(7.25)	129.0(8.6)	108.8(7.25)	45.0(3.0)	130.5(8.7)	168.0(11.2)
δ_v deg	−76	−80	−76	−75	−80	−85
v km/s	420	420	450	420	420	420
θ_i deg	90.5	56.4	81.8	72.6	95.3	124.2
θ_f deg	61.8	78.2	19.6	76.0	60.5	55.6
(O) ΔV_∞ mm/s	3.92±0.3	−4.6±1.0	13.46±0.01	−2±1	1.80±0.03	0.02±0.01
(P) ΔV_∞ mm/s	3.92±0.1	−4.60±0.6	13.40±0.1	−0.99±1.0	1.77±0.3	0.025±0.03

Table 1: Earth flyby parameters from [1] for spacecraft Galileo (GLL: flybys I and II), NEAR, Cassini, Rosetta and MESSENGER (M'GER). V_∞ is the average osculating hyperbolic asymptotic speed, α and δ are the right ascension and declination of the incoming (i) and outgoing (f) osculating asymptotic velocity vectors, and (O) ΔV_∞ is the putative "excess speed" anomaly deduced by assuming that the speed of light is isotropic in modeling the doppler shifts, as in (4). The observed (O) ΔV_∞ values are from [1], and after correcting for atmospheric drag in the case of GLL-II, and thruster burn in the case of Cassini. (P) ΔV_∞ is the predicted "excess speed", using (7), taking account of the known light speed anisotropy and its effect upon the doppler shifts, using α_v and δ_v as the right ascension and declination of the 3-space flow velocity, having speed v, which has been taken to be 420 km/s in all cases, except for NEAR, see Fig. 3. The ± values on (P) ΔV_∞ indicate changes caused by changing the declination by 5% — a sensitivity indicator. The angles θ_i and θ_f between the 3-space velocity and the asymptotic initial/final SV velocity V are also given. The observed doppler effect is in exceptional agreement with the predictions using (7) and the previously measured 3-space velocity. The flyby doppler shift is thus a new technique to accurately measure the dynamical 3-space velocity vector, albeit retrospectively from existing data. Note: By fine tuning the α_v and δ_v values for each flyby a perfect fit to the observed (O) ΔV_∞ is possible. But here we have taken, for simplicity, the same values for GLL-I and NEAR.

However expanding (1) we obtain, for the same Doppler shift*

$$V_{i\infty} \equiv \frac{\Delta f_i}{f}\cdot\frac{c}{2} = \frac{f_i - f}{f}\cdot\frac{c}{2} = \left(1+\frac{v_i^2}{c^2}\right)V+\dots \qquad (5)$$

where V is the actual asymptotic speed. Similarly after the flyby we obtain

$$V_{f\infty} \equiv -\frac{\Delta f_f}{f}\cdot\frac{c}{2} = -\frac{f_f - f}{f}\cdot\frac{c}{2} = \left(1+\frac{v_f^2}{c^2}\right)V+\dots \qquad (6)$$

and we see that the "asymptotic" speeds $V_{i\infty}$ and $V_{f\infty}$ must differ, as indeed first noted in the data by [3]. We then obtain the expression for the so-called flyby anomaly

$$\begin{aligned}\Delta V_\infty &= V_{f\infty} - V_{i\infty} = \frac{v_f^2 - v_i^2}{c^2}V + \dots \\ &= \frac{v^2}{c^2}\left(\cos(\theta_f)^2 - \cos(\theta_i)^2\right)V_\infty + \dots \qquad (7)\end{aligned}$$

where here $V \approx V_\infty$ to sufficient accuracy, where V_∞ is the average of $V_{i\infty}$ and $V_{f\infty}$, The existing data on $\mathbf{v}$ permits *ab initio* predictions for ΔV_∞, and as well a separate least-squares-fit to the individual flybys permits the determination of the average speed and direction of the 3-space velocity, relative to the Earth, during each flyby. These results are all remarkably consistent with the data from the 11 previous laboratory experiments that studied $\mathbf{v}$. Note that whether the 3-space velocity is $+\mathbf{v}$ or $-\mathbf{v}$ is not material to the analysis herein, as the flyby effect is 2nd order in v.

3 Earth flyby data analaysis

Eqn. (7) permits the speed anomaly to be predicted as the direction and speed v of the dynamical 3-space is known, as shown in Fig. 3. The first determination of its direction was reported by Miller [8] in 1933, and based on extensive observations during 1925/1926 at Mt.Wilson, California, using a large gas-mode Michelson interferometer. These observations confirmed the previous non-null observations by Michelson and Morley [7] in 1887. The general characteristics of $\mathbf{v}(\mathbf{r},t)$ are now known following the detailed analysis of the experiments noted above, namely its average speed, and removing the Earth orbit effect, is some 420±30km/s,

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* We ignore terms of order vV/c^2 within the parentheses, as in practice they are much smaller than the v^2/c^2 terms.

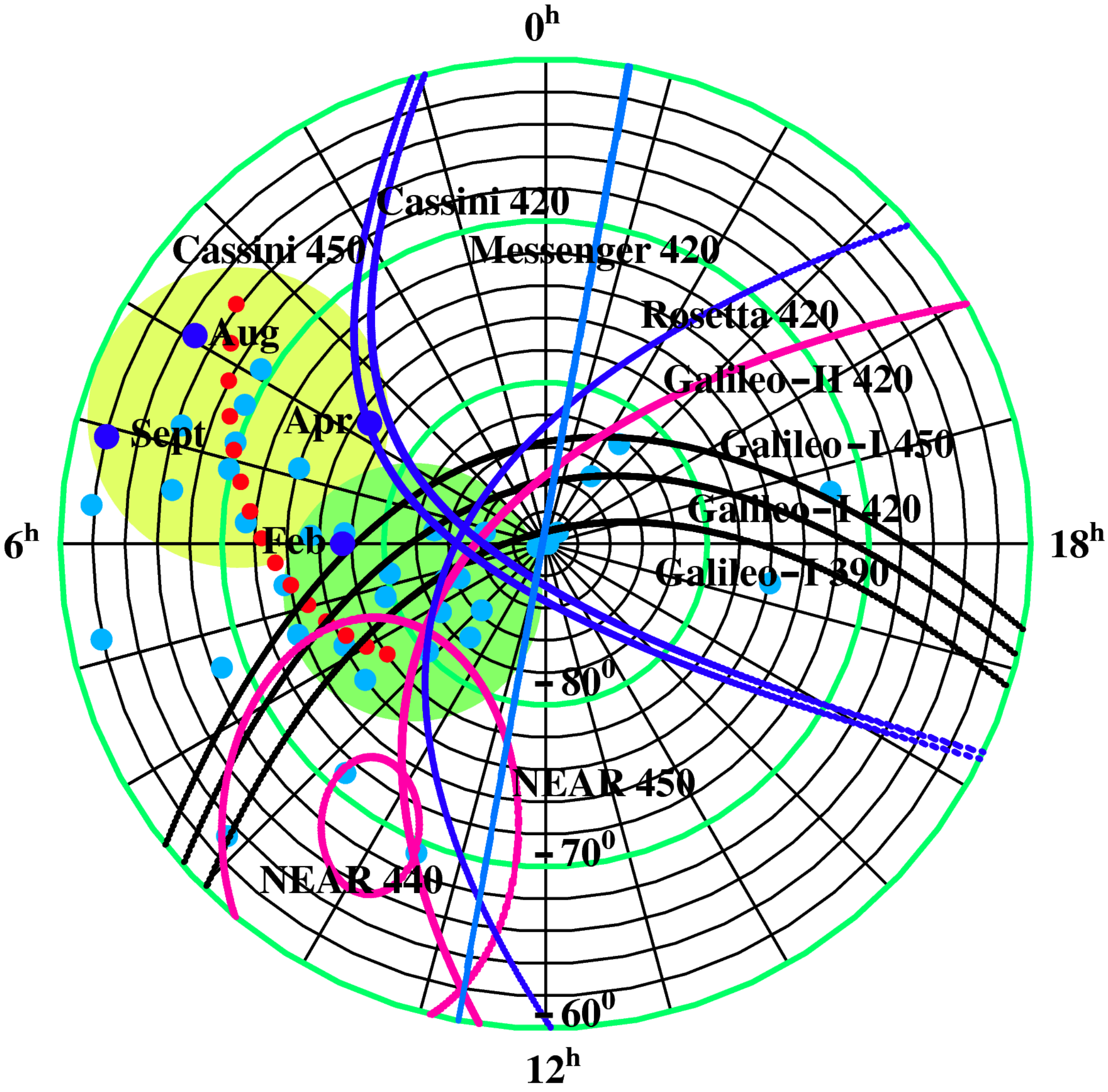

Fig. 3: Southern celestial sphere with RA and Dec shown. The 4 dark blue points show the consolidated results from the Miller gas-mode Michelson interferometer [8] for four months in 1925/1926, from [22]. The sequence of red points show the running daily average RA and Dec trend line, as determined from the optical fiber interferometer data in [17], for every 5 days, beginning September 22, 2007. The light-blue scattered points show the RA and Dec for individual days from the same experiment, and show significant turbulence/wave effects. The curved plots show iso-speed ΔV_∞ "anomalies": for example for $v = 420$ km/s the RA and Dec of $\mathbf{v}$ for the Galileo-I flyby must lie somewhere along the "Galileo-I 420" curve. The available spacecraft data in Table 1, from [1], does not permit a determination of a unique $\mathbf{v}$ during that flyby. In the case of "Galileo-I" the curves are also shown for 420±30 km/s, showing the sensitivity to the range of speeds discovered in laboratory experiments. We see that the "Galileo-I" December flyby has possible directions that overlap with the December data from the optical fiber interferometer, although that does not exclude other directions, as the wave effects are known to be large. In the case of NEAR we must have $v \geqslant 440$ km/s otherwise no fit to the NEAR ΔV_∞ is possible. This demonstrates a fluctuation in v of at least +20 km/s on that flyby day. This plot shows the remarkable concordance in speed and direction from the laboratory techniques with the flyby technique in measuring $\mathbf{v}$, and its fluctuation characteristics. The upper-left coloured disk (radius = 8°) shows concordance for September/August interferometer data and Cassini flyby data (MESSENGER data is outside this region — but has very small ΔV_∞ and large uncertainty), and the same, lower disk, for December/January/February/March data (radius = 6°). The moving concordance effect is undertsood to be caused by the earth's orbit about the Sun, while the yearly average of 420±30 km/s is a galaxy related velocity. Directions for each flyby $\mathbf{v}$ were selected and used in Table 1.

from direction right ascension $\alpha_v = 5.5 \pm 2^{\mathrm{hr}}$, declination $\delta_v = 70 \pm 10^\circ$ S — the center point of the Miller data in Fig. 3, together with large wave/turbulence effects, as illustrated in Fig. 4. Miller's original calibration technique for the interferometer turned out to be invalid [22], and his speed of approximately 208 km/s was recomputed to be 420±30 km/s in [19, 22], and the value of 420 km/s is used here as shown in Table 1. The direction of $\mathbf{v}$ varies throughout the year due to the Earth-orbit effect and low frequency wave effects. A more recent determination of the direction was reported in [17] using an optical-fiber version of the Michelson interferometer, and shown also in Fig. 3 by the trend line and data from individual days. Directions appropriate to the date of each flyby were approximately determined from Fig. 3.

The SC data in Table 1 shows the values of V_∞ and ΔV_∞ after determining the osculating hyperbolic trajectory, as discussed in [1], as well as the right ascension and declination of the asymptotic SC velocity vectors $\mathbf{V}_{i\infty}$ and $\mathbf{V}_{f\infty}$. In computing the predicted speed "anomaly" ΔV_∞ using (7) it is only necessary to compute the angles θ_i and θ_f between the dynamical 3-space velocity vector and these SC incoming and outgoing asymptotic velocities, respectively, as we assume here that $|v| = 420$ kms, except for NEAR as discussed in Fig. 3 caption. So these predictions are essentially *ab initio* in that we are using 3-space velocities that are reasonably well known from laboratory experiments. The observed Doppler effects are in exceptional agreement with the predictions using (7) and the previously measured 3-space velocity. The flyby anomaly is thus a new technique to accurately measure the dynamical 3-space velocity vector, albeit retrospectively from existing data.

4 New gravitational waves

Light-speed anisotropy experiments have revealed that a dynamical 3-space exists, with the speed of light being c, in vacuum, only with respect to to this space: observers in motion "through" this 3-space detect that the speed of light is in general different from c, and is different in different directions. The dynamical equations for this 3-space are now known and involve a velocity field $\mathbf{v}(\mathbf{r}, t)$, but where only relative velocities are observable locally — the coordinates $\mathbf{r}$ are relative to a non-physical mathematical embedding space. These dynamical equations involve Newton's gravitational constant G and the fine structure constant α. The discovery of this dynamical 3-space then required a generalisation of the Maxwell, Schrödinger and Dirac equations. The wave effects already detected correspond to fluctuations in the 3-space velocity field $\mathbf{v}(\mathbf{r}, t)$, so they are really 3-space turbulence or wave effects. However they are better known, if somewhat inappropriately, as "gravitational waves" or "ripples" in "spacetime". Because the 3-space dynamics gives a deeper understanding of the spacetime formalism we now know that the metric of the induced spacetime, merely a mathematical construct having no ontological significance, is related to $\mathbf{v}(\mathbf{r}, t)$ according to [21, 22, 27]

$$ds^2 = dt^2 - \frac{(d\mathbf{r} - \mathbf{v}(\mathbf{r}, t)dt)^2}{c^2} = g_{\mu\nu}\, dx^\mu dx^\nu . \qquad (8)$$

The gravitational acceleration of matter, a quantum effect, and of the structural patterns characterising the 3-space, are given by [21, 23]

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \qquad (9)$$

and so fluctuations in $\mathbf{v}(\mathbf{r}, t)$ may or may not manifest as a gravitational acceleration. The flyby technique assumes that the SC trajectories are not affected — only the light speed anisotropy is significant. The magnitude of this turbulence depends on the timing resolution of each particular experiment, and was characterised to be sub-mHz in frequency by Cahill and Stokes [14]. Here we have only used asymptotic osculating hyperbolic trajectory data from [1]. Nevertheless even this data suggests the presence of wave effects. For example the NEAR data requires a speed in excess of 440 km/s, and probably closer to 450 km/s, whereas the other flybys are consistent with the average of 420 km/s from laboratory experiments. So here we see flyby evidence of fluctuations in the speed v.

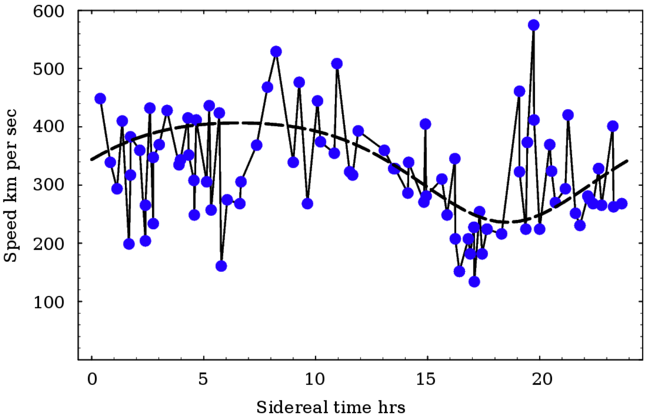

Fig. 4: Speeds v_P, of the 3-space velocity $\mathbf{v}$ projected onto the horizontal plane of the Miller gas-mode Michelson interferometer, plotted against local sidereal time in hours, for a composite day, with data collected over a number of days in September 1925. The data shows considerable fluctuations, from hour to hour, and also day to day, as this is a composite day. The dashed curve shows the non-fluctuating best-fit variation over one day, as the Earth rotates, causing the projection onto the plane of the interferometer of the velocity of the average direction of the space flow to change. The maximum projected speed of the curve is 417 km/s, and the min/max occur at approximately 5 hrs and 17 hrs sidereal time (right ascension); see Fig. 3 for September. Analysing Millers's extensive data set from 1925/26 gives average speed, after removing earth orbit effect, of 420±30 km/s, and the directions for each month shown in Fig. 3.

Data exists for each full flyby, and analysis of that data using the new Doppler shift theory will permit the study and characterisation of the 3-space wave turbulence during each flyby: essentially the flybys act as gravitational wave detectors. These gravitational waves are much larger than predicted by general relativity, and have different properties.

5 Pioneer 10/11 anomalies

The Pioneer 10//11 spacecraft have been exploring the outer solar system since 1972/73. The spacecraft have followed escape hyperbolic orbits near the plane of the ecliptic, after earlier planet flybys. The Doppler shift data, using (2), have revealed an unexplained anomaly beyond 10 AU [28]. This manifests as an unmodelled increasing blue shift $\frac{d}{dt}(\frac{\Delta f}{f}) = (2.92 \pm 0.44) \times 10^{-18}$ s/s^2, corresponding to a constant inward sun-directed acceleration of $a = \frac{dV}{dt} = (8.74 \pm 1.33) \times 10^{-8}$ cm/s^2, averaged from Pioneer 10 and Pioneer 11 data. However the Doppler-shift data from these spacecraft has been interpreted using (2), instead of (1), in determining the speed, which in turn affects the distance data. Essentially this implies that the spacecraft are attributed with a speed that is too large by $\frac{v^2}{c^2} V_D$, where V_D is the speed determined using (2). This then implies that the spacecraft are actually closer to the Sun by the distance $\frac{v^2}{c^2} R_D$, where R_D is the distance determined using (2). This will then result in a computed spurious inward acceleration, because the gravitational pull of the Sun is actually larger than modelled, for distance R_D. However this correction to the Doppler-shift analysis appears not to be large enough to explain the above mention acceleration anomaly. Nevertheless re-analysis of the Pioneer 10/11 data should be undertaken using (1).

6 Conclusions

The spacecraft earth flyby anomalies have been resolved. Rather than actual relative changes in the asymptotic inward and outward speeds, which would have perhaps required the invention of a new force, they are instead direct manifestations of the anisotropy of the speed of light, with the Earth having a speed of some 420±30 km/s relative to a dynamical 3-space, a result consistent with previous determinations using laboratory experiments, and dating back to the Michelson-Morley 1887 experiment, as recently reanalysed [18, 19, 21]. The flyby data also reveals, yet again, that the 3-space velocity fluctuates in direction and speed, and with results also consistent with laboratory experiments. Hence we see a remarkable concordance between three different laboratory techniques, and the newly recognised flyby technique. The existing flyby data can now be re-analysed to give a detailed charaterisation of these gravitational waves. The detection of the 3-space velocity gives a new astronomical window on the galaxy, as the observed speeds are those relevant to galactic dynamics. The dynamical 3-space velocity effect also produces very small vorticity effects when passing the Earth, and these are predicted to produce observable effects on the GP-B gyroscope precessions [29].

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The Neutrosophic Logic View to Schrödinger's Cat Paradox, Revisited

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The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot. To make this proposition clear, we revisit our previous paper by offering an illustration using modified coin tossing problem, known as Parrondo's game.

1 Introduction

The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. In this article we argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot.

In the preceding article we have discussed how Neutrosophic logic view can offer an alternative method to solve the well-known problem in Quantum Mechanics, i.e. the Schrödinger's cat paradox [1, 2], by introducing indeterminacy of the outcome of the observation.

In other article we also discuss possible re-interpretation of quantum measurement using Unification of Fusion Theories as generalization of Information Fusion [3, 4, 5], which results in proposition that one can expect to neglect the principle of "excluded middle"; therefore Bell's theorem can be considered as merely tautological. [6] This alternative view of Quantum mechanics as Information Fusion has also been proposed by G. Chapline [7]. Furthermore this Information Fusion interpretation is quite consistent with measurement theory of Quantum Mechanics, where the action of measurement implies information exchange [8].

In the first section we will discuss basic propositions of Neutrosophic probability and Neutrosophic logic. Then we discuss solution to Schrödinger's cat paradox. In subsequent section we discuss an illustration using modified coin tossing problem, and discuss its plausible link to quantum game.

While it is known that derivation of Schrödinger's equation is heuristic in the sense that we know the answer to which the algebra and logic leads, but it is interesting that Schrödinger's equation follows logically from de Broglie's *grande loi de la Nature* [9, p.14]. The simplest method to derive Schrödinger's equation is by using simple wave as [9]:

$$\frac{\partial^2}{\partial x^2}\,\mathtt{exp}(ikx) = -k^2 \cdot \mathtt{exp}(ikx)\,. \tag{1}$$

By deriving twice the wave and defining:

$$k = \frac{2\pi m v}{h} = \frac{mv}{\hbar} = \frac{p_x}{\hbar}\,, \tag{2}$$

where p_x, $\hbar$ represents momentum at x direction, and rationalised Planck constants respectively.

By introducing kinetic energy of the moving particle, T, and wavefunction, as follows [9]:

$$T = \frac{mv^2}{2} = \frac{p_x^2}{2m} = \frac{\hbar^2}{2m}\,k^2\,, \tag{3}$$

and

$$\psi(x) = \mathtt{exp}(ikx)\,. \tag{4}$$

Then one has the time-independent Schrödinger equation from [1, 3, 4]:

$$-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\,\psi(x) = T \cdot \psi(x)\,. \tag{5}$$

It is interesting to remark here that by convention physicists assert that "the wavefunction is simply the *mathematical function* that describes the wave" [9]. Therefore, unlike the wave equation in electromagnetic fields, one should not consider that equation [5] has any physical meaning. Born suggested that the square of wavefunction represents the probability to observe the electron at given location [9, p.56]. Although Heisenberg rejected this interpretation, apparently Born's interpretation prevails until today.

Nonetheless the founding fathers of Quantum Mechanics (Einstein, De Broglie, Schrödinger himself) were dissatisfied with the theory until the end of their lives. We can summarize the situation by quoting as follows [9, p.13]:

> "The interpretation of Schrödinger's wave function (and of quantum theory generally) remains a matter of continuing concern and controversy among scientists who cling to philosophical belief that the natural world is basically logical and deterministic."

Furthermore, the "pragmatic" view of Bohr asserts that for a given quantum measurement [9, p.42]:

> "A system does not possess objective values of its physical properties until a measurement of one of them is made; the act of measurement is asserted to force the system into an eigenstate of the quantity being measured."

In 1935, Einstein-Podolsky-Rosen argued that the axiomatic basis of Quantum Mechanics is incomplete, and subsequently Schrödinger was inspired to write his well-known cat paradox. We will discuss solution of his cat paradox in subsequent section.

2 Cat paradox and imposition of boundary conditions

As we know, Schrödinger's deep disagreement with the Born interpretation of Quantum Mechanics is represented by his cat paradox, which essentially questioning the "statistical" interpretation of the wavefunction (and by doing so, denying the physical meaning of the wavefunction). The cat paradox has been written elsewhere [1, 2], but the essence seems quite similar to coin tossing problem:

> "Given $p = 0.5$ for each side of coin to pop up, we will never know the state of coin before we open our palm from it; unless we know beforehand the "state" of the coin (under our palm) using ESP-like phenomena. Prop. (1)."

The only difference here is that Schrödinger asserts that the state of the cat is half alive and half dead, whereas in the coin problem above, we can only say that we don't know the state of coin until we open our palm; i.e. the state of coin is *indeterminate* until we open our palm. We will discuss the solution of this problem in subsequent section, but first of all we shall remark here a basic principle in Quantum Mechanics, i.e. [9, p.45]:

> "Quantum Concept: The first derivative of the wavefunction Ψ of Schrödinger's wave equation must be single-valued everywhere. As a consequence, the wavefunction itself must be single-valued everywhere."

The above assertion corresponds to quantum logic, which can be defined as follows [10, p.30; 11]:

$$P \vee Q = P + Q - PQ\,. \tag{6}$$

As we will see, it is easier to resolve this cat paradox by releasing the aforementioned constraint of "single-valuedness" of the wavefunction and its first derivative. In fact, nonlinear fluid interpretation of Schrödinger's equation (using the level set function) also indicates that the physical meaning of wavefunction includes the notion of multivaluedness [12]. In other words, one can say that observation of spin-half electron at location x does not exclude its possibility to pop up somewhere else. This counter-intuitive proposition will be described in subsequent section.

3 Neutrosophic solution of the Schrödinger cat paradox

In the context of physical theory of information [8], Barrett has noted that "there ought to be a set theoretic language which applies directly to all quantum interactions". This is because the idea of a bit is itself straight out of *classical set theory*, the definitive and unambiguous assignment of an element of the set {0,1}, and so the assignment of an information content of the photon itself is fraught with the same difficulties [8]. Similarly, the problem becomes more adverse because the fundamental basis of conventional statistical theories is the same classical set {0,1}.

For example the Schrödinger's cat paradox says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory. In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function, which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Therefore the question can be summarized as follows [1]:

> "How to describe a particle ζ in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\zeta \in P_1$ and $\zeta \in \neg P_1$ is a true contradiction, with respect to Quantum Concept described above."

Now we will discuss some basic propositions in Neutrosophic logic [1].

3a Non-standard real number and subsets

Let T,I,F be standard or non-standard real subsets $\subseteq]^{-}0, 1^{+}[$,

with sup T = t_sup, inf T= t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup + i_sup + f_sup,
n_inf = t_inf + i_inf + f_inf.

Obviously, t_sup, i_sup, f_sup $\leqslant 1^{+}$; and t_inf, i_inf, f_inf $\geqslant {}^{-}0$, whereas n_sup $\leqslant 3^{+}$ and n_inf $\geqslant {}^{-}0$. The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single element; finite or infinite; union or intersection of various subsets etc. They may also overlap. These real subsets could represent the relative errors in determining t, i, f (in the case where T, I, F are reduced to points).

For interpretation of this proposition, we can use modal logic [10]. We can use the notion of "world" in modal logic, which is semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^{+}$ if A is "true in all possible worlds." (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named *necessary truth*, as opposed to possible truth), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a "specific" world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each "world" is dynamic, depending on an ensemble of parameters, we introduce the sub-category "conjuncture" within it to reflect a particular state of the world.

In a formal way, let's consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the Neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it "relative falsehood". Also, the Neutrosophic indeterminacy value $NL_i(A) = 1$ if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".

3b Neutrosophic probability definition

Neutrosophic probability is defined as: "Is a generalization of the classical probability in which the chance that an event A occurs is t% true — where t varies in the subset T, i% indeterminate — where i varies in the subset I, and f% false — where f varies in the subset F. One notes that NP(A) = (T, I, F)". It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

The universal set, endowed with a Neutrosophic probability defined for each of its subset, forms a Neutrosophic probability space.

3c Solution of the Schrödinger's cat paradox

Let's consider a neutrosophic set a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^-0$ and 1^+. For example: $x\,(0.5,\ 0.2,\ 0.3)$ belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is *undecidable*); or $y\,(0,\ 0,\ 1)$ belongs to A (which normally means y is not for sure in A); or $z\,(0,\ 1,\ 0)$ belongs to A (which means one does know absolutely nothing about z's affiliation with A).

More general, $x\,((0.2\text{–}0.3),\ (0.40\text{–}0.45) \cup [0.50\text{–}0.51],\ \{0.2,\ 0.24,\ 0.28\})$ belongs to the set A, which means:

- with a probability in between 20-30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- with a probability of 20% or 24% or 28% x is not in A;
- the indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $\text{n\_sup} = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [1, 2], given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i.e. we don't know if it is dead or alive), then herein the Neutrosophic logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We have described Neutrosophic solution of the Schrödinger's cat paradox. Alternatively, one may hypothesize four-valued logic to describe Schrödinger's cat paradox, see Rauscher *et al.* [13, 14].

In the subsequent section we will discuss how this Neutrosophic solution involving "possible truth" and "indeterminacy" can be interpreted in terms of coin tossing problem (albeit in modified form), known as Parrondo's game. This approach seems quite consistent with new mathematical formulation of game theory [20].

4 An alternative interpretation using coin toss problem

Apart from the aforementioned pure mathematics-logical approach to Schrödinger's cat paradox, one can use a well-known neat link between Schrödinger's equation and Fokker-Planck equation [18]:

$$D\,\frac{\partial^2 p}{\partial z^2} - \frac{\partial \alpha}{\partial z}\,p - \alpha\,\frac{\partial p}{\partial z} - \frac{\partial p}{\partial t} = 0\,. \qquad (7)$$

A quite similar link can be found between relativistic classical field equation and non-relativistic equation, for it is known that the time-independent Helmholtz equation and Schrödinger equation is formally identical [15]. From this reasoning one can argue that it is possible to explain Aharonov effect from pure electromagnetic field theory; and therefore it seems also possible to describe quantum mechanical phenomena without postulating the decisive role of "observer" as Bohr asserted. [16, 17]. In idiomatic form, one can expect that quantum mechanics does not have to mean that "the Moon is not there when nobody looks at".

With respect to the aforementioned neat link between Schrödinger's equation and Fokker-Planck equation, it is interesting to note here that one can introduce "finite difference" approach to Fokker-Planck equation as follows. First, we can define local coordinates, expanded locally about a point (z_0, t_0) we can map points between a real space (z, t) and an integer or discrete space (i, j). Therefore we can sample the space using linear relationship [19]:

$$(z, t) = (z_0 + i\lambda, t_0 + j\tau)\,, \tag{8}$$

where λ is the sampling length and τ is the sampling time. Using a set of finite difference approximations for the Fokker-Planck PDE:

$$\frac{\partial p}{\partial z} = A_1 = \frac{p\,(z_0 + \lambda, t_0 - \tau) - p\,(z_0 - \lambda, t_0 - \tau)}{2\lambda}\,, \tag{9}$$

$$\frac{\partial^2 p}{\partial z^2} = 2A_2 = = \frac{p\,(z_0 - \lambda, t_0 - \tau) - 2p\,(z_0, t_0 - \tau) + p\,(z_0 + \lambda, t_0 - \tau)}{\lambda^2}\,, \tag{10}$$

and

$$\frac{\partial p}{\partial t} = B_1 = \frac{p\,(z_0, t_0) - p\,(z_0, t_0 - \tau)}{\tau}\,. \tag{11}$$

We can apply the same procedure to obtain:

$$\frac{\partial \alpha}{\partial z} = A_1 = \frac{\alpha\,(z_0 + \lambda, t_0 - \tau) - \alpha\,(z_0 - \lambda, t_0 - \tau)}{2\lambda}\,. \tag{12}$$

Equations (9–12) can be substituted into equation (7) to yield the required finite partial differential equation [19]:

$$p\,(z_0, t_0) = a_{-1} \cdot p\,(z_0 - \lambda, t_0 - \tau) - a_0 \cdot p\,(z_0, t_0 - \tau) + + a_{+1} \cdot p\,(z_0 + \lambda, t_0 - \tau)\,. \tag{13}$$

This equation can be written in terms of discrete space by using [8], so we have:

$$p_{i,j} = a_{-1} \cdot p_{i-1,j-1} + a_0 \cdot p_{i,j-1} + a_{+1} \cdot p_{i+1,j-1}\,. \tag{14}$$

Equation (14) is precisely the form required for Parrondo's game. The meaning of Parrondo's game can be described in simplest way as follows [19]. Consider a coin tossing problem with a biased coin:

$$p_{head} = \frac{1}{2} - \varepsilon\,, \tag{15}$$

where ε is an external bias that the game has to "overcome". This bias is typically a small number, for instance 1/200. Now we can express equation (15) in finite difference equation (14) as follows:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon\right) \cdot p_{i-1,j-1} + 0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon\right) \cdot p_{i+1,j-1}\,. \tag{16}$$

Furthermore, the bias parameter can be related to an applied external field.

With respect to the aforementioned Neutrosophic solution to Schrödinger's cat paradox, one can introduce a new "indeterminacy" parameter to represent conditions where the outcome may be affected by other issues (let say, apparatus setting of Geiger counter). Therefore equation (14) can be written as:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + + a_0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}\,, \tag{17}$$

where unlike the bias parameter ($\sim$1/200), the indeterminacy parameter can be quite large depending on the system in question. For instance in the Neutrosophic example given above, we can write that:

$$\eta \sim 0.2 - 0.3 = k \left(\frac{d}{t}\right)^{-1} = k \left(\frac{t}{d}\right) \leqslant 0.50. \tag{18}$$

The only problem here is that in original coin tossing, one cannot assert an "intermediate" outcome (where the outcome is neither A nor B). Therefore one shall introduce modal logic definition of "possibility" into this model. Fortunately, we can introduce this possibility of intermediate outcome into Parrondo's game, so equation (17) shall be rewritten as:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + + (2\eta) \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}\,, \tag{19}$$

For instance, by setting $\eta \sim 0.25$, then one gets the finite difference equation:

$$p_{i,j} = (0.25 - \varepsilon) \cdot p_{i-1,j-1} + (0.5) \cdot p_{i,j-1} + + (0.25 + \varepsilon) \cdot p_{i+1,j-1}\,, \tag{20}$$

which will yield more or less the same result compared with Neutrosophic method described in the preceding section.

For this reason, we propose to call this equation (19): *Neutrosophic-modified Parrondo's game.* A generalized expression of equation [19] is:

$$p_{i,j} = (p_0 - \varepsilon - \eta) \cdot p_{i-1,j-1} + (z\eta) \cdot p_{i,j-1} + + (p_0 + \varepsilon - \eta) \cdot p_{i+1,j-1}\,, \tag{21}$$

where p_0, z represents the probable outcome in standard coin tossing, and a real number, respectively. For the practical meaning of η, one can think (by analogy) of this indeterminacy parameter as a variable that is inversely proportional to the "thickness ratio" (d/t) of the coin in question. Therefore using equation (18), by assuming $k = 0.2$, coin thickness = 1.0 mm, and coin diameter $d = 50$ mm, then we get $d/t = 50$, or $\eta = 0.2\,(50)^{-1} = 0.004$, which is negligible. But if we use a thick coin (for instance by gluing 100 coins altogether), then by assuming $k = 0.2$, coin thickness = 100 mm, and coin diameter $d = 50$ mm, we get $d/t = 0.5$, or $\eta = 0.2\,(0.5)^{-1} = 0.4$, which indicates that chance to get outcome neither A nor B is quite large. And so forth.

It is worth noting here that in the language of "modal logic" [10, p.54], the "intermediate" outcome described here is given name 'possible true', written $\Diamond A$, meaning that "it is not necessarily true that not-A is true". In other word, given that the cat cannot be found in location x, does not have to mean that it shall be in y.

Using this result (21), we can say that our proposition in the beginning of this paper (Prop. 1) has sufficient reasoning; i.e. it is possible to establish link from Schrödinger wave equation to simple coin toss problem, albeit in modified form. Furthermore, this alternative interpretation, differs appreciably from conventional Copenhagen interpretation.

It is perhaps more interesting to remark here that Heisenberg himself apparently has proposed similar thought on this problem, by introducing "potentia", which means "*a world devoid of single-valued actuality but teeming with unrealized possibility*" [4, p.52]. In Heisenberg's view an atom is certainly real, but its attributes dwell in an existential limbo "halfway between an idea and a fact", a quivering state of attenuated existence. Interestingly, experiments carried out by J. Hutchison seem to support this view, that a piece of metal can come in and out from existence [23].

In this section we discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo's game. Further observation and theoretical study is recommended to explore more implications of this plausible link.

5 Concluding remarks

In the present paper we revisit the Neutrosophic logic view of Schrödinger's cat paradox. We also discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo's game.

It is recommended to conduct further experiments in order to verify and explore various implications of this new proposition, including perhaps for the quantum computation theory.

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A Classical Model of Gravitation

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A classical model of gravitation is proposed with time as an independent coordinate. The dynamics of the model is determined by a proposed Lagrangian. Applying the canonical equations of motion to its associated Hamiltonian gives conservation equations of energy, total angular momentum and the z component of the angular momentum. These lead to a Keplerian orbit in three dimensions, which gives the observed values of perihelion precession and bending of light by a massive object. An expression for gravitational redshift is derived by accepting the local validity of special relativity at all points in space. Exact expressions for the GEM relations, as well as their associated Lorentz-type force, are derived. An expression for Mach's Principle is also derived.

1 Introduction

The proposed theory is based on two postulates that respectively establish the dynamics and kinematics of a system of particles subject to a gravitational force. The result is a closed particle model that satisfies the basic experimental observations of the force.

The details of applications and all derivations are included in the doctoral thesis of the author [1].

2 Postulates

The model is based on two postulates:

Postulate 1: The **dynamics** of a system of particles subject to gravitational forces is determined by the Lagrangian,

$$L = -m_0(c^2+v^2)\exp\frac{R}{r}\,, \tag{1}$$

where m_0 is *gravitational rest mass* of a test body moving at velocity $\mathbf{v}$ in the vicinity of a massive, central body of mass M, $\gamma = 1/\sqrt{1-v^2/c^2}$, $R = 2GM/c^2$ is the Schwarzschild radius of the central body.

Postulate 2: Special Relativity (SR) is valid instantan-eously and locally at all points in the reference system of the central massive body. This gives the **kinematics** of the system.

3 Conservation equations

Applying the canonical equations of motion to the Hamiltonian, derived from the Lagrangian, leads to three conservation equations:

$$\begin{aligned} E &= m_0c^2\,\frac{e^{R/r}}{\gamma^2} = \text{total energy} = \text{constant}\,, && (2)\\ \mathbf{L} &= e^{R/r}\,\mathbf{M}\,, && (3)\\ &= \text{total angular momentum} = \text{constant}\,, &&\\ L_z &= e^{R/r}\,m_0r^2\sin^2\theta\,\dot{\phi}\,, && (4)\\ &= z \text{ component of } \mathbf{L} = \text{constant}\,, && \end{aligned}$$

where $\mathbf{M} = (\mathbf{r}\times m_0\mathbf{v})$. Equations (2), (3) and (4) give the quadrature of motion:

$$\frac{d\Psi}{du} = \pm\left[\frac{e^{2Ru}}{L^2} - u^2 - \frac{Ee^{Ru}}{L^2}\right]^{-1/2}, \tag{5}$$

where $u = 1/r$, $L = |\mathbf{L}|$ and Ψ is defined by

$$|\mathbf{M}| = m_0r^2\,\frac{d\Psi}{dt}\,. \tag{6}$$

Expanding the exponential terms to second degree yields a differential equation of generalized Keplerian form,

$$\frac{d\Psi}{du} = (au^2+bu+c)^{-1/2}, \tag{7}$$

where

$$\left.\begin{aligned} u &= \frac{1}{r}\\ a &= \frac{R^2(4-E)}{2L^2} - 1\\ b &= \frac{R(2-E)}{L^2}\\ c &= \frac{1-E}{L^2} \end{aligned}\right\}, \tag{8}$$

and the convention $m_0 = c = 1$ was used.

Integrating (7) gives the orbit of a test particle as a generalized conic,

$$u = K(1+\epsilon\cos k\Psi)\,, \tag{9}$$

where the angles are measured from $\Psi = 0$, and

$$k = (-a)^{\frac{1}{2}}\,, \tag{10}$$

$$K = -\frac{b}{2a}\,, \tag{11}$$

$$\epsilon = \left(1-\frac{4ac}{b}\right)^{\frac{1}{2}}. \tag{12}$$

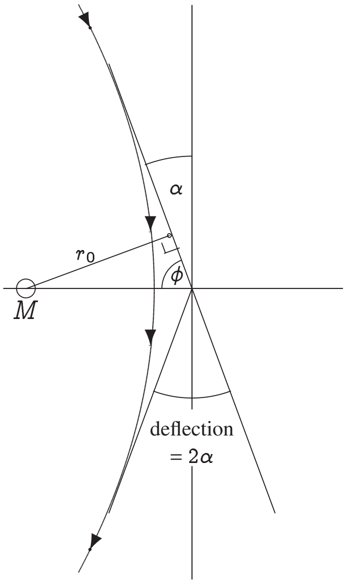

Fig. 1: Deflection of light.

4 Gravitational redshift

Assuming the validity of $\gamma d\tau = dt$ of SR at each point in space and taking frequencies as the inverses of time, (2) yields

$$\nu = \nu_0\, e^{-R/2r} \qquad (\nu_0 = \text{constant}), \tag{13}$$

which, to first approximation in $\exp(-R/2r)$, gives the observed gravitational redshift.

5 Perihelion precession

In the case of an ellipse ($\epsilon < 1$), the presence of the coefficient k causes the ellipse not to be completed after a cycle of $\theta = 2\pi$ radians, i.e. the perihelion is shifted through a certain angle. This shift, or precession, can be calculated as (see Appendix 9):

$$\Delta\phi = \frac{3\pi R}{\bar{a}\,(1-\epsilon^2)}\,, \tag{14}$$

where $\bar{a}$ is the semi-major axis of the ellipse. This expression gives the observed perihelion precession of Mercury.

6 Deflection of light

We define a photon as a particle for which $v = c$. From (2) it follows that $E = 0$ and the eccentricity of the conic section is found to be (see Appendix 9)

$$\epsilon = \frac{r_0}{R}\,, \tag{15}$$

where r_0 is the impact parameter. Approximating r_0 by the radius of the sun, it follows that $\epsilon > 1$. From Fig. 1 we see that the trajectory is a hyperbola with total deflection equal to $2R/r_0$. This is in agreement with observation.

7 Lorentz-type force equation

The corresponding force equation is found from the associated Euler-Lagrange equations:

$$\dot{\mathbf{p}} = \mathbf{E}m + m_0\mathbf{v}\times\mathbf{H}\,, \tag{16}$$

where

$$\mathbf{p} \;=\; m_0\,\dot{\mathbf{r}} = m_0\,\mathbf{v}\,, \tag{17}$$

$$m \;=\; \frac{m_0}{\gamma^2}\,, \tag{18}$$

$$\mathbf{E} \;=\; -\,\hat{\mathbf{r}}\,\frac{GM}{r^2}\,, \tag{19}$$

$$\mathbf{H} \;=\; \frac{GM\,(\mathbf{v}\times\mathbf{r})}{c^2r^3}\,. \tag{20}$$

The force equation shows the deviation from Newton's law of gravitation. The above equations are analogous to the gravitoelectromagnetic (GEM) equations derived by Mashhoon [2] as a lowest order approximation to Einstein's field equations for $v \ll c$ and $r \gg R$.

8 Mach's Principle

An *ad hoc* formulation for Mach's Principle has been presented as [3, 4]

$$G \cong \frac{Lc^2}{M}\,, \tag{21}$$

where: L = radius of the universe,
M = mass of the universe ≅ mass of the distant stars.

This relation can be found by applying the energy relation of (2) to the system of Fig. 2.

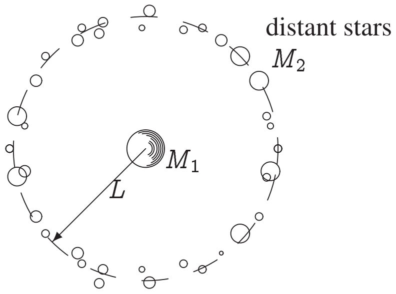

Fig. 2: Mutual gravitational interaction between a central mass M_1 and the distant stars of total mass M_2.

The potential at M_2 due to M_1 is $\Phi_1 = GM_1/L = R_1c^2/2L$ and the potential of the shell at M_1 is $\Phi_2 = GM_2/L = R_2c^2/2L$. Furthermore, since M_1 and M_2 are in relative motion, the value of γ will be the same for both of them. Applying (2) to the mutual gravitational interaction between the shell of distant stars and the central body then gives

$$E = M_1c^2\exp\frac{R_2}{L} = M_2c^2\exp\frac{R_1}{L}\,.$$

Since $L > R_2 \gg R_1$ we can realistically approximate the exponential to first order in R_2/L. After some algebra we get $R_2 \approx L$, which gives the Mach relation,

$$\frac{2GM_2}{Lc^2} \approx 1\,.$$

9 Comparison with General Relativity

The equations of motion of General Relativity (GR) are approximations to those of the proposed Lagrangian. This can be seen as follows.

The conservation equations of (2), (3) and (4) can also be derived from a generalized metric,

$$ds^2 = e^{-R/r}\,dt^2 - e^{R/r}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right). \quad (22)$$

Comparing this metric with that of GR,

$$ds^2 = \left(1 - \frac{R}{r}\right) dt^2 - \frac{1}{1-\frac{R}{r}}\,dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \quad (23)$$

we note that (23) is a first order approximation to the time and radial coefficients, and a zeroth order approximation to the angular coefficients of (22). It implies that all predictions of GR will be accommodated by the Lagrangian of (1) within the orders of approximation.

Comparing (5) with the corresponding quadrature of GR,

$$\frac{d\theta}{du} = \pm\left[\frac{1-E}{J^2} + \frac{uRE}{J^2} - u^2 + Ru^3\right]^{-1/2}, \quad (24)$$

we note that it differs from the Newtonian limit, or the Keplerian form of (7), by the presence of the Ru^3 term. The form of this quadrature does not allow the conventional Keplerian orbit of (9).

Appendix

A.1 Precession of the perihelion

After one revolution of 2π radians, the perihelion of an ellipse given by the conic of (9) shifts through an angle $\Delta\phi = \frac{2\pi}{k} - 2\pi$ or, from (10), as

$$\Delta\phi = 2\pi\left[(-a)^{-1/2} - 1\right], \quad (25)$$

where a is given by (8). The constants of motion E and L are found from the boundary conditions of the system, i.e. $du/d\theta = 0$ at $u = 1/r_-$ and $1/r_+$, where r_+ and r_- are the maximum and minimum radii respectively of the ellipse. We find [1]

$$\left.\begin{aligned} E &\approx 1 + \frac{R}{2\bar{a}} \\ \frac{R^2}{L^2} &\approx \frac{2R}{\bar{a}\,(1-\epsilon^2)} \end{aligned}\right\}, \quad (26)$$

where $\bar{a} = (r_+ + r_-)/2$ is the semi-major axis of the approximate ellipse. Substituting these values in (8) gives

$$a = \frac{3R}{\bar{a}(1-\epsilon^2)} - 1\,. \quad (27)$$

Substituting this value in (25) gives (14).

A.2 Deflection of light

We first have to calculate the eccentricity ϵ of the conic for this case,

$$\epsilon = \left(1 - \frac{4ac}{b^2}\right)^{1/2}.$$

For a photon, setting $v = c$ in (8) gives

$$\epsilon^2 = \left[-1 + \frac{L^2}{R^2}\right]. \quad (28)$$

At the distance of closest approach, $r = r_0 = 1/u_0$, we have $d\theta/du = 0$, so that from (5):

$$L^2 = \frac{e^{2Ru_0}}{u_0^2} = r_0^2\, e^{2R/r_0}. \quad (29)$$

From (28) and (29), and ignoring terms of first and higher order in R/r_0, we find

$$\epsilon \approx \frac{r_0}{R}\,. \quad (30)$$

For a hyperbola $\cos\phi = 1/\epsilon$, so that (see Fig. 1):

$$\begin{aligned} \sin\alpha &= 1/\epsilon \\ \Rightarrow \alpha &\approx 1/\epsilon \\ \Rightarrow 2\alpha &\approx 2R/r_0 = \text{total deflection.} \end{aligned}$$

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On the Origin of the Dark Matter/Energy in the Universe and the Pioneer Anomaly

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Einstein's special relativity is a theory rich of paradoxes, one of which is the recently discovered *Relativistic Invariant Mass Paradox*. According to this Paradox, the relativistic invariant mass of a galaxy of moving stars exceeds the sum of the relativistic invariant masses of the constituent stars owing to their motion relative to each other. This excess of mass is the mass of *virtual matter* that has no physical properties other than positive relativistic invariant mass and, hence, that reveals its presence by no means other than gravity. As such, this virtual matter is the dark matter that cosmologists believe is necessary in order to supply the missing gravity that keeps galaxies stable. Based on the Relativistic Invariant Mass Paradox we offer in this article a model which quantifies the anomalous acceleration of Pioneer 10 and 11 spacecrafts and other deep space missions, and explains the presence of dark matter and dark energy in the universe. It turns out that the origin of dark matter and dark energy in the Universe lies in the Paradox, and that the origin of the Pioneer anomaly results from neglecting the Paradox. In order to appreciate the physical significance of the Paradox within the frame of Einstein's special theory of relativity, following the presentation of the Paradox we demonstrate that the Paradox is responsible for the extension of the kinetic energy theorem and of the additivity of energy and momentum from classical to relativistic mechanics. Clearly, the claim that the acceleration of Pioneer 10 and 11 spacecrafts is anomalous is incomplete, within the frame of Einstein's special relativity, since those who made the claim did not take into account the presence of the Relativistic Invariant Mass Paradox (which is understandable since the Paradox, published in the author's 2008 book, was discovered by the author only recently). It remains to test how well the Paradox accords with observations.

1 Introduction

Einstein's special relativity is a theory rich of paradoxes, one of which is the *Relativistic Invariant Mass Paradox*, which was recently discovered in [1], and which we describe in Section 5 of this article. The term mass in special relativity usually refers to the rest mass of an object, which is the Newtonian mass as measured by an observer moving along with the object. Being observer's invariant, we refer the Newtonian, rest mass to as the *relativistic invariant mass*, as opposed to the common *relativistic mass*, which is another name for energy, and which is observer's dependent. Lev B. Okun makes the case that the concept of relativistic mass is no longer even pedagogically useful [2]. However, T. R. Sandin has argued otherwise [3].

As we will see in Section 5, the Relativistic Invariant Mass Paradox asserts that the resultant relativistic invariant mass m_0 of a system S of uniformly moving N particles exceeds the sum of the relativistic invariant masses m_k, $k = 1, \dots, N$, of its constituent particles, $m_0 > \sum_{k=1}^{N} m_k$, since the contribution to m_0 comes not only from the masses m_k of the constituent particles of S but also from their speeds relative to each other. The resulting excess of mass in the resultant relativistic invariant mass m_0 of S is the mass of virtual matter that has no physical properties other than positive relativistic invariant mass and, hence, that reveals itself by no means other than gravity. It is therefore naturally identified as the *mass* of *virtual* dark matter that the system S possesses. The presence of dark matter in the universe in a form of virtual matter that reveals itself only gravitationally is, thus, dictated by the Relativistic Invariant Mass Paradox of Einstein's special theory of relativity. Accordingly, (i) the fate of the dark matter particle(s) theories as well as (ii) the fate of their competing theories of modified Newtonian dynamics (MOND [4]) are likely to follow the fate of the eighteenth century phlogiston theory and the nineteenth century luminiferous ether theory, which were initiated as *ad hoc* postulates and which, subsequently, became obsolete.

Dark matter and dark energy are *ad hoc* postulates that account for the observed missing gravitation in the universe and the late time cosmic acceleration. The postulates are, thus, a synonym for these observations, as C. Lämmerzahl, O. Preuss and H. Dittus had to admit in [5] for their chagrin. An exhaustive review of the current array of dark energy theories is presented in [6].

The Pioneer anomaly is the anomalous, unmodelled acceleration of the spacecrafts Pioneer 10 and 11, and other spacecrafts, studied by J. D. Anderson *et al* in [7] and summarized by S. G. Turyshev *et al* in [8]. In [7], Anderson *et al* compared the measured trajectory of a spacecraft against its theoretical trajectory computed from known forces acting on the spacecraft. They found the small, but significant discrepancy known as the anomalous, or unmodelled, acceleration directed approximately towards the Sun. The inability to explain the Pioneer anomaly with conventional physics has contributed to the growing interest about its origin, as S. G. Turyshev, M. M. Nieto and J. D. Anderson pointed out in [9]. It is believed that no conventional force has been overlooked [5] so that, seemingly, new physics is needed. Indeed, since Anderson *et al* announced in [7] that the Pioneer 10 and 11 spacecrafts exhibit an unexplained anomalous acceleration, numerous articles appeared with many plausible explanations that involve new physics, as C. Castro pointed out in [10].

However, we find in this article that no new physics is needed for the explanation of both the presence of dark matter/energy and the appearance of the Pioneer anomaly. Rather, what is needed is to cultivate the Relativistic Invariant Mass Paradox, which has recently been discovered in [1], and which is described in Section 5 below.

Accordingly, the task we face in this article is to show that the Relativistic Invariant Mass Paradox of Einstein's special relativity dictates the formation of dark matter and dark energy in the Universe and that, as a result, the origin of the Pioneer anomaly stems from the motions of the constituents of the Solar system relative to each other.

2 Einstein velocity addition vs. Newton velocity addition

The improved way to study Einstein's special theory of relativity, offered by the author in his recently published book [1], enables the origin of the dark matter/energy in the Universe and the Pioneer anomaly to be determined. The improved study rests on analogies that Einsteinian mechanics and its underlying hyperbolic geometry share with Newtonian mechanics and its underlying Euclidean geometry. In particular, it rests on the analogies that Einsteinian velocity addition shares with Newtonian velocity addition, the latter being just the common vector addition in the Euclidean 3-space $\mathbb{R}^3$.

Einstein addition $\oplus$ is a binary operation in the ball $\mathbb{R}^3_c$ of $\mathbb{R}^3$,

$$\mathbb{R}^3_c = \{\mathbf{v} \in \mathbb{R}^3 : \|\mathbf{v}\| < c\} \tag{1}$$

of all relativistically admissible velocities, where c is the speed of light in empty space. It is given by the equation

$$\mathbf{u}\oplus\mathbf{v} = \frac{1}{1+\frac{\mathbf{u}\cdot\mathbf{v}}{c^2}}\left\{\mathbf{u} + \frac{1}{\gamma_{\mathbf{u}}}\mathbf{v} + \frac{1}{c^2}\frac{\gamma_{\mathbf{u}}}{1+\gamma_{\mathbf{u}}}(\mathbf{u}\cdot\mathbf{v})\mathbf{u}\right\} \tag{2}$$

where $\gamma_{\mathbf{u}}$ is the gamma factor

$$\gamma_{\mathbf{v}} = \frac{1}{\sqrt{1-\frac{\|\mathbf{v}\|^2}{c^2}}} \tag{3}$$

in $\mathbb{R}^3_c$, and where $\cdot$ and $\|\ \|$ are the inner product and norm that the ball $\mathbb{R}^3_c$ inherits from its space $\mathbb{R}^3$. Counterintuitively, Einstein addition is neither commutative nor associative.

Einstein gyrations $\mathrm{gyr}[\mathbf{u},\mathbf{v}] \in Aut(\mathbb{R}^3_c, \oplus)$ are defined by the equation

$$\mathrm{gyr}[\mathbf{u},\mathbf{v}]\mathbf{w} = \ominus(\mathbf{u}\oplus\mathbf{v})\oplus(\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w})) \tag{4}$$

for all $\mathbf{u},\mathbf{v},\mathbf{w} \in \mathbb{R}^3_c$, and they turn out to be automorphisms of the Einstein groupoid $(\mathbb{R}^3_c, \oplus)$. We recall that a groupoid is a non-empty space with a binary operation, and that an automorphism of a groupoid $(\mathbb{R}^3_c, \oplus)$ is a one-to-one map f of $\mathbb{R}^3_c$ onto itself that respects the binary operation, that is, $f(\mathbf{u}\oplus\mathbf{v}) = f(\mathbf{u})\oplus f(\mathbf{v})$ for all $\mathbf{u},\mathbf{v} \in \mathbb{R}^3_c$. To emphasize that the gyrations of the Einstein groupoid $(\mathbb{R}^3_c, \oplus)$ are automorphisms of the groupoid, gyrations are also called gyroautomorphisms.

Thus, $\mathrm{gyr}[\mathbf{u},\mathbf{v}]$ of the definition in (4) is the gyroautomorphism of the Einstein groupoid $(\mathbb{R}^3_c, \oplus)$, generated by $\mathbf{u},\mathbf{v} \in \mathbb{R}^3_c$, that takes the relativistically admissible velocity $\mathbf{w}$ in $\mathbb{R}^3_c$ into the relativistically admissible velocity $\ominus(\mathbf{u}\oplus\mathbf{v})\oplus(\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w}))$ in $\mathbb{R}^3_c$.

The gyrations, which possess their own rich structure, measure the extent to which Einstein addition deviates from commutativity and associativity as we see from the following identities [1, 11, 12]:

$\mathbf{u}\oplus\mathbf{v} = \mathrm{gyr}[\mathbf{u},\mathbf{v}](\mathbf{v}\oplus\mathbf{u})$	Gyrocommutative Law
$\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w}) = (\mathbf{u}\oplus\mathbf{v})\oplus\mathrm{gyr}[\mathbf{u},\mathbf{v}]\mathbf{w}$	Left Gyroassociative
$(\mathbf{u}\oplus\mathbf{v})\oplus\mathbf{w} = \mathbf{u}\oplus(\mathbf{v}\oplus\mathrm{gyr}[\mathbf{u},\mathbf{v}]\mathbf{w})$	Right Gyroassociative
$\mathrm{gyr}[\mathbf{u},\mathbf{v}] = \mathrm{gyr}[\mathbf{u}\oplus\mathbf{v},\mathbf{v}]$	Left Loop Property
$\mathrm{gyr}[\mathbf{u},\mathbf{v}] = \mathrm{gyr}[\mathbf{u},\mathbf{v}\oplus\mathbf{u}]$	Right Loop Property

Einstein addition is thus regulated by its gyrations so that Einstein addition and its gyrations are inextricably linked. Indeed, the Einstein groupoid $(\mathbb{R}^3_c, \oplus)$ forms a group-like mathematical object called a *gyrocommutative gyrogroup* [13], which was discovered by the author in 1988 [14]. Interestingly, Einstein gyrations are just the mathematical abstraction of the relativistic *Thomas precession* [1, Sec. 10.3].

The rich structure of Einstein addition is not limited to its gyrocommutative gyrogroup structure. Einstein addition admits scalar multiplication, giving rise to the Einstein gyrovector space. The latter, in turn, forms the setting for the Beltrami-Klein ball model of hyperbolic geometry just as vector spaces form the setting for the standard model of Euclidean geometry, as shown in [1].

Guided by the resulting analogies that relativistic mechanics and its underlying hyperbolic geometry share with classical mechanics and its underlying Euclidean geometry, we are able to present analogies that Newtonian systems of particles share with Einsteinian systems of particles in Sections 3 and 4. These analogies, in turn, uncover the Relativistic Invariant Mass Paradox in Section 5, the physical significance of which is illustrated in Section 6 in the frame of Einstein's special theory of relativity. Finally, in Sections 7 and 8 the Paradox reveals the origin of the dark matter/energy in the Universe as well as the origin of the Pioneer anomaly.

3 Newtonian systems of particles

In this section we set the stage for revealing analogies that a Newtonian system of N particles and an Einsteinian system of N particles share. In this section, accordingly, as opposed to Section 4, $\mathbf{v}_k$, $k = 0, 1, \dots, N$, are Newtonian velocities in $\mathbb{R}^3$, and m_0 is the Newtonian resultant mass of the constituent masses m_k, $k = 1, \dots, N$ of a Newtonian particle system S.

Accordingly, let us consider the following well known classical results, (6) – (8) below, which are involved in the calculation of the Newtonian resultant mass m_0 and the classical center of momentum (CM) of a Newtonian system of particles, and to which we will seek Einsteinian analogs in Section 4. Thus, let

$$S = S(m_k, \mathbf{v}_k, \Sigma_0, N)\,, \qquad \mathbf{v}_k \in \mathbb{R}^3 \tag{5}$$

be an isolated Newtonian system of N noninteracting material particles the k-th particle of which has mass m_k and Newtonian uniform velocity $\mathbf{v}_k$ relative to an inertial frame Σ_0, $k = 1, \dots, N$. Furthermore, let m_0 be the resultant mass of S, considered as the mass of a virtual particle located at the center of mass of S, and let $\mathbf{v}_0$ be the Newtonian velocity relative to Σ_0 of the Newtonian CM frame of S. Then,

$$1 = \frac{1}{m_0}\sum_{k=1}^{N} m_k \tag{6}$$

and

$$\left.\begin{aligned} \mathbf{v}_0 &= \frac{1}{m_0}\sum_{k=1}^{N} m_k \mathbf{v}_k \\ \mathbf{u} + \mathbf{v}_0 &= \frac{1}{m_0}\sum_{k=1}^{N} m_k(\mathbf{u} + \mathbf{v}_k) \end{aligned}\right\}, \tag{7}$$

$\mathbf{u}, \mathbf{v}_k \in \mathbb{R}^3$, $m_k > 0$, $k = 0, 1, \dots, N$. Here m_0 is the Newtonian mass of the Newtonian system S, supposed concentrated at the center of mass of S, and $\mathbf{v}_0$ is the Newtonian velocity relative to Σ_0 of the Newtonian CM frame of the Newtonian system S in (5).

It follows from (6) that m_0 in (6) – (7) is given by the Newtonian resultant mass equation

$$m_0 = \sum_{k=1}^{N} m_k\,. \tag{8}$$

The derivation of the second equation in (7) from the first equation in (7) is immediate, following (i) the distributive law of scalar-vector multiplication, and (ii) the simple relationship (8) between the Newtonian resultant mass m_0 and its constituent masses m_k, $k = 1, \dots, N$.

4 Einsteinian systems of particles

In this section we present the Einsteinian analogs of the Newtonian expressions (5) – (8) listed in Section 3. The presented analogs are obtained in [1] by means of analogies that result from those presented in Section 2.

In this section, accordingly, as opposed to Section 3, $\mathbf{v}_k$, $k = 0, 1, \dots, N$, are Einsteinian velocities in $\mathbb{R}^3_c$, and m_0 is the Einsteinian resultant mass, yet to be determined, of the masses m_k, $k = 1, \dots, N$, of an Einsteinian particle system S.

In analogy with (5), let

$$S = S(m_k, \mathbf{v}_k, \Sigma_0, N), \qquad \mathbf{v}_k \in \mathbb{R}^3_c \tag{9}$$

be an isolated Einsteinian system of N noninteracting material particles the k-th particle of which has invariant mass m_k and Einsteinian uniform velocity $\mathbf{v}_k$ relative to an inertial frame Σ_0, $k = 1, \dots, N$. Furthermore, let m_0 be the resultant mass of S, considered as the mass of a virtual particle located at the center of mass of S (calculated in [1, Chap. 11]), and let $\mathbf{v}_0$ be the Einsteinian velocity relative to Σ_0 of the Einsteinian center of momentum (CM) frame of the Einsteinian system S in (9). Then, as shown in [1, p. 484], the relativistic analogs of the Newtonian expressions in (6) – (8) are, respectively, the following Einsteinian expressions in (10) – (12),

$$\left.\begin{aligned} \gamma_{\mathbf{v}_0} &= \frac{1}{m_0}\sum_{k=1}^{N} m_k \gamma_{\mathbf{v}_k} \\ \gamma_{\mathbf{u}\oplus\mathbf{v}_0} &= \frac{1}{m_0}\sum_{k=1}^{N} m_k \gamma_{\mathbf{u}\oplus\mathbf{v}_k} \end{aligned}\right\} \tag{10}$$

and

$$\left.\begin{aligned} \gamma_{\mathbf{v}_0}\mathbf{v}_0 &= \frac{1}{m_0}\sum_{k=1}^{N} m_k \gamma_{\mathbf{v}_k}\mathbf{v}_k \\ \gamma_{\mathbf{u}\oplus\mathbf{v}_0}(\mathbf{u}\oplus\mathbf{v}_0) &= \frac{1}{m_0}\sum_{k=1}^{N} m_k \gamma_{\mathbf{u}\oplus\mathbf{v}_k}(\mathbf{u}\oplus\mathbf{v}_k) \end{aligned}\right\}, \tag{11}$$

$\mathbf{u}, \mathbf{v}_k \in \mathbb{R}^3_c$, $m_k > 0$, $k = 0, 1, \dots, N$. Here m_0,

$$m_0 = \sqrt{\left(\sum_{k=1}^{N} m_k\right)^2 + 2\sum_{\substack{j,k=1\\ j<k}}^{N} m_j m_k (\gamma_{\ominus\mathbf{v}_j\oplus\mathbf{v}_k} - 1)} \tag{12}$$

is the relativistic invariant mass of the Einsteinian system S, supposed concentrated at the relativistic center of mass of S

(calculated in [1, Chap. 11]), and $\mathbf{v}_0$ is the Einsteinian velocity relative to $\Sigma_{\mathbf{0}}$ of the Einsteinian CM frame of the Einsteinian system S in (9).

5 The relativistic invariant mass paradox of Einstein's special theory of relativity

In analogy with the Newtonian resultant mass m_0 in (8), which follows from (6), it follows from (10) that the Einsteinian resultant mass m_0 in (10) – (11) is given by the elegant Einsteinian resultant mass equation (12), as shown in [1, Chap. 11].

The Einsteinian resultant mass equation (12) presents a Paradox, called the *Relativistic Invariant Mass Paradox*, since, in general, this equation implies the inequality

$$m_0 > \sum_{k=1}^{N} m_k \tag{13}$$

so that, paradoxically, the invariant resultant mass of a system may exceed the sum of the invariant masses of its constituent particles.

The paradoxical invariant resultant mass equation (12) for m_0 is the relativistic analog of the non-paradoxical Newtonian resultant mass equation (8) for m_0, to which it reduces in each of the following two special cases:

(*i*) The Einsteinian resultant mass m_0 in (12) reduces to the Newtonian resultant mass m_0 in (8) in the limit as $c \to \infty$; and

(*ii*) The Einsteinian resultant mass m_0 in (12) reduces to the Newtonian resultant mass m_0 in (8) in the special case when the system S is rigid, that is, all the internal motions in S of the constituent particles of S relative to each other vanish. In that case $\ominus\mathbf{v}_j \oplus \mathbf{v}_k = \mathbf{0}$ so that $\gamma_{\ominus\mathbf{v}_j \oplus \mathbf{v}_k} = 1$ for all $j, k = 1, N$. This identity, in turn, generates the reduction of (12) to (8).

The second equation in (11) follows from the first equation in (11) in full analogy with the second equation in (7), which follows from the first equation in (7) by the distributivity of scalar multiplication and by the simplicity of (8). However, while the proof of the latter is simple and well known, the proof of the former, presented in [1, Chap. 11], is lengthy owing to the lack of a distributive law for the Einsteinian scalar multiplication (see [1, Chap. 6]) and the lack of a simple relation for m_0 like (8), which is replaced by (12). Indeed, the proof of the former, that the second equation in (11) follows from the first equation in (11), is lengthy, but accessible to undergraduates who are familiar with the vector space approach to Euclidean geometry. However, in order to follow the proof one must familiarize himself with a large part of the author's book [1] and with its "gyrolanguage", as indicated in Section 2.

It is therefore suggested that interested readers may corroborate numerically (using a computer software like MATLAB) the identities in (10) – (12) in order to gain confidence in their validity, before embarking on reading several necessary chapters of [1].

6 The physical significance of the paradox in Einstein's special theory of relativity

In this section we present two classically physical significant results that remain valid relativistically owing to the Relativistic Invariant Mass Paradox, according to which the relativistic analog of the classical resultant mass m_0 in (8) is, paradoxically, the relativistic resultant mass m_0 in (12).

To gain confidence in the physical significance that results from the analogy between

(*i*) the Newtonian resultant mass m_0 in (8) of the Newtonian system S in (5) and

(*ii*) the Einsteinian invariant resultant mass m_0 in (12) of the Einsteinian system S in (9)

we present below two physically significant resulting analogies. These are:

(1) *The Kinetic Energy Theorem* [1, p. 487]: According to this theorem,

$$K = K_0 + K_1\,, \tag{14}$$

where

(*i*) K_0 is the relativistic kinetic energy, relative to a given observer, of a virtual particle located at the relativistic center of mass of the system S in (9), with the Einsteinian resultant mass m_0 in (12); and

(*ii*) K_1 is the relativistic kinetic energy of the constituent particles of S relative to its CM; and

(*iii*) K is the relativistic kinetic energy of S relative to the observer.

The Newtonian counterpart of (14) is well known; see, for instance, [15, Eq. (1.55)]. The Einsteinian analog in (14) was, however, unknown in the literature since the Einsteinian resultant mass m_0 in (12) was unknown in the literature as well till its first appearance in [1]. Accordingly, Oliver D. Johns had to admit for his chagrin that "The reader (of his book; see [15, p. 392]) will be disappointed to learn that relativistic mechanics does not have a theory of collective motion that is as elegant and complete as the one presented in Chapter 1 for Newtonian mechanics."

The proof that m_0 of (12) is compatible with the validity of (14) in Einstein's special theory of relativity is presented in [1, Theorem 11.8, p. 487].

(2) *Additivity of Energy and Momentum*: Classically, energy and momentum are additive, that is, the total energy and the total momentum of a system S of particles is, respectively, the sum of the energy and the sum of momenta of its constituent particles. Consequently,

> also the resultant mass m_0 of S is additive, as shown in (8). Relativistically, energy and momentum remain additive but, consequently, the resultant mass m_0 of S is no longer additive. Rather, it is given by (12), which is the relativistic analog of (8).
>
> The proof that m_0 of (12) is compatible with the additivity of energy and momentum in Einstein's special theory of relativity is presented in [1, pp. 488–491].

Thus, the Einsteinian resultant mass m_0 in (12) of the Einsteinian system S in (9) is the relativistic analog of the Newtonian resultant mass m_0 in (8) of the Newtonian system S in (5). As such, it is the Einsteinian resultant mass m_0 in (12) that is responsible for the extension of the validity of (14) and of the additivity of energy and momentum from classical to relativistic mechanics.

However, classically, mass is additive. Indeed, the Newtonian resultant mass m_0 equals the sum of the masses of the constituent particles, $m_0 = \sum_{k=1}^{N} m_k$, as we see in (8). Relativistically, in contrast, mass is not additive. Indeed, the Einsteinian resultant mass m_0 may exceed the sum of the masses of the constituent particles, $m_0 \geqslant \sum_{k=1}^{N} m_k$, as we see from (12). Accordingly, from the relativistic viewpoint, the resultant mass m_0 in (12) of a galaxy that consists of stars that move relative to each other exceeds the sum of the masses of its constituent stars. This excess of mass reveals its presence only gravitationally and, hence, we identify it as the mass of dark matter. Dark matter is thus virtual matter with positive mass, which reveals its presence only gravitationally. In particular, the dark mass m_{dark} of the Einsteinian system S in (9), given by (16) below, is the mass of virtual matter called the dark matter of S. To contrast the real matter of S with its virtual, dark matter, we call the former *bright* (or, luminous, or, baryonic) matter. The total mass m_0 of S, which can be detected gravitationally, is the composition of the bright mass m_{bright} of the real, bright matter of S, and the dark mass m_{dark} of the virtual, dark matter of S. This mass composition, presented in (15)–(17) in Section 7 below, quantifies the effects of dark matter.

7 The origin of the dark matter

Let

$$m_{bright} = \sum_{k=1}^{N} m_k \tag{15}$$

and

$$m_{dark} = \sqrt{2 \sum_{\substack{j,k=1 \\ j<k}}^{N} m_j m_k (\gamma_{\ominus\mathbf{v}_j \oplus \mathbf{v}_k} - 1)} \tag{16}$$

so that the Einsteinian resultant mass m_0 in (12) turns out to be a composition of an ordinary, bright mass m_{bright} of real matter and a dark mass m_{dark} of virtual matter according to the equation

$$m_0 = \sqrt{m_{bright}^2 + m_{dark}^2} \tag{17}$$

The mass m_{bright} in (15) is the Newtonian resultant mass of the particles of the Einsteinian system S in (9). These particles reveal their presence gravitationally, as well as by radiation that they may emit and by occasional collisions.

In contrast, the mass m_{dark} in (16) is the mass of virtual matter in the Einsteinian system S in (9), which reveals its presence only gravitationally. In particular, it does not emit radiation and it does not collide. As such, it is identified with the dark matter of the Universe.

In our expanding universe, with accelerated expansion [16], relative velocities between some astronomical objects are significantly close to the speed of light c. Accordingly, since gamma factors $\gamma_{\mathbf{v}}$ approach ∞ when their relative velocities $\mathbf{v} \in \mathbb{R}_c^3$ approach the speed of light, it follows from (16) that dark matter contributes an increasingly significant part of the mass of the universe.

8 The origin of the dark energy

Under different circumstances dark matter may appear or disappear resulting in gravitational attraction or repulsion. Dark matter increases the gravitational attraction of the region of each stellar explosion, a supernova, since any stellar explosion creates relative speeds between objects that were at rest relative to each other prior to the explosion. The resulting generated relative speeds increase the dark mass of the region, thus increasing its gravitational attraction. Similarly, relative speeds of objects that converge into a star vanish in the process of star formation, resulting in the decrease of the dark mass of a star formation region. This, in turn, decreases the gravitational attraction or, equivalently, increases the gravitational repulsion of any star formation inflated region. The increased gravitational repulsion associated with star formation results in the accelerated expansion of the universe, first observed in 1998; see [6, p. 1764], [17] and [18, 19]. Thus, according to the present special relativistic dark matter/energy model, the universe accelerated expansion is a late time cosmic acceleration that began at the time of star formation.

9 The origin of the Pioneer anomaly

The Einsteinian resultant mass m_0 of our Solar system is given by the composition (17) of the bright mass m_{bright} and the dark mass m_{dark} of the Solar system. The bright mass m_{bright} of the Solar system equals the sum of the Newtonian masses of the constituents of the Solar system. Clearly, it is time independent. In contrast, the dark mass m_{dark} of the Solar system stems from the speeds of the constituents of the Solar system relative to each other and, as such, it is time dependent.

The Pioneer 10 and 11 spacecrafts and other deep space missions have revealed an anomalous acceleration known as the Pioneer anomaly [7, 8]. The Pioneer anomaly, described in the introductory section, results from an unmodelled acceleration, which is a small constant acceleration on top of which there is a smaller time dependent acceleration. A brief summary of the Pioneer anomaly is presented by K. Tangen, who asks in the title of [20]: "Could the Pioneer anomaly have a gravitational origin?"

Our answer to Tangen's question is affirmative. Our dark matter/energy model, governed by the Einsteinian resultant mass m_0 in (15) – (17), offers a simple, elegant model that explains the Pioneer anomaly. The motion of any spacecraft in deep space beyond the Solar system is determined by the Newtonian law of gravity where the mass of the Solar system is modelled by the Einsteinian resultant mass m_0 in (17) rather than by the Newtonian resultant mass m_0 in (8). It is the contribution of the dark mass m_{dark} to the Einsteinian resultant mass m_0 in (15) – (17) that generates the Pioneer anomaly.

Ultimately, our dark matter/energy model, as dictated by the paradoxical Einsteinian resultant mass m_0 in (12), will be judged by how well the model accords with astrophysical and astronomical observations. Since our model is special relativistic, only uniform velocities are allowed. Hence, the model can be applied to the solar system, for instance, under the assumption that, momentarily, the solar system can be viewed as a system the constituents of which move uniformly.

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A Critical Analysis of Universality and Kirchhoff's Law: A Return to Stewart's Law of Thermal Emission

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It has been advanced, on experimental (P.-M. Robitaille, *IEEE Trans. Plasma Sci.*, 2003, v. 31(6), 1263–1267) and theoretical (P.-M. Robitaille, *Progr. Phys.*, 2006, v. 2, 22–23) grounds, that blackbody radiation is not universal and remains closely linked to the emission of graphite and soot. In order to strengthen such claims, a conceptual analysis of the proofs for universality is presented. This treatment reveals that Gustav Robert Kirchhoff has not properly considered the combined effects of absorption, reflection, and the directional nature of emission in real materials. In one instance, this leads to an unintended movement away from thermal equilibrium within cavities. Using equilibrium arguments, it is demonstrated that the radiation within perfectly reflecting or arbitrary cavities does not necessarily correspond to that emitted by a blackbody.

1 Introduction

Formulated in 1858, Stewart's Law [1] states that when an object is studied in thermal equilibrium, its absorption is equal to its emission [1]. Stewart's formulation leads to the realization that the emissive power of any object depends on its temperature, its nature, and on the frequency of observation. Conversely, Gustav Kirchhoff [2–4] reaches the conclusion that the emissive power of a body is equal to a universal function, dependent only on its temperature and the frequency of interest, and independent of its nature and that of the enclosure. He writes: "*When a space is surrounded by bodies of the same temperature, and no rays can penetrate through these bodies, every pencil in the interior of the space is so constituted, with respect to its quality and intensity, as if it proceeded from a perfectly black body of the same temperature, and is therefore independent of the nature and form of the bodies, and only determined by the temperature*" (see [4], p. 96–97).

At the same time, Max Planck, in his *Theory of Heat Radiation*, reminds us that: "*. . . in a vacuum bounded by totally reflecting walls any state of radiation may persist*" (see [5], §51). Planck is aware that a perfect reflector does not necessarily produce blackbody radiation in the absence of a perfect absorber [6]. It is not simply a matter of waiting a sufficient amount of time, but rather the radiation will "*persist*" in a non-blackbody, or arbitrary, state. Planck re-emphasizes this aspect when he writes: "*Every state of radiation brought about by such a process is perfectly stationary and can continue infinitely long, subject, however, to the condition that no trace of an emitting or absorbing substance exists in the radiation space. For otherwise, according to Sec. 51, the distribution of energy would, in the course of time, change through the releasing action of the substance irreversibly, i.e., with an increase of the total entropy, into the stable distribution corresponding to black radiation*" (see [5], §91). Planck suggests that if an absorbing substance is present, blackbody radiation is produced. Such a statement is not supported scientifically. In fact, a perfect absorber, such as graphite or soot, is required [6–8].

Recently, I have stated [6–8] that cavity radiation was not universal and could only assume the normal distribution (i.e. that of the blackbody) when either the walls of the cavity, or the objects it contains, were perfectly absorbing. These ideas are contrary to the expressed beliefs of Kirchhoff and Planck. Therefore, they deserve further exposition by revisiting Kirchhoff's basis for universality. In combination with a historical review of blackbody radiation [8], such an analysis demonstrates that claims of universality were never justified [6–8].

2.1 Kirchhoff's first treatment of his law

Kirchhoff's first presentation of his law [2] involved two plates, C and c, placed before one another (see Fig. 1). Neither plate was perfectly absorbing, or black. Behind each plate, there were mirrors, R and r, which ensured that all the radiation remained between the plates. Kirchhoff assumed that one of the plates, c, was made of a special material which absorbed only one wavelength and transmitted all others. This assumption appears to have formed the grounds for the most strenuous objections relative to Kirchhoff's first derivation [9–11]. Kirchhoff moved to insist (see [9] for a treatment in English) that, under these conditions, at a certain temperature and wavelength, all bodies had the same ratio of emissive and absorptive powers.

The fallacy with Kirchhoff's argument lays not only in the need for a special material in the second plate, c, as so many have hinted [9–11]. The most serious error was that he did not consider the reflection from the plates themselves. He treated

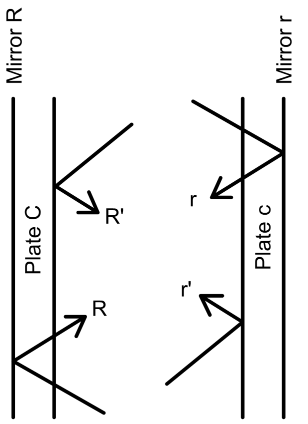

Fig. 1: Schematic representation of Kirchhoff's first proof [2]. C and c represented objects of a specified nature (see text). R and r corresponded to perfectly reflecting mirrors. Note that Kirchhoff had neglected the reflection from the surfaces of C and c denoted as R' and r'.

the reflection as coming only from the mirrors placed behind the plates. But this dealt with the problem of transmission, not reflection. As a result, Kirchhoff ignored the reflection produced by the surfaces of the plates.

The total radiation leaving from the surface of each plate, given thermal equilibrium, is obtained, not only by its emission, E (or e), but rather by the *sum* of its emission, E (or e), and reflection, R' (or r'). It is only when the plates are black that surface reflection can be neglected. Consequently, if Kirchhoff insists that surface reflection itself need not be addressed ($R' = r' = 0$), he simply proves that the ratio of emission to absorption is the same for all blackbodies, not for all bodies. The entire argument, therefore, is flawed because Kirchhoff ignored the surface reflection of each plate, and is considering all reflection as originating from the perfectly reflecting mirrors behind the plates. A proper treatment would not lead to universality, since the total radiation from plate C was $E + R'$ not simply E, where R' denotes the reflection from surface C (see Fig. 1). Similarly, the total radiation from plate c was $e + r'$, not simply e, where r' denotes the reflection from surface c. The mirrors, R and r, are actually dealing only with transmission through plates C and c. The conceptual difficulty when reviewing this work is that Kirchhoff apparently treats reflection, since mirrors are present. In fact, he dismisses the issue. The mirrors cannot treat the reflection off the surfaces of C and c. They deal with transmission. Kirchhoff's incorrect visualization of the effect of reflection is also a factor in his second proof.

2.2 Kirchhoff's second treatment of his law

Kirchhoff's second treatment of his law [3, 4] is much more interesting conceptually and any error will consequently be more difficult to locate. The proof is complex, a reality recognized by Stewart in his *Reply*: "*I may remark, however, that the proof of the Heidelburg Professor is so very elaborate that I fear it has found few readers either in his own country or in this*" [12].

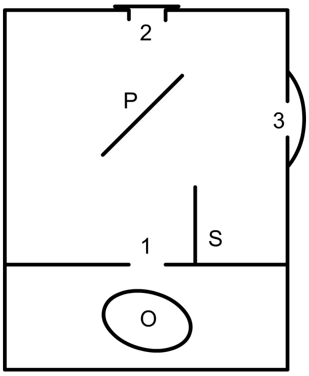

Fig. 2: Schematic representation of Kirchhoff's second proof [3, 4]. The cavity contained three openings, labeled 1, 2, and 3. There was also a plate, P, which was perfectly transmitting for the frequency and polarization of interest, and perfectly reflecting for all others. While the existence of such a plate can be the source of objections relative to Kirchhoff's proof [10], the discussion in this work does not center on the nature of the plate. Idealized objects can be assumed as valid as they represent (more or less) mathematical extensions of physical observations (see text). A black screen, S, was used to prevent radiation from traveling directly between openings 1 and 3. An object, which was either perfectly absorbing or arbitrary, was placed in the enclosure located behind opening 1. The key to Kirchhoff's proof relied on rapidly changing the covering of opening 3, from a perfect concave mirror to a perfectly absorbing surface. In Kirchhoff's initial presentation, the entire cavity was perfectly absorbing [3, 4]. However, Kirchhoff extended his result to be independent of the nature of the walls, making it acceptable to consider the entire cavity as perfectly reflecting (see text).

Kirchhoff began by imagining a cavity whose walls were perfectly absorbing (see Fig. 2). In the rear of the cavity was an enclosure wherein the objects of interest were placed. There were three openings in the cavity, labeled 1, 2, and 3. He conceived that openings 2 and 3 could each be sealed with a perfectly absorbing surface. As a result, when Kirchhoff did this, he placed his object in a perfectly absorbing cavity [6]. He eventually stipulated that the experiment was independent of the nature of the walls, in which case the cavity could be viewed as perfectly reflecting [6]. Yet, as has been previously highlighted [6], the scenario with the perfectly reflecting cavity required, according to Planck, the introduction of a minute particle of carbon [5, 8]. Hence, I have argued that Kirchhoff's analysis was invalid on this basis alone [6]. By carefully considering Kirchhoff's theoretical constructs, the arguments against blackbody radiation, within a perfect reflecting enclosure, can now be made from a slightly different perspective.

Kirchhoff's analysis of his cavity (see Fig. 2) was ingenious. He set strict conditions for the positions of the walls which linked the openings 1 and 2, and which contained opening 3. The key was in the manner wherein opening 3 was handled. Kirchhoff permitted opening 3 to be covered either with a perfect absorber or with a perfect concave mirror. He then assumed that equilibrium existed in the cavity and that he could instantaneously change the covering at opening 3. Since equilibrium was always preserved, Kirchhoff could then treat the rays within the cavity under these two different conditions and, hence, infer the nature of the radiation within the cavity at equilibrium.

Kirchhoff initially demonstrated that, if the enclosed object and the cavity were perfectly absorbing, the radiation was denoted by the universal function of blackbody radiation. He then replaced the object with an arbitrary one, and concluded, once again, that the radiation was black. Kirchhoff's presentation was elegant, at least when the cavity was perfectly absorbing. The Heidelburg Professor extended his findings to make them independent of the nature of the walls of the enclosure, stating that the derivation was valid, even if the walls were perfectly reflecting. He argued that the radiation within the cavity remained blackbody radiation. Let us revisit what Kirchhoff had done.

Since the walls can be perfectly reflecting, this state is adopted for our analysis. Opening 3 can once again be covered, either by a concave mirror or by a perfectly absorbing surface. An arbitrary object, which is not a blackbody, is placed in the cavity. The experiment is initiated with the perfect concave mirror covering opening 3. As shown in Section 3.1.2, under these conditions, the cavity contains radiation whose nature depends not on the cavity, but on the object. This radiation, in fact, is not black. This can be seen, if the object was taken as perfectly reflecting. The arbitrary radiation is weaker at all frequencies. Thus, when an arbitrary object is placed in the enclosure, the intensity of the radiation within the cavity, at any given frequency, does not correspond to that predicted by the Planckian function (see Section 3.1.2). However, when opening 3 is covered by a perfectly absorbing substance, the radiation in the cavity becomes black (see Sections 3.1.2 and 3.2). The emission from the object is that which the object emits and which it reflects. The latter originates from the surface of opening 3 (see Section 3.2). When the perfect absorber is placed over opening 3, the entire cavity appears to hold blackbody radiation. Therefore, by extending his treatment to the perfect reflector, Kirchhoff is inadvertently jumping from one form of cavity radiation (case 1: the concave mirror, object radiation) to another (case 2: the perfect absorber, blackbody radiation) when the covering on opening 3 is changed. At that moment, the cavity moves out of equilibrium.

Thus, Kirchhoff's proof is invalid. This is provided, of course, that the test began with the perfect concave mirror covering opening 3. Only under these circumstances would Kirchhoff's proof fail. Nonetheless, the experimental proof cannot be subject to the order in which manipulations are executed. This is because the validity of equilibrium arguments is being tested. Consequently, nothing is independent of the nature of the walls. This is the lesson provided to us by Balfour Stewart in his treatise when he analyzes radiation in a cavity temporarily brought into contact with another cavity [8]. Dynamic changes, not equilibrium, can be produced in cavities, if reflectors are used. This is the central error relative to Kirchhoff's second attempt at universality [3, 4].

There are additional minor problems in Kirchhoff's presentation [3, 4]. In §13 of his proof [3, 4], Kirchhoff is examining an arbitrary object within a perfectly absorbing cavity. It is true that the resultant cavity radiation will correspond to a blackbody, precisely because the walls are perfectly absorbing (see Section 3.1.1). However, Kirchhoff states: "*the law §3 is proved under the assumption that, of the pencil which falls from surface 2 through opening 1 upon the body C, no finite part is reflected by this back to the surface 2; further, that the law holds without limitation, if we consider that when the condition is not fulfilled, it is only necessary to turn the body C infinitely little in order to satisfy it, and that by such a rotation the quantities E and A undergo only and [sic] infinitely small change*" (see [4], p. 92). Of course, real bodies can have diffuse reflection. In addition, rotation does not ensure that reflection back to surface 2 will not take place. Real bodies also have directional spectral emission, such that the effect of rotation on E and A is not necessarily negligible. These complications are of little significance within a perfectly absorbing cavity. The radiation within such enclosures is always black (see Section 3.1.1). Conversely, the problems cannot be dismissed in the perfect reflector and the entire proof for universality, once again, is invalid.

For much of the 19th century, the understanding of blackbody radiation changed little, even to the time of Planck [11]. No laboratory proof of Kirchhoff's Law was ever produced, precisely because universality could not hold. Only theoretical arguments prevailed [10]. Yet, such findings cannot form the basis for a law of physics. Laws stem from experiments and are fortified by theory. They are not born *de novo*, using mathematics without further validation. It is not possible to ensure that black radiation exists, within a perfectly reflecting cavity, without recourse at least to a carbon particle [6, 8]. In fact, this is the route which Planck utilized in treating Kirchhoff's Law [5, 8].

3 Thermal equilibrium in cavities

A simple mathematical treatment of radiation, under conditions of thermal equilibrium, begins by examining the fate of the total incoming radiation, Γ, which strikes the surface of an object. The various portions of this radiation are either absorbed (A), reflected (R), or transmitted (T) by the object. If normalized, the sum of the absorbed, reflected, or transmitted radiation is equal to $\alpha + \rho + \tau = 1$. Here, absorptivity, α, corresponds to the absorbed part of the incoming radiation/total incoming radiation. Similarly, the reflectivity, ρ, is the reflected part of the incoming radiation/total incoming radiation. Finally, the transmissivity, τ, involves the transmitted part of the incoming radiation/total incoming radiation. If all objects under consideration are fully opaque, then $1 = \alpha + \rho$.

Stewart's Law [1] states that, under conditions of thermal equilibrium, the ability of an object to absorb light, α, is exactly equal to its ability to emit light, ε. Nonetheless, for this presentation, Stewart's Law is not assumed to be valid [1]. The question arises only in the final Section 4.2, when two objects are placed within a perfectly reflecting cavity. Emissivity, ε, is standardized relative to lamp-black [8] and, for such a blackbody, it is equal to 1. For a perfect reflector, the emissivity, ε, is 0. All other objects hold values of emissivity between these two extremes. If thermal equilibrium is not established, then ε and α are not necessarily equal [8].

If a cubical cavity is considered with walls P^1, P^2, P^3, P^4, P^5 (top surface), and P^6 (bottom surface), the following can be concluded at thermal equilibrium: since P^1 and P^3 are equal in area and opposite one another, then the total radiation from these walls must be balanced, $\Gamma_{p1} - \Gamma_{p3} = 0$. Similarly, $\Gamma_{p2} - \Gamma_{p4} = 0$ and $\Gamma_{p5} - \Gamma_{p6} = 0$. As such, $\Gamma_{p1} = \Gamma_{p3}$ and $\Gamma_{p2} = \Gamma_{p4}$. If one considers pairs of adjacent walls, then $(\Gamma_{p1} + \Gamma_{p2}) - (\Gamma_{p3} + \Gamma_{p4}) = 0$. It is possible to conclude that $\Gamma_{p1} = \Gamma_{p2} = \Gamma_{p3} = \Gamma_{p4}$ and, using symmetry, it can finally be concluded that $\Gamma_{p1} = \Gamma_{p2} = \Gamma_{p3} = \Gamma_{p4} = \Gamma_{p5} = \Gamma_{p6}$. Consequently, with normalization, $\Gamma_c = \frac{1}{6}(\Gamma_{p1} + \Gamma_{p2} + \Gamma_{p3} + \Gamma_{p4} + \Gamma_{p5} + \Gamma_{p6})$. For an opaque cavity, the total radiation coming from the cavity, Γ_T, is given by $\Gamma_T = \varepsilon_c\Gamma_c + \rho_c\Gamma_c = \varepsilon_c\Gamma_c + (1 - \alpha_c)\Gamma_c$. This states that the total emission from the cavity must be represented by the sum of its internal emission and reflection. If the cavity is constructed from perfectly absorbing walls, $\alpha_c = 1$, $\rho_c = 0$, yielding $\Gamma_T = \varepsilon_c\Gamma_c$. The cavity is black and ε_c must now equal 1, by necessity. Stewart's Law [1] has now been proved for blackbodies. If the cavity is made from perfectly reflecting walls, at thermal equilibrium, $\varepsilon_c\Gamma_c + (1 - \alpha_c)\Gamma_c = 0$. There is also no source of radiation inside the cavity ($\varepsilon_c = 0$) and $(1 - \alpha_c)\Gamma_c = 0$, leading explicitly to $\Gamma_c = 0$. Because $\Gamma_c = 0$, the total radiation monitored $\Gamma_T = \varepsilon_c\Gamma_c + \rho_c\Gamma_c = 0$.

These conclusions can be extended to perfectly absorbing and reflecting cavities of rectangular (or arbitrary) shapes. The central point is that a perfectly reflecting cavity can sustain no radiation, a first hint that universality cannot be valid. Planck only obtains blackbody radiation, in such cavities, by invoking the action of a carbon particle [6, 8]. This special case will be treated in Sections 3.1.1 and 3.2.

3.1 An object in a perfect cavity

At thermal equilibrium, the total emission from the surface of the object, Γ_{so}, is equal to that from the surface of the cavity, Γ_{sc}. When normalizing, the total emission, Γ_T, will therefore be as follows: $\Gamma_T = \frac{1}{2}\Gamma_{so} + \frac{1}{2}\Gamma_{sc}$. The total radiation from the surface of the object is equal to that which it emits plus that which it reflects, $\Gamma_{so} = [\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}} + \rho_{\mathrm{o}}\Gamma_c]$, and similarly for the surface of the cavity, $\Gamma_{sc} = [\varepsilon_c\Gamma_c + \rho_c\Gamma_{\mathrm{o}}]$. Therefore, at equilibrium, $[\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}} + \rho_{\mathrm{o}}\Gamma_c] = [\varepsilon_c\Gamma_c + \rho_c\Gamma_{\mathrm{o}}]$ or $\Gamma_{\mathrm{o}}[\varepsilon_{\mathrm{o}} - \rho_c] = \Gamma_c[\varepsilon_c - \rho_{\mathrm{o}}]$. Solving for either Γ_{o} or Γ_c, we obtain that $\Gamma_{\mathrm{o}} = \Gamma_c\frac{[\varepsilon_c - \rho_{\mathrm{o}}]}{[\varepsilon_{\mathrm{o}} - \rho_c]}$ and $\Gamma_c = \Gamma_{\mathrm{o}}\frac{[\varepsilon_{\mathrm{o}} - \rho_c]}{[\varepsilon_c - \rho_{\mathrm{o}}]}$.

3.1.1 An arbitrary object in a perfectly absorbing cavity

In such a case $\varepsilon_c = 1$, $\rho_c = 0$. Since $\Gamma_T = \frac{1}{2}\Gamma_{so} + \frac{1}{2}\Gamma_{sc}$, then $\Gamma_T = \frac{1}{2}\left(\varepsilon_{\mathrm{o}}\Gamma_c\frac{[\varepsilon_c - \rho_{\mathrm{o}}]}{[\varepsilon_{\mathrm{o}} - \rho_c]} + \rho_{\mathrm{o}}\Gamma_c\right) + \frac{1}{2}\left(\varepsilon_c\Gamma_c + \rho_c\Gamma_c\frac{[\varepsilon_c - \rho_{\mathrm{o}}]}{[\varepsilon_{\mathrm{o}} - \rho_c]}\right)$. It is readily shown that $\Gamma_T = \Gamma_c$. Note that no use of Stewart's Law [1] was made in this derivation. In any case, when an object is placed within a cavity, which is perfectly absorbing, the emitted spectrum is independent of the object and depends only on the nature of the cavity. A blackbody spectrum is produced. This was the condition which prevailed over much of the 19th century when cavities were often lined with soot [8]. If the radiation was *independent of the nature of the walls, or of the object*, it was because the walls were coated with this material [8].

3.1.2 An arbitrary object in a perfectly reflecting cavity

In such a case $\varepsilon_c = 0$, $\rho_c = 1$. Since $\Gamma_T = \frac{1}{2}\Gamma_{so} + \frac{1}{2}\Gamma_{sc}$, then $\Gamma_T = \frac{1}{2}\left(\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}} + \rho_{\mathrm{o}}\Gamma_{\mathrm{o}}\frac{[\varepsilon_{\mathrm{o}} - \rho_c]}{[\varepsilon_c - \rho_{\mathrm{o}}]}\right) + \frac{1}{2}\left(\varepsilon_c\Gamma_{\mathrm{o}}\frac{[\varepsilon_{\mathrm{o}} - \rho_c]}{[\varepsilon_c - \rho_{\mathrm{o}}]} + \rho_c\Gamma_{\mathrm{o}}\right)$. It is readily shown that $\Gamma_T = \Gamma_{\mathrm{o}}$. Note, once again, that no use of Stewart's Law [1] was made in this derivation. When an object is placed within a cavity which is perfectly reflecting, the emitted spectrum is determined only by the object and is independent of the nature of the cavity. If the object is perfectly absorbing, like a carbon particle [6, 8], a blackbody spectrum will be obtained. Furthermore, if an arbitrary object is placed within a cavity, which is perfectly reflecting, the emitted spectrum *is dependent only on the nature of the object*. One observes *object radiation*, not blackbody radiation, because the object was never black *a priori*. This is the condition which Kirchhoff has failed to realize when he extended his treatment to be independent of the nature of the walls in his 1860 proof [3, 4], as seen in Section 2.

3.1.3 An arbitrary object in an arbitrary cavity

Consider such a general case. Since $\Gamma_T = \frac{1}{2}\Gamma_{so} + \frac{1}{2}\Gamma_{sc}$, then $\Gamma_T = \frac{1}{2}\left(\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}} + \rho_{\mathrm{o}}\Gamma_{\mathrm{o}}\frac{[\varepsilon_{\mathrm{o}} - \rho_c]}{[\varepsilon_c - \rho_{\mathrm{o}}]}\right) + \frac{1}{2}\left(\varepsilon_c\Gamma_{\mathrm{o}}\frac{[\varepsilon_{\mathrm{o}} - \rho_c]}{[\varepsilon_c - \rho_{\mathrm{o}}]} + \rho_c\Gamma_{\mathrm{o}}\right)$ or alternatively, we have $\Gamma_T = \frac{1}{2}\left(\varepsilon_{\mathrm{o}}\Gamma_c\frac{[\varepsilon_c - \rho_{\mathrm{o}}]}{[\varepsilon_{\mathrm{o}} - \rho_c]} + \rho_{\mathrm{o}}\Gamma_c\right) + \frac{1}{2}\left(\varepsilon_c\Gamma_c + \rho_c\Gamma_c\frac{[\varepsilon_c - \rho_{\mathrm{o}}]}{[\varepsilon_{\mathrm{o}} - \rho_c]}\right)$. In this case, the expressions cannot be further simplified and the initial form, $\Gamma_T = \frac{1}{2}\Gamma_{so} + \frac{1}{2}\Gamma_{sc}$, can be maintained. Therefore, the total radiation emitted from such a cavity is a mixture *depending on both the characteristics of the object and the walls of the cavity.*

This highlights that cavities do not always contain black radiation and that universality is invalid [6–8].

3.2 An arbitrary object and a carbon particle in a perfectly reflecting cavity

If thermal equilibrium exists between an opaque object, o, a carbon particle, p, and a cavity, c, then $[\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}}+\rho_{\mathrm{o}}\Gamma_p+\rho_{\mathrm{o}}\Gamma_c]-$ $-[\varepsilon_p\Gamma_p+\rho_p\Gamma_{\mathrm{o}}+\rho_p\Gamma_c]+[\varepsilon_c\Gamma_c+\rho_c\Gamma_{\mathrm{o}}-\rho_c\Gamma_p]=0$. Since the cavity is perfectly reflecting, $\Gamma_c=0$, $\varepsilon_c=0$, and $\rho_c=1$, yielding, $\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}}+\rho_{\mathrm{o}}\Gamma_P-\varepsilon_p\Gamma_p-\rho_p\Gamma_{\mathrm{o}}+\Gamma_{\mathrm{o}}-\Gamma_p=0$, and with rearrangement, $(\varepsilon_{\mathrm{o}}+\rho_{\mathrm{o}}-1)\Gamma_p-\varepsilon_p\Gamma_p+(1-\rho_p)\Gamma_{\mathrm{o}}=0$. If we take Stewart's Law ($\varepsilon_p=\alpha_p$; $\varepsilon_{\mathrm{o}}=\alpha_{\mathrm{o}}$) as valid [1], we can see that $\varepsilon_{\mathrm{o}}+\rho_{\mathrm{o}}=1$, and then $(1-\rho_p)\Gamma_{\mathrm{o}}=\varepsilon_p\Gamma_p$, leading directly to $\Gamma_{\mathrm{o}}=\Gamma_p$. Alternatively, we may notice that, by definition, $\rho_{\mathrm{o}}=1-\alpha_{\mathrm{o}}$ and $\rho_p=1-\alpha_p$, then, $\Gamma_{\mathrm{o}}=\frac{(\varepsilon_p-\varepsilon_{\mathrm{o}}+\alpha_{\mathrm{o}})}{\alpha_p}\Gamma_p$. If we take the particle to be black, we can simplify to $\Gamma_{\mathrm{o}}=(1-\varepsilon_{\mathrm{o}}+\alpha_{\mathrm{o}})\Gamma_p$. Therefore, if we then observe the radiation in the cavity and find it to be black, since the particle is also black, Stewart's law is verified. This is because Γ_{o} will be black and equal to Γ_p only when $\varepsilon_{\mathrm{o}}=\alpha_{\mathrm{o}}$.

The problem can be examined from a slightly different angle in order to yield a little more insight, but the same conclusions hold. Because the objects are in a perfect reflector, then the radiation coming off their surfaces can be expressed as $\Gamma_{so}=\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}}+\rho_{\mathrm{o}}\Gamma_p$ and $\Gamma_{sp}=\varepsilon_p\Gamma_p+\rho_p\Gamma_{\mathrm{o}}$. Given thermal equilibrium, the production of radiation from each object must be equal, $\Gamma_{so}=\Gamma_{sp}$, and thus $\varepsilon_{\mathrm{o}}\Gamma_{\mathrm{o}}+\rho_{\mathrm{o}}\Gamma_p=$ $=\varepsilon_p\Gamma_p+\rho_p\Gamma_{\mathrm{o}}$. Consequently, $\Gamma_{\mathrm{o}}=\frac{[\varepsilon_p-\rho_{\mathrm{o}}]}{[\varepsilon_{\mathrm{o}}-\rho_p]}\Gamma_p$ (see Section 3.1). If the particle is black, $\varepsilon_p=1$ and $\rho_p=0$, and $\Gamma_{\mathrm{o}}=\frac{(1-\rho_{\mathrm{o}})}{\varepsilon_{\mathrm{o}}}\Gamma_p$. As a result of thermal equilibrium, the object must be producing a total emission which appears black in nature. Γ_{o} must equal Γ_p. All solutions involve $\rho_{\mathrm{o}}+\varepsilon_{\mathrm{o}}=1$, which as stated above, is a proof of Stewart's Law ($\varepsilon_{\mathrm{o}}=\alpha_{\mathrm{o}}$). The object takes the appearance of a blackbody through the sum of its emission and reflection. The presence of completely black radiation within a cavity filled in this manner constitutes an explicit verification of Stewart's Law [1], as mentioned above. Since such cavities are known to be black, Stewart's Law has been proven. In fact, we have returned to the first portion of Section 3.1.2. The effect is the same as if the walls of the cavity were perfectly absorbing. This is the point Planck failed to realize when he placed the carbon particle within the perfectly reflecting cavity and gave it a catalytic function [5, 6, 8].

4 Conclusions

Nearly 150 years have now passed since Gustav Robert Kirchhoff first advanced his Law of Thermal Radiation. Kirchhoff's Law [2–4] was far reaching. Its universal nature had a profound effect on the scientists of the period. At the time, many of these men were trying to discover the most general laws of nature. Hence, the concept of universality had great appeal and became ingrained in the physics literature. As a result, Kirchhoff's Law has endured, despite controversy [10], until this day. Recently, I have questioned universality [6, 7]. It is doubtful that Kirchhoff's Law can long survive the careful discernment of those physicists who wish to further pursue this issue.

At the same time, Kirchhoff's Law seems inseparably tied to Max Planck's equation [13]. As such, could a reevaluation of Kirchhoff's ideas compromise those of Max Planck [13]? In the end, it is clear that this cannot be the case [8]. Planck's solution to the blackbody problem remains valid for cavities which are perfectly absorbing. Thus, physics loses nothing of the Planck and Boltzmann constants, h and k, which were born from the study of heat radiation [1, 8]. That blackbody radiation loses universal significance also changes nothing, in fact, relative to the mathematical foundations of quantum theory. However, the same cannot be said relative to experimental findings [8]. In the end, the physics community may well be led to reconsider some of these positions [8].

Balfour Stewart [1] preceded Kirchhoff [2–4] by nearly two years in demonstrating, under equilibrium, the equality between absorptivity and emissivity. Stewart's treatment, unlike Kirchhoff's, does not lead to universality [1, 8, 9, 14] but, rather, shows that the emissive power of an object is dependent on its nature, its temperature, and the frequency of observation. This is true even within cavities, provided that they do not contain a perfect absorber. It is only in this special circumstance that the nature of the object is eliminated from the problem. Yet, this is only because the nature of the carbon itself controls the situation. Stewart also properly treats emission and reflection in his *Treatise* [14]. Despite popular belief to the contrary [9], Stewart's interpretation is the correct solution. Conversely, Kirchhoff's formulation, not only introduced error, but provided justification for setting temperatures inappropriately. I have repeatedly expressed concern in this area [6–8]. It can be argued that Stewart's analysis lacked mathematical sophistication [9]. Stewart himself [12] counters the point [8]. Nonetheless, it is doubtful that the important consequences of Stewart's work can continue to be ignored. Justice and the proper treatment of experimental data demand otherwise.

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Dedication

This work is dedicated to the memory of my beloved mother, Jacqueline Alice Roy (May 12, 1935 – December 02, 1996).

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Blackbody Radiation and the Carbon Particle

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Since the days of Kirchhoff, blackbody radiation has been considered to be a universal process, independent of the nature and shape of the emitter. Nonetheless, in promoting this concept, Kirchhoff did require, at the minimum, thermal equilibrium with an enclosure. Recently, the author stated (P.-M. Robitaille, *IEEE Trans. Plasma Sci.*, 2003, v. 31(6), 1263–1267; P.-M. Robitaille, *Progr. in Phys.*, 2006, v. 2, 22–23), that blackbody radiation is not universal and has called for a return to Stewart's law (P.-M. Robitaille, *Progr. in Phys.*, 2008, v. 3, 30–35). In this work, a historical analysis of thermal radiation is presented. It is demonstrated that soot, or lampblack, was the standard for blackbody experiments throughout the 1800s. Furthermore, graphite and carbon black continue to play a central role in the construction of blackbody cavities. The advent of universality is reviewed through the writings of Pierre Prévost, Pierre Louis Dulong, Alexis Thérèse Petit, Jean Baptiste Joseph Fourier, Siméon Denis Poisson, Frédérick Hervé de la Provostaye, Paul Quentin Desain, Balfour Stewart, Gustav Robert Kirchhoff, and Max Karl Ernst Ludwig Planck. These writings illustrate that blackbody radiation, as experimentally produced in cavities and as discussed theoretically, has remained dependent on thermal equilibrium with at least the smallest carbon particle. Finally, Planck's treatment of Kirchhoff's law is examined in detail and the shortcomings of his derivation are outlined. It is shown once again, that universality does not exist. Only Stewart's law of thermal emission, not Kirchhoff's, is fully valid.

1 Introduction

If real knowledge is to be derived from an equation, it is often necessary to reassess the experiments that gave it life. A thorough evaluation of these developments, relative to Planck's equation [1, 2], can be found in Hans Kangro's *Early History of Planck's Radiation Law* [3]. Kangro reminds us of the need to study important milestones relative to physical ideas: "*Only concern with details appearing in sources reveals — often unexpectedly — what has really happened historically, and allowed something to be divined from that history as to 'how it really happened'*" [3; p. 3]. He then sets forth a fascinating account of the history of the law [1, 2] which gave birth to modern physics. Kangro's work [3] is unique for its balance relative to experimental methods and theoretical foundations. It covers, in considerable detail, the period from Kirchhoff to Planck [3]. Hoffmann's work [4] is also valuable since it is short, well written, and reviews the experiments from which Planck formulated his equation [1, 2]. Kuhn's text [5] centers on the theoretical basis of Planck's law. It has been the subject of substantial justified criticism, primarily for advancing that Planck was not the first to introduce quantized processes [6–8]. It is by using such works, and the collection of the scientific literature, that we may revisit the days of Planck [9–16] and judge, with perhaps greater insight than our forefathers, the soundness of the claims on which universality in blackbody radiation rests.

At the onset, it should be emphasized that the validity of Planck's equation [1, 2], as a mathematical solution to the blackbody problem, is not being disputed in any way. The accuracy and merit of Planck's equation [1, 2] has been established beyond question. Nonetheless, two aspects of Planck's formulation are being brought to the forefront. First, that Planck [1, 2, 9–16], Einstein [17, 18], and all of physics have yet to ascribe a direct physical process for the production of blackbody radiation [19]. That is to say, blackbody radiation remains unlinked to a specific and identifiable physical entity (such as the nucleus, the electron, etc). Second, that blackbody radiation is not universal, contrary to what Kirchhoff has concluded [20–22] and Planck believed [1, 2, 9].

I have previously stated that Kirchhoff's law [20–22], and, as a necessary result, Planck's law [1, 2] and blackbody radiation, are not universal in nature [23–25]. Kirchhoff's conclusions hold only for objects in thermal equilibrium with a perfectly absorbing enclosure [23]. Under these conditions, Kirchhoff's cavities act, in essence, as transformers of light [23]. Any object placed within them will give a total emission which is the sum of its own emission and the reflection of the emission from the cavity wall. Consequently, the entire cavity appears black [23, 25]. Outside the restrictions imposed by such a cavity, universality does not exist [23–25]. As for Kirchhoff's law, it holds only under very limited experimental conditions: the walls of these cavities, or the objects they contain, must be perfectly absorbing (see [25] for a proof).

Otherwise, Kirchhoff's law in its widest sense (i.e. universality) does not hold [23]. However, that section of Kirchhoff's law specifically addressing the equality between emissivity and absorptivity at equilibrium is valid. This is Stewart's law [26], not Kirchhoff's [20–22], as will be seen below.

In Planck's words (see [9; §44]), Kirchhoff's law of thermal emission holds that: "*With these assumptions, according to equations (46), (45), and (43), Kirchhoff's law holds,* $E/A = I = d\sigma \cos\theta \, d\Omega \, K_\nu \, d\nu$, *i.e., the ratio of the emissive power to the absorbing power of any body is independent of the nature of the body*". The implications of Kirchhoff's law are best summarized in the words of its originator: "*When a space is surrounded by bodies of the same temperature, and no rays can penetrate through these bodies, every pencil in the interior of the space is so constituted, with respect to its quality and intensity, as if it proceeded from a perfectly black body of the same temperature, and is therefore independent of the nature and form of the bodies, and only determined by the temperature… In the interior of an opaque glowing hollow body of given temperature there is, consequently, always the same brightness whatever its nature may be in other respects*" [22; §17]. Kirchhoff's law states that, for all bodies, the ratio of emissive to absorbing power is a function of only wavelength and temperature, given thermal equilibrium with an enclosure. All that Kirchhoff knew about his universal function, in 1859, was that its value was zero in the visible range at low temperatures, non-zero at high temperatures, and non-zero at the longer wavelengths at all temperatures [3; p. 7]. Planck [1, 2], in 1900, eventually defined the function on the right side of Kirchhoff's law [20–22].

Given thermal equilibrium within an enclosure, Kirchhoff's law [20–22] states that the ability of an object to emit a photon is equal to its ability to absorb one. This aspect of Kirchhoff's work [20–22], properly called Stewart's law [25, 26], is not being questioned. If equilibrium holds, the equality between emissivity and absorptivity has been experimentally demonstrated (see [25] for a complete discussion). It is only when objects are permitted to radiate freely, that equality may fail. Discussions on this issue have been published [27–29]. It has been argued that the equality between absorptivity and emissivity may, in fact, still be applicable for freely radiating bodies, provided that "*the distribution over material states is the equilibrium condition*" [27]. At the same time, it should be realized that, under all non-equilibrium conditions, these laws collapse [20–22, 25, 26].

The vast experimental knowledge relative to thermal emission reveals that virtually all materials fall far short of exhibiting blackbody behavior. Yet, Max Thiesen, a pupil of Kirchhoff, in 1900 stated that: "*we have become accustomed to treat radiation independently of the emitting body*" and therefore, this radiation should "*be designated simply as black radiation*" [3; p. 184]. Experimental reality illustrates that nothing in nature behaves like a blackbody. Kirchhoff's statement that: "*In the interior of an opaque glowing hollow body of given temperature there is, consequently, always the same brightness whatever its nature may be in other respects*" [22; Brace, p. 97] is incorrect without much further consideration. Even graphite and soot produce the desired result only over a limited range of conditions. It remains true that "*different bodies … radiate different kinds of heat*" as published in the first issue of *Nature* in 1869 [30]. An examination of thermal emissivity plots is sufficient to confirm these statements [31]. Not a single object in nature is a blackbody. Hence, it is reasonable to wonder why this concept has so captivated physics. In studying blackbody radiation, it will be demonstrated that radiation within an enclosed body is not necessarily black [25], as Kirchhoff's law erroneously dictates [20–22].

If this subject matter remains important after all these years [1, 2, 20–22], it is because so much of physics, and more specifically astrophysics, is tied to the concept of universality in blackbody radiation. Agassi highlights the importance of Kirchhoff's law for astrophysics: "*Browsing through the literature, one may find an occasional use of Kirchhoff's law in some experimental physics, but the only place where it is treated at all seriously today is in the astrophysical literature*" [32]. As a result, in astrophysics, if a thermal spectrum is observed which displays, or even approximates, a Planckian (or normal) distribution, temperatures are immediately inferred. For this reason, the fall of universality heralds, in the most profound and far-reaching manner, a new dawn in this sub-discipline. Should universality be reconsidered, there are significant consequences for our models of the Sun and relative to the temperatures of the stars [33–35]. The validity of the ~ 3 K microwave background temperature would be questioned [36–41] and with it, perhaps, the entire framework of cosmology [33, 42]. Kirchhoff's law of thermal emission [20–22] may well be the simplest law in physics, but it is clear that, upon its validity, rests the very foundation of modern astrophysics.

Given these facts, it is unusual that Planck has advanced an equation [1, 2] which remains unlinked to any real physical process or object. Sadly, it is somewhat as a result of Kirchhoff's law that Planck remained unable to link his equation to a physical cause. The problem was an extremely serious one for Planck, and the fact that his hands were tied by universality is no more evident than in the helplessness he displays in the following quotation: "*On the contrary, it may just as correctly be said that in all nature there is no process more complicated than the vibrations of black radiation. In particular, these vibrations do not depend in any characteristic manner on the special processes that take place in the centers of emission of the rays, say on the period or the damping of emitting particles; for the normal spectrum is distinguished from all other spectra by the very fact that all individual differences caused by the special nature of the emitting substances are perfectly equalized and effaced. Therefore to attempt to draw conclusions concerning the special properties of the parti-*

cles emitting the rays from the elementary vibrations in the rays of the normal spectrum would be a hopeless undertaking" [9; §111].

Yet, it is primarily universality that makes this task a "hopeless undertaking". Planck, in fact, realized that vibrating atoms, electrons, or particles of some sort, must be responsible for the process of thermal emission. He specifically believed that the answer might be found by studying the electron and devoted much of his life to this topic [5; pp. 133–134, 198–199, 245]. But, unfortunately, Planck never makes the link to a real physical species, and the electron itself is not the proper lone candidate. Planck's belief that the answer lay in electron theory is explicitly contained in his letter to Paul Ehrenfest on July 6, 1905 in which he states: "*But perhaps it is not out of the question to make progress in the following way. If one assumes that resonator oscillations are produced by the motion of electrons…*" [5; p. 132]. Lorenz had already been successful, in deriving the radiation equation for long wavelengths (the Rayleigh-Jeans solution), using the analysis of electrons [5; p. 190].

Surprisingly, the real solution to the blackbody radiation problem has never been discovered [19]. Even Albert Einstein, in 1909, expressed frustration in this regard in a letter to H. A. Lorentz: "*I cherish the hope that you can find the right way, if indeed you find the reasons given in the paper for the untenability of the current foundations to be at all valid. But if you should deem those reasons to be invalid, then your counterarguments could perhaps furnish the key to the real solution of the radiation problem*" [18; p. 105]. The problem was never solved. As late as 1911, Einstein continues to express his frustration to Lorentz: "*I am working on the case of damped resonators; it involves quite a lot of calculation. The case of the electrons in the magnetic field, which I already mentioned in Brussels, is interesting, but not as much as I had thought in Brussels. Electrons in a spatially variable magnetic field are oscillators with variable frequency. If one neglects the radiation, then statistical mechanics yields the distribution law at every location if it is known at one location. If that location is field-free, then Maxwell's distribution holds there; from this one concludes it must hold everywhere. This leads of course to Jean's formula. Nevertheless, to me the thing seems to show that mechanics does not hold even in the case of the electron moving in the magnetic field. I am telling you this as an argument against the view that mechanics ceases to hold at the point where more than two things interact with each other. Anyway, the h-disease looks ever more hopeless*" [18; p. 228]. Blackbody radiation was never linked to a direct physical process. Yet, according to Kuhn, Einstein pointed out that "*not only the vibrations of electrons but also those of charged ions must, contribute to the blackbody problem*" [5; p. 210]. Nonetheless, Kuhn goes on to write that by the early 1910s "*while the nature of Planck's oscillators and of the corresponding emission process remained a mystery, the black-body problem could provide no further clues to physics*" [5; p. 209]. In 1910, Peter Debye, derives Planck's law by quantizing the vibration modes of the electromagnetic field without recourse to oscillators [5; p. 210]. Albert Einstein would soon obtain it using his coefficients [17]. But the nature of the emitter was not identified [19]. In fact, in both cases, physics moved increasingly outside the realm of physical reality and causality.

Astrophysics believes that nothing of known physical origin is needed to obtain a blackbody spectrum. All that is required is a mathematical construct involving photons in thermal equilibrium and this, well outside the confines of a solid enclosure, as demanded by the experimental constraints surrounding blackbody radiation. Astrophysics has no need of the physical lattice, of some physical species vibrating within the confines of a structural physical assembly. But, if a thermal spectrum is to be produced, it is precisely this kind of physical restriction which must exist [19, 23]. However, as long as the idea that blackbody radiation is independent of the nature of the walls prevails, there can be no correction of this situation. It is the very formation of Kirchhoff's law [20–22] which must be brought into question, if any progress is to be made toward linking Planck's equation [1, 2] to the physical world and if astrophysics is to reform the manner in which it treats data. For these reasons, we now embark on the review of the findings which led to the concept of universality. Overwhelming evidence will emerge (see also [23–25]) that this concept is erroneous and should be reconsidered.

2 Experimental production of black radiation

2.1 The 19th century and the lampblack standard

Wedgwood published his delightful analysis on the production of light from heated substances in 1792 [43]. The works are noteworthy and pleasant to read because 1) they define the "state of the art" just prior to the 19th century, 2) they examine a plethora of substances, and 3) they possess wonderful historical descriptions of antecedent works. The experiments contained therein are nothing short of elegant for the period. Even at this time, the emission within a cylinder, either polished or blackened (presumably covered with lampblack), had already gained the attention of science [43]. Wedgwood realized that it did not matter, if heat entered the substance of interest through light, or through friction [43]. Much was already known about thermal radiation, but confusion remained.

The experimental aspects of the science of thermal radiation really began with the release of Leslie's *An Experimental Inquiry into the Nature and Propagation of Heat* [44]. In this classic work, Leslie describes how all objects emit light, but also that they have very different emissive powers, even at the same temperature [44; pp. 81, 90, 110]. This was well understood throughout the 19th century [45, 46]. Leslie opens his work as follows: "*The object I chiefly proposed, was to discover the nature, and ascertain the properties of* *what is termed Radiant Heat. No part of physical science appeared so dark, so dubious and neglected*" [44; p. X]. Ironically, Leslie's last sentence rings somewhat true, even 200 years later.

Using reflectors made of tin, Leslie analyzed radiation emitted from the sides of a cube made of "block tin". At least one side was kept polished, one side was often coated with lampblack, and the other two were used to place miscellaneous substances, like tin foil, colored papers, or pigments [44; p. 8]. In order to maintain a constant temperature, the cubes were filled with water. The key to Leslie's experiments was a differential thermometer. By positioning various faces of the cube towards the reflector and placing his thermometer at the focal point, he soon discovered that polished metals give much less radiant heat than soot. He also realizes that the power to absorb or emit heat is somehow conjoined [44; p. 24]. It is interesting that, in his very first experiment, Leslie examines lampblack. It would become, for the rest of the 1800s, the means by which radiation would be calibrated.

Lampblack, the oxidation product of oil lamps, was not only a suitable material for coating surfaces and generating blackbodies over the course of the 1800s, it rapidly became the standard of radiation. By 1833, the Reverend Baden Powell, whose son was to form the Scouting movement, already writes that: "*all experimenters have usually blackened their thermometer*" [47; p. 276]. In 1848, G. Bird notes how lampblack has become a reference standard in the study of emission [48; p. 516]. Stewart refers repeatedly to lampblack invoking that soot had become the standard by which all radiation was to be measured: "*The reason why lampblack was chosen as the standard is obvious; for, it is known from Leslie's observations, that the radiating power of a surface is proportional to its absorbing power. Lampblack, which absorbs all the rays that fall upon it, and therefore possesses the greatest possible absorbing power, will possess also the greatest possible radiating power*" [26; §4]. He directly refers to lampblack heat [49; p. 191]. His experiments with lampblack are covered below in the context of the theoretical formulation of the law of radiation. Silliman's work is particularly valuable in that it was completed in 1861 [50]. It not only gives a well written and thorough account of the current state of knowledge in heat radiation, but it restates the central role of lampblack: "*Lampblack is the only substance which absorbs all the thermal rays, whatever be the source of heat*" [50; p. 442].

Langley re-emphasizes the extensive use of lampblack in his paper on solar and lunar spectra: "*I may reply that we have lately found an admirable check upon the efficiency of our optical devices in the behavior of that familiar substance lampblack, which all physicists use either on thermometers, thermopiles, or bolometers*" [51]. In 1893, Clerke writes of the "*lampblack standard*" in her tremendous work on the history of Astronomy [52; p. 271]. Tillman, in the 4th edition of his *Elementary Lessons in Heat*, summarizes well the belief that prevailed throughout the 1800s: "*Lampblack is the most perfect absorber and radiator, it being devoid of both reflecting and diffusive power. Its absorbing power is also most nearly independent of the source of heat. It absorbs all rays nearly alike, the luminous as well as the dark ones. Lampblack is accordingly taken as the standard surface of absorption, absorbing in the greatest degree every variety of ray which fall upon it. It is consequently, also , when hot, the typical radiator, giving out the maximum amount of heat which any substance at the same temperature could possibly give out; moreover, it gives out the maximum amount of each kind of heat that can be given out by any body at that temperature*" [46; p. 92]. Tillman does recall Langley's discovery that, in the infrared, lampblack was nearly transparent [51]. In any event, the role of lampblack in thermal radiation was well established by the end of the 19th century.

In his textbook on physics, published for the 7th time in 1920, Watson provides an elaborate description of the use of lampblack in coating both thermometers and surfaces for the study of comparative emission between objects [53]. He describes the lampblack standard as follows: "*Lampblack, although it does not absorb quite the whole of the incident radiation, yet possesses the property of absorbing very nearly, if not quite, the same proportion of the incident radiation whatever the wave-length, and so this substance is taken as a standard*" [53; p. 301].

A review of the blackbody literature for the 19th century reveals that blackbodies were produced either from graphite itself or from objects covered with lampblack (soot) or paints, which contained soot or bone black [54]. That is not to say that other substances were not used. Kangro [3] outlines an array of studies where experimentalists, over a small region of the spectrum, used different materials (platinum black, copper oxide, iron oxide, thorium oxide, etc). Nonetheless, graphite and soot take precedence over all other materials, precisely because their absorbance extends over such a wide range of wavelengths. Conversely, all other materials exhibit disadvantages, either because of their suboptimal emissivity, or due to their limited frequency ranges [31]. There are problems in visualizing the infrared, even with platinum black. Kangro explains: "*They (Lummer and Kurlbaum) changed to a platinum box as being more easily heated electrically and better suited to exact temperature measurement, then they used a platinum roll and finally a platinum cylinder the interior of which was blackened with iron oxide, and also divided by diaphragms the whole enclosed in a large asbestos cylinder*" [3; p. 159]. They also report "*the defective absorption of long wavelengths by Platinum black with which their bolometers were coated*" as a possible source of error [3; p. 159]. Lummer and Kurlbaum made their 1898 cavity from platinum blackened with a mixture of chromium, nickel, and cobalt oxide [4]. Nonetheless, in order to properly visualize the longest wavelengths, the method of residual rays, developed by Rubens, was utilized [4]. These were critical experiments for Planck. Yet, since platinum black could not reach elevated temperatures, in 1903, Lummer and Pringsheim would design a new blackbody with graphite walls [4]. This design has endured, essentially unchanged, until the present day [4].

2.2 The 19th century and the general state of knowledge

In 1833, Powell gave his excellent report on radiant heat [47]. By this time, the amount of radiation was known to be inversely related to conductive power [47; p. 266]. The more an object conducted thermal radiation, the better it acted as a reflector and the worst it was as an emitter/absorber. Based on the experiments of William Ritchie [55], it was also known that the absorptive power of a substance was directly related to its emissive power [47, p. 265]. Prévost's theory on thermal equilibrium, the famous *Theory of Exchanges* [56–58] was understood [47; p. 261]. Herschel's studies with infrared radiation were complete and the blocking action of glass was established [47; pp. 269–272]. While Herschel had discovered infrared radiation in 1800 [59], it was not until Langley, that infrared radiation could be accurately monitored [51]. At the time, Langley observed that lampblack was very nearly transparent to infrared radiation. Using prisms, it was also known that, on opposite sides of the spectrum, there existed "isothermal points" [47; p. 296]. Prisms played an important role in the early classification of the quality of light and heat by separation into colors [47; pp. 291–296]. Interestingly, Powell takes a sidestep relative to liquids and writes in his conclusion: "*In liquids, it has been disputed whether there can be radiation; and they are worse conductors than solids*" [47; p. 300]. Silliman notes that, even at the time of Kirchhoff, there remained some debate as to the relation between absorptive and emissive powers [50; p. 441], with de la Provostaye, Desains, and Melloni highlighting that these were not always equivalent. Given this general state of knowledge during the 19th century, we now move to the most important areas of experimentation, Prévost's *Theory of Exchanges* [56–58] and cavity radiation at thermal equilibrium.

2.3 The 19th century and cavity radiation

Pierre Prévost advanced his powerful *Theory of Exchanges* just as the 19th century came to life [56–58]. In formulating his law, Prévost invokes the enclosure: "*. . . I will suppose the two portions to be enclosed in an empty space, terminated on all sides by impenetrable walls*" [56; in Brace, p. 5]. He then moves to develop his *Theory of Exchanges* [56–58]. This theory was critical to Kirchhoff's thinking when the concept of universality was formulated [20–22]. As such, it is important to understand how Prévost's theory was viewed, not simply at the time of its formulation, but in the days of Kirchhoff. This knowledge can be gained by examining Balfour Stewart's summary of Prévost's theory. Stewart recounts the central ideas of equilibrium with an enclosure in his *Treatise* [49]. He summarizes Prévost's findings as follows: "*1. If an enclosure be kept at a uniform temperature, any substance surrounded by it on all sides will ultimately attain that temperature. 2. All bodies are constantly giving out radiant heat, at a rate depending upon their substance and temperature, but independent of the substance or temperature of the bodies that surround them. 3. Consequently when a body is kept at uniform temperature it receives back just as much heat as it gives out*" [49; p. 215].

With Prévost, nearly 70 years before Kirchhoff, the real study of cavity radiation began. At the same time, the understanding of cavity radiation really grew near the 1820s. This was when the experimental work of Dulong and Petit [60] with cavities took place. Simultaneously, theoretical studies of heat were being forged by Fourier [61–67] and Poisson [68, 69]. Fourier's works are particularly important in that they represent the most far-reaching theoretical analysis of heat and cavities in this time frame.

The paper by Dulong and Petit [60] is a major milestone in experimental science and it is difficult to do it justice in a brief treatment. Thus, let us concentrate not on the first section dealing with the measurements of temperatures, the dilatation of solids, and the specific heats of materials, but rather on the second section. This section addresses the laws of cooling derived within an enclosure. Of course, Kirchhoff's law of thermal emission [20–22] deals with radiation under equilibrium conditions. Conversely, the results of Dulong and Petit examine a dynamic process [60]. While they do not directly apply, the studies by Dulong and Petit form the experimental basis for the works that follow and are crucial to understanding cavity radiation. Dulong and Petit recognized the importance of distinguishing the effects of gas particles and radiative emission in cooling [60]. By examining the cooling of water and liquids in enclosures of varying shapes, they conclude that the rate of cooling is independent of the shape of the walls of the enclosure, on its size, and on the nature of the liquid [60; p. 245]. Note how this conclusion is reminiscent of Kirchhoff's law [20–22]. Importantly, they observe that the rate of cooling is dependent on the state of the surface of the enclosure [60; p. 245].

Dulong and Petit continue their inquiry into the laws of cooling by building a copper enclosure, the inner surface of which they cover with lampblack [60; p. 247]. They place a thermometer at the center of the enclosure. The outer surface of the thermometer is either silvered or left in its glassy state [60; p. 250]. Using a pump, a balloon (containing various gases of interest), and a barometer attached to the enclosure, they deduce the law of cooling. Dulong and Petit accomplished their goal by varying the gas pressure within the enclosure while monitoring the drop in temperature of the previously heated thermometer. Initially, ignoring the effect of gases and working near vacuum, they quickly realize that the rate of cooling depends on the nature of the thermometer surface, and this even within the blackened cavity [60; p. 260]. The rates of cooling of the two thermometers were proportional to one another, not equal [60; p. 260]. They arrive at a simple general law of cooling that applies to all bodies [60; p. 263]. Finally, by repeating the same experiments with gases at different pressures, they derive a law of cooling with two terms depending on radiation and the effect of the gas. They infer that the first term depends on the nature, the size, and the absolute temperature of the enclosure, while the second term depends only on the characteristics of the gas [60; p. 288]. Dulong and Petit's work is not revisited in a substantial manner until de la Provostaye and Desain publish their Mémoires [70–75].

De la Provostaye and Desain published their second *Memoir on the Radiation of Heat* in 1848, more than 10 years before the formulation of Kirchhoff's law of thermal emission [71]. The authors open their work by stating (all translations from French were made by the author): "*We must know how the quantity of heat emitted by a surface of a determined size depends on its temperature, its proper nature, its state, on the direction of the emission*" [71; p. 358]. They then highlight: "*but that we (scientists) have not, up to this day, introduced into the solution questions of equilibrium and of movement of the heat*" [71; p. 358].

The authors revisit Dulong and Petit's experiments with gases using a half liter cylinder, blackened interiorly with lampblack (*noir de fumée*), in which they can introduce gases. They were never able to confirm the exact relation of Dulong and Petit and, therefore, present a more elaborate equation to describe the law of cooling [71; p. 369]. The paper contains a relevant caveat in that the authors report that it is not always easy to obtain a black surface, even with lampblack paste. They resort to the flame of a lamp to resurface the object of interest in order that its emission becomes truly independent of angle of observation [71; p. 398]. However, the bulk of our concern is relative to their work on the approach towards thermal equilibrium within an enclosure [71; pp. 406–431].

They recall that Fourier has proved: "*1) that within a blackened enclosure without reflective power, equilibrium is established from element to element, 2) that the equilibrium is maintained in the same manner if we restore to one of the elements a reflective power, as long as we admit, in the first instance, that the absorbing and reflecting powers are complementary; and in the second place, that the emissive power is equal to the absorptive power, 3) that the same will hold, if we restore a certain reflective power to all the elements*" [71; p. 406].

De la Provostaye and Desain highlight that the enclosure must be blackened for Fourier's conclusions to hold, but the latter does not always specifically state if his cavity is blackened interiorly. Nonetheless, Fourier's derivations make the assumption that the wall of the enclosure follows Lambert's law [66]. As such, the objects can be viewed as placed within a perfectly absorbing cavity. De la Provostaye and Desain make the point as follows: "*The demonstration supposes, what the author* (Fourier) *seems in fact to have recognized for himself (Annales de Chimie et de Physiques, tome XXVII, page 247* (see [66]) *in his last Memoires, that the radiating body is stripped of all reflective power. It would therefore be not at all general. . .*" [71; p. 408].

De la Provostaye and Desain begin their studies by placing a hypothetical thermometer in a spherical cavity and make no assumptions other than stating that diffuse reflection does not occur. They permit, therefore, that both the cavity and the thermometer can sustain normal reflection and emission. Assuming that reflective power does not depend on the angle of incidence, they permit the rays to travel throughout the cavity and follow the progression of the rays over time, until equilibrium is reached. The authors conclude that the radiation inside such a cavity will not follow Lambert's law [71; p. 414]. The result is important because it directly contradicts Kirchhoff's assertion that the radiation inside all cavities must be black [20–22]. They then restrict their treatment to the consideration of angles below 60° or 70°, in order to reach a simplified form for the laws of cooling.

Like Dulong and Petit [60], de la Provostaye and Desain [70–75] are not concerned exclusively with thermal equilibrium, but rather, they are examining the velocity of cooling, the path to equilibrium. They provide important insight into the problem, as the following excerpt reveals: "*When in an blackened enclosure with an invariable temperature* t*, we introduce a thermometer at the same temperature and a body either warmer or colder, but maintained always at the same degree* T*, the thermometer will warm or cool, and, following the reciprocal exchanges of heat, it will attain a final temperature* θ*, whose value, function of* T *and* t*, depends also on the emissive power* E' *of its surface, of that* E *of the source, and of their forms, sizes and reciprocal distances*" [71; p. 424].

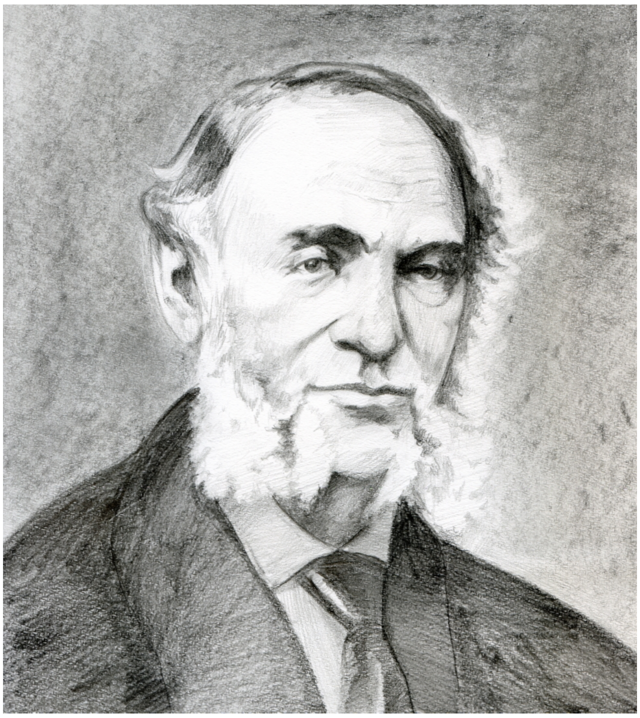
Siegel [76] highlights appropriately that de la Provostaye and Desains defined the emissive power E of a body as a fraction of the radiant emission of the blackbody where $f(t)$ is the emission of the blackbody, and the emission of the body is $Ef(t)$ [74; p. 431]. In contrast, Kirchhoff defines emission simply as E, which, in fact, corresponds to de la Provostaye and Desain's $Ef(t)$ [76]. Consequently, the universal function $f(t)$ is incorporated into Kirchhoff's law, even when it does not seem to be the case [76].

3 Cavity radiation

3.1 The Stewart-Kirchhoff dispute

Balfour Stewart [26] preceded Kirchhoff [20–22] by at least 2 years in the treatment of radiation at thermal equilibrium. Both Kirchhoff and Stewart built on the idea, initially advanced by Prévost [56–58], and expanded upon by Fourier [61–67], Poisson [68, 69], Dulong [60], Petit [60], de la Provostaye [70–75], Desains [70–75], and surely others, that thermal equilibrium existed between objects at the same temperature in the presence of confinement [49; p. 196]. The Stewart-Kirchhoff conflict is one of the darkest moments in

Gustav Robert Kirchhoff (12 March 1824 – 17 October 1887)



Balfour Stewart (1 November 1828 – 19 December 1887)

the history of science and it has been the subject of an excellent review [76]. This public quarrel is worth revisiting, not only because it is a powerful example of how science must not be performed, but also because it is very likely that the dispute between these men, and the international involvement of their collaborators [76] was directly responsible for the persistence of universality. If Stewart and Kirchhoff had better communicated, Kirchhoff might have yielded and the erroneous concept of universality, might have been retracted.

However, nationalistic passions were inflamed to such a measure that reason and scientific truth were moved to secondary positions. The animosity between Germany and British scientists would eventually reach the boiling point when, in 1914, Planck and 92 other learned men signed the *Appeal to the Cultured Peoples of the World* [16; pp. 70–ff]. Planck apparently signed the *Appeal* without examining its contents. Wien, for his part, insisted that British scientists "*appropriated discoveries made in Germany, confused truth and falsehood, argued in bad faith, and . . . that England was the worst enemy of the Reich*" [16; p. 72]. He urged that German scientists avoid, as much as possible, publication in British journals [16; p. 72]. Planck, for his part, refused to sign Wien's manifesto [16; pp. 70–ff]. While the Stewart-Kirchhoff affair cannot bear all the responsibility for these tragic developments, and while other scientific battles also raged [76], it is relatively certain that the situation played an early role in the building of such misconceptions.

The papers from Stewart and Kirchhoff which caused this conflict were all published in *The London, Edindurgh, and Dublin Philosophical Magazine and Journal of Science*. Kirchhoff was able to have access to the English literature, primarily through the assistance of F. Guthrie and Henry E. Roscoe. The latter translated many of Kirchhoff's works into English for *Philosophical Magazine*. Roscoe had studied and published with Bunsen who, in turn, eventually became Kirchhoff's key collaborator.

Stewart opens the discourse by publishing, in 1858, "*An account of some experiments on radiant heat, involving an extension of Prévost's Theory of Exchanges*" [26]. It will be discovered below that, in fact, it is Stewart's work which reached the proper conclusion, not Kirchhoff's [25]. Yet, Stewart's *Account* [26] has been forgotten, in large part, because, unlike Kirchhoff's papers [20–22], it did not arrive at universality as Seigel emphasizes [76].

The battle really begins when F. Guthrie translates Kirchhoff's paper and places it in *Philosophical Magazine* [21], the journal where Stewart's work had appeared just two years earlier. Kirchhoff is rapidly criticized for failure to cite prior work, not only relative to Stewart, but relative to other seminal discoveries [76]. With the aid of Roscoe [77–80], he publishes in 1863, "*Contributions towards the history of spectrum analysis and of the analysis of the solar atmosphere*" [81] in which he seems to dismiss the importance of Stewart's contributions. Kirchhoff writes: "*This proof cannot be a strict one, because experiments which have only taught us concerning more and less, cannot strictly teach us concerning equality*" [81]. Kirchhoff highlights that Stewart is not treating an enclosure in his experiments, but extends his conclusions to these objects [81]. In the end, Kirchhoff's *Contributions* [81] is not impolite… but it is tough.

For Stewart, Kirchhoff's *Contributions* [81] is viewed as an attack which must be immediately countered [82]. Stewart opens his rebuttal by stating: "*In the course of his remarks the learned author has reviewed in a somewhat disparaging manner some researches of mine on radiant heat, in consequence of which I am forced to reply, although very unwillingly, and desiring much to avoid a scientific controversy, especially with Professor Kirchhoff as an opponent*" [82]. In his own defense, Stewart then adds: "*nor did I omit to obtain the best possible experimental verification of my views, or to present this to men of science as the chief feature, grounding theory upon the experiments, rather than deducing the experiments from the theory*" [82]. This powerful charge by Stewart, in the end, forms the entire argument against Kirchhoff's proof [82]. Kirchhoff's results can never be validated by experiments, and Stewart, as an expert in heat radiation, must have recognized this to be the case [82].

Stewart closes his defense as follows: "*Although I preceded Kirchhoff nearly two years in my demonstration, I did not hesitate to acknowledge that his solution had been independently obtained; but, as a general principle, I cannot consent to admit that when a man of science has proved a new law and is followed by another who from the same premises deduces the same conclusions, the latter is justified in depreciating the labours of the former because he conceives that his solution is more complete. Will Kirchhoff himself willingly forego his own claims in favour of any one who shall in the future ages devise (if this be possible) a simpler and more convincing demonstration than that which has been given us by the Hiedelberg Professor? I feel, Sir, that, as an historian of science, you will acknowledge the justice of these remarks, and join me in regretting that one who has so eminently distinguished himself in original investigation should have chosen to superadd to his functions as a discoverer those of a severe and hostile critic upon the labours of those men who have worked at the same subject with himself, and by all of whom he has been treated with the utmost possible consideration*" [82].

The Stewart-Kirchhoff dispute reached such a magnitude that Kirchhoff, it seems, never again publishes in *Philosophical Magazine*, even though Bunsen, for his part, continues to utilize the journal. Stewart remained at a profound disadvantage, as he did not benefit from a relationship similar to that between Kirchhoff, Bunsen, and Roscoe. Roscoe would reprint Kirchhoff's infamous *Contributions* [81] in his *Spectrum Analysis* [80; pp. 115–122]. However, in this version [80; pp. 115–122], all text referring to Stewart has been removed without comment. It is impossible to understand Roscoe's motivation for the attenuated version. Roscoe may have suffered for having translated the letter. Alternatively, Kirchhoff's *Contributions* [81] might not fit in its entirety within the context of the other lectures. In any event, Roscoe‘s *Spectrum Analysis* is a strange ode to Kirchhoff, which lacks broad scientific review. Regrettably, it seems that Roscoe made no attempt to reconcile the Kirchhoff-Stewart matter through proper and continuing scientific discourse.

In the end, Kirchhoff and Stewart each fell short of the mark. However, Kirchhoff's error was more serious [20–22], since it has theoretical consequences to this day. As for Balfour Stewart, had he presented a better theoretical case [26], the course of physics may have followed a different path. Kirchhoff, for example, correctly highlighted that Stewart's proof should not use the index of refraction, but rather, the square of the index [81]. Stewart conceded the point [76, 82]. For Kirchhoff, Stewart's proof was possibly true, not necessarily true [81]. Siegel elegantly clarified Kirchhoff's concerns [76]. These shortcomings in Stewart's derivation hinder the search for truth. Finally, had nationalistic sentiments not been aroused [76], it might have been easier to resolve the conflict.

3.2 Balfour Stewart

In examining Stewart's writings [26, 49, 82–85], we discover, as Brace highlights, *"the comprehensiveness of his mind and the originality of his genius*" [83; p. 72]. Many of Stewart's [26, 82–85] ideas are contained in his *Elementary Treatise on Heat* [49] and the later reflects his positions at the end of his life. As such, our discussion will begin first with the examination of this work and close with the review of his 1858 and 1859 papers [26, 83].

By the time Stewart writes his *Treatise*, he clearly recognizes that all substances display at least selective absorption of light [49; p. 191]. He comments on the probable identity of heat and light and writes: "*The facts detailed in this chapter all tend to shew that radiant light and heat are only varieties of the same physical agent, and also that when once the spectrum of a luminous object has been obtained, the separation of the different rays from one another is physically complete; so that if we take any region of the visible spectrum, its illuminating and heating effect are caused by precisely the same rays*" [49; p. 195]. He continues: "*Furthermore, we have reason to suppose that the physical distinction between different parts of the spectrum is one of wave length, and that rays of great wave length are in general less refracted than those of small wave length*" [49; p. 196].

Stewart's thoughts with respect to radiation within a cavity are important, not only because they provide us insight into the proper analysis of the enclosures, but also because they clearly outline what was known just prior to Planck. Stewart's comments relative to these experiments are summarized once again in his *Treatise*: "*… let us for our present purpose imagine to ourselves a chamber of the following kind. Let the walls which surround this chamber be kept at a constant temperature, say 100°C, and let them be covered with lampblack — a substance which reflects no heat, or at least* *very little; — also let there be a thermometer in the enclosure. It is well known that this thermometer will ultimately indicate the temperature of the surrounding walls… Suppose that the outside of the bulb of the thermometer of last article is covered with tinfoil, so that its reflecting power is considerable. Now according to the Theory of Exchanges this thermometer is constantly radiating heat towards the lampblack, but it is receiving just as much heat as it radiates. Let us call radiation of lampblack 100, and suppose that 80 of these 100 rays which strike the thermometer are reflected back from its tinfoil surface, while the remaining 20 are absorbed. Since therefore the thermometer is absorbing 20 rays, and since nevertheless its temperature is not rising, it is clear that it must be also radiating 20 rays, that is to say, under such circumstances its absorption and radiation must be equal to one another. If we now suppose the outside of the bulb to be blackened instead of being covered with tinfoil, the thermometer will absorb nearly all the 100 rays that fall upon it, and just as in the previous case, since its temperature is not rising, it must be radiating 100 rays. Thus we see that when covered with tinfoil it only radiated 20 rays, but when blackened it radiates 100. The radiation from a reflecting metallic surface ought therefore, if our theory be true, to be much less than from a blackened one. This has been proved experimentally by Leslie, who shewed that good reflectors of heat are bad radiators. Again, we have seen that in the case of the bulb covered with tinfoil 80 of the 100 rays which fell upon it were reflected back, and we have also seen that 20 were radiated by the bulb. Hence the heat reflected plus the heat radiated by this thermometer in the imaginary enclosure (author underscoring text) will be equal to 100, that is to say, it will be equal to the lampblack radiation from the walls of the enclosure. We may generalize this statement by saying that in an enclosure of constant temperature the heat reflected plus the heat radiated by any substance will be equal to the total lampblack radiation of that temperature, and this will be the case whether the reflecting substance be placed inside the enclosure or whether it form a part of the walls of the enclosure*” [49; pp. 199–201].

Stewart reaches this conclusion for an enclosure whose walls have been covered with lampblack [49]. In that case, the heat inside the enclosure will correspond to that from lampblack, as I have shown [25]. In the pages which follow [49], Stewart goes on to explain that his law holds, in a manner which is independent of the nature of the walls, provided that both radiation and reflection are included. He also illustrates independence relative to wall shape. Importantly, he invokes the work of de la Provostaye and Desains with silver and lampblack to demonstrate that the total radiation inside an enclosure containing a silver surface will also be equal to 100, where 2.2 parts arise from the emission of silver itself and 97 parts from the reflection of lampblack. Stewart realizes that the value of 100 is only achieved in the presence of lampblack. The nature of the wall was immaterial simply because lampblack was always present. In fact, it appears that Stewart was actually contemplating enclosures which contain both reflective surfaces and absorbing ones, as seen in his section 227: “*It has already been stated (Art. 204) that the stream of radiant heat continually proceeding through an enclosure of which the walls are kept at a constant temperature depends only on the temperature of the walls, and not on the nature of the various substances of which they are composed; the only difference being that for metals this stream is composed partly of radiated and partly also of reflected heat, while for lampblack it is composed wholly of radiated heat. This may be expressed by saying that this stream depends upon or is a function of the temperature, and of it alone; but there is the following very important difference between a reflecting and lampblack surface, as representing this stream of radiant heat. It is only when a reflecting surface forms part of a complete enclosure of the same temperature as itself, that the radiated and reflected heat from this surface together represent the whole stream of heat; for if we bring it for a moment into another enclosure of lower temperature, the reflected heat is altered, and although the radiation will for a short time continue nearly constant, yet this radiation will not represent the whole stream of heat due to the temperature of the surface. On the other hand, if a lampblack surface be placed in the above position, since the stream of heat which flows from it is entirely independent of the reflexion due to neighboring bodies, the heat which it radiates when brought for a moment into an enclosure of lower temperature than itself will truly represent the stream of radiant heat due to the temperature of the lampblack*” [49; pp. 221–222]. One can see that reflecting materials provide very different conditions than lampblack within enclosures. That is, within an enclosure under dynamic conditions, objects which are partially or fully reflecting cannot indefinitely support black radiation. They simply emit their own radiation and reflect the heat incident upon their surface. Through this discussion, Stewart demonstrates that thermal equilibrium would be disturbed when a perfect absorber is replaced with a reflector, bringing about dynamic rather than equilibrium conditions. This was an important insight relative to the analysis which I recently provided [25] of Kirchhoff's second proof [21, 22].

In order to examine the velocity of temperature change, Stewart invokes a thin copper globe lined with lampblack: “*Having now considered the law of cooling as representing with much accuracy the quantity of heat given out by a black substance at different temperatures, we come next to the relation between the temperature and the quality or nature of the heat given out. And here we may remark that the laws which connect the radiation of a black body with its temperature, both as regards to the quantity and the quality of the heat given out, hold approximately for bodies of indefinite thickness which are not black, — thus, for instance, they would hold for a metallic surface, which would represent very nearly a lampblack surface, with the radiation diminished a certain number of times. These laws would not, however, hold* *exactly for a white surface, such as chalk; for this substance behaves like lampblack with respect to rays of low temperature, while it is white for rays of high temperature, and the consequence of this will be that its radiation will increase less rapidly than that of a lampblack surface. In like manner, these laws will not hold exactly for coloured surfaces*" [49; p. 230]. Note how these statements are directly contradictory to what Kirchhoff requires. For Stewart, there is no universality and this is a major distinction between his work and that of his adversary [25].

With regards *specifically* to a black surface, Stewart writes (see page 231): "*1. The spectrum of the radiant heat and light given out by a lampblack surface is continuous, embracing rays of all refrangibilities between certain limits on either side… 2. We have reason to think that as the temperature rises, the spectrum of a black substance is extended in the direction of greatest refrangibility, so as to embrace more and more of the violet and photographic rays*" [49; p.231]. Stewart goes on to discuss thin plates of glass and explains how they cannot be compared to lampblack, as their radiation with increasing temperature will be substantially different [49; p. 232].

It is clear that if scientists of the period coated the walls of their enclosure with lampblack, that emission would be independent of the nature of the walls themselves, precisely because lampblack was coating these walls. After all, Stewart fully realizes that silver, for instance, has a total emission much below lampblack [49; pp. 201–206]. Stewart used an enclosure coated with lampblack to arrive at the following laws: "*1. The stream of radiant heat is the same throughout, both in quantity and quality; and while it depends on the temperature it is entirely independent of the materials or shape of the enclosure. 2. This stream is unpolarized. 3. The absorption of a surface in such an enclosure is equal to its radiation and this holds for every kind of heat*" [49; p. 206]. That is how the concept of independence of the nature of the walls entered the literature. Nothing, in fact, was independent. The walls were simply coated with lampblack [49; pp. 201–206]. This was such an obvious part of these experiments, during the 19th century, that it is likely that most scientists, unlike Balfour Stewart, simply neglected to report their common practice. As a result, future generations who followed the theoretical avenues of Kirchhoff, actually came to believe that the nature of the walls was unimportant and the vital role of the soot coating was forgotten.

Stewart's law stated that absorption was equal to radiation for every kind of heat [26, 49, 76, 82]. This was true under equilibrium conditions. However, Kirchhoff objected [81] to this formulation by Stewart [26], since he believed that Stewart had inappropriately extended the results of his experimental finding to include equality whereas proportionality was all that had been proven [76, 81]. In any event, the fact remains that Stewart's conclusion [26, 49, 82], not Kirchhoff's [20–22], was correct. It alone was supported by the experimental findings and, unlike Kirchhoff's law [20–22], made no claims of universality [76].

The central portion of Stewart's proof considers a continuous plate of rock salt positioned between two plates covered with lampblack [26; §12]. The idea is both simple and powerful. Stewart immediately reaches the result that "*the absorption of a plate equals, its radiation, and that for every description of heat*" [26; §19]. Then, Stewart considers radiation internal to a substance: "*Let AB, and BC be two contiguous, equal, and similar plates in the interior of a substance of indefinite extent, kept at a uniform temperature*" [26; §20]. Stewart is invoking the same restriction found for thermal equilibrium with an enclosure. However, he moves to the interior of a body, apparently in order to avoid dealing with surface reflection [82]. Seigel [76] highlights this point. Kirchhoff believes that Stewart has not properly treated the enclosure [81]. The point is weak as Stewart's entire treatment is based on the ideas of Prévost [55–57].

Stewart is clearly working within the confines of Prévost's *Theory of Exchanges* [26, 56–58]. Considering the equilibrium between lampblack and an arbitrary surface at thermal equilibrium, he writes "*… hence the total quantity of heat radiated and reflected which leaves the surface… (is) the same as if the substance had been lampblack, the only difference being, that, in the case of lampblack, all this heat is radiated, whereas in other substances only part is radiated, the remainder being reflected heat*" [26; §31]. He continues: "*Although we have considered only one particular case, yet this is quite sufficient to make the general principle plain. Let us suppose we have an enclosure whose walls are of any shape, or any variety of substances (all at a uniform temperature), the normal or statical condition will be, that the heat radiated and reflected together, which leaves any portion of the surface, shall be equal to the radiated heat which would have left that same portion of the surface, if it had been composed of lampblack… Let us suppose, for instance, that the walls of this enclosure were of polished metal, then only a very small quantity of heat would be radiated; but this heat would be bandied backwards and forwards between the surfaces, until the total amount of radiated and reflected heat together became equal to the radiation of lampblack*" [26; §32]. These passages are quite similar to Kirchhoff's with the distinction that universality is never invoked. Stewart realizes that the lampblack surface within the enclosure is essential.

Stewart's manner of addressing the problem is lacking, as Siegel highlights [76], especially for Kirchhoff [81]. A review of this work [76] provides a sufficient discussion. Stewart advances an initial attempt at the correct solution to the radiation puzzle, but the presentation was not sufficient, at least for his adversary. Surprisingly, in his *Reply* to Kirchhoff in 1863, Stewart seems embarrassed [76] relative to reflection writing: "*I shall only add that it was attempted, as far as possible, to disengage the proof, theoretical and experimental, from the embarrassment of considering surface reflexion*"

[82]. If reflection is neglected, however, almost by definition, the radiation must be black [25]. Consequently, all attempts to address the issue devoid of surface reflection can never yield the proper conclusion relative to the existence of universality. Stewart reaches the proper answer because he does include reflection in his papers [26, 83] and within his *Treatise* [49]. Within an enclosure containing a lampblack surface and another object, he reminds us that "*the reflection plus the radiation of the body at any temperature equals the lampblack radiation at that temperature*" [83; §44]. The proper consideration of reflection is key [25] and though Stewart may have had weaknesses in his presentation, he did ascertain the truth.

3.3 Gustav Kirchhoff and his law

It can be said that Kirchhoff's law of thermal emission [20–22], through its claims of the universal nature of radiation within enclosures, represents one of the most profound dismissals of experimental science in the history of physics. The great mass of experimental evidence speaks against universality of radiation within cavities. Cavity radiation only assumes the normal distribution (i.e. that of the blackbody) when either the walls of the cavity, or at least one of the objects it contains, are perfectly absorbing [23, 25]. In fact, the proof that Kirchhoff's law does not hold, in its universal form, does not require extensive mathematical or experimental arguments, only simple ones [23–25].

Schirrmacher [86] emphasizes that, at the time Planck formulated his law, a solid proof of Kirchhoff's remained absent. Furthermore, he highlights that, as late as 1912, Hilbert was arguing that Kirchhoff's law still lacked proof [86]. Hilbert makes this statement in spite of Planck's attempt to prove the law in his *Theory of Heat Radiation* [9]. Schirrmacher also outlines that nearly all attempts to advance universality were met with a refutation [86; p. 16]. Sadly, these corrections never prevailed.

De la Provostaye was one of the first to offer an analysis of cavity radiation following Kirchhoff, in 1863 [87]. In his work, de la Provostaye deduces that the radiation within a perfectly absorbing cavity must be black [87]. He also infers that a cavity, a portion of whose walls are perfectly absorbing, and which contains an object of arbitrary emittance and reflectance, must also contain normal (or blackbody) radiation [87]. Like Kirchhoff, he attempts to extend his findings to a perfectly reflecting cavity. At first, he concedes that a fully reflecting cavity must be devoid of radiation. At this point, de la Provostaye should have ceased as the question was resolved; but strangely … he continues. Prompted perhaps by the quest for Kirchhoff's universality [20–22], he permits radiation to enter the perfectly reflecting cavity and immediately moves to show that such radiation must be black [87]. As a result, de la Provostaye stumbles in a manner quite similar to Kirchhoff and his paper does not, in fact, form a refutation of Kirchhoff's law [87]. De la Provostaye simply objected that Kirchhoff, by introducing perfect reflectors, essentially dictated the result which he sought [86].

De la Provostaye's analysis of cavity radiation is particularly important, because he was an expert in the subject. He had dealt with enclosures on an experimental basis and must have known from the work of his own hands, that Kirchhoff's law could not hold, in its universal form. This is why he presents the second case discussed above where at least a portion of the cavity walls remained perfectly absorbing. De la Provostaye did overreach in his conclusions [87] in a manner not dissimilar from Kirchhoff [20–22].

In any event, de la Provostaye's theoretical objections relative to the absence of a perfectly reflecting mirror was not the central problem for Kirchhoff [25]. While many followed de la Provostaye's initial objection, refutations always seemed to be based on arguments such as perfectly reflecting mirrors do not exist, neither do perfectly diathermanous (or transparent) bodies, or bodies which can only absorb one wavelength. Such idealized substances are utilized in various proofs of Kirchhoff's law [86]. Unfortunately, since Kirchhoff's law is based on a theoretical extension of experimental reality, the fact that idealized objects do not exist is not sufficient to overturn Kirchhoff's position [25]. Hence, the law has prevailed, even though experimental reality is well established against its claims as de la Provostaye and Stewart must have realized.

The only way to refute Kirchhoff's law is to show that some section of its treatment either fails to consider an essential aspect of physical reality or that, through its derivation, Kirchhoff himself violates the thermal equilibrium, which he required as a precondition [25]. Both of these complications have been brought to the forefront [25]. Kirchhoff's law is not valid for two reasons: first, the importance of reflection is not properly included and second, Kirchhoff's model gives rise, under certain conditions, to a violation of thermal equilibrium [25].

Physics is in a difficult position relative to Kirchhoff's law, since the modern relationship between radiation and absorption, under equilibrium conditions, is based upon this work. At the same time, Kirchhoff's claims of universality given enclosure are strictly invalid [25]. A perfect absorber must be present. The only means of rectifying this situation is to finally acknowledge the merit of Stewart's contributions [26, 49, 83].

3.4 Max Planck and cavity radiation

3.4.1 Whence the carbon particle

In the first preface of his book *The Theory of Heat Radiation* Planck mentions that he has "*deviated frequently from the customary methods of treatment, wherever the matter presented or considerations regarding the form of presentation seems to call for it, especially in deriving Kirchhoff's laws…*" [9; p. xi]. Yet, when one reads Planck's text, the precise nature of the deviations cannot be ascertained and the origin of the carbon particle remains a mystery. Since the exposition deals with Kirchhoff, one could be led to assume that the idea came from Kirchhoff [23]. Planck, after all, was a strict theoretician. He relied on experimentalists to give him insight in the particle used for the generation of blackbody radiation. Still, we are never told specifically that Kirchhoff invoked the carbon particle [23]. It is certain that, at the time of Kirchhoff, virtually all blackbodies were covered with lampblack. Hence, radiation in a cavity whose inner walls were coated with lampblack would have been observed to be independent of the nature of the walls. This simple observation may well have prompted Kirchhoff and Planck to reach for physically profound statements relative to universality while minimizing the role of soot.

The origin of the carbon particle is surely of historical interest. However, with regards to physics, its existence causes concern, not its historical origin. How a particle of carbon entered the perfectly reflecting cavity and involved the actions of Kirchhoff, Planck, or another scientist, alters nothing relative to the consequences for universality [23]. What remain critical are Kirchhoff's claims that blackbody radiation was independent of the nature of the walls of the cavity, whether these were absorbing, transparent or reflecting to radiation, provided that thermal equilibrium was maintained [21, 22]. Planck's invocation of the carbon particle [9] shatters all these arguments [23, 25] and, as such, it is important to repeat the many words of Planck relative to the need for a tiny piece of carbon.

We begin by recalling how Planck himself was well aware that real blackbodies are formed using lampblack. Nothing here is independent of the nature of the walls: "*Now, since smooth non-reflecting surfaces do not exist … it follows that all approximately black surfaces which may be realized in practice (lampblack, platinum black)…*" [9; §11]. Relative to the carbon particle itself, the first key passages come at the end of Part I: "*Thus far all the laws derived in the preceding sections for diathermanous media hold for a definite frequency, and it is to be kept in mind that a substance may be diathermanous for one color and adiathermanous for another. Hence the radiation of a medium completely enclosed by absolutely reflecting walls is, when thermodynamic equilibrium has been established for all colors for which the medium has a finite coefficient of absorption, always the stable radiation corresponding to the temperature of the medium such as is represented by the emission of a black body. Hence this is briefly called "black" radiation. On the other hand, the intensity of colors for which the medium is diathermanous is not necessarily the stable black radiation, unless the medium is in a state of stationary exchange of radiation with an absorbing substance*" [9; §50]. Planck recognizes that the presence of a perfectly absorbing substance is required within the perfect reflector. If this condition is not fulfilled, Planck reminds us immediately that: "*… in a vacuum bounded by totally reflecting walls any state of radiation may persist*" [9; §51]. As such, Planck is fully aware that the perfect reflector can never produce blackbody radiation in the absence of a perfect absorber. It is not simply a matter of waiting a sufficient amount of time, but rather, the radiation will *persist* in a non-blackbody or arbitrary state. He re-emphasizes this aspect clearly "*Every state of radiation brought about by such a process is perfectly stationary and can continue infinitely long, subject, however, to the condition that no trace of an emitting or absorbing substance exists in the radiation space. For otherwise, according to Sec. 51, the distribution of energy would, in the course of time, change through the releasing action of the substance irreversibly, i.e., with an increase of the total entropy, into the stable distribution corresponding to black radiation*" [9; §91].

Planck soon brings the carbon particle front and center: "*But as soon as an arbitrarily small quantity of matter is introduced into the vacuum, a stationary state of radiation is gradually established. In this the radiation of every color which is appreciably absorbed by the substance has intensity K_ν corresponding to the temperature of the substance and determined by the universal function (42) for $q = c$, the intensity of radiation of the other colors remaining intermediate. If the substance introduced is not diathermanous for any color, e.g., a piece of carbon however small, there exists at the stationary state of radiation in the whole vacuum for all colors the intensity K_ν of black radiation corresponding to the temperature of the substance. The magnitude of K_ν regarded as a function of υ gives the spectral distribution of black radiation in a vacuum, or the so-called normal energy spectrum, which depends on nothing but the temperature. In the normal spectrum, since it is the spectrum of emission of a black body, the intensity of radiation of every color is the largest which a body can emit at that temperature at all*" [9; §51].

"*It is therefore possible to change a perfectly arbitrary radiation, which exists at the start in the evacuated cavity with perfectly reflecting walls under consideration, into black radiation by the introduction of a minute particle of carbon. The characteristic feature of this process is that the heat of the carbon particle may be just as small as we please, compared with the energy of radiation contained in the cavity of arbitrary magnitude. Hence, according to the principle of conservation of energy, the total energy of radiation remains essentially constant during the change that takes place, because the changes in the heat of the carbon particle may be entire neglected, even if its changes in temperature would be finite. Herein the carbon particle exerts only a releasing (auslösend) action. Thereafter the intensities of the pencils of different frequencies originally present and having different frequencies, directions, and different states of polarization change at the expense of one another, corresponding to the passage of the system from a less to a more stable state of radiation or from a state of smaller to a state of larger entropy. From a thermodynamic point of view this process is perfectly analogous, since the time necessary for the process is not essential,* *to the change produced by a minute spark in a quantity of oxy-hydrogen gas or by a small drop of liquid in a quantity of supersaturated vapor. In all these cases the magnitude of the disturbance is exceedingly small and cannot be compared with the magnitude of the energies undergoing the resultant changes, so that in applying the two principles of thermodynamics the cause of the disturbance of equilibrium, viz., the carbon particle, the spark, or the drop, need not be considered. It is always a case of a system passing from a more or less unstable into a more stable state, wherein, according to the first principle of thermodynamics, the energy of the system remains constant, and, according to the second principle, the entropy of the system increases*" [9; §52]. Planck views the carbon particle simply as a catalyst. He does not recognize that it has a vital function as a perfect absorber. This is a critical oversight, as demonstrated in my review of thermal equilibrium within a perfectly reflecting cavity containing a carbon particle [25].

Planck invokes the carbon particle repeatedly throughout his text. This issue is so central to the discussion at hand that all these sections must be brought forth. He writes: "*For the following we imagine a perfectly evacuated hollow cylinder with an absolutely tight-fitting piston free to move in a vertical direction with no friction. A part of the walls of the cylinder, say the rigid bottom, should consist of a black body, which temperature T may be regulated arbitrarily from the outside. The rest of the walls including the inner surface of the piston may be assumed to be totally reflecting. Then, if the piston remains stationary and the temperature, T, constant, the radiation in the vacuum will, after a certain time, assume the character of black radiation (Sec. 50) uniform in all directions. The specific intensity, K, and the volume density, u, depend only on the temperature, T, and are independent of the volume, V, of the vacuum and hence the position of the piston*" [9; §61].

"*Let us also consider a reversible adiabatic process. For this it is necessary not merely that the piston and the mantle but also that the bottom of the cylinder be assumed as completely reflecting, e.g., as white. Then the heat furnished on compression or expansion of the volume of radiation is $Q = 0$ and the energy of radiation changes only by the value pdV of the external work. To insure, however, that in a finite adiabatic process the radiation shall be perfectly stable at every instant, i.e., shall have the character of black radiation, we may assume that inside the evacuated cavity there is a carbon particle of minute size. This particle, which may be assumed to possess an absorbing power differing from zero for all kinds of rays, serves merely to produce stable equilibrium of the radiation in the cavity (Sec. 51 et seq.) and thereby to ensure the reversibility of the process, while its heat contents may be taken as so small compared with the energy of radiation, U, that the addition of heat required for an appreciable temperature change of the particle is perfectly negligible. Then at every instant in the process there exists absolutely stable equilibrium of radiation and the radiation has the temperature of the particle in the cavity. The volume, energy, and entropy of the particle may be entirely neglected*" [9; §68].

"*Let us finally, as a further example, consider a simple case of an irreversible process. Let the cavity of volume V, which is elsewhere enclosed by absolutely reflecting walls, be uniformly filled with black radiation. Now let us make a small hole through any part of the walls, e.g., by opening of a stopcock, so that the radiation may escape into another completely evacuated space, which may also be surrounded by rigid, absolutely reflecting walls. The radiation will at first be of a very irregular character; after some time, however, it will assume a stationary condition and will fill both communicating spaces uniformly, its total volume being, say, V'. The presence of a carbon particle will cause all conditions of black radiation to be satisfied in the new state*" [9; §69].

"*If the process of irreversible adiabatic expansion of the radiation from the volume V to the volume V' takes place as just described with the single difference that there is no carbon particle present in the vacuum, after the stationary state of radiation is established, as will be the case after a certain time on account of the diffuse reflection from the walls of the cavity, the radiation in the new volume V' will not any longer have the character of black radiation, and hence no definite temperature … If a carbon particle is afterwards introduced into the vacuum, absolutely stable equilibrium is established by a second irreversible process, and, the total energy as well as the total volume remaining constant, the radiation assumes the normal energy distribution of black radiation and the entropy increases to the maximum value S'…*" [9; §70].

"*Hence, on subsequent introduction of a carbon particle into the cavity, a finite change of the distribution of energy is obtained, and simultaneously the entropy increases further to the value S' calculated in (82)*" [9; §103].

Throughout *The Theory of Heat Radiation*, Planck invokes the carbon particle as a vital determinant of blackbody radiation. Only in the section of the derivation of Wien's law does he try to minimize the importance of his catalyst. However, in this case, the derivation starts with the presence of a blackbody spectrum *a priori*. One could argue that Planck goes through great pains to explain that he does not need the particle when, in fact, he has already invoked it to produce the radiation he requires as a starting point. The discussion is well worth reading precisely for the number of times that the carbon particle is utilized: "*The starting point of Wien's displacement law is the following theorem. If the black radiation contained in a perfectly evacuated cavity with absolutely reflecting walls is compressed or expanded adiabatically and infinitely slowly, as described above in Sec. 68, the radiation always retains the character of black radiation, even without the presence of a carbon particle. Hence the process takes place in an absolute vacuum just as was calculated in Sec. 68 and the introduction, as a precaution, of a carbon particle is shown to be superfluous. But this is true only in this special* *case, not at all in the case described in Sec. 70…*" [9; §71].

"*Let the completely evacuated hollow cylinder, which is at the start filled with black radiation, be compressed adiabatically and infinitely slowly to a finite fraction of the original volume. If, now, the compression being completed, the radiation were no longer black, there would be no stable thermodynamic equilibrium of the radiation (Sec. 51). It would then be possible to produce a finite change at constant volume and constant total energy of radiation, namely, the change to the absolutely stable state of radiation, which would cause a finite increase of entropy. This change could be brought about by the introduction of a carbon particle, containing a negligible amount of heat as compared with the energy of radiation. This change, of course, refers only to the spectral density of the radiation* u_v*, whereas the total density of the energy u remains constant. After this has been accomplished, we could, leaving the carbon particle in the space, allow the cylinder to return adiabatically and infinitely slowly to its original volume and then remove the carbon particle. The system will then have passed through a cycle without any external changes remaining. For heat has been neither added nor removed, and the mechanical work done on compression has been regained on expansion, because the latter, like the radiation pressure, depends only on the total density u of the energy of radiation, not on its spectral distribution. Therefore, according to the first principle of thermodynamics, the total energy of radiation is at the end just the same as at the beginning, and hence also the temperature of the black radiation is again the same. The carbon particle and its changes do not enter into the calculation, for its energy and entropy are vanishingly small compared with the corresponding quantities of the system. The process has therefore been reversed in all details; it may be repeated any number of times without any permanent change occurring in nature. This contradicts the assumption, made above, that a finite increase in entropy occurs; for such a finite increase, once having taken place, cannot in any way be completely reversed. Therefore no finite increase in entropy can have been produced by the introduction of the carbon particle in the space of radiation, but the radiation was, before the introduction and always, in the state of stable equilibrium*" [9; §71].

In reading these sections, it is almost as if Planck has entered into a duel with the carbon particle. He tries to minimize its role, even though it is strictly necessary to his success. In any event, as I have shown [25], when Planck (or Kirchhoff) places the carbon particle inside the perfectly reflecting cavity, it is as if the entire cavity had been lined with soot [23]. Thermal equilibrium arguments are powerful, and one of their interesting aspects is that equilibrium does not depend on the extent of the interacting surfaces. This affects only the amount of time required to reach equilibrium, not the nature of the radiation present under equilibrium conditions. Planck's catalyst is a perfect absorber, and therefore, given equilibrium, it controls the entire situation. The carbon particle does not simply lead to a distribution of radiation which would have occurred even in its absence.

3.4.2 Planck's derivation of Kirchhoff's law

Planck's derivation of Kirchhoff's law, as presented in *The Theory of Heat Radiation* [9; pp. 1–45], brings the reader to universality, precisely because reflection is not fully considered. Planck's exposition is elegant and involves two distinct parts. The first deals with radiation within an object [9; §4–26] and is eerily similar to Stewart's formulation [26, 82]. The second examines radiation between "*two different homogeneous isotropic substances contiguous to each other … and enclosed in a rigid cover impermeable to heat*" [9; §35–39]. By combining these two parts, Planck arrives at a relationship which is independent of the nature of the materials in a manner consistent with his belief in universality.

A cursory examination of this derivation [9; pp. 1–45], suggests that universality must be valid. Planck seems to properly include reflection, at least when discussing the interface between two separate materials [9; §35–39]. He arrives with ease at Kirchhoff's law, $q^2(\varepsilon_v/\alpha_v) = q^2 K_v$, [9; Eq. 42], involving the square of the velocity of propagation, q, the coefficient of emission, ε_v, the coefficient of absorption, α_v, and the universal function, K_v. This relationship simplifies to the familiar form $\varepsilon_v/\alpha_v = K_v$. *The Theory of Heat Radiation* focuses, later, on the definition of the universal function, which of course, is the right side of Planck's famous equation [1, 2]:

$$\frac{\varepsilon_v}{\alpha_v} = \frac{2hv^3}{c^2}\,\frac{1}{e^{hv/kT}-1}\,.$$

Unfortunately, there is a difficulty at the very beginning of the Planck's elucidation of Kirchhoff's law.

In order to arrive at universality [20–22], Planck first examines the equilibrium of radiation within an object. He begins by considering only the emission from a single element $d\tau$ internal to the object and in so doing, is deliberately ignoring reflection. Planck writes, in deriving Eq. (1), that the "*total energy in a range of frequency from* v *to* $v + dv$ *emitted in the time dt in the direction of the conical element* $d\Omega$ *by a volume element* $d\tau$" [9; §6] is equal to $dt\,d\tau\,d\Omega\,dv\,2\varepsilon_v$. This will lead directly to Kirchhoff's law. If Planck had properly weighed that the total radiation coming from the element $d\tau$ was equal to the *sum* of its emission and reflection, he would have started with $dt\,d\tau\,d\Omega\,dv\,2(\varepsilon_v + \rho_v)$, which would not lead to universality.

Planck moves on to examine absorption, by imagining two elements $d\sigma$ and $d\sigma'$ which are exchanging radiation within the same substance [9; §20]. Finally, he views the total "*space density of radiation*" in a sphere at the center of which is a volume element, v, receiving radiation from a small surface element, $d\sigma$ [9; §22]. In the end, by combining his results for emission and absorption, Planck demonstrates that within an individual substance, $K_v = \varepsilon_v/\alpha_v$. He writes the powerful conclusion that "*in the interior of a medium in a state of thermodynamic equilibrium the specific intensity of radiation of a certain frequency is equal to the coefficient of emission divided by the coefficient of absorption of the medium for this frequency*" [9; §26]. This was the flaw in his presentation. Had Planck fully included reflection, he would have obtained $K_v = (\varepsilon_v + \rho_v)/(\alpha_v + \rho_v)$.

Yet, this is only the first portion of Planck's walk to universality. In order to extend his deduction to all substances, he must first bring two differing materials in contact with one another. He accomplishes this correctly in §35–38. Properly treating reflection in this case, he is led, as was seen above, to $q^2(\varepsilon_v/\alpha_v) = q^2 K_v$ [9; §38], a statement of universality. The equation becomes completely independent of the nature of the substance. But if Planck had properly executed the first portion of his proof [9; §1–26], he would have been led, for every substance, once again to $K_v = (\varepsilon_v + \rho_v)/(\alpha_v + \rho_v)$.

In hindsight, there are many problems with Planck's derivation. In the first section of his proof, he moves to the inside of an object. He advances that thermal equilibrium is achieved internally, not through conduction and the vibration of atoms, but rather through radiation. While it is true, as Planck believes, that in a state of thermal equilibrium there can be no net conduction, it cannot be said that there can be no conduction. In fact, modern condensed matter physics would surely argue that thermal equilibrium within objects is sustained through conduction, not radiation. Planck like Stewart before him [26, 76, 82] invokes internal radiation as a central component of his proof. He does so precisely to avoid dealing with reflection. He assumes that the volume elements $d\tau$, $d\sigma$ and $d\sigma'$ can sustain only emission, not reflection. In so doing, he predetermines the outcome he seeks, beginning as we have seen with his equation (1) [9; §6].

3.5 Graphite, carbon-black, and the modern age

Graphite and soot, whose commercial forms include carbon black [88] and black carbon [89], continue to be at the center of nearly all blackbody experiments conducted by the National Bureau of Standards and other laboratories. Nonetheless, certain metal blacks [88], namely platinum black and gold black [90–92], have a narrow range of uses as absorbers, especially at long wavelengths. Platinum black is usually prepared by electroplating the surface with platinum. Gold black is particularly interesting as a material. It is produced, by vaporizing the metal onto a substrate until thin gold films are generated. In this sense, the conductivity of gold is being structurally limited and the resulting material is black. In the end, the metal blacks are used primarily in the infrared, and their applications, while important, even in the days following Planck, are somewhat limited.

It remains the overwhelming case that the walls of many cavities are still made from graphite [93–97]. However, if they are made of alternate materials (i.e. brass [98], copper [99], clay [93]), they are either blackened, or smoked with soot [98], or they are covered with black paint [93, 96, 98–104]. Some of these paints have proprietary contents. Nonetheless, it is relatively certain that they all contain the carbon black pigment [105, 106]. For instance, the author has been able to verify that Aeroglaze Z306 and Z302 both contain carbon black (private communication, Robert Hetzell, Lord Corporation, Erie, PA). The same can be ascertained relative to Nextel Velvet coating P/N101-C10 black. It is true that carbon black, with its extremely high carbon content remains the premium black pigment [105]. Graphite and soot (carbon black, black carbon) continue to absolutely dominate all work with experimental blackbodies.

Even fixed point blackbodies [95] which operate at the freezing points of elements such as gold [95], aluminum, zinc, and tin [100] rely either on graphite [107] walls or cavities coated with black paints. In these fixed point blackbodies, the metal freezing/melting point ensures that the entire surface of the emitter can be temporarily maintained at a unique temperature. Interestingly, the metals themselves appear to be relatively innocuous or transparent to emission by the graphitic, or carbon lined, surfaces of the cavity.

There are restrictions on the quality of freezing point blackbody cavities, and these have been outlined by Geist [108]: "*How well the actual radiance approaches the ideal radiance in a given blackbody is often referred to in a qualitative manner as the quality of the blackbody...The principle restriction on the concept of quality...is that it can only be defined for radiation from blackbodies with wall materials whose thermal radiative parameters are independent of wavelength. One important class of freezing point blackbody for which this is not a serious restriction is the class whose cavity walls are constructed from graphite.*" A mathematical treatment of laboratory blackbodies reveals that the production of a cavity whose performance will yield a high quality blackbody is not a trivial task [109].

In any event, it remains clear that whether a blackbody is designed to operate at the freezing point of an element or not, graphite [31, 107], or soot (carbon black [105, 106], or black carbon [89]) continue to dominate this field.

4 Conclusion

Through the exposition of Kirchhoff's law, we have been able to highlight that universality does not hold in cavity radiation. The great bulk of experimental evidence leads to this conclusion. Indeed, if blackbody radiation was universal, there would be no need for the National Bureau of Standards to utilize graphite or soot in order to study such processes. The absence of cavities made of arbitrary walls (without any trace of a perfect absorber) is the best physical proof that universality does not hold. Our laboratories require carbon. Nothing further is needed to shatter Kirchhoff's belief. Nonetheless, even the simplest of mathematical considerations suffices to illustrate the point [25]. Perfectly reflecting cavities, containing no objects, emit no radiation [25]. Perfectly reflecting cavities which contain objects emit radiation which is characteristic of these objects [25]. Thus, if a carbon particle is placed within a perfectly reflecting cavity, the cavity will be black, irrespective of the size of the particle. This is a testament to the power of thermal equilibrium; but if the particle is small, it may take some time to reach this equilibrium. Perfectly absorbing cavities emit normal, or blackbody radiation [23, 25]. In such a cavity, the proper description of the radiation from an arbitrary object is $(\varepsilon_v + \rho_v)/(\alpha_v + \rho_v) = f(T, v)$ [25]. This equation echoes Stewart [26, 49, 82]. Conversely, Kirchhoff incorrectly advanced $\varepsilon_v/\alpha_v = f(T, v)$, leading to universality [20–22].

Consequently, when examining blackbody radiation, we are not dealing with a phenomenon of universal significance. Rather, we are dealing with a physical process which is extremely limited in its applications. Blackbodies are made of solids, and specifically relative to practical blackbodies, they are made of graphite. Nature knows no equivalent as is well demonstrated by the review of thermal emissivity tables [31]. Yet, even in the case of radiation from graphite, the physical cause of the process remains remarkably unknown to modern science. The physical species producing blackbody emission has not been concretely identified [19, 23].

If Planck's law [1, 2] has not been linked to a physical species, it is in part certain that the formulation of Kirchhoff's law [20–22], in its creation of universality, hindered the process. At the same time, there is a fundamental difficulty in providing a complete physical picture relative to thermal emission. This is because the nature of the oscillators, at the heart of thermal radiation, can change depending on the physical nature of the material being examined. The thermal emission profiles of metals are highly affected by their conduction electrons, at least in the sense that their presence acts to prevent emission and favor reflection. For each opaque material, a unique emission profile exists [31] and the answer to these problems will most likely involve the use of computational tools, not simple algebraic solutions. It may well be that entire lattices will have to be represented and processed in digital forms, in order to yield meaningful results. Yet, some thermal emission profiles, which provide Planck-like behavior, such as graphite, the microwave background (only apparent Planckian behavior), and the emission of the photosphere (only apparent Planckian behavior), may be capable of being solved analytically. A solution for one of these is likely to have broad implications for the others. At the same time, only graphite will remain truly Planckian in nature, as it is the only one restricted to a solid. The microwave background and the photosphere produce only apparent Planckian spectra. Since their physical sources are not solids, their relevant internal bonds (if any) are weak, and they support convection processes which alter the validity of the temperatures they report [33].

For graphite or soot

$$\frac{\varepsilon_v}{\alpha_v} \sim \frac{2hv^3}{c^2}\,\frac{1}{e^{hv/kT}-1}$$

as Planck derived [1, 2]. Conversely, for the Sun and the microwave background, we can write that

$$\frac{\varepsilon_v}{\alpha_v} \sim \frac{2hv^3}{c^2}\,\frac{1}{e^{hv/kTapp}-1}\,,$$

where T_{app} is constant. $T_{app} = T/\iota$, where T is the real temperature of the source and ι is a variable, with temperature dependence, whose value is ~1,000 for the photosphere and ~100 for the microwave background [33]. Thus, the real temperature of the photosphere is ~1,000 times higher than the currently accepted temperature [34, 35]. Similarly, the temperature for the source of the microwave background is ~100 times higher than the measured value [33, 39, 40]. These complications arise because we are dealing with non-solids outside the confines of enclosure [23, 33].

If a Planckian approach is used to analyze graphite, the carbon nucleus can be viewed as the mass and the carbon-carbon bond as the spring in an oscillator scenario [1, 2]. If the microwave background is confirmed to be from an oceanic source [33, 36–42], then the oscillators might be entire water molecules, linked through weak hydrogen bonding, vibrating within a fleeting lattice. In this regard, it remains interesting that water can become completely black. This occurs, for instance, when shock waves from nuclear explosions propagate in the sea. For the photosphere, if a hydrogen-based condensed Sun is contemplated [34, 35], the vibration of protons within a fleeting lattice field will have to be considered. In this case, the electrons might simply occupy conduction bands. Nonetheless, the nuclei should be viewed as being confined to a distinct condensed structure which, though fleeting, is being maintained, perhaps only by the need to sustain the quantum mechanical requirements to produce the conduction bands. Physicists versed in the properties of condensed liquid metallic hydrogen might consider these questions. Only the future can reveal how mankind moves forward on linking a given physical species to a center of emission.

With the loss of the universal function, the proper treatment of materials will involve the long recognized fact that the ratio of the emission, e, of an object to its absorption, a, is equal to a complex function dependent on its temperature, T, its nature, N, (its shape, the roughness of its surface, its specific heat, etc.), and the wavelengths of interest, namely $e/a = f(T, N, \lambda)$. Also, e and a, individually, are functions of these parameters, otherwise, as Agassi highlights [30], spectroscopy would be impossible. The aforementioned equation can be simplified to Kirchhoff's formulation $e/a = f(T, \lambda)$ only within a perfectly absorbing enclosure or within an enclosure where a perfect absorber is also present. In all these cases, the object never truly becomes a blackbody. Along with its own emission, it simply reflects radiation in the cavity and appears to hold blackbody properties. It is difficult to envision how this scenario is of any use in modern physics.

The physics community has persisted in upholding Kirchhoff's law of thermal emission even though it has been refuted both recently [23–25] and in the past (see [86] for a discussion of the controversy surrounding Kirchhoff's law). This has occurred despite the fact that graphite and soot are uniquely positioned in all blackbody work with cavities. Nonetheless, some of this hesitance may be due to a certain respect, even reverence, for Kirchhoff and his work. In part, there is also the proximity to Planck himself. Such concerns are unjustified, in that even if Kirchhoff's law loses its universal status, nothing changes relative to Planck's derivation. Planck's law [1, 2] simply becomes devoid of universal significance. It maintains its value relative to the treatment of radiation within perfectly absorbing enclosures and within perfectly reflecting enclosures which contain a perfect absorber. Of course, Planck's equation will no longer extend to simple perfectly reflecting enclosures.

At the same time, the merit of k and h, at the heart of Planck's law, is not altered. The great changes simply involve the interdict of extending the laws of thermal emission [1, 2, 110, 111], without modification, to objects which are not solids [33–42] or enclosed within perfectly absorbing cavities [23–25].

Despite these facts, it may well be that physics remains unwilling to pronounce itself relative to the invalidity of Kirchhoff's treatment until the consequences of the error become so great that society demands retraction. The reassignment of the microwave background to the Earth [33, 36–42] should eventually provide sufficient motivation to act. On that day, a new age in astrophysics will spring forth [34, 35] and we may finally begin to write the long-awaited ode to Balfour Stewart.

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Dedication

This work is dedicated to my sister Hélène, her husband Gervais Bédard, their children (Sonia, Karl, and Geneviève) and their grandchildren (Megan Gagné, Raphaël Turcotte†, and Théogènes Turcotte) on this, their 30th wedding anniversary (May 22, 1978).

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The Rôle of the Element Rhodium in the Hyperbolic Law of the Periodic Table of Elements

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The rôle of the element rhodium as an independent affirmation of calculations by the Hyperbolic Law and validity of all its relations is shown herein. The deviation in calculation by this method of the atomic mass of heaviest element is 0.0024%, and its coefficient of scaling 0.001–0.005%.

1 Introduction

The method of rectangular hyperbolas assumes that their peaks (i.e. vertices) should be determine with high accuracy. For this purpose the theorem of Lagrange and the coefficient of scaling calculated by the Author for transition from the system of coordinates of the image of a hyperbola, standard practice of the mathematician, and used in chemistry, are utilized. Such an approach provides a means for calculating the parameters of the heaviest element in the Periodic Table of D. I. Mendeleyev [1].

In the first effect of the Hyperbolic Law it is shown that to each direct hyperbola corresponds an adjacent hyberbola: they intersect on the line $Y = 0.5$ at a point the abscissa of which is twice the atomic mass of an element [2]. This fact is clearly illustrated for Be, Ca, Cd in Fig. 1.

Upon close examination of the figure deeper relationships become apparent:

- From the centre of adjacent hyperbolas ($X = 0, Y = 1$) the secants have some points of crossing, the principal of which lie on the line $Y = 0.5$ and on the virtual axes (peaks);
- The secants intersect a direct hyperbola in two points, with gradual reduction of a segment with the increase in molecular mass;
- Behind the virtual axis of adjacent hyperbolas the secants cut a direct hyperbola in only one point;
- In conformity therewith, the magnitude of the abscissa, between a secant and a point of intersection of hyperbolas on the line $Y = 0.5$, also changes;
- For the element rhodium the secant becomes a tangent and also becomes the virtual axis of adjacent hyperbolas.

2 Mathematical motivation

On the basis of the presented facts, we have been led to calculations for 35 elements to establish the laws for the behavior of secants. The results are presented in the table for the following parameters:

- Atomic numbers of elements and their masses;
- Calculated coordinates of peaks of elements (the square root of the atomic mass and coefficient of scaling 20.2895 are used);
- Abscissas of secants on the line $Y = 0.5$ are deduced from the equation of a straight lines by two points

$$\frac{(X - X_1)}{(X_2 - X_1)} = \frac{(Y - Y_1)}{(Y_2 - Y_1)} \quad \text{(column 6);}$$

- Points of intersection of direct and adjacent hyperbolas (column 7);
- Difference between the abscissas in columns 6 and 7 (column 8);
- Tangent of an inclination of a secant from calculations for column 6.

According to columns 6 and 7 in Fig. 2, dependences which essentially differ from each other are obtained. Abscissas of secants form a curve of complex form which can describe with high reliability (size of reliability of approximation $R^2 = 1$) only a polynomial of the fifth degree. The second dependency has a strictly linear nature ($Y = 2X$), and its straight line is a tangent to a curve at the point (102.9055, 205.811). For clarity the representation of a curve has been broken into two parts: increases in molecular mass (Fig. 3) and in return — up to hydrogen, inclusive (Fig. 4). The strongly pronounced maximum for elements B, C, N, O, F, Ne is observed.

At the end of this curve there is a very important point at which the ordinate is equal to zero, where (the line of rhodium in the table) the data of columns 6 and 7 coincide.

Thus it is unequivocally established that for rhodium the secant, tangent and the virtual axis for an adjacent hyperbola are represented by just one line, providing for the first time a means to the necessary geometrical constructions on the basis of only its atomic mass (**the only one in the Periodic Table**), for the proof of the Hyperbolic Law.

Graphical representation of all reasoning is reflected in Fig. 5 from which it is plain that the point with coordinates (205.811, 0.5) is the peak of both hyperbolas, and the peaks of Ca and Ta are on both sides of it. Below are the calculations for the basic lines of rhodium on these data:

1. A secant: —

$$\frac{(X-0)}{(205.811-0)}=\frac{(Y-1)}{(0.5-1)}\,,$$

whence

$$Y=-0.0024294134X+1\,.$$

At $Y=0$, $X=411.622$; in this case coordinates of peak will be: $X=205.811$, $Y=0.5$.

2. A tangent:— the equation of a direct hyperbola,

$$Y=\frac{102.9055}{X}\,,$$

its derivative at $X=205.811$, so

$$Y'=-\frac{102.9055}{205.811^2}=-0.0024294134\,,$$

$$Y-0.5=-0.0024294134X+0.5\,.$$

Finally,

$$Y=-0.0024294134X+1\,;$$

at $Y=0$, $X=411.622$.

3. A normal: — (the virtual axis),

$$Y=0.0024294134X\,;$$

at $Y=1$, $X=411.622$.

Here are the same calculations for the tabulated data presented:

1. A secant: —

$$\frac{X}{205.82145}=\frac{(Y-1)}{(0.4999746-1)}\,,$$

whence

$$Y=-0.0024294134X+1\,;$$

$$Y=1\,,\quad X=411.622\,.$$

2. A tangent: —

$$Y=\frac{102.9055}{X}\,,$$

the fluxion at $X=205.821454$,

$$Y'=-\frac{102.9055}{205.82145^2}=-0.0024291667\,,$$

so

$$Y-0.4999746=-0.0024291667(X-205.82145)\,,$$

whence

$$Y=-0.0024291667X+0.99994928\,,$$

$$Y=0\,,\quad X=411.6429\,.$$

3. A normal: —

$$Y=0.0024291667X\,;$$

$$Y=1\,,\quad X=411.6638\,.$$

3 Comparative analysis calculations

For a secant the results are identical with the first set of calculations above, whereas for a tangent and normal there are some deviations, close to last element calculated.

By the first set of calculations above its atomic mass is 411.622; hence the deviation is 411.663243 − 411.622 = = 0.041243 (0.01%). By the second set the size of a tangent and a normal are close to one another (an average of 411.65335) and have a smaller deviation: 411.663243 − − 411.65335 = 0.009893 (0.0024%). This is due to the tangent of inclination of the virtual axis of a direct hyperbola in the first set is a little high.

Using rhodium (Fig. 5) we can check the propriety of a choice of coefficient of scaling. It is necessary to make the following calculations for this purpose:

- Take the square root of atomic mass of rhodium ($X = = Y = 10.1442348$);
- Divide X_0 by X of the peak (205.811/10.1442348 = = 20.2885);
- Divide $Y = 10.1442348$ by Y_0 of the peak (0.5): also gives 20.2885;
- The difference by X and Y with the coefficient obtained, 20.2895, yielding the same size at 0.001 or 0.005%.

Formulae for transition from one system of coordinates to another have been given in the first paper of this series.

Using data for peaks, from the table, we get the following results:

Coordinates of peak

$$X_0=205.8215,\quad Y_0=0.49997,$$

$$X=Y=10.1442348,$$

then

$$\frac{X_0}{X}=20.2895,\quad \frac{Y}{Y_0}=20.2897,$$

i. e. absolute concurrence (maximum difference of 0.0009%).

4 The rôle of the element Rhodium

However, all these insignificant divergences do not belittle the most important conclusion: that the validity of the Hyperbolic Law is estabished because the data calculated above completely coincide with calculations for rhodium is proved, based only on its atomic mass.

All the calculations for the table were necessary in order to find a zero point for rhodium, for which it is possible to do so without calculating the secant, but using only its atomic mass, thereby verifying the Hyperbolic Law.

How to get the correct choice of abscissa of a secant is depicted in Fig. 6 (using beryllium as an example) where instead of its tabulated value, 35.7434, the value equal to twice the point of intersection (36.0488) has been used. Here we

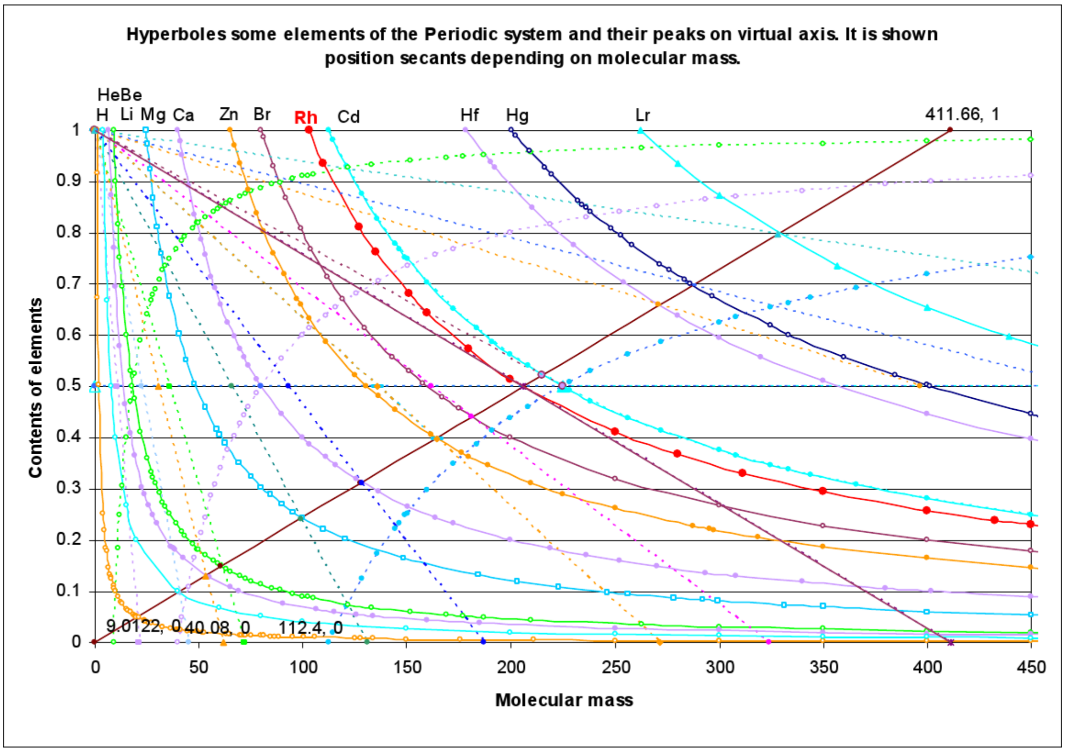

Fig. 1

tried to make a start from any fixed point not calculated (similar to the case for rhodium). It has proved to be impossible and has led to a mistake in the definition of the peak. In Fig. 7 the geometrical constructions for beryllium on the basis of correct settlement of data are given.

5 Conclusions

Previously we marked complexity of a choice of peak of a hyperbola of an element in the coordinates, satisfying the conditions $Y \leqslant 1$, $K \leqslant X$, as on an axis of ordinates the maximum value being a unit whilst the abscissa can take values in the hundreds of units. The problem has been solved by means of the theorem of Lagrange and the coefficient of scaling deduced. On the basis thereof our further conclusions depended, so it was very important to find a method not dependent on our calculations and at the same time allowing unequivocally to estimate the results. Owing to properties of the virtual axis of an rectangular hyperbola on which peaks of all elements lie, it is enough to have one authentic point.

Analyzing the arrangement of the virtual axes of direct and adjacent hyperbolas, we have paid attention to their point of intersection (205.83, 0.5), the abscissa of which is exactly half of atomic mass of the last element. As secants from the centre $X = 0$, $Y = 1$ cut direct hyperbolas any way (Fig. 1), we have been led to necessary calculations and have obtained a zero point at which the secant coincides with a tangent and the valid axis. The divergence with tabular data is in the order of 0.004%–0.009%.

Thus rhodium provides an independent verification of the method of rectangular hyperbolas for the Periodic Table of elements of D. I. Mendeleyev.

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1	2	3	4	5	6	7	8	9
El.	**No.**	**At. mass**	**X_0 peak**	**Y_0 peak**	**Abs. secant**	**Cross. hyperb.**	**$\Delta = 6 - 7$**	**$\tan a$, secant**
H	1	1.0079	20.3695	0.04948	10.715	2.0158	8.6992	−0.046664
He	2	4.0026	40.5992	0.0986	22.5163	8.0052	14.5111	−0.0222
Li	3	6.941	53.4543	0.12985	30.7155	13.882	16.8335	−0.01628
Be	4	9.0122	60.9097	0.14976	35.7434	18.0244	17.719	−0.014
B	5	10.811	66.712	0.162055	39.80692	21.622	18.18492	−0.01256
C	6	12.0107	70.3162	0.1708	42.4	24.0214	18.3786	−0.0117923
N	7	14.0067	75.9345	0.184458	46.5546	28.0134	18.5412	−0.01074
O	8	15.9994	81.1565	0.197143	50.5423	31.9988	18.5435	−0.009893
F	9	18.9984	88.4362	0.21483	56.3163	37.9968	18.3195	−0.008878
Ne	10	20.1797	91.1441	0.2214	58.5311	40.3594	18.1717	−0.0085425
Mg	12	24.305	100.0274	0.242983	66.0669	48.61	17.4569	−0.007568
S	16	32.065	114.89125	0.27909	79.6849	64.13	15.5549	−0.006273
Ca	20	40.078	128.4471	0.31202	93.3508	80.156	13.1948	−0.005356
Cr	24	51.9961	146.3042	0.3554	113.484	103.9922	9.4918	−0.004406
Zn	30	65.409	164.093	0.3986	136.428	130.818	5.61	−0.003665
Br	35	79.904	181.366	0.44057	162.0982	159.808	2.29	−0.003085
Zr	40	91.224	193.7876	0.47074	183.075	182.448	0.627	−0.002731
Mo	42	95.94	198.7336	0.482757	192.1085	191.88	0.2285	−0.002603
Rh	**45**	**102.906**	**205.82145**	**0.4999746**	**205.811**	**205.811**	**0**	**−0.00242941**
Cd	48	112.411	215.1175	0.52256	225.26	224.822	0.458	−0.00221946
Ba	56	137.327	237.7658	0.577573	281.428	274.654	6.774	−0.001777
Nd	60	144.242	243.6785	0.591936	298.5785	288.484	10.09455	−0.0016746
Sm	62	150.36	248.7926	0.60436	314.417	300.72	13.7	−0.00159
Dy	66	162.5	258.6414	0.628283	347.9	325	22.9	−0.001437
Yb	70	173.04	266.8976	0.64834	379.48	346.08	33.4	−0.0013176
Hf	72	178.49	271.068	0.65847	396.843	356.98	39.863	−0.00126
Ta	73	180.948	272.928	0.663	404.923	361.896	43.027	−0.0012348
Re	75	186.207	276.8658	0.67255	422.7646	372.414	50.35	−0.0011827
Ir	77	192.217	281.2984	0.68332	444.1376	384.434	59.704	−0.0011258
Hg	80	200.59	287.3598	0.698	475.8318	401.18	74.6518	−0.00105
At	85	210	294.0228	0.71423	514.44	420	94.44	−0.000972
Fr	87	223	302.9868	0.736	573.85	446	127.85	−0.00087
Th	90	232.038	309.0658	0.75077	620.0472	464.07612	155.971	−0.000806
Am	95	243	316.282	0.7683	682.53	486	196.53	−0.0007326
Es	99	252	322.0858	0.7824	740.0874	504	236.0874	−0.0006756

a) columns 4 and 5 contain coordinates of peaks of rectangular hyperbolas of elements;

b) in a column 6 are presented abscissas the secants which are starting with the peak center (0,1) up to crossings with line $Y = 0.5$; at prolongation they cross the valid axis in points peaks;

c) in a column 7 are resulted abscissa points of crossing of a direct and adjacent hyperbola each element presented here;

d) the column 8 contains a difference between sizes of 6 and 7 columns;

e) in a column 9 tangents of a corner of an inclination of secants are resulted; at an element "rhodium" this line crosses an axis X in a point with abscissa, equal 411.622, and its position coincides with tangent in peak; $411.66 - 411.62 = 0.04$ or nearly so 0.01% from atomic mass.

Table 1: Results of calculations for some elements of the Periodic Table

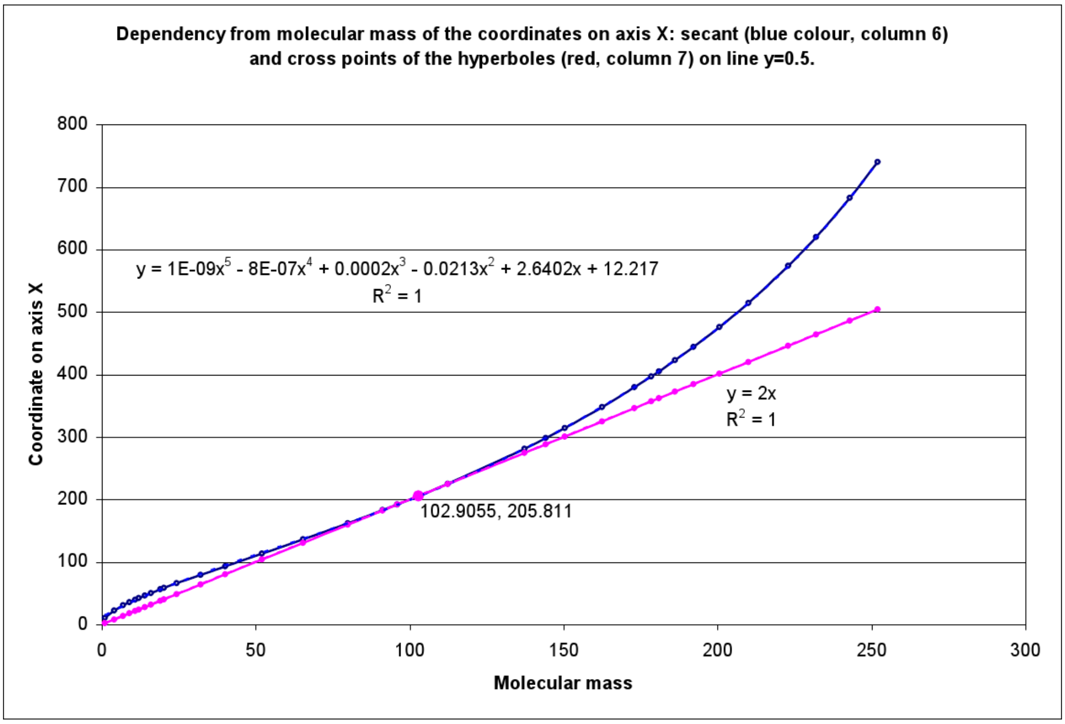
Dependency from molecular mass of the coordinates on axis X: secant (blue colour, column 6) and cross points of the hyperboles (red, column 7) on line y=0.5.
y = 1E-09x^5 - 8E-07x^4 + 0.0002x^3 - 0.0213x^2 + 2.6402x + 12.217
R^2 = 1
y = 2x
R^2 = 1
102.9055, 205.811
Coordinate on axis X
Molecular mass
800
700
600
500
400
300
200
100
0
0
50
100
150
200
250
300

Fig. 2

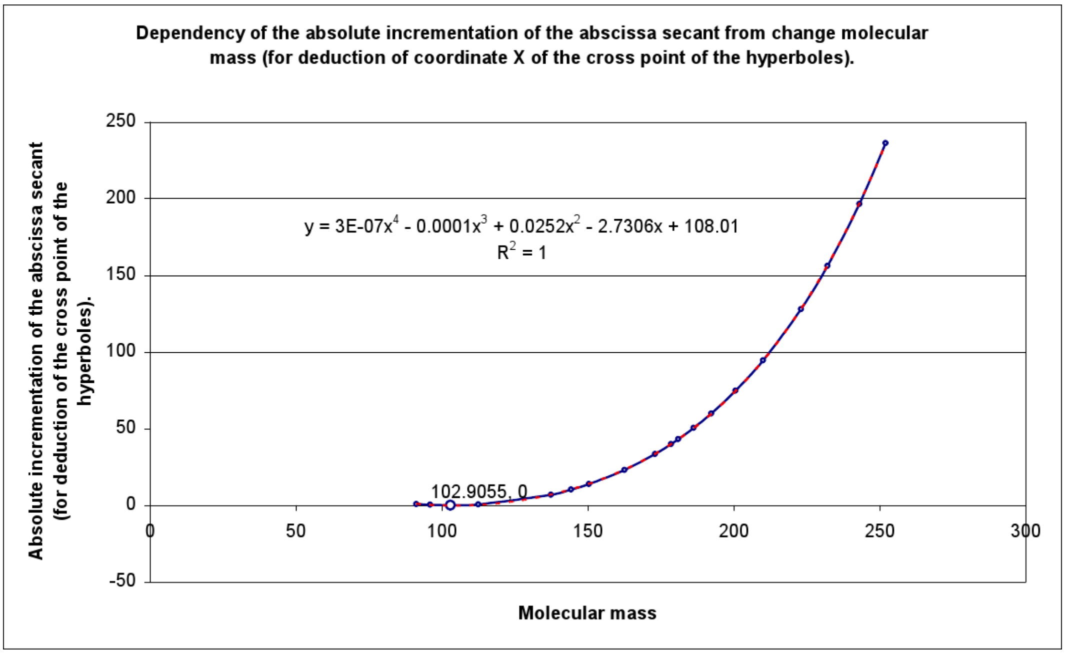
Dependency of the absolute incrementation of the abscissa secant from change molecular mass (for deduction of coordinate X of the cross point of the hyperboles).
y = 3E-07x^4 - 0.0001x^3 + 0.0252x^2 - 2.7306x + 108.01
R^2 = 1
102.9055, 0
Absolute incrementation of the abscissa secant (for deduction of the cross point of the hyperboles).
Molecular mass
250
200
150
100
50
0
-50
0
50
100
150
200
250
300

Fig. 3

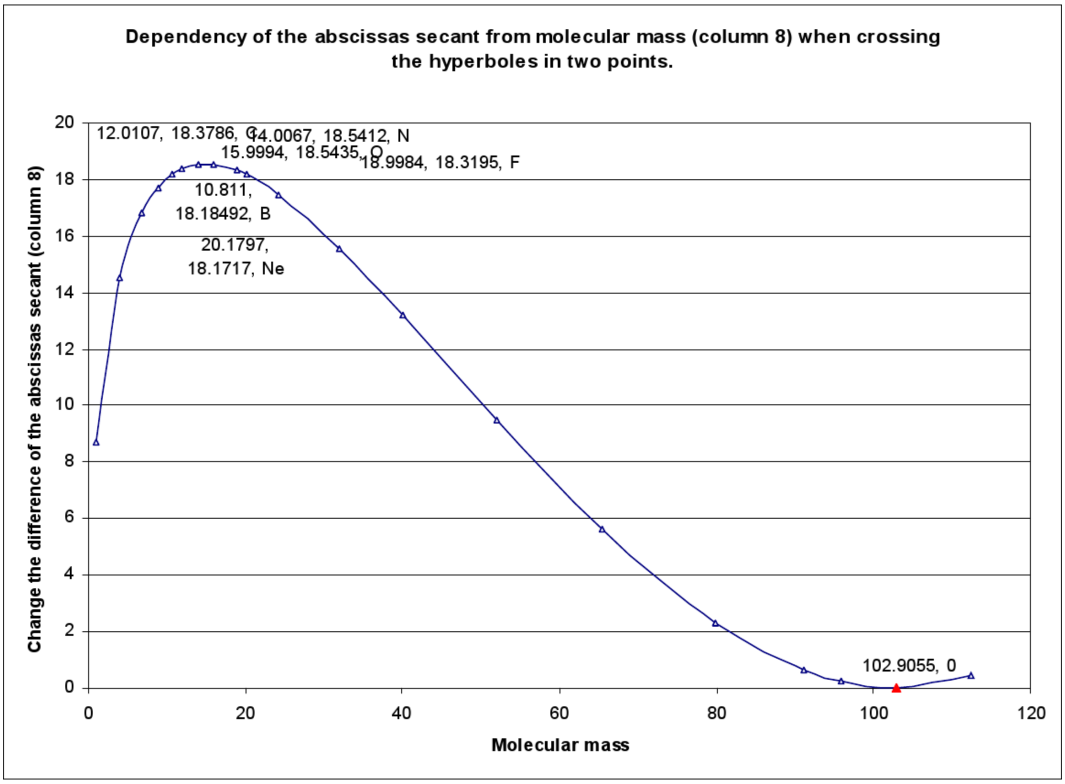
Dependency of the abscissas secant from molecular mass (column 8) when crossing the hyperboles in two points.
Change the difference of the abscissas secant (column 8)
Molecular mass
0
2
4
6
8
10
12
14
16
18
20
0
20
40
60
80
100
120
12.0107, 18.3786, C
14.0067, 18.5412, N
15.9994, 18.5435, O
18.9984, 18.3195, F
10.811, 18.18492, B
20.1797, 18.1717, Ne
102.9055, 0

Fig. 4

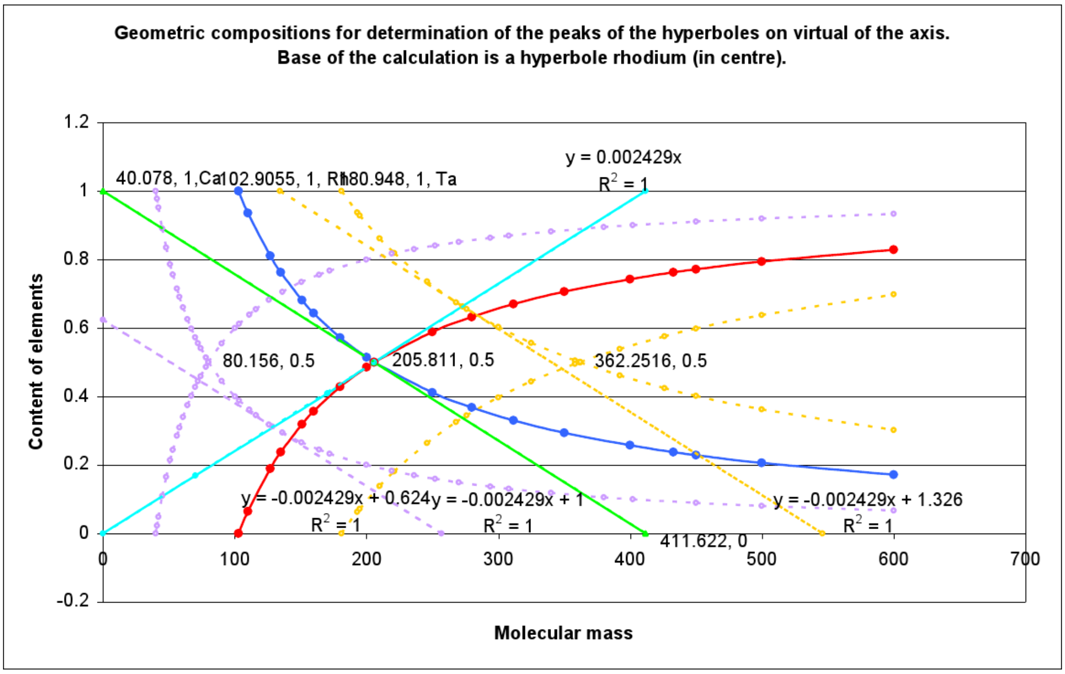
Geometric compositions for determination of the peaks of the hyperboles on virtual of the axis. Base of the calculation is a hyperbole rhodium (in centre).
Content of elements
Molecular mass
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
100
200
300
400
500
600
700
40.078, 1,Ca
102.9055, 1, Rh
180.948, 1, Ta
y = 0.002429x
R² = 1
80.156, 0.5
205.811, 0.5
362.2516, 0.5
y = -0.002429x + 0.624
R² = 1
y = -0.002429x + 1
R² = 1
y = -0.002429x + 1.326
R² = 1
411.622, 0

Fig. 5

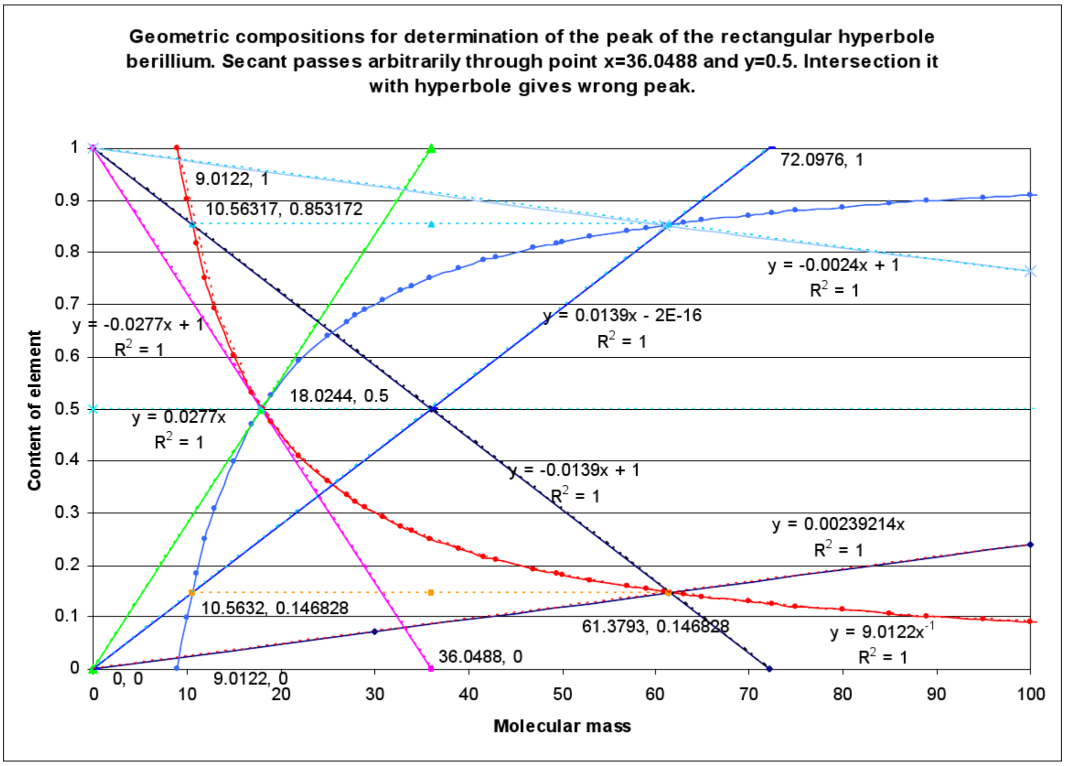
Geometric compositions for determination of the peak of the rectangular hyperbole berillium. Secant passes arbitrarily through point x=36.0488 and y=0.5. Intersection it with hyperbole gives wrong peak.

Fig. 6

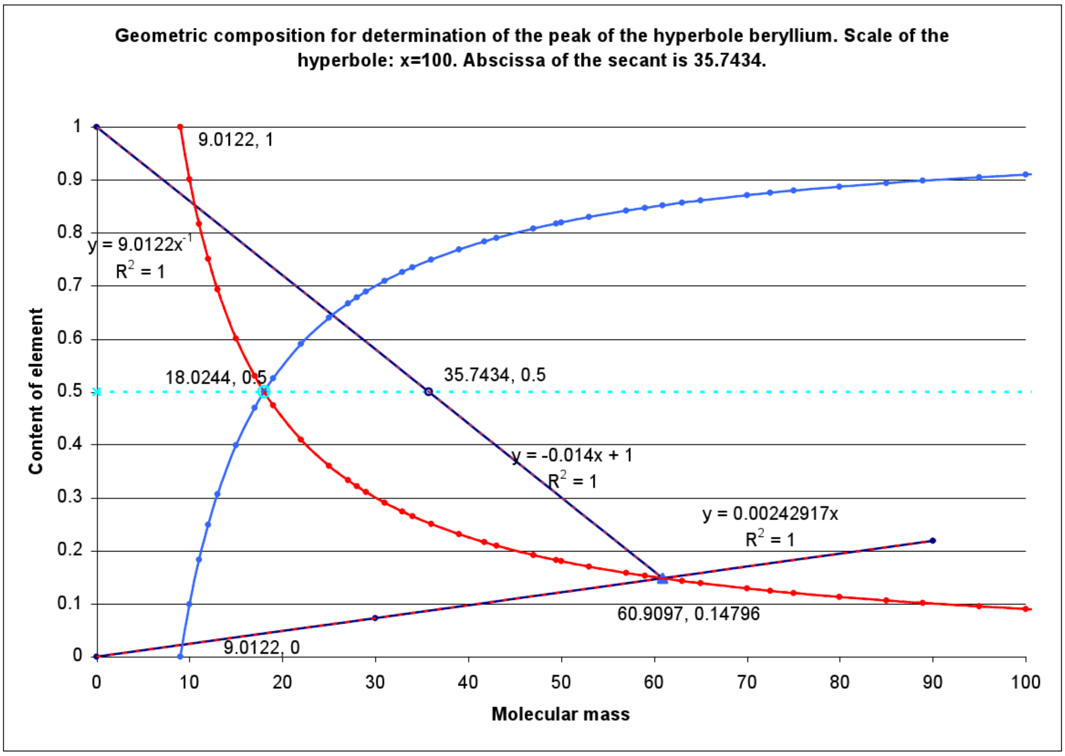
Geometric composition for determination of the peak of the hyperbole beryllium. Scale of the hyperbole: x=100. Abscissa of the secant is 35.7434.

Fig. 7

What Gravity Is. Some Recent Considerations

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It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. The present article describes a non-exhaustive list of such gravitation theories for the purpose of inviting further and more clear discussions.

1 Introduction

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. Of course, our purpose here is not to say the last word on this interesting issue.

2 Newtonian and non-relativistic approaches

Since the days after Newton physicists argued what is the meaning of "action at a distance" (Newton term) or "spooky action" (Einstein term). Is it really possible to imagine how an apple can move down to Earth without a medium whatsoever?

Because of this difficulty, from the viewpoint of natural philosophy, some physicists maintained (for instance Euler with his impulsion gravity), that there should be "pervasive medium" which can make the attraction force possible. They call this medium "ether" though some would prefer this medium more like "fluid" instead of "solid". Euler himself seems to suggest that gravitation is some kind of "external force" acting on a body, instead of *intrinsic* force:

> "gravity of weight: It is a power by which all bodies are forced towards the centre of the Earth" [3].

But the Michelson-Morley experiment [37] opened the way for Einstein to postulate that ether hypothesis is not required at all in order to explain Lorentz's theorem, which was the beginning of Special Relativity. But of course, one can ask whether the Michelson-Morley experiment really excludes the so-called ether hypothesis. Some experiments after Michelson seem to indicate that "ether" is not excluded in the experiment setup, which means that there is Earth absolute motion [4, 5].

To accept that gravitation is external force instead of intrinsic force implies that there is distinction between gravitation and inertial forces, which also seem to indicate that inertial force can be modified externally via electromagnetic field [6].

The latter notion brings us to long-time discussions in various physics journals concerning the electromagnetic nature of gravitation, i.e. whether gravitation pulling force have the same properties just as electromagnetic field is described by Maxwell equations. Proponents of this view include Tajmar and de Matos [7, 8], Sweetser [9]. And recently Rabounski [10] also suggests similar approach.

Another version of Euler's hypothesis has emerged in modern way in the form of recognition that gravitation was carried by a boson field, and therefore gravitation is somehow related to low-temperature physics (superfluid as boson gas, superconductivity etc.). The obvious advantage of superfluidity is of course that it remains frictionless and invisible; these are main features required for true ether medium — i.e. no resistance will be felt by objects surrounded by the ether, just like the passenger will not feel anything inside the falling elevator. No wonder it is difficult to measure or detect the ether, as shown in Michelson-Morley experiment. The superfluid Bose gas view of gravitation has been discussed in a series of paper by Consoli et al. [11], and also Volovik [12].

Similarly, gravitation can also be associated to superconductivity, as shown by de Matos and Beck [29], and also in Podkletnov's rotating disc experiment. A few words on Podkletnov's experiment. Descartes conjectured that there is no gravitation without rotation motion [30]. And since rotation can be viewed as solution of Maxwell equations, one can say that there is no gravitation separated from electromagnetic field. But if we consider that equations describing superconductivity can be viewed as mere generalization of Maxwell equations (London field), then it seems we can find a modern version of Descartes' conjecture, i.e. *there is no gravitation without superconductivity rotation*. This seems to suggest the significance of Podkletnov's experiments [31, 32].

3 Relativistic gravitation theories

Now we will consider some alternative theories which agree with both Newton theory and Special Relativity, but differ either slightly or strongly to General Relativity. First of all, Einstein's own attempt to describe gravitation despite earlier gravitation theories (such as by Nordstrom [1]) has been inspired by his thought-experiment, called the "falling elevator" experiment. Subsequently he came up with conjecture that there is proper metric such that a passenger inside the elevator will not feel any pulling gravitation force. Therefore gravitation can be replaced by certain specific-chosen metric.

Now the questions are twofold: (a) whether the proper-metric to replace gravitation shall have non-zero curvature or it can be flat-Minkowskian; (b) whether the formulation of General relativity is consistent enough with Mach principle from where GTR was inspired. These questions inspired heated debates for several decades, and Einstein himself (with colleagues) worked on to generalize his own gravitation theories, which implies that he did find that his theory is not complete. His work with Strauss, Bergmann, Pauli, etc. (Princeton School) aimed toward such a unified theory of gravitation and electromagnetism.

There are of course other proposals for relativistic gravitation theories, such as by Weyl, Whitehead etc. [1]. Meanwhile, R. Feynman and some of his disciples seem to be more flexible on whether gravitation shall be presented in the General-Relativity "language" or not.

Recently, there is also discussion in online forum over the question: (a) above, i.e. whether curvature of the metric surface is identical to the gravitation. While most physicists seem to agree with this proposition, there is other argument suggesting that it is also possible to conceive General Relativity even with zero curvature [13, 14].

Of course, discussion concerning relativistic gravitation theories will not be complete without mentioning the PV-gravitation theory (Puthoff et al. [15]) and also Yilmaz theory [16], though Misner has discussed weaknesses of Yilmaz theory [17], and Yilmaz et al. have replied back [18]. Perhaps it would be worth to note here that General Relativity itself is also not without limitations, for instance it shall be modified to include galaxies' rotation curve, and also it is actually theory for one-body problem only [2], therefore it may be difficult to describe interaction between bodies in GTR.

Other possible approaches on relativistic gravitation theories are using the fact that the "falling-elevator" seems to suggest that it is possible to replace gravitation force with certain-chosen metric. And if we consider that one can find simplified representation of Maxwell equations with Special Relativity (Minkowski metric), then the next logical step of this "metrical" (some physicists prefer to call it "geometrodynamics") approach is to represent gravitation with yet another special relativistic but with extra-dimension(s). This was first conjectured in Kaluza-Klein theory [19]. Einstein himself considered this theory extensively with Strauss etc. [20]. There are also higher-dimensional gravitation theories with 6D, 8D and so forth.

In the same direction, recently these authors put forth a new proposition using Carmeli metric [21], which is essentially a "phase-space" relativity theory in 5-dimensions.

Another method to describe gravitation is using "torsion", which is essentially to introduce torsion into Einstein field equations. See also torsional theory developed by Hehl, Kiehn, Rapoport etc. cited in [21].

It seems worth to remark here, that relativistic gravitation does not necessarily exclude the possibility of "aether" hypothesis. B. Riemann extended this hypothesis by assuming (in 1853) that the gravitational aether is an incompressible fluid and normal matter represents "sinks" in this aether [34], while Einstein discussed this aether in his Leiden lecture *Ether and Relativity*.

A summary of contemporary developments in gravitation theories will not be complete without mentioning Quantum Gravity and Superstring theories. Both are still major topics of research in theoretical physics and consist of a wealth of exotic ideas, some or most of which are considered controversial or objectionable. The lack of experimental evidence in support of these proposals continues to stir a great deal of debate among physicists and makes it difficult to draw definite conclusions regarding their validity [38]. It is generally alleged that signals of quantum gravity and superstring theories may occur at energies ranging from the mid or far TeV scale all the way up to the Planck scale.

Loop Quantum Gravity (LQG) is the leading candidate for a quantum theory of gravitation. Its goal is to combine the principles of General Relativity and Quantum Field Theory in a consistent non-perturbative framework [39]. The features that distinguish LQG from other quantum gravity theories are: (a) background independence and (b) minimality of structures. Background independence means that the theory is free from having to choose an apriori background metric. In LQG one does not perturb around any given classical background geometry, rather arbitrary fluctuations are allowed, thus enabling the quantum "replica" of Einstein's viewpoint that gravity is geometry. Minimality means that the general covariance of General Relativity and the principles of canonical quantization are brought together without new concepts such as extra dimensions or extra symmetries. It is believed that LQG can unify all presently known interactions by implementing their common symmetry group, the four-dimensional diffeomorphism group, which is almost completely broken in perturbative approaches.

The fundamental building blocks of String Theory (ST) are one-dimensional extended objects called strings [40, 41]. Unlike the "point particles" of Quantum Field Theories, strings interact in a way that is almost uniquely specified by mathematical self-consistency, forming an allegedly valid quantum theory of gravity. Since its launch as a dual resonance model (describing strongly interacting hadrons), ST has changed over the years to include a group of related superstring theories (SST) and a unifying picture known as the M-theory. SST is an attempt to bring all the particles and their fundamental interactions under one umbrella by modeling them as vibrations of super-symmetric strings.

In the early 1990s, it was shown that the various superstring theories were related by dualities, allowing physicists to map the description of an object in one superstring theory to the description of a different object in another superstring theory. These relationships imply that each of SST represents a different aspect of a single underlying theory, proposed by E. Witten and named M-theory. In a nut-shell, M-theory combines the five consistent ten-dimensional superstring theories with eleven-dimensional supergravity. A shared property of all these theories is the holographic principle, that is, the idea that a quantum theory of gravity has to be able to describe physics occurring within a volume by degrees of freedom that exist on the surface of that volume. Like any other quantum theory of gravity, the prevalent belief is that true testing of SST may be prohibitively expensive, requiring unprecedented engineering efforts on a large-system scale. Although SST is falsifiable in principle, many critics argue that it is un-testable for the foreseeable future, and so it should not be called science [38].

One needs to draw a distinction in terminology between string theories (ST) and alternative models that use the word "string". For example, Volovik talks about "cosmic strings" from the standpoint of condensed matter physics (topological defects, superfluidity, superconductivity, quantum fluids). Beck refers to "random strings" from the standpoint of statistical field theory and associated analytic methods (spacetime fluctuations, stochastic quantization, coupled map lattices). These are not quite the same as ST, which are based on "brane" structures that live on higher dimensional spacetime.

There are other contemporary methods to treat gravity, i.e. by using some advanced concepts such as group(s), topology and symmetries. The basic idea is that Nature seems to prefer symmetry, which lead to higher-dimensional gravitation theories, Yang-Mills gravity etc.

Furthermore, for the sake of clarity we have omitted here more advanced issues (sometimes they are called "fringe research"), such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al. [35]), antigravity (see for instance Naudin's experiment) etc. [36].

4 Wave mechanical method and diffraction hypothesis

The idea of linking gravitation with wave mechanics of Quantum Mechanics reminds us to the formal connection between Helmholtz equation and Schrödinger equation [22].

The use of (modified) Schrödinger equation has become so extensive since 1970s, started by Wheeler-DeWitt (despite the fact that the WDW equation lacks observation support). And recently Nottale uses his scale relativistic approach based on stochastic mechanics theory in order to generalize Schrödinger equation to describe wave mechanics of celestial bodies [23]. His scale-relativity method finds support from observations both in Solar system and also in exo-planets.

Interestingly, one can also find vortex solution of Schrödinger equation, and therefore it is worth to argue that the use of wave mechanics to describe celestial systems implies that there are vortex structure in the Solar system and beyond. This conjecture has also been explored by these authors in the preceding paper. [24] Furthermore, considering formal connection between Helmholtz equation and Schrödinger equation, then it seems also possible to find out vortex solutions of Maxwell equations [25, 26, 27]. Interestingly, experiments on plasmoid by Bostick et al. seem to vindicate the existence of these vortex structures [28].

What's more interesting in this method, perhaps, is that one can expect to to consider gravitation and wave mechanics (i.e. Quantum Mechanics) in equal footing. In other words, the quantum concepts such as ground state, excitation, and zero-point energy now can also find their relevance in gravitation too. This "classical" implications of Wave Mechanics has been considered by Ehrenfest and also Schrödinger himself.

In this regards, there is a recent theory proposed by Gulko [33], suggesting that matter absorbs from the background small amounts of energy and thus creates a zone of reduced energy, and in such way it attracts objects from zones of higher energy.

Another one, by Glenn E. Perry, says that gravity is diffraction (due to the changing energy density gradient) of matter or light as it travels through the aether [33].

We can remark here that Perry's Diffraction hypothesis reminds us to possible production of energy from physical vacuum via a small fluctuation in it due to a quantum indeterminancy (such a small oscillation of the background can be suggested in any case because the indeterminancy principle). On the average the background vacuum does not radiate — its energy is constant. On the other hand, it experiences small oscillation. If an engine built on particles or field interacts with the small oscillation of the vacuum, or at least "senses the oscillation, there is a chance to get energy from them. Because the physical vacuum is eternal capacity of energy, it is easy to imagine some possible techniques to be discovered in the future to extract this energy.

Nonetheless, diffraction of gravity is not a "new hot topic" at all. Such ideas were already proposed in the 1920's by the founders of relativity. They however left those ideas, even unpublished but only mentioned in memoirs and letters. The main reason was that (perhaps) almost infinitely small energy which can be extracted from such background per second. (In the mean time, there are other vaious proposals suggesting that it is possible to 'extract' energy from gravitation field).

About Glenn Perry and his theory. There is a drawback that that matter he called "aether" was not properly determined by him. In such a way like that, everything can be "proven". To produce any calculation for practical purpose, we should have exact data on the subject of this calculation, and compare it with actual experiments.

On the other hand, such an idea could be put into another field — the field of Quantum Mechanics. That is, to study diffraction not gravitational radiation (gravitational waves which is so weak that not discovered yet), but waves of the field of the gravitational force — in particular those can be seismic-like waves travelling in the cork of the Earth (we mean not the earthquakes) but in the gravitational field of the planet. These seismic-like oscillations (waves) of the gravitational force are known to science, and they aren't weak: everyone who experienced an earthquake knows this fact.

Other hint from wave aspect of this planet is known in the form of Schumann resonance, that the Earth produces vibration at very-low frequency, which seems to support the idea that planetary mass vibrates too, just as hypothesized in Wave Mechanics (de Broglie's hypothesis). Nonetheless, there are plenty of things to study on the large-scale implications of the Wave Mechanics.

5 Concluding remarks

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. Of course, our purpose here is not to say the last word on this interesting issue. For the sake of clarity, some advanced subjects have been omitted, such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al.), antigravity etc. As to the gravitation research in the near future, it seems that there are multiple directions which one can pursue, with which we're not so sure. The only thing that we can be sure is that everything changes (Heraclitus of Ephesus), including how we define "what the question is" (Wheeler's phrase), and also what we mean with "metric", "time", and "space". Einstein himself once remarked that 'distance' itself is merely an illusion.

Acknowledgment

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Models for Quarks and Elementary Particles — Part III: What is the Nature of the Gravitational Field?

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The first two parts of this article series dealt with the questions: What is a quark? and What is mass? While the present models lead to a *physical idea* of the mass, the geometrical theory of the general relativity only shows the *effect* of mass. From the physical idea of mass, from the idea of the resultant vector (EV) as electric flux $\notin$ and from the ideas relating to the magnetic monopole (MMP) it follows that the gravitational field is an electrical field. The share of the electrical gravitational flux $\notin_\Gamma$ on the entire electrical flux $\notin$ of a quark is determined from Newton's empirical gravitational constant G. The superposition of the $\notin_\Gamma$-fluxes of two quark collectives produces the gravitational force effect between two quark collectives. Gravitational fields reach infinitely far according to our current ideas. Connected with the quark oscillations hinted in the Parts I and II this results in the idea of the $\notin$-$\notin_\Gamma$-flux spreading with infinite speed, having enormous consequences.

1 Introduction

In Parts I and II separate reference is made to the most productive assumptions or ideas relating to the development of the models. In Part I the formal assumptions/ideas are shown, which include the vectors in the constellation of the outer product of a vector with certain angular movements. At the end of Part II it transpires that the locus loop created by the EV is a physical central-symmetrical sinus oscillation in the mass-affected three-quark particle. Other productive ideas are the orthogonal, hyperbolic space with two real axes and an imaginary axis as well as the identification of the formal EV with a physical meaning. The EV identified as electrical flux $\notin$ with the dimension [Vm] results in the idea of the MMP. The absolute number of $\notin$ amounts to $\notin\ = 1.8095\times10^{-8}$ [Vm] according to the network of constants, see [1, page 143]. The massless MMP is an important idea to recognise on the one hand what mass is and on the other hand to develop the quark structure of the massless photon(-likes) from the quark composition of the electron.

2 The meaning of the "fountain"

In Part II the model idea for the composition of the MMP with the surrounding electrical field is shown with Fig. 1. Thus, the decisive physical components of a quark are introduced with Part II, not considering the dynamics of these components in mass-affected and massless particles. Relating to Fig. 1 it was not explained what the $\notin_\Gamma$-field is. This is done now.

During the course of the development of the models attempts were made to look behind the facade of Newton's gravitational equation wherein obviously there was no shortage of incorrect estimates, one way streets and wrong tracks. Newton's gravitational constant G included in the equation is one of the many independent quantities of the standard model of physics to be determined empirically.

In Part II it is shown what mass is. The route there commences with the equations of $E = m \times c^2$ and $E = h \times \nu$, resulting in equation 1 of Part II, which can also be described as equation (8–II) of [1]: $m = \frac{elt}{2^e\alpha\times\lambda_C\times c^2}$. If this form is introduced in Newton's gravitational equation $K_\Gamma = G \times \frac{m_a\times m_b}{l^2}$, G can be determined with the correct dimension [m^5/VAs^5]:

$$G = \frac{4^e\alpha^2 \times c^4 \lambda_{Ca}\lambda_{Cb}}{elt \times n_1 \times n_2}\,. \tag{1}$$

In it $^e\alpha$ are the fine structure constant, λ_C the Compton wavelengths of the elementary particles involved and elt see below. On the route to clarifying the gravitational equation the aim is to find what the quantities n_1 and n_2 are and how large they are. If equation (1) is solved for n_1 and n_2 and the Compton wavelengths of the nucleons (as mass-richest elementary particles) are substituted for λ_C, the empirical numerical value $\sqrt{n_1 \times n_2} = n_i = 3.939\times10^{18}$ is obtained.

The n thus are gigantic numbers. What do these gigantic numbers stand for?

At this point it is highly productive to use Fig. 1 of Part II. Visible is the MMP that occurs with highest frequencies, which is enclosed by the electrical source flux $\notin$. Here, by far the predominant part of this source flux $\notin$ is closely connected with the magnetic flux Φ (Maxwell). Only the minute share $\notin_\Gamma$ of the total flux $\notin$ leads to the outside. This share is expressed in the simple relationship:

$$\notin_\Gamma = \frac{1}{n} \times \notin\ . \tag{2}$$

If the gigantic numbers n are substituted in the equation (2) (see [1, page 172, equation (8–XIII)]), it follows:

$$\notin_\Gamma = \frac{1}{3.939\times10^{18}} \times \notin =$$
$$= 2.539\times10^{-19} \times 1.8095\times10^{-8} = 4.594\times10^{-27} \text{ [Vm]}.$$

This is the minute share of the $\notin$-field $\notin_\Gamma$ ($\notin_\Gamma$-field or gravitational field), leaving the quarks of a three-quark particle (3QT). $\notin_\Gamma$ is shown as a symbolic line in Fig. 1, Part II.

In addition to Newton's gravitational equation there are further important equations of physics with a similar structure, such as the equations of Coulomb (elec. charges), Rydberg (spectral series) and Schrödinger (waves). These equations are different forms of the universal equation from [1, page 157]:

$$elt \times elt \times n_1 \times n_2 = a \times b \times elt_a \times elt_b \,. \tag{3}$$

In it the universal constant elt has the dimension [VAsm], [1, page 141]. It can be composed of many kinds of constants, e.g.: $elt = {}^N h \times c$ [VAsm] with ${}^N h = h \times 2^e \alpha$.

Equation (3) can be paraphrased with some considerations in a further equation (4) , which can be written next to the equations of Coulomb (charges), Newton (gravitation), Rydberg (spectral series) and Schrödinger (waves): With $elt = K \times l^2$ and according to [1, Fig. 8–1c], $elt = \notin^2 \times \varepsilon_0$ it follows from equation (3):

$$K = \frac{\varepsilon_0}{n_1 \times n_2} \times \frac{a\notin \times b\notin}{l^2} \,. \tag{4}$$

If the relationship $\notin_\Gamma = \frac{1}{n} \times\notin$ of equation (2) is substituted in equation (4) and if some more considerations are examined, the following is obtained:

$$K = \varepsilon_0 \times \frac{a\notin_\Gamma \times b\,\notin_\Gamma}{l^2} \,, \tag{4a}$$

$$K = \frac{\varepsilon_0}{0.8\,\pi} \times \frac{a\notin_\Gamma \times b\notin_\Gamma}{l^2} \,. \tag{4b}$$

Thus the following is realised:

1. The meaning of the gigantic numbers n_1 and n_2 in Newton's empirical, gravitational constant G analysed with equation (1) is seen as follows. With the product of the inverse of the number n_i and of the electrical source flux $\notin$ the minute fraction of the electrical source flux, that is to say $\notin_\Gamma$, of each "3QT" is described, where $\notin_\Gamma$ is leaving the quarks of a "3QT". The minute fraction of $\notin$ accounts for the $\notin_\Gamma$-field of a quark or a "3QT";
2. The quantity of said fraction of the $\notin$-field of a "3QT" is $\frac{1}{3.939\times10^{18}} = 2.539\times10^{-19}$ or inverted $3.939\times10^{18}\times$ $\times \notin_\Gamma = \notin$. $\notin_\Gamma$ has the empirical value $\notin_\Gamma = 1.8095\times10^{-8}$ [Vm] $\times 2.539\times10^{-19} = 4.594\times10^{-27}$ [Vm] as absolute number. These numbers apply to our galactic environment;
3. The equations (4a) and (4b) signify that the superposition of the $\notin_\Gamma$-fields of two quarks or two quark collectives (a and b) produces the gravitational force effect between two quark collectives;
4. These considerations have made the "gravitation" a superposition of physical namely electrical $\notin_\Gamma$-fields of highest frequency!

3 Some aspects relating to the $\notin_\Gamma$-fields

In Part II it is explained by means of Shapiro's experiments how electrical fields and thus the gravitational fields influence the photon(-likes). This physical substantiation for example for the reduction of the speed of light ("refractive index of the vacuum") is to be preferred compared to an substantiation through the geometrical theory of the general relativity.

Gravitational fields reach infinitely far according to our current ideas. The loci of the quarks (sinus oscillations) of which we and our environment consist, are traversed within 10^{-20} (electrons) to 10^{-25} (nucleons) seconds. This means the $\notin_\Gamma$-field of a quark expands into infinity and contracts again within this absurdly short time. The propagation speed of the $\notin_\Gamma$-field is thus infinitely large. (Of course this has an effect on large research projects as e.g. LISA with which the allegedly wave-shaped and light-speed propagation of the gravitational field according to the standard physics is to be investigated.)

The infinitely fast propagation of the $\notin_\Gamma$-field has "natural" consequences everywhere. If the composition of the quarks according to Fig. 1 of Part II applies — which is assumed in these models — the electrical field $\notin$ enclosing the MMPs also expands at infinite speed. This means the $\notin$-fields of the mass-affected particles occur instantaneously. The range of the $\notin$-fields is approximately congruent with the range within the Maginpar or the range of the $\notin$-fields is congruent with the confinement. The confinement located inside a particle is marked off from the outer range by a spherical shell around the coordinate centre with approximately the radius of the Maginpar. ***No causality applies any longer in the small range of the $\notin$-field within the confinement!***

The infinitely fast propagation of the $\notin$-field undoubtedly also influences the uncertainty principle. The latter is valid for the range outside the confinement and therefore for electromagnetic processes. In the outer range with causality — with Δt between two events — applies e.g. $\Delta t \times \Delta E = h$ or $\Delta x \times \Delta p = h$.

Inside the confinement the ranges for the toroidal magnetic field Φ and the electric source field $\notin$ are distinguished, where $\Delta t = 0$ applies because of the instantaneous propagation of the $\notin$-field. Some relation for the interior of the confinement corresponding to the uncertainty principle looks different; the input quantities are certain: ${}^N\Theta \times \nu \times \lambda = {}^N h$. The product from inertia quantum ${}^N\Theta$ times frequency ${}^N\Theta \times \nu$

corresponds to the impulse p or Δp and λ corresponds to the x or Δx. (Otherwise ${}^N\Theta = {}^N h/c$ is the definition equation for the natural constant ${}^N\Theta$.)

Entirely different aspects are touched by the infinitely fast propagation of the $\notin_\Gamma$-field, which are merely mentioned here but not discussed: A) The infinitely fast propagation of the $\notin_\Gamma$-field revitalises the Mach principle according to which the local behaviour of matter is based on the influences of the remainder of the universe. B) The universal structure of galactic chains and dark bubbles and the synchronised creation of galaxies are based on the infinitely fast propagation. C) According to the models the centres of the galaxies are quantum objects. The considerations relating to causality and uncertainty also apply to these. D) The Planck length, [1, page 178], is determined through the interaction of MMP and $\notin$-$\notin_\Gamma$-field. E) The experiments of A. Zeilinger for teleportation are based on the infinitely fast propagation of the $\notin$-field in the rapidly enlarging confinement of polarisation-entangled photons (12QT).

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Models for Quarks and Elementary Particles — Part IV: How Much Do We Know of This Universe?

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Essential laws and principles of the natural sciences were discovered at the high aggregation levels of matter such as molecules, metal crystals, atoms and elementary particles. These principles reappear in these models in modified form at the fundamental level of the quarks. However, the following is probably true: since the principles apply at the fundamental level of the quarks they also have a continuing effect at the higher aggregation levels. In the manner of the law of mass action, eight processes for weak interaction are formulated, which are also called Weak Processes here. Rules for quark exchange of the reacting elementary particles are named and the quasi-Euclidian or complex spaces introduced in Part I associated with the respective particles. The weak processes are the gateway to the "second" strand of this universe which we practically do not know. The particles with complex space, e.g. the neutrino, form this second strand. According to the physical model of gravitation from Part III the particles of both strands have $\notin_{\Gamma}$-fields and are thus subject to the superposition, which results in the attraction by gravity of the particles of both strands. The weak processes (7) and (8) offer a fair chance for the elimination of highly radioactive waste.

1 Introduction

The first parts of this series of papers have headline questions which are answered within the scope of the models [1]: I) What is a quark? II) What is mass? III) What is the nature of the gravitational field?

Which of the three questions will a physicist representing the current standard model be able to answer positively without hesitation? The standard model of physics combines huge quantities of analyses, conformities with natural laws and theories. However, too many independent quantities that can only be captured empirically still enter the standard model of physics and inconsistencies between individual theories are known. For this reason, theoreticians are looking for new physics especially in the field of the strings, loops and branes; however, they have been unable to establish any reference to reality. The standard model of cosmology has the general theory of relativity (GTR) as thread, wherein the GTR is a geometrical and not a physical theory. Despite this deficit the mainstream of cosmologists is absolutely convinced of the big bang model which is based on the GTR, wherein the big bang is a central part of the standard model of cosmology. The physical model of gravitation presented in Part III opens up a new interpretation of our universe. The perspectives of Part III render a Part V for cosmology — the utmost level of organisation — unnecessary. But there is a Part V in preparation concerning the magnetic load, which leads to the undermost level of organisation of our universe. Although many relationships are better recognizable with this model than in the past, there is certainly a lot we do not know of our universe.

2 The weak interaction

The equations of the weak interaction which in the following are also called "Weak Processes" are the central content of the present Part IV. Physics books present equations relating to the weak interaction. These equations are considered correct although the authors have no exact idea of what a quark is, although they are uncertain as to the mass possessed for instance by a neutrino, although they should have doubts in the uniformity of so-called "elementary particles", although they are looking for additional particles that could be included in the equations.

An often-quoted equation in the literature is formulated thus:

$$p^{+} + \bar{\nu}_{e} \rightarrow n^{0} + e^{+}. \tag{1}$$

According to Table 1 of Part I, each of the four elementary particles involved is a three-quark particle (3QT). If this is used to make a quark equation — which cannot happen in the standard model of physics — according to the models to date equation (1) must read as follows:

$${\underbrace{uu}}_{||}\, d + {\underbrace{\bar{d}\bar{d}}}_{\perp}\, \bar{u} \rightarrow {\underbrace{dd}}_{||}\, u + {\underbrace{\bar{d}\bar{d}}}_{\perp}\, \bar{d}\,.$$

As can be seen, the quarks on both sides do not agree in ***number and type***. If the left side is correct, an $\bar{u}$ and an u are missing on the right, instead there are a d and an $\bar{d}$ too many on the right; the charge balance would be correct as in equation (1).

The literature equation (1) cannot be corrected because it is wrong. To get onto the right track here are some fundamental remarks concerning equations with particles.

From [1], Chapter 8.5, page 202: The (A) law of mass action, the (B) Pauli principle, the (C) superconductivity and the (D) uncertainty principle were found at higher aggregation levels of the particle world and applied to (A) molecules, (B) atoms, (C) metal crystals and (D) elementary particles. All four can be found again in these models in modified form at the fundamental level of the quarks, e.g. in the following (A) weak processes or with the (B) configurations of the nuclei in [1], Chapter 7.5 or in the (C) "fountain", Fig. 1 in Part I, or in the definition of the natural constant of the (D) inertia quantum ${}^{N}\Theta$, see penultimate paragraph of Part III. Probably the effect of such laws and principles has to be seen differently: ***Since they apply at the fundamental level they continue to have an effect also at the higher aggregation levels.***

The following is an example using the (B) Pauli's principle. The Pauli principle states for a complete atom — i.e. for a higher aggregation level — that a shell (K, L, M etc. with the sub-shells s, p, d etc.) of the atomic shell cannot be occupied by two electrons.

In Part I, Table 1 in line A shows the particles $\underbrace{dd}_{||}d \equiv e_e$ and $\underbrace{uu}_{||}u \equiv \Delta_\Delta$ for the fundamental level of the quarks. In addition, Fig. 12 in Part I shows the loci for a $\underbrace{dd}-Zk$. (A definition of the "dual-coordination" or briefly "Zk" is given in Part I, page 74, paragraph 5.) If the locus of a third d-quark were to be placed in the level of this Zk, either space I or space III would be occupied with two loci. Such double occupancy is demanded for the particles e_e and Δ_Δ by the $_{||}$ symbol. According to the Pauli principle this means at the fundamental level of the quarks that the particles e_e and Δ_Δ are prohibited, see Table 2! Allowed are only the electron $\underbrace{dd}_{\perp}d \equiv e^-$ and the deldopon $\underbrace{uu}_{\perp}u \equiv (\Delta^{++})$, where each quark assumes a different position.

Another example relates to the (A) law of mass action. This law primarily applies to the fundamental quark equations, but was initially discovered by us by means of the chemical reactions at the high aggregation level. The equation of a chemical reaction is formulated in the same manner as a fundamental quark equation. All constituents entering a fundamental reaction again come out of the reaction in a changed composition. Nothing disappears or is added. In this regard, some of the equations for the weak interaction offered in physics books are totally unsatisfactory, since the particles on both sides of the equations lack a common basis. This is also evident from the above equation (1): for the nucleons there is the quark representation in the standard model, not for the leptons.

3 The eight weak processes

Reading the following is not easy, the subject however highly interesting for the understanding of our universe. The comments regarding the equations are intended to facilitate this understanding.

Eight processes with the construction

Starting particle → $(Quarkpool)$ → Reaction products

$$\mathrm{p}^+ + \mathrm{e}^- \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \mathrm{n}^0 + \nu_\mathrm{e} \tag{2}$$

$$\underbrace{uu}_{||}d + \underbrace{dd}_{\perp}d \rightarrow \begin{pmatrix} uu_{||}\, d\vdots dd_{\perp}d \\ \uparrow \text{ ---------} \uparrow \end{pmatrix} \rightarrow \underbrace{dd}_{||}u + \\ + \underbrace{dd}_{\perp}u \tag{2a}$$

Space type qeR qeR → → → → qeR koR

$$\mathrm{n}^0 + \nu_\mathrm{e} \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \mathrm{p}^+ + \mathrm{e}^- \tag{3}$$

$$\underbrace{dd}_{||}u + \underbrace{dd}_{\perp}u \rightarrow \begin{pmatrix} dd_{||}\, u\vdots dd_{\perp}u \\ \uparrow \text{ ---------} \uparrow \end{pmatrix} \rightarrow \underbrace{uu}_{||}d + \\ + \underbrace{dd}_{\perp}d \tag{3a}$$

Space type qeR koR → → → → qeR qeR

The equations (2) and (3) count among the best known of the weak interaction. For the formulations according to the standard model the common basis of the particles mentioned above is absent. As quark equation (2a) and (3a), they correspond to the characteristics of the law of mass action. Details for a "quark pool" are included in the quark equations. This quark pool stands for the physical process of the reaction of the particles involved which requires a finite time and during which exchange processes take place. The signs within the brackets explain this exchange. During both the above processes a quark from the Zk of the baryon/nucleon involved is exchanged for the singular quark of the lepton, while the quark from the Zk of the baryon does not belong to the $\mathrm{u}_{||}\mathrm{d}$-group.

It can also be seen that the structure symbols in the equations are retained. A $_{||}$ and a $_{\perp}$ symbol each are present on the left and on the right side of the equation. This is to be correlated with the retention of the baryon and lepton number of the standard model. This means there are fixed rules for the reactions during the weak processes.

In each third line for each reaction the space type **qeR** or **koR**, see [5], Part I, page 72/73, of the elementary particle is noted. If two particles from "our" quasi-Euclidian space (qeR) react with each other the probability of the reaction substantially depends on a resonance possibility, i.e. the size of the particles MAGINPARs. In addition to this probability for a reaction there is obviously also a second one. This depends on the space type. This means, two particles with the same space type react with each other with far greater probability than particles with different space type.

We are aware of this in the case of the hugely plentiful neutrinos with the complex space type koR which hit the particles of earth with the space type qeR with only an extremely

3QT	dd d	dd u	uu d	uu u
Locus level of singular quark parallel (||) to the locus level of the Zk ‡	e_e ‡	n^0	p^+	Δ_Δ ‡
Space type of particle	*koR1* ‡	**qeR1**	**qeR1**	*koR1* ‡
Locus level of singular quark vertically ($\perp$) on locus level of Zk	e^-	ν_e	$?^+$	(Δ^{++})
Space type of particle	**qeR2**	*koR2*	*koR2*	**qeR2**

koR ≡ "complex" space, qeR ≡ quasi-Euclidian space.
The number 1 or 2 designates the number of the ρ-rotation levels per particle.
‡Elementary particles prohibited by the Pauli principle.

Table 2: Space structures of the elementary particles

low probability. The probabilities for a reaction are called MAGINPAR and space type probability. All eight weak processes are characterized in that at least one particle of a process has the space type koR. Thus the space type probability applies to the eight here treated processes which is why we talk about the "weak" interaction.

$$p^+ \quad + \quad ?^+ \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad n^\circ \quad + \quad (\Delta^{++}) \tag{4}$$

$$\underbrace{uu}{}_{||}\, d + \underbrace{uu}{}_{\perp}\, d \rightarrow \left(\begin{array}{c} uu_{||}\, d\vdots uu_{\perp} d \\ \uparrow \text{ ---------} \uparrow \end{array}\right) \rightarrow \underbrace{dd}{}_{||}\, u + $$

$$+ \underbrace{uu}{}_{\perp}\, u \tag{4a}$$

Space type qeR koR → → → → qeR qeR

$$n^\circ \quad + \quad (\Delta^{++}) \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow p^+ \quad + \quad ?^+ \tag{5}$$

$$\underbrace{dd}{}_{||}\, u + \underbrace{uu}{}_{\perp}\, u \rightarrow \left(\begin{array}{c} dd_{||}\, u\vdots uu_{\perp} u \\ \uparrow \text{ ---------} \uparrow \end{array}\right) \rightarrow \underbrace{uu}{}_{||}\, d + $$

$$+ \underbrace{uu}{}_{\perp}\, d \tag{5a}$$

Space type qeR qeR → → → → qeR koR

A hypothesis (here the models under consideration) establishes new predictions/expansions unknown to date for the (physical) teaching applicable to that point, which have to be verified. Such predictions are made by Table 1 and the still to follow Table 2 with some of the particles noted there, which also occur in the equations (4) and (5). For the sake of brevity the particles of the Tables that have not been found yet will not be further commented upon at this point. Reference is only made to the respective exchange of the quarks in the quark pool, which corresponds to the fixed rules for the reactions mentioned above.

To facilitate the association of the space types with the individual elementary particles Table 2 is inserted.

The best known equation to describe the "β-decay" is the following:

$$n^0 \quad \rightarrow p^+ + e^- + \bar{\nu}_e \tag{6}$$

Under the aspect of the standard theories such an equation is possible because four totally independent particles are present, the electric charges involved are correct and the n^0 has the greatest mass/energy so that it can decay into the three other particles of lower energy. Under the aspect of the models developed here the two sides of this process cannot be brought into line even from the number of the quarks involved. The right side of the equation comprises nine quarks, the left side three quarks. In other words six quarks have to be added to the left side, while a 6QT or boson is obvious.

The following arguments speak for the photon-like ν-gamma ($\nu - \gamma$) as trigger of the process — incompletely — described with the above non-equation:

1. The particle is not yet known which is why it is not named so far on the left side of the process;
2. Because of its space type koR the particle — based on ν_e (Table 2) — is difficult to discover;
3. ν-gamma brings with it the necessary number and type of quarks and of structure signs $_{||}$ respectively $_{\perp}$.

The almost known "β-decay" according to the standard model as fully formulated weak process according to these models then becomes the following as particle and quark equation:

$$n^0 + \nu - \gamma \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow p^+ + e^- + \bar{\nu}_e \tag{8}$$

$$\underbrace{dd}{}_{||}\, u + \underbrace{dd}{}_{\perp}\, \underline{\underline{u\bar{u}}}{}_{\perp}\, \underbrace{\bar{d}\bar{d}} \rightarrow \left(\begin{array}{l} d\ from\ n^0 \Leftrightarrow \\ u\ from\ Bk \end{array}\right) \rightarrow$$

$$\rightarrow \underbrace{uu}{}_{||}\, d + d_{\perp}\, \underbrace{dd} + \bar{u}_{\perp}\, \underbrace{\bar{d}\bar{d}} \tag{8a}$$

Space type qeR koR → → →

→qeR qeR koR

The central part $\underline{\underline{u\bar{u}}}$ of the $\nu-\gamma$ within the quark-equation (8a) is called a "binding coordination", briefly "Bk".

The fixed rules for the quark reactions need only be modified slightly for the reaction type (8) relative to the reaction type (2) and (3) or (4) and (5):

- A baryon each reacts with a photon-like 6QT (instead of 3QT lepton).
- From the original particles, a formally singular quark each (not anti-quark) of a lepton and now part of a Bk in the photon-like is exchanged for a Zk-quark (not from the u||d-group) of the baryon.
- In addition to the type and number of the quarks involved the type and number of the structure signs $_{||}$ and $_{\perp}$ now agree on both sides of equation (8) as well.

Since equation (8) relative to the non-equation (6) has been explained, equation (7) is now added where e-γ (our "normal" photon) has to be additionally considered compared with the standard version.

$$\mathrm{p}^+ + \mathrm{e}-\gamma \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \mathrm{n}^0 + \mathrm{e}^+ + \nu_{\mathrm{e}} \tag{7}$$

$$\underbrace{uu}\,_{||}\,d + \underbrace{dd}\,_{\perp}\,\underline{\underline{d\bar{d}}}\,_{\perp}\;\underbrace{\bar{d}\bar{d}} \rightarrow \left(\begin{array}{l} u\ from\ p^+ \Leftrightarrow \\ d\ from\ Bk \end{array}\right) \rightarrow$$

$$\rightarrow \underbrace{dd}\,_{||}\,u + \underbrace{\bar{d}\bar{d}}\,_{\perp}\bar{d} + \underbrace{dd}\,_{\perp}u \tag{7a}$$

Space type qeR qeR → → →

→ qeR qeR koR

Since the following equations (9) and (10) contain particles not yet found from the systematic of Table 1 and 2 here they will not be further commented upon. In structure they correspond to the type of the equations (7) and (8) and complete the set of the weak processes according to these models:

$$\mathrm{p}^+ +?-\gamma \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \mathrm{n}^0 + (\Delta^{2+}) +?^+ \tag{9}$$

$$\underbrace{uu}\,_{||}\,d + \underbrace{uu}\,_{\perp}\,\underline{\underline{d\bar{d}}}\,_{\perp}\;\underbrace{\bar{u}\bar{u}} \rightarrow \left(\begin{array}{l} u\ from\ p^+ \Leftrightarrow \\ d\ from\ Bk \end{array}\right) \rightarrow$$

$$\rightarrow \underbrace{dd}\,_{||}\,u + \underbrace{uu}\,_{\perp}u + \underbrace{\bar{u}\bar{u}}\,_{\perp}\bar{d} \tag{9a}$$

$$\mathrm{n}^0 + (\Delta) - \gamma \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \mathrm{p}^+ +?^+ + (\Delta^{2-}) \tag{10}$$

$$\underbrace{dd}\,_{||}\,u + \underbrace{uu}\,_{\perp}\,\underline{\underline{u\bar{u}}}\,_{\perp}\;\underbrace{\bar{u}\bar{u}} \rightarrow \left(\begin{array}{l} d\ from\ n^0 \Leftrightarrow \\ u\ from\ Bk \end{array}\right) \rightarrow$$

$$\rightarrow \underbrace{uu}\,_{||}\,d + d\,_{\perp}\underbrace{uu} + \bar{u}\,_{\perp}\underbrace{\bar{u}\bar{u}} \tag{10a}$$

The weak processes are the gateway to the "second" strand of this universe. The particles having a complex space (koR) form this second strand. "Our" particles with quasi-Euclidian space (qeR) from the "first" strand overlap those from the second strand without problems, which is why the spaces also overlap without problems. (What is a "space" being created in our imagination?) The "spaces" do not interact with each other.

In contrast with this, the physical = electric $\notin_{\Gamma}$-fields from qeR and koR interact very well with each other so that their superposition results in the mutual attraction, see [1], page 186, line 18. Measured by the undiscovered particles of Tables 1 and 2 there is much to be discovered behind the gate to the "second" strand of this universe. Judging by the ratio of the gravitational effects of the visible matter and the dark matter what can be discovered behind the gate is a multiple of what we already know.

4 The Meaning of the Weak Processes (7) and (8)

Equations (7) and (8) contain some fascinating technical potential. H. Stumpf deals with nuclear reaction rates of the electroweak interaction [2] and at the end of his paper he refers to L. I. Urutskoev and other Russian authors, who perform experiments regarding this item. The potential of those works includes finding new routes for the elimination of highly radioactive waste. In a few years this waste from hundreds of disused nuclear reactors will pile up in many states of our earth. The final storage of this waste is not clarified and costs for a long time storage with e. g. sarcophagus as in Tschernobyl would be enormous. The duration of storage follows from natural β-decay half-life periods of different elements or their isotopes which can last for up to 1.5×10^{24} years for $^{128}_{52}\mathrm{Te}$, [4], page 34, which mankind cannot live to see.

Equation (7) respectively (7a) demonstrates, that the protons of radioactive elements can have resonance and can react with very short waved photons (e$-\gamma$) into neutrons, positrons and neutrinos. Thereby the structure and the therewith combined beat of the photon shown in Part II, page 77, left column, point (2) and the storage of the photon in an electron (resonance), Part II, page 77, right column, penultimate paragraph are called to mind.

Equation (7) is confirmed by two aspects of the above mentioned experiments of L. I. Urutskoev et al. [3]. First aspect: The central incidents of the experiments are electric discharges between metallic foils in vessels filled with various fluids, [2], page 455. My interpretation is, that by the discharges those short waved e$-\gamma$ of process (7) are generated, which can have resonance and reaction with the protons of the (radioactive) elements. Second aspect: The possibility of "low-energy nuclear transformation" is reported in [3]. If an electron and a visible photon have a comparable COMPTON-wavelength and therefore have resonance, then the photon has an energy of multiplier 10^5 less than the electron, [1], Chapter 8.2.3, page 163. With weak interaction nuclei emit short waved e$-\gamma$ in the range of a few keV up to a few MeV. That means nuclei are in the position to have resonance with those short waved e$-\gamma$. If such short waved e$-\gamma$ arrives at a nucleus and hit (a neutron or) a proton then there is the possibility for the "low-energy" exchange of quarks in a quark-pool according to the rules of page 72, middle of right column, and page 74, upper part left column. By the exchange of quarks in accordance with equation (7) the proton transforms into a neutron and by this a new element respectively a new isotope takes shape. New elements respectively isotopes were detected by the authors of [3].

Following are comments on the peculiarities of the weak process (7):

1. The Standard Model of Physics treats the β-"decay" as statistical phenomenon or as happening by chance. The model under consideration especially the weak process (7) presents a dosed bombardment of protons by e$-\gamma$. The transformation of the protons into products of reaction happens not by chance instead the reaction is determined by the efficiency of the law of mass action.
2. Without the knowledge of the weak process (7) Urutskoev et al. with exotic experiments strive for the realisation of reactions according to this process. With knowledge of equation (7) different experiments are possible:
   Possibly one could observe the weak process cease when the bombardment of protons by photons, which can have resonance, is prevented completely. Nevertheless the "radioactive decay" of a specimen with an outer screening could continue because a radiation could be released "from the interior" of this specimen. The latter could stem from the less probable but possible opposite reaction of equation (7): $\mathrm{n}^0 + \mathrm{e}^+ + \nu_e \rightarrow \mathrm{p}^+ + \mathrm{e}-\gamma$. The e$-\gamma$ originating in the interior of the atomic nucleus would be absorbed after flying a very short distance in the specimen because of a high probability of resonance. By this the weak process (7) would be caused "from the interior".
3. The weak process (7) cannot be observed in nature, [4], S. 38.

Following are comments on the peculiarities of the weak process (8):

1. The very common but not applicable non-equation (6) claims that the neutrons of radioactive elements would "decay" into protons, electrons and anti-neutrinos. As with equation (7) the law of mass action is valid with equation (8);
2. By the exchange of quarks according equation (8) a new element respectively a new isotope takes shape. The problem is, until now we still do not know the $\nu-\gamma$ because of its complex space koR and beyond this we cannot shield it from the outside or handle it at all. From that point of view we would be dependent on the sun, on space or on nuclear reactors as generators for $\nu-\gamma$ of whatever intensity and wavelength to shorten half-life periods by chance.

Eventually a possibility on the basis of the opposite reaction of the weak process (8) will be revealed. Those $\nu-\gamma$ originating from $\mathrm{p}^+ + \mathrm{e}^- + \bar{\nu}_e \rightarrow \mathrm{n}^0 + \nu-\gamma$ would be absorbed after flying the shortest distance because of a high probability of resonance. By this the weak process (8) would be released "from the interior". The opposite reaction of the weak process (8) should be reinforced by proper conditions in such a way that the reaction rates are of sufficient size.

In summary: Though till now we do not know the $\nu-\gamma$-radiation so far and, much less, we can control it, there is hope to transform the neutrons of radioactive elements by $\nu-\gamma$ via the opposite reaction of equation (8). The construction of some technical apparatus for short waved e$-\gamma$-radiation as e.g. X-rays of 10^3 to 10^6 eV is feasible. By the reaction of proton and e$-\gamma$ (photon) according to the weak process (7) natural, partly very long time half-life periods can be shortened down to seconds using a technical apparatus! The use of both types of radiation, $\nu-\gamma$ and e$-\gamma$, would be decisive steps for the elimination of highly radioactive waste.

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The Generalized Conversion Factor in Einstein's Mass-Energy Equation

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Einstein's September 1905 paper is origin of light energy-mass inter conversion equation ($L = \Delta mc^2$) and Einstein speculated $E = \Delta mc^2$ from it by simply replacing L by E. From its critical analysis it follows that $L = \Delta mc^2$ is only true under special or ideal conditions. Under general cases the result is $L \propto \Delta mc^2$ ($E \propto \Delta mc^2$). Consequently an alternate equation $\Delta E = Ac^2 \Delta M$ has been suggested, which implies that energy emitted on annihilation of mass can be equal, less and more than predicted by $\Delta E = \Delta mc^2$. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally 20–60 MeV less than Q-value predicted by Δmc^2. The mass of particle Ds (2317) discovered at SLAC, is more than current estimates. In many reactions including chemical reactions $E = \Delta mc^2$ is not confirmed yet, but regarded as true. It implies the conversion factor than c^2 is possible. These phenomena can be explained with help of generalized mass-energy equation $\Delta E = Ac^2 \Delta M$.

1 Introduction

Mass energy inter-conversion processes are the oldest in nature and constitute the basis of various phenomena. Before Einstein's work, Newton [1] stated that "Gross bodies and light are convertible into one another…". Einstein derived light energy-mass inter-conversion equation for Newton's perception as $L = \Delta mc^2$. Before Einstein scientists such as S. Tolver Preston [2] Olinto De Pretto [3], Fritz Hasenohrl [4, 5] Frederick Soddi [6] contributed to the topic.

Einstein's derivation of $L = \Delta mc^2$ (from which Einstein speculated $E = \Delta mc^2$), is true under special conditions (where selective values of variables are taken). Under general conditions (when all possible values of parameters are taken) equations like $L = 0.0011 \Delta mc^2$, $L = 0.999988 \Delta mc^2$ etc. are obtained i.e. $L \propto \Delta mc^2$. Thus conversion factor other than c^2 is possible in Einstein's derivation. Further the generalized mass–energy equation $\Delta E = Ac^2 \Delta M$, is derived, and $E = \Delta mc^2$ is special case of the former depending upon value of A (depends upon the characteristics conditions of the process). Thus apart from theoretical limitations, $E = \Delta mc^2$ has experimental limitations e.g. sometimes experimental results differ from it and in many cases it is not confirmed. Under such cases $\Delta E = Ac^2 \Delta M$ is widely useful and applicable. The fission fragments result from U^{235} and Pu^{239} have total kinetic energy 20–60 MeV less than Q-value (200 MeV) of reaction predicted by $\Delta E = \Delta mc^2$ [7–9]. Palano [10] has confirmed that mass of particle Ds (2317) has been found more than current estimates based upon $\Delta E = \Delta mc^2$. Also $\Delta E = \Delta mc^2$ does not give consistent results in explaining the binding energy, as it violates the universal equality of masses of nucleons.

All these facts can be explained by $\Delta E = Ac^2 \Delta M$ with value of A less or more than one. $\Delta E = \Delta mc^2$ is not confirmed in many processes such chemical reactions, atom bomb explosions, volcanic reactions etc. Whatever may be the case $\Delta E = Ac^2 \Delta M$ is capable of explaining the phenomena. Thus conversion factor other than c^2 is possible, in Einstein's September 1905 derivation and confirmed experimentally also.

2 Einstein's light energy — mass equation $L = \Delta mc^2$ and its hidden aspects

Einstein [11] perceived that let there be a luminous body at rest in co-ordinate system (x, y, z). The system (ξ, η, ζ) is in uniform parallel translation w.r.t. system (x, y, z); and origin of system (ξ, η, ζ) moves along x-axis with relative velocity v. Let a system of plane light waves have energy ℓ relative to system (x, y, z), the ray direction makes angle ϕ with x-axis of the system (ξ, η, ζ). The quantity of light measured in system (ξ, η, ζ) has the energy [11, 12].

$$\ell * = \ell \, \frac{\left(1 - \frac{v}{c} \cos\phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

Einstein has given Eq. (1) in his paper known as Special Theory of Relativity [12] and called Eq. (1) as Doppler principle for any velocities whatever.

Let E_0 and H_0 are energies in coordinate system (x, y, z) and system (ξ, η, ζ) before emission of light energy, further E_1 and H_1 are the energies of body in the both systems after it emits light energy. Thus Einstein wrote various equations as Energy of body in system (x, y, z)

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L; \tag{2}$$

Energy of body in system (ξ, η, ζ)

$$H_0 = H_1 + 0.5\beta L \left[\left(1 - \frac{v}{c}\cos\phi\right) + \left(1 + \frac{v}{c}\cos\phi\right)\right] \tag{3}$$

where $\beta = 1/[1 - v^2/c^2]^{1/2}$;

$$H_0 = H_1 + \beta L; \tag{4}$$

or

$$(H_0 - E_0) - (H_1 - E_1) = L\,(\beta - 1)\,. \tag{5}$$

Einstein calculated, kinetic energy of body before emission of light energy, $K_0(m_b v^2/2)$ and kinetic energy of body after emission of light energy, $K(m_a v^2/2)$ as

$$K_0 - K = L\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \tag{6}$$

Einstein considered the velocity in classical region thus applying binomial theorem,

$$K_0 - K = L\biggl(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + {} + \frac{15v^6}{48c^6} + \frac{105v^8}{384c^8} + \ldots - 1\biggr). \tag{7}$$

Further Einstein quoted [16] "Neglecting magnitudes of fourth and higher orders, we may place"

$$K_0 - K = L\,\frac{v^2}{2c^2} \tag{8}$$

$$M_b\frac{v^2}{2} - M_a\frac{v^2}{2} = L\,\frac{v^2}{2c^2} \tag{9}$$

or

$$L = (M_b - M_a)\,c^2 = \Delta m c^2, \tag{10}$$

or Mass of body after emission (M_a) = Mass of body before emission $(M_b - L/c^2)$.

Now replacing L (light energy) by E (total energy or every energy) Einstein wrote

$$E = (M_b - M_a)\,c^2 = \Delta m c^2 \tag{11}$$

or Mass of body after emission (M_a) = Mass of body before emission $(M_b - E/c^2)$.

Thus Einstein derived that conversion factor between mass and light energy is precisely equal to c^2, this aspect is elaborated by Fadner [13]. But Einstein's this derivation has been critically discussed by many such a Planck [14], Stark [15], Ives [16], Stanchel [17], Okun [18] and N. Hamdan [19] etc. At the same time in some references [20, 21] it is expressed that Einstein has taken hints to derive equation $E = \Delta m c^2$ and from existing literature without acknowledging the work of preceding scientists. Max Born [22] has expressed that Einstein should have given references of existing literature.

Thus Einstein's work on the topic has been critically analyzed by scientists since beginning, in views of its scientific and procedural aspects.

3 The conversion factor between mass-energy other than c^2 is also supported by Einstein's derivation under general conditions

As already mentioned Einstein's September 1905 derivation of $\Delta L = \Delta m c^2$ is true under special or ideal conditions (selected values of parameters is taken) only, this aspect is studied critically with details by the author [23–36] discussing those aspects which have not been raised earlier. Thus the value of conversion factor other than c^2 is also supported from Einstein's derivation under general conditions (all possible values of variables). The law or phenomena of interconversion of mass and energy holds good in all cases for all bodies and energies under all conditions.

In the derivation of $\Delta L = \Delta m c^2$ there are FOUR variables e.g.

(a) Number of waves emitted,

(b) l magnitude of light energy,

(c) Angle ϕ at which light energy is emitted and

(d) Uniform velocity, v.

Einstein has taken special values of parameters and in general for complete analysis the derivation can be repeated with all possible values of parameters i.e. under general conditions taking in account the momentum conservation (which is discussed in next sub-section).

(A) The body can emit large number of light waves but Einstein has taken only TWO light waves emitted by luminous body.

(B) The light waves emitted may have different magnitudes but Einstein has taken EQUAL magnitudes

(C) Body may emit large number of light waves of different magnitudes of energy making DIFFERENT ANGLES (other than 0° and 180°) assumed by Einstein.

(D) Einstein has taken velocity in classical region ($v \ll c$) has not at all used velocity in relativistic region. If velocity is regarded as in relativistic region (v is comparable with c), then equation for relativistic variation of mass with velocity i.e.

$$M_{rel} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{12}$$

is taken in account. It must be noted that before Einstein's work this equation was given by Lorentz [37, 38] and firstly confirmed by Kaufman [39] and afterwards more convincingly by Bucherer [40]. Einstein on June 19, 1948 wrote a letter to Lincoln Barnett [41] and advocated abandoning relativistic mass and suggested that is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass.

It is strange suggestion as Einstein has used relativistic mass in his work including in the expression of relativistic kinetic energy [12] from which rest mass energy is derived.

(E) In addition Einstein has assumed that body remains at rest before and after emission of light energy. But the body may be at rest i.e. $v = 0$, velocity may be in classical region and velocity may be in relativistic region ($v \sim c$), the law of inter-conversion of mass and energy holds good under all conditions.

In electron-positron annihilation, the material particles are in motion before and after annihilation. In materialization of energy, a gamma ray photon is converted to electron positron pair, which move in opposite directions to conserve momentum. In nuclear fission and fusion particles remain in motion in the process of mass energy inter conversion. The thermal neutron which causes fission has velocity 2185 m/s.

4 $L \propto \Delta mc^2$ is mathematically consistent in Einstein's derivation, under general conditions

Under general conditions (all possible values of variables) the value of conversion factor other than c^2 can be easily justified mathematically in Einstein's derivation [23–36]. This aspect is not touched by the preceding authors [13–21].

(a) In Einstein's derivation if one wave is regarded as to form angle 0.5° rather than 0° then

$$\begin{aligned} H_0 = H_1 + 0.5\beta L \times \\ \times \left[\left(1 - \frac{v}{c}\cos 0.5^\circ\right) + \left(1 - \frac{v}{c}\cos 180^\circ\right)\right], \end{aligned} \tag{13}$$

or

$$H_0 = H_1 + \beta L\left(1 + 0.000018038\,\frac{v}{c}\right),$$

or

$$K_0 - K = 0.000019038\,lL\,\frac{v}{c} + L\,\frac{v^2}{2c^2}\,,$$

$$\Delta m\,(M_b - M_a) = 0.000038077\,\frac{L}{cv} + \frac{L}{c^2}\,, \tag{14}$$

or

$$L = \frac{\Delta mc^2}{1141} = 0.000876\Delta mc^2, \tag{15}$$

$$\Delta L \propto \Delta mc^2.$$

Further, M_a (mass after emission of light energy) $= M_b$ (mass before emission of light energy): $0.000038077L/cv = L/c^2$ in (14).

According to Einstein if body emits two light waves of energy $0.5L$ each in opposite directions then decrease in mass is given by Eq. (10) i.e. $\Delta m = L/c^2$ and in this case decrease in mass is $(0.000038077L/cv + L/c^2)$ thus there is no definite value of decrease in mass in Einstein's derivation. In this case decrease in mass is more than as predicted by Einstein, hence again the conversion factor other than c^2 is confirmed i.e. $\Delta L \propto \Delta mc^2$. Like this there are many examples of this type.

(b) The central equation in Einstein's derivation is Eq. (1) and binomial theorem is equally applicable to it at any stage i.e. in the beginning or end. Einstein applied binomial theorem in the end and obtained $L = \Delta mc^2$, but the same equation is not obtained if binomial theorem is applied in the beginning. The binomial theorem is simply a mathematical tool and its application at any stage should not affect results i.e. make or mar equation $L = \Delta mc^2$.

The reason is that typical nature of derivation and Eq. (1) is different from other relativistic equations. The energy is scalar quantity and independent of direction but Eq. (1) is directional in nature due to angle ϕ. In contrast if binomial theorem is applied to Relativistic Kinetic Energy in the beginning or at the end then result is same i.e. classical form of kinetic energy $(m_{rest}v^2/2)$. So there is inconsistency in applications in this case.

Applying binomial theorem to Eq. (1) and repeating the calculations as Einstein did, altogether different results are obtained,

$$\ell^* = \ell\left(1 - \frac{v}{c}\cos\phi\right)\left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \ldots\right). \tag{16}$$

Here $v/c \ll 1$, hence v^2/c^2 and higher terms can be neglected. Thus

$$\ell^* = \ell\left(1 - \frac{v}{c}\cos\phi\right)$$

or

$$(H_0 - E_0) - (H_1 - E_1) = 0\,,$$

or

$$K_b - K_a = 0\,,$$

or

$$\frac{1}{2}\,M_b v^2 - \frac{1}{2}\,M_a v^2 = 0\,,$$

or

$$\begin{aligned} &\text{Mass of body before emission } (M_b) = \\ &= \text{Mass of body after emission } (M_a). \end{aligned} \tag{17}$$

Thus light energy is being emitted, but under this condition Einstein's this derivation does not provide any relationship (equality or proportionality) between mass annihilated and energy created. Similar is the situation if velocity $v = 0$. Hence Einstein's derivation gives decrease in mass of body equal to L/c^2 only under certain conditions. Thus in this case derivation is not valid.

Sr. No	Values of various parameters	Whether$L = \Delta mc^2$ or $L \propto \Delta mc^2$
1	$0.5L$, $0.5L$, $\phi = 0^\circ$, $\phi = 180^\circ$	$L = \Delta mc^2$
2	$0.5L$, $0.5L$, $\phi = 0.5^\circ$, $\phi = 180^\circ$	$L = \Delta mc^2/901$ or $L \propto \Delta mc^2$
3	$0.5001L$, $0.49999L$, $\phi = 0^\circ$, $\phi = 180^\circ$	$L = 0.9999988\Delta mc^2$ or $L \propto \Delta mc^2$
4	$0.5L$, $0.5L$, $\phi = 0^\circ$, $\phi = 180^\circ$ but $v = 0$	No relation between L and Δm
5	$0.5L$, $0.5L$, $\phi = 0^\circ$, $\phi = 180^\circ$ but Binomial Theorem is applied in beginning.	No relation between L and Δm
6	For other energies than light	Equations not considered

Table 1: Einstein's Sep 1905 derivation gives $L = \Delta mc^2$ under certain conditions and $L \propto \Delta mc^2$ under general conditions

(c) Let the body emits two light waves of slightly different energies i.e. $0.5001L$ and $0.4999L$ in opposite directions and other parameters remain the same as assumed by Einstein. In this case

$$H_0 = H_1 + 0.4999\beta L\left(1 - \frac{v}{c}\cos 0^\circ\right) + + 0.5001\beta L\left(1 - \frac{v}{c}\cos 180^\circ\right). \tag{18}$$

Now proceeding in the same way as Einstein did

$$K_0 - K = 0.0002L\frac{v}{c} + L\frac{v^2}{2c^2} \tag{19}$$

or

$$\Delta m = \text{Mass of body before emission}(M_b) - \text{Mass of body after emission}(M_a),$$

$$= 0.0004\frac{L}{cv} + \frac{L}{c^2} \tag{20}$$

or

$$M_a = 0.004\frac{L}{cv} - \frac{L}{c^2} + M_b$$

or

$$L = \frac{\Delta mc^2}{\left(0.0004\frac{c}{v} + 1\right)}.$$

The velocity v is in classical region, say 10 m/s,

$$L = \Delta mc^2\left[0.000083\right], \tag{21}$$

$$\Delta L \propto \Delta mc^2.$$

Thus, $\Delta E \propto \Delta mc^2$. Hence conversion factor other than c^2 follows from Einstein's derivation under general conditions.

(d) Energy emitted in various reactions. In his September 1905 paper Einstein derived Eq. (10) i.e. $\Delta L = \Delta mc^2$ and then replaced L (light energy) by E (total energy) and speculated

$$\Delta E = \Delta mc^2. \tag{11}$$

In Eq. (11) E stands for all possible energies of the universe e.g.: (i) sound energy, (ii) heat energy, (iii) chemical energy, (iv) nuclear energy, (v) magnetic energy, (vi) electrical energy, (vii) energy emitted in form of invisible radiations, (viii) energy emitted in cosmological and astrophysical phenomena, (ix) energy emitted volcanic reactions, (x) energies co-existing in various forms etc., etc.

Now Eq. (1) i.e.

$$\ell^* = \ell\,\frac{\left(1 - \frac{v}{c}\cos\phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is put forth for light energy by Einstein in June 1905 paper (ℓ^* is light energy in moving frame), it is not meant for other possible energies as quoted above.

Einstein never justified Eq. (1) for all the energies cited above. The parameters used in Einstein's equation are defined for light energy only, not for all the energies. Thus by this derivation $L = \Delta mc^2$ is derived under special conditions for light energy only and replacing L by E in Eq. (10) is not justified.

There are evidences that Einstein worked hurriedly in other case also e.g. in theory of static universe the introduction of cosmological constant proved to be incorrect and Einstein accepted the mistake later as quoted by Gamow [42]. The various cases when $\Delta E \propto \Delta mc^2$ is justified are shown in Table 1.

5 Conservation of momentum in general cases

The momentum is conserved irrespective of the fact that body remains at rest or recoils or tends to recoil after emission of light energy [43]. The law of conservation of momentum can be used to calculate the velocity of recoil in this case also. Let the body of mass 10 kg emits two waves of energy in visible region of wavelength 5000Å it corresponds to energy 7.9512×10^{-19} J. This energy is emitted in two waves i.e., as obvious, $0.5001L$ (3.97639512×10^{-19} J) and $0.4999L$ (3.97480488×10^{-19} J). Applying the conservation of momentum [43] the recoil velocity, recoil momentum and recoil kinetic energy comes out to be -5.3×10^{-32} m/s, 5.3×10^{-31} kg×m/s and 1.404×10^{-62} J respectively. This recoil velocity (V_r) will change the uniform velocity v as $V_r + v$, but it will not make any difference to final result of change in mass as in Eq. (21), due to negligible value of V_r [27]. Hence in the law of conservation of momentum is obeyed in this case also.

6 Experimental feasibility with conversion factors other than c^2

(a) Dirac [44] was one of the first physicists to suggest that, in connection with his theory of large numbers, fundamental dimensional constants may vary in time during the expansion of the universe. The idea of variation of the speed of light is suggested in various cosmological models [45, 46] and has been the subject of attention by physicists in investigations of extra dimensions, strings and branes [47]. Webb [48] has reported variations in fine structure constant over cosmological time scales and hence variations in c. This suggestion implies $\Delta E \propto \Delta mc^2$.

(b) Einstein has derived $L = \Delta mc^2$ (conversion factor between mass and energy is precisely equal to c^2) under the extremely special or ideal conditions , which are even difficult to attain practically. The work of scientists before Einstein also justifies $\Delta E \propto \Delta mc^2$.

This discussion does not confront with existing experimental situation but addresses those theoretical and experimental issues for which $\Delta E = \Delta mc^2$ is not analyzed yet. The mass energy inter-conversion equation, with conversion factor equal to c^2 i.e. $\Delta E = \Delta mc^2$ has been confirmed in nuclear physics and is also basis of nuclear physics. Even elementary units of atomic mass (1 amu) or and energy (eV) are based upon it. Thus it will remain standard in measurements as seven days in a week; its validity in this regard is not doubted at all.

The aim is to discuss experimentally those phenomena in which $\Delta E = \Delta mc^2$ is not applied yet. The mass energy conversion processes are weird in nature and all have not been at all studied in view of $\Delta E = \Delta mc^2$. The conversion factor other than c^2 is discussed for such elusive cases, not for those it is already confirmed. Hence there is no confrontation with the established experimental situation at all, but aim is to open a mathematical front ($\Delta E \propto \Delta mc^2$) for numerous experimentally unstudied phenomena in nature. This development can be discussed as below.

7 Most abundant chemical reactions

(i) **Unconfirmed chemical reactions.** When Einstein derived $E = \Delta mc^2$, chemical reactions were the most abundant sources of energy in nature. Till date $E = \Delta mc^2$ is not confirmed in the chemical reaction and the reason cited for this is that equipments are not enough sensitive [49, 50]. Consider burning of 1kg straw or paper or petrol in controlled way i.e. in such a way that masses, ashes, gases and energy produced can be estimated. Even if 0.001 kg or 1gm of matter is annihilated then energy equal to 9×10^{13} J (can drive a truck of mass 1000 kg to distance of 9×10^7 km) will be produced. Until the equation is not confirmed in such reactions, then scientifically $E = \Delta mc^2$ may not be regarded as precisely true in such cases. It is equally possible that energy emitted may be less than predicted by $E = \Delta mc^2$ i.e. $E \propto \Delta mc^2$ is feasible, it is an open possibility unless ruled out.

Reactions in nuclear physics

(ii) **Less efficiency:** The efficiency of the nuclear weapons as well as nuclear reactors is far less than the theoretical value predicted by $E = \Delta mc^2$. Robert Serber (member of first American team entered Hiroshima and Nagasaki in September 1945 to assess loses), has indicated [51] that the efficiency of "Little Boy" weapon (U^{235}, 49 kg) that was used against Hiroshima was about 2% only. It is assumed that all the atoms don't undergo fission, thus material is wasted. But no such waste material is specifically measured quantitatively. Thus the waste material (nuclear reactor or weapon) must be measured and corresponding energy be calculated, and it must quantitatively explain that why efficiency is less. It may require the measurements of all types of energies (may co-exist in various forms) in the processes and experimental errors. Until such experiments are specifically conducted and $E = \Delta mc^2$ is confirmed, $\Delta E \propto \Delta mc^2$ is equally feasible.

(iii) **Less energy:** In laboratory it is confirmed [7, 52, 53] that using thermal neutrons the total kinetic energy (TKE) of fission fragments that result from of U^{235} and Pu^{239} is 20–60 MeV less than Q-value (200 MeV) of reaction predicted by $\Delta E = \Delta mc^2$. This observation is nearly four decades old. Bakhoum [7] has explained it on the basis of equation $H = mv^2$ (energy emitted is less than $E = \Delta mc^2$). Hence here $E \propto \Delta mc^2$ is justified.

(iv) **More mass:** Palano [10] has confirmed that mass of particle Ds (2317) has been found more than current estimates based upon $\Delta E = \Delta mc^2$. Thus in this case $E \propto \Delta mc^2$ is justified.

(v) **Binding energy and mass defect in deuteron:** There are two inherent observations [23, 28, 29] about nucleus: firstly, masses of nucleons are fundamental constants, i.e. they are the same universally (inside and outside the nucleus in all cases); and secondly nuclei possess Binding Energy ($BE = \Delta mc^2$) owing to a mass defect. To explain these observations, in the case of the deuteron ($BE = 2.2244$ MeV), the mass defect of nucleons must be 0.002388 amu or about 0.11854% of the mass of nucleons, i.e., nucleons must be lighter in the nucleus. This is not experimentally justified, as masses of nucleons are universal constants. Thus observations and predictions based upon $\Delta E = \Delta mc^2$ are not justified, hence $\Delta E \propto \Delta mc^2$ is equally feasible.

8 Mathematical form of extended equation

Until $E = \Delta mc^2$ is not precisely confirmed experimentally in ALL CASES , it is equally feasible to assume that the energy emitted may be less than $E = \Delta mc^2$ (or $E \propto \Delta mc^2$). It does not have any effect on those cases where $E = \Delta mc^2$ is confirmed, it simply scientifically stresses confirmation of $E = \Delta mc^2$ in all cases. Also when reactants are in bulk amount and various types of energies are simultaneously emitted and energies may co-exist. Thus both the possibilities are equally probable until one is not specifically ruled out. In view of weirdness in reactions emitting energy in universe, some theoretical inconsistencies in the derivation and non-availability of data, one can explore the second possibility even as a postulate. All the equations in science are regarded as confirmed when specifically justified in all experiments time and again. The reactions involving inter-conversion of mass and energy are utmost diverse, weird and new phenomena are being added regularly, thus $E = mc^2$ needs to be confirmed in all cases. Thus in general, in view of above proportionality it may be taken in account as

$$dE \propto c^2 dm\,.$$

The above proportionality $dE \propto c^2 dm$ can be changed into equation by introducing a constant of proportionality. The inception of proportionality constant is consistent with centuries old perception of constant of proportionality in physics since days of Aristotle and Newton. In second law of motion ($F = kma$) the value of constant of proportionality, k is always unity (like universal constant) i.e. $F = ma$. When more and more complex phenomena were studied or values of constants of proportionality were determined then it showed dependence on the inherent characteristics of the phenomena. In case constant of proportionality varies from one situation to other then it is known as co-efficient of proportionality e.g. co-efficient of thermal conductivity or viscosity etc. Thus removing the proportionality between dE and $c^2 dm$, we get

$$dE = Ac^2 dm\,, \qquad (22)$$

where$A$ is (a co-efficient) used to remove that sign of proportionality; it depends upon inherent characteristics of the processes in which conversion of mass to energy takes place and it is dimensionless. It has nature precisely like Hubble's constant (50 and 80 kilometers per second-Megaparsec, Mpc) or coefficient of viscosity (1.05×10^{-3} poise to 19.2×10^{-6} poise) or co-efficient of thermal conductivity (0.02 $\mathrm{Wm^{-1}K^{-1}}$ to 400 $\mathrm{Wm^{-1}K^{-1}}$) etc. Thus, in fact Hubble's constant may be regarded Hubble's variable constant or Hubble's coefficient, as it varies from one heavenly body to other. If "A" is equal to one, then we will get $dE = dmc^2$ i.e. same as Einstein's equation.

In Eq. (22) "A" is regarded as conversion factor as it describes feasibility and extent of conversion of mass into energy. For example out of bulk mass, the mass annihilated to energy is maximum in matter-antimatter annihilation, apparently least in chemical reactions, undetermined in volcanic reactions and cosmological reactions. It (the co-efficient A) depends upon the characteristic conditions of a particular process. It may be constant for a particular process and varies for the other depending upon involved parameters or experimental situation. Thus "A" cannot be regarded as universal constant, just like universal gravitational constant G and k in Newton's Second Law of Motion. The reason is that mass energy inter-conversion are the bizarre processes in nature and not completely studied.

Now consider the case that when mass is converted into energy. Let in some conversion process mass decreases from M_i(initial mass) to M_f (final mass), correspondingly energy increases from E_i (initial energy) to E_f (final energy). The Eq. (22) gives infinitesimally small amount of energy dE created on annihilation of mass dm. To get the net effect the Eq. (22) can be integrated similarly Einstein has obtained the relativistic form of kinetic energy in June 1905 paper [18]

$$\int dE = Ac^2 \int dm\,,$$

$$\text{Initial limit of mass} = M_i,\ \text{Initial limit of Energy} = E_i\,,$$

$$\text{Final limit of mass} = M_f,\ \text{Final limit of Energy} = E_f\,.$$

Initially when mass of body is M_i, then E_i is the initial energy of the system. When mass (initial mass, M_i) is converted into energy by any process under suitable circumstances the final mass of system reduces to M_f. Consequently, the energy of system increases to E_f the final energy. Thus M_f and E_f are the quantities after the conversion. Hence, Eq. (22) becomes

$$E_f - E_i = Ac^2\,(M_f - M_i) \qquad (23)$$

or

$$\Delta E = Ac^2 \Delta m \qquad (24)$$

$$\text{Energy evolved} = Ac^2\ \text{(decrease in mass)}. \qquad (25)$$

If the characteristic conditions of the process permit then whole mass is converted into energy i.e. after the reaction no mass remains ($M_f = 0$)

$$\Delta E = -\,Ac^2 M_i \qquad (26)$$

In this case energy evolved is negative implies that energy is created at the cost of annihilation of mass and the process is exo-energic nature (energy is emitted which may be in any form). Energy is scalar quantity having magnitude only, thus no direction is associated with it.

Thus the generalized mass-energy equivalence may be stated as

> "The mass can be converted into energy or vice-versa under some characteristic conditions of the process, but conversion factor may or may not always be c^2 (9×10^{16} $\mathrm{m^2/s^2}$) or c^{-2}."

9 Applications of generalized mass energy inter conversion equation $\Delta E = Ac^2 \Delta m$

(i) It is already mentioned in section (3) that if 0.001 kg or 1gm of matter is annihilated then energy equal to 9×10^{13} J (can drive a truck of mass 1000 kg to distance of $9{\times}10^7$ km) will be produced. Such or similar predictions are not experimentally confirmed and energy emitted can be found less than predictions.

Let the energy observed is $4.5{\times}10^{13}$ J corresponding to mass annihilated 0.001 kg , then value of A from $\Delta E = A c^2 \Delta m$ will be 0.5 i.e.

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{4.5{\times}10^{13}\,\text{J}}{9{\times}10^{16}} = 0.5. \tag{27}$$

Thus in this case mass energy inter-conversion equation becomes

$$\Delta E = 0.5c^2 \Delta m\,. \tag{28}$$

(ii) Let the TKE of fission fragments of U^{235} and Pu^{239} is 175 MeV (as experimentally it is observed less), instead of expected 200 MeV. It can be explained with help of $\Delta E = A c^2 \Delta m$ with value of A is equal to 0.875 i.e.

$$A = \frac{\Delta E}{c^2} \Delta m = \frac{175}{200} = 0.875\,. \tag{29}$$

Thus energy of fission fragments of U^{235} and Pu^{239} is given by

$$\Delta E = 0.875c^2 \Delta m\,. \tag{30}$$

Thus value of A less than one is justified experimentally in this case.

(iii) The anomalous observation of excess mass of Ds(2317) can be understood with help of $\Delta E = A c^2 \Delta m$, as mass of the observed particle is found more [10] than predictions of $E = \Delta m c^2$. In this case value of A will be less than one. For understanding consider energy equal to 10^6 J is converted into mass, then corresponding mass must be $1.11{\times}10^{-11}$ kg. We are considering the case that mass is found more than this. Let the mass be $1.12{\times}10^{-11}$ kg. The value of A this case is 0.992, as calculated from $\Delta E = A c^2 \Delta m$ i.e.

$$A = \frac{10^6}{10.8{\times}10^5} = 0.992\,. \tag{31}$$

Thus in this mass energy inter conversion equation becomes

$$\Delta E = 0.992c^2 \Delta m \quad \text{or} \quad \Delta m = 1.008 \Delta E\,. \tag{32}$$

Thus corresponding to small energy more mass is emitted.

(vi) $\Delta E = A c^2 \Delta m$ is useful in explaining the binding energy (2.2244MeV or $3.55904{\times}10^{-13}$ J), mass defect (0.002388 amu or $2.388{\times}10^{-3}$ amu) and universal equality of mass of nucleons ($m_n = 1.008664$ amu, $m_p = 1.006082$ amu). Obviously neutron and protons contribute equally towards the mass defect (0.001194 amu), then mass of neutron inside nucleus must be 1.00747 amu (mass outside nucleus i.e. in Free State is 1.008664 amu). Similarly corresponding mass of proton in the nucleus must be 1.006082 amu (mass of proton outside nucleus 1.007274 amu). But decrease in mass of nucleons inside nucleus is not justified, as masses of nucleons are universally same [23, 28, 29].

Thus mass defect of deuteron must be **infinitesimally small**, only then masses of nucleons are same inside **nucleus and outside nucleus.** Also binding energy must be 2.2244 MeV as experimentally observed. Both these experimentally confirmed facts can be explained with help of $\Delta E = A c^2 \Delta m$.

Let in this case the mass defect is negligibly small i.e. $2.388{\times}10^{-13}$ amu or $3.9653{\times}10^{-40}$ kg. Then value of A (coefficient of proportionality or mass energy inter conversion coefficient) is 10^{10} i.e. for annihilation of infinitesimally small mass exceptionally large amount of energy is liberated. Thus

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{3.5634{\times}10^{-13}}{9{\times}10^{16}{\times}3.9653{\times}10^{-40}} = 10^{10}\,, \tag{33}$$

$$\Delta E = 10^{10} c^2 \Delta m\,. \tag{34}$$

(v) Webb [48] has reported results for time variability of the fine structure constant or Sommerfeld fine structure constant (α) using absorption systems in the spectra of distant quasars. The variation in magnitude of alpha has been observed as

$$\frac{\Delta\alpha}{\alpha} = \frac{(\alpha_{then} - \alpha_{now})}{\alpha_{now}} = -\,0.9{\times}10^{-5}. \tag{35}$$

According to CODATA currently accepted value of alpha (α_{now}) is $7.297352{\times}10^{-3}$. Hence from Eq. (35),

$$\alpha_{then} = 0.007296\,. \tag{36}$$

Now corresponding to the reduced value of α ($\alpha_{then} = 0.007296$) the the speed of light can be determined from equation

$$c_{then} = \frac{e^2}{2\alpha_{then}\varepsilon h} \tag{37}$$

as $2.994{\times}10^8$ m/s (where all terms have usual meanings). Currently accepted value of the speed of light is $2.99729{\times}10^8$ m/s.

To explain the energy emitted with this value of the speed of light is the value A ($\Delta E = A c^2 \Delta M$)

$$A = \frac{c^2}{c^2_{then}} = 1.001\,. \tag{38}$$

Thus in this case mass energy inter conversion equation becomes

$$\Delta E = 1.001c^2 \Delta m\,. \tag{39}$$

Hence $\Delta E \propto \Delta m c^2$ has both experimental and theoretical support, with emergence of new experimental data its significance will increase.

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On the Necessity of Aprioristic Thinking in Physics

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The thinking which encompasses both reasoning-in-itself and reasoning-for-itself, called "aprioristic thinking" by Hegel, is the freest form of thinking. This form of thinking is imparted to the physical sciences by philosophy. Only under this condition can physics obtain deeper scientific knowledge.

In the beginning of the last century, the renowned scientist Anri Bergson [1] gave an advanced notice: "We experience now one of the greatest crises; all our thinking, all ethics, all life, all our spiritual and moral existence are in a condition of intellectual fermentation. . . ". This fermentation, according to the opinion of the known philosopher Edmund Husserl [2], occurs due to installation dominant in positivistic and naturalistic philosophy. This installation of ordinary consciousness contrasts the human consciousness and being to each other, and, therefore, not taking into account consciousness, can lead to more crisis the European sciences. As pointed out by Husserl, the sciences about the nature can be founded only by means of phenomenology, as a strict philosophy, which is oriented towards a first-hand experience of consciousness. Though many years have already passed since then, as these scientists have written, resolute turn in this question is not yet present. Even, in spite of the fact that in one of the achievements of modern physics — in quantum physics — the consciousness of the observer has found a place for itself. In the interpretation of quantum mechanics, the most important upshot of this for physicists is that this problem is related to the problem of consciousness — an interdisciplinary problem concerning not only physicists, but also philosophers, psychologists, physiologists and biologists. Its solution will result in deeper scientific knowledge. But all the same, for some reason, scientists very often in case of scientific cognition neglect questions of the interaction between our consciousness and the surrounding world. If we wish to reach fuller scientific knowledge, we should not deal with physical phenomena and thinking (consciousness) itself separately. The well-known physicist Wigner [3] maintains that the separation between our perception and the laws of nature is no more than simplification. And though we are convinced that it has a harmless character, to nevertheless merely forget about it should not be the case. It is clear that deeper scientific knowledge should include in itself a problem of the theory of cognition — a problem of the origin of knowledge and a logical substantiation of the relevant system of knowledge.

In deciding upon this problem, the cognition theory considers the connection between "I", my consciousness and an external world, and says that the decision is concealed in the interaction between sensuality and reason. Reason transforms our feelings into thoughts and it means that the representations are replaced with concepts. If science does not wish to be, as it was described by Hegel [4], a simple unit of data then, of course, it should have concepts and should operate with them. But, if science also does not wish to be positivistic (all sciences, except philosophy, are positivistic) then it should have a rational basis and beginning. Only in this case, does the sole purpose (affair) of science become the concept of the concept. (Hegel has distinguished between the sciences as follows: 1) sciences, as a simple unit of data, 2) the extremely positive sciences, 3) positive sciences, 4) philosophy. Positivism of a physical science is that it does not know that its definitions are final).

Physics, certainly, has a rational basis which is intimately connected with philosophy too. But what prevents a physical science from becoming a "mere" philosophy? Hegel has elaborated on the notion of a positivistic side of the sciences. In physics, this positivism is characterized by the lack of knowledge that its definitions are final and therefore there is no transition into the higher sphere. This finiteness is connected with the finiteness of the cognition (feeling, belief, authority of others, and authority of external and internal contemplation).

However, it is perhaps meant so to happen, as described by Hegel, that thoughtful contemplation, lowering casual conditions and organizing everything, will present the general outline before a detailed intellectual exposition. It is clear then that an intellectual physical science will picture a rational science of Nature in the form of an image which is the external image of Nature. This image is called a physical picture of the world, or, as called by Max Planck [5], the world of a physical science. Planck has explained further about it: ". . . We are compelled to recognize behind the sensual world the second, real world which leads independent existence independent of the person, — the world which we not can comprehend directly, but we comprehend via the sensual world, via known symbols which he informs us, as if we would consider a interesting subject only through the glasses, optical properties of which are absolutely unknown for us".

Thus, according to Planck, there are three worlds: the real world, the sensual world and the world of a physical science or a physical picture of the world. The real world is the world outside us, it exists irrespective of our understanding of its laws, i.e. irrespective of our consciousness and therefore it is the objective world. The sensual world is our world because we perceive it through our bodies of perception: eyes, hearing, charm etc., and it is subjective (it is possible to tell that it is illusion). A physical picture of the world is the world in which can be reflected both real and the sensual world. This world is a bridge for us with which help we study the world around. Reflection of the real world in the world of a physical science is a physical picture of the real world; it is also possible to describe the quantum world and the science studying this world is the quantum physics. The reason why the real world is the quantum world is because the so-called world of atoms and electrons, as Planck has given above, exists independently of the person. Reflection of the sensual world in a physical picture of the world is a physical picture of the sensual world (the classical world) and the corresponding science is the classical physics. Thus, only in case of the thoughtful contemplation can the physics can be concerned with the philosophy of nature.

But when will it be possible to tell, whether the physical science is not simply concerned with philosophy, and even enters into it, to a certain extent it? Based on a well-known classification of all sciences by Hegel, the nature philosophy is a science about an idea in another-being. Hegel has thus said: "what is real, is reasonable", referring to understanding in the context of the reality of a reasonable idea. Such a reality is the maintenance of Hegel's philosophy. Hegel writes that phenomena, being unstable (random) and existing in continuous fluidity, are in contrast to the idea and do not enter into it. Therefore Hegel takes the idea as the maintenance of his philosophy. In the ancient time, Plato too spoke about ideas [2]. He wrote: "In a horse, in the house or in the fine woman there is nothing real. The reality is concluded as a universal type (idea) of a horse, the house, the fine woman" [6]. Plato confirms the continuous fluidity of all existing forms and asks the question: can the philosophy be within continuous and chaotic fluidity? As a result, the human knowledge is possible only under the condition of the existence of steady ideas, and with the help of it, is possible to distinguish between things based on fluid validity and to plan in it any logical order. Hegel understands that an idea will be steady, if it will be the reality of a "reasonable". After all, only reason is steady, absolute. But this is not only because it is so ingenious to define ideas in the way Hegel did it. In "Metaphysics", Aristotle, criticizing Plato, asserts that the idea of a thing explains nothing in the thing itself, even provided that the idea relates to the thing, as found for example, in the fact that whiteness concerns a white subject. Aristotle did not actually deny the independent existence of ideas, but attributed to them the existence within things themselves. Namely, Hegel's idea — the reality of the "reasonable" — satisfies Aristotle's requirement. Because, in such determination, the idea is taken from the reality itself. But against Hegel's reality the mind at once acts. The mind says to us that ideas are no existing chimeras. If science does not want to conceptualize its concept then it, of course, will agree with the mind. Then, very figuratively, it is described by Hegel as follows: just as meal process is ungrateful to the meal (simply eats it, not giving instead of anything), similarly, thinking process will be ungrateful to aposteriori experience, and will simply give nothing in exchange. In order to receive something from thinking process, it is necessary to make the thinking itself by the subject of thinking. Reflection transforms our representations into concepts. And further reflections of concepts transform concepts into concepts, i.e. it becomes clear as a concept. Only under such conditions can the science understand its concept. However, only in philosophy do we find that the subject of thinking is the thinking itself (for example, for the mathematician, it is numbers, spaces etc.). The thinking, opposing with itself to itself, is the reasoning-for-itself. Process thinking nevertheless is inside and consequently it is the reasoning-in-itself. As a result, the "in itself" and "for itself" reasoning is the most substantial form of free thinking and it is defined by Hegel, as aprioristic thinking. Only by aprioristic thinking can the generality and authenticity be found. Namely, in this thinking, philosophy informs the maintenance of empirical sciences. The obligation of the sciences is not to refuse this process, because it is a very noble act for a science to reach the concept of the concept. But the mind, objecting again, speaks to us: "But what it can give to the physics? ". At all times, there have been physicists who, knowing about the finiteness of the knowledge of their science, have spoken about deeper scientific knowledge [8–15]. They envision when it will be possible to speak about the physicist and about the consciousness of the observer simultaneously.

Hegel has very interestingly written: "In the physicist we too get acquainted with the general, with essence, the only distinction between physics and the philosophy of nature is that the philosophy of nature leads up us to the comprehension of the true forms of the concept of natural things". But doesn't it mean that in deeper scientific cognition the physical science has transited into a higher circle which is not present in physics because of its positivism? And the answer to this question is, of course, yes, it does. Thus, only under the condition of deeper scientific knowledge can we claim that the physical science is the philosophy of nature (in the sense that, for example, the apple is a fruit).

Hegel defines the philosophy of nature, as a science about an idea in its another being. As he writes, in philosophy we do not learn anything else, except ideas, but the ideas exist here as exterior forms. An exterior form of an idea is its another being. Because the being of an idea (reasoning-in-itself and reasoning-for-itself) takes place in the reason itself. Nature receives its exterior, that exterior which we see, in the exterior process of an idea. In fact, Hegel's slogan "what is reasonable, is real" is confirmed.

Unwittingly, we could as well resolve one more problem. The maintenance of philosophy, as Hegel writes, is an idea which excludes from itself, the phenomenon, chance. But the maintenance of physics is Nature, its phenomena. At the same time we may ask, “when can the physical science become the philosophy of nature?” All becomes clear when we agree with Hegel, that Nature is connected with an idea, in the sense that it is an idea in its another being. The laws of Nature, discovered by our thinking about physics, are also ideas – reasonables of reality.

Thus, as in the past, philosophy will continue to play an important role related to the necessity for the sciences to enter a higher level. Only in this case can the sciences avoid the crisis about which Husserl has always warned us. As Bergson continues that which has been said in the beginning of this article: “. . . The new system, more general, wider should become the doctrine for many decades and even centuries. These new principles should direct all our life on a new way on which the mankind will approach to cognition of true and to happiness increase at the Earth”.

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Potential Energy Surfaces of the Even-Even $^{230\text{-}238}$U Isotopes

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Nuclear structure of $^{230\text{-}238}$U isotopes hav been studied in the frame work of the interacting boson approximation model $(IBM-1)$. The contour plot of the potential energy surfaces, $V(\beta,\gamma)$, shows that all nuclei are deformed and have rotational characters, $SU(3)$. Levels energy spectra belonging to the gsb, β , γ bands, electromagnetic transition rates $B(E1)$ and $B(E2)$, quadrupole moment Q_0, deformation parameterare β_2 and the strength of the electric monopole transitions $X(E0/E2)$ are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

1 Introduction

The observation of a large quadrupole moments to $^{230\text{-}238}$U isotopes had led to the suggestion that these nuclei might be deformed and have to be confirmed by the measurement of their nuclear properties as well as the observation of their rotational band structures. It is noticed that the level schemes of uranium isotopes are characterized by the existence of two bands of opposite parity and lay in the region of octupole deformations. The primary evidence for this octupole deformaton comes from the parity-doublet bands, fast electric transition $(E1)$ between the negative and positive parity bands and the low-lying 1^-, 0_2^+ and 2_2^+ excitation energy states. Many authors have studied $^{230\text{-}238}$U isotopes theoretically using different models. The relativistic Mean Field Model has employed [1–4] to obtain the densities of the cluster and daughter nuclei. Also, a systematic α-decay properties of the even-even heavy and superheavy nuclei have been investigated. The energy of the deformed nuclei in the actinide region has been determined in the frame work of the macroscopic — microscopic approach. The Yukawa folding procedure has used [5] together with the Liquid Drop Model [6].

The properties of the states of the alternating parity bands in actinides are analyzed within the Cluster Model. The model has been used successfully in calculating levels energy, quadrupole moments and half-lives of cluster radioactivity. A comparison was mad between the predicted data [7–13] and the calculated values by other models and show good agreement.

The band heads, energy spacings within bands and a number of interband as well as intraband $B(E2)$ transition rates are well reproduced [14] for all actinide nuclei using the Exactly Separable Davidson (ESD) solution of the Bohr Hamiltonian.

The potential energy surfaces are calculated [15] to ^{230}U using the most advanced asymmetric two-center shell model that are added to the Yukawa-plus-exponential model.

Until now scarce informations are available about the actinide region in general and this is due to the experimental difficulties associated with this mass region. In the present article we used the Interacting Boson Model $(IBM-1)$ which is a theoretical model and differ than all the previous models used with the actinid nuclei. The aim of the present work is to process calculation for the follows:

1. For the potential energy surfaces, $V(\beta,\gamma)$, for all $^{230\text{-}238}$U nuclei;
2. For levels energy;
3. For the electromagnetic transition rates $B(E1)$ and also calculation for $B(E2)$;
4. For the electric quadrupole moment Q_0;
5. For the deformation parameter β_2;
6. For the strength of the electric monopole transitions $X(E0/E2)$.

2 (IBA-1) model

2.1 Level energies

The IBA-1 model was applied to the positive and negative parity low-lying states in even-even $^{230\text{-}238}$U isotopes. The proton, π, and neutron, ν, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d-bosons. Creation $(s^\dagger d^\dagger)$ and annihilation $(s\tilde{d})$ operators are for s and d bosons. The Hamiltonian [16] employed for the present calculation is given as:

$$\begin{aligned} H = {} & EPS \cdot n_d + PAIR \cdot (P \cdot P) + \\ & + \frac{1}{2}\, ELL \cdot (L \cdot L) + \frac{1}{2}\, QQ \cdot (Q \cdot Q) + \\ & + 5OCT \cdot (T_3 \cdot T_3) + 5HEX \cdot (T_4 \cdot T_4)\,, \end{aligned} \tag{1}$$

nucleus	EPS	$PAIR$	ELL	QQ	OCT	HEX	$E2SD(eb)$	$E2DD(eb)$
^{230}U	0.2000	0.000	0.005	−0.0150	0.0000	0.0000	0.2060	−0.6094
^{232}U	0.2000	0.000	0.0050	−0.0150	0.0000	0.0000	0.1890	−0.5591
^{234}U	0.2000	0.0000	0.0044	−0.0150	0.0000	0.0000	0.1782	−0.5271
^{236}U	0.2000	0.0000	0.0055	−0.0150	0.0000	0.0000	0.1720	−0.5088
^{238}U	0.2000	0.0000	0.0057	−0.0150	0.0000	0.0000	0.1630	−0.4822

Table 1: Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

where

$$P \cdot p = \frac{1}{2} \left[\begin{array}{l} \left\{ (s^\dagger s^\dagger)_0^{(0)} - \sqrt{5} (d^\dagger d^\dagger)_0^{(0)} \right\} x \\ \left\{ (ss)_0^{(0)} - \sqrt{5} (\tilde{d}\tilde{d})_0^{(0)} \right\} \end{array} \right]_0^{(0)}, \tag{2}$$

$$L \cdot L = -10\sqrt{3} \left[(d^\dagger \tilde{d})^{(1)} x \, (d^\dagger \tilde{d})^{(1)} \right]_0^{(0)}, \tag{3}$$

$$Q \cdot Q = \sqrt{5} \left[\begin{array}{l} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + +\tilde{d} s)^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} \end{array} \right]_0^{(0)}, \tag{4}$$

$$T_3 \cdot T_3 = -\sqrt{7} \left[(d^\dagger \tilde{d})^{(2)} x \, (d^\dagger \tilde{d})^{(2)} \right]_0^{(0)}, \tag{5}$$

$$T_4 \cdot T_4 = 3 \left[(d^\dagger \tilde{d})^{(4)} x \, (d^\dagger \tilde{d})^{(4)} \right]_0^{(0)}. \tag{6}$$

In the previous formulas, n_d is the number of boson; $P \cdot P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL, QQ, OCT, HEX is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator [16] employed in this study is given by:

$$T^{(E2)} = E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \frac{1}{\sqrt{5}} E2DD \cdot (d^\dagger \tilde{d})^{(2)}. \tag{7}$$

The reduced electric quadrupole transition rates between $I_i \to I_f$ states are given by

$$B(E_2, I_i - I_f) = \frac{[< I_f \parallel T^{(E_2)} \parallel I_i >]^2}{2I_i + 1}. \tag{8}$$

3 Results and discussion

3.1 The potential energy surface

The potential energy surfaces [17], $V(\beta, \gamma)$, for uranium isotopes as a function of the deformation parameters β and γ have been calculated using :

$$\begin{aligned} E_{N_\Pi N_\nu}(\beta,\gamma) &= <N_\pi N_\nu; \beta\gamma \, | H_{\pi\nu} | N_\pi N_\nu; \beta\gamma> = \\ &= \zeta_d (N_\nu N_\pi) \beta^2 (1+\beta^2) + \beta^2 (1+\beta^2)^{-2} \times \\ &\times \left\{ k N_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu) \beta \cos 3\gamma] \right\} + \\ &+ \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu (N_\nu - 1) \left(\frac{1}{10} c_0 + \frac{1}{7} c_2 \right) \beta^2 \right\}, \end{aligned} \tag{9}$$

where

$$\bar{X}_\rho = \left(\frac{2}{7} \right)^{0.5} X_\rho \quad \rho = \pi \text{ or } \upsilon \,. \tag{10}$$

The calculated potential energy surfaces, $V(\beta, \gamma)$, for uranium series of isotopes are presented in Fig. 1 and Fig. 2. It shows that all nuclei are deformed and have rotational-like characters. The two wells on both oblate and prolate sides are not equal but the prolate is deeper in all nuclei.. The energy and electromagnetic transition rates are calculated cosidering uranium series of isotopes a rotational-like nuclei.

3.2 Energy spectra

IBA-1 model has been used in calculating the energy of the positive and negative parity low -lying levels of uranium series of isotopes. In many deformed actinide nuclei the negative parity bands have been established and these nuclei are considered as an octupole deformed. A simple means to examine the nature of the band is to consider the ratio R which for octupole band, $R \succ 1$, and defined as [18]:

$$R = \frac{E(I+3) - E(I-1)_{NPB}}{E(I) - E(I-2)_{GSB}}. \tag{11}$$

In the present calculations all values of R for uranium series of isotopes are $\succ$ 1, and we treated them as octupole deformed nuclei.

A comparison between the experimental spectra [19–23] and our calculations, using values of the model parameters given in Table 1 for the ground and octupole bands, are illustrated in Fig. 3. The agreement between the calculated levels energy and their correspondence experimental values for all uranium nuclei are reasonable, but slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which

I_i^+ I_f^+	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
0_1 Exp. 2_1	9.70(12)	10.0(10)	10.66(20)	11.61(15)	12.09(20)
0_1 Theor. 2_1	9.7128	10.0163	10.6479	11.6506	12.1143
2_1 0_1	1.9426	2.0033	2.1296	2.3301	2.4229
2_2 0_1	0.0107	0.0113	0.0104	0.0095	0.0081
2_2 0_2	1.2419	1.3677	1.5411	1.7598	1.8855
2_3 0_1	0.0190	0.0131	0.0099	0.0082	0.0066
2_3 0_2	0.0027	0.0095	0.0131	0.0144	0.0139
2_3 0_3	0.0245	0.0085	0.0031	0.0013	0.0007
2_4 0_3	0.7577	0.8679	1.0291	1.2308	1.3730
2_4 0_4	0.0508	0.0415	0.1309	0.0710	0.0022
4_1 2_1	2.7740	2.8443	3.0182	3.3014	3.4336
4_1 2_2	0.0699	0.0480	0.0352	0.0276	0.0213
4_1 2_3	0.0046	0.0019	0.0010	0.0007	0.0005
6_1 4_1	3.0183	3.0849	3.2707	3.5790	3.7256
6_1 4_2	0.0706	0.0532	0.0412	0.0333	0.0260
6_1 4_3	0.0128	0.0066	0.0039	0.0026	0.0018
8_1 6_1	3.0670	3.1387	3.3335	3.6548	3.8121
8_1 6_2	0.0618	0.0503	0.0415	0.0351	0.0381
8_1 6_3	0.0201	0.0117	0.0073	0.0049	0.0034
10_1 8_1	2.9919	3.0827	3.2910	3.6237	3.7930
10_1 8_2	0.0510	0.0439	0.0383	0.0340	0.0280

Table 2: Table 2: Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2\, b^2$).

I_i^- I_f^+	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
1_1 0_1	0.1353	0.1602	0.1824	0.2071	0.2294
1_1 0_2	0.0531	0.0512	0.0492	0.0475	0.0449
3_1 2_1	0.2509	0.2811	0.3075	——	——
3_1 2_2	0.0811	0.0763	0.0711	——	——
3_1 2_3	0.0013	0.0002	0.0000	——	——
5_1 4_1	0.3628	0.3913	——	——	—
5_1 4_2	0.0862	0.0831	——	——	—
5_1 4_3	0.0020	0.0006	——	——	—
7_1 6_1	0.4809	0.5064	——	——	——
7_1 6_2	0.0816	0.0811	——	——	——
9_1 8_1	0.6043	0.6267	——	——	——
9_1 8_2	0.0736	0.0749	——	——	——

Table 3: Table 3: Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu\, e^2 b$).

$nucleus$	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
Q_0	9.920	10.020	10.340	10.800	11.020
β_2	0.263	0.264	0.272	0.282	0.286

Table 4: Table 4: The calculated electric quadrupole moment Q_0 and deformation parameter β_2.

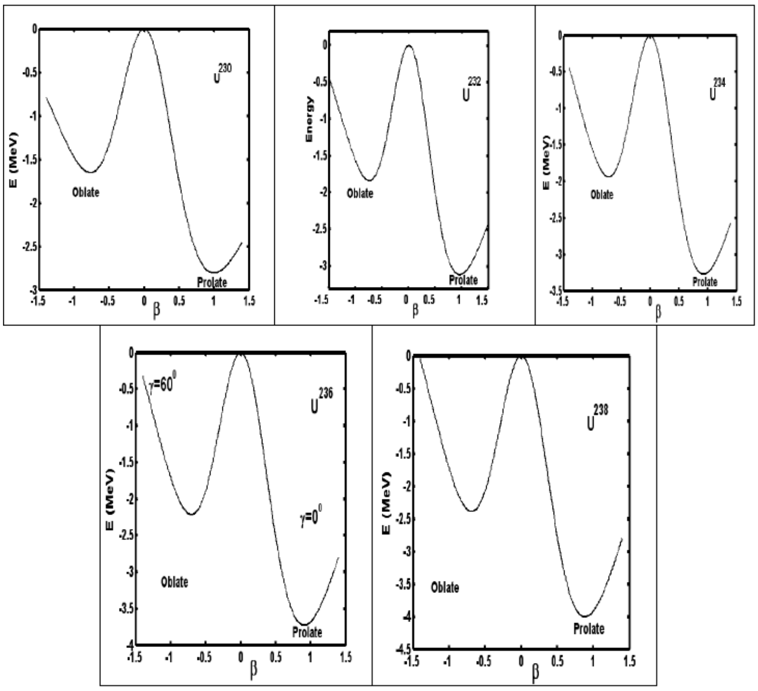

Fig. 1: Potential energy surfaces for $^{230\text{-}238}$U nuclei at γ= 0° (prolate) and 60° (oblate).

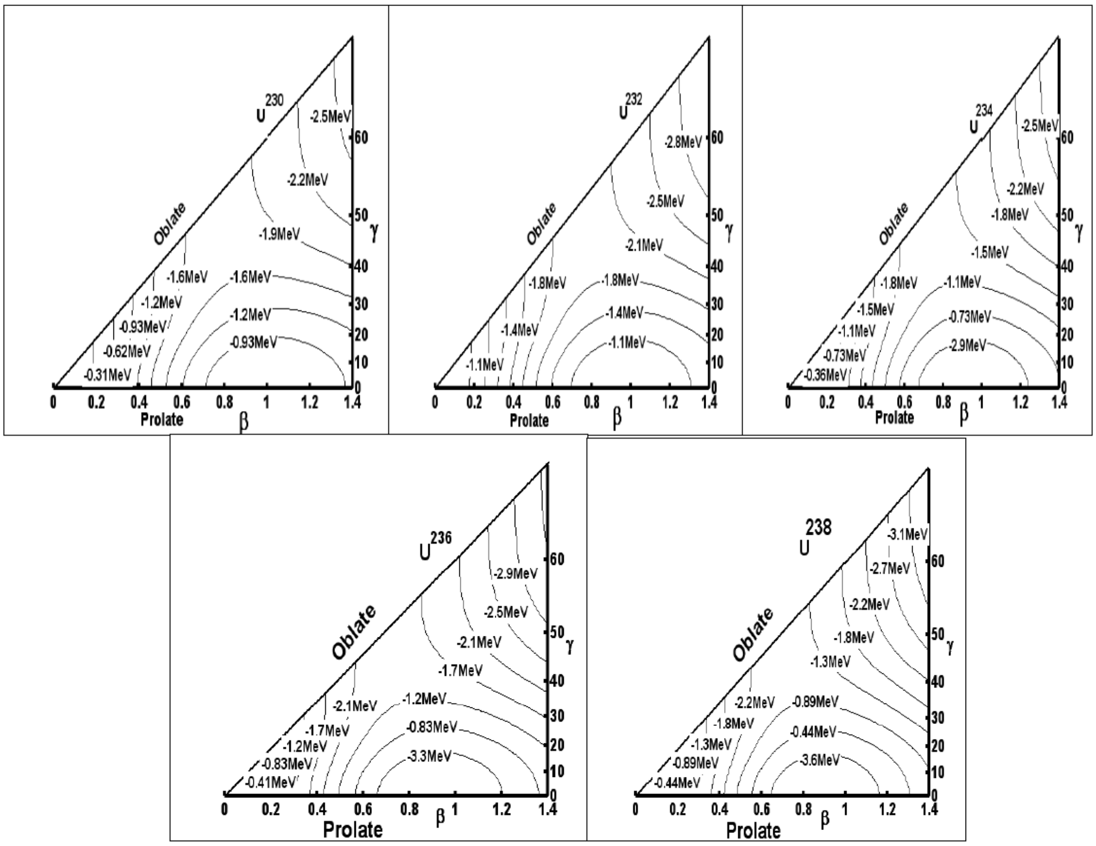

Fig. 2: Contour plot of the potential energy surfaces for $^{230\text{-}238}$U nuclei.

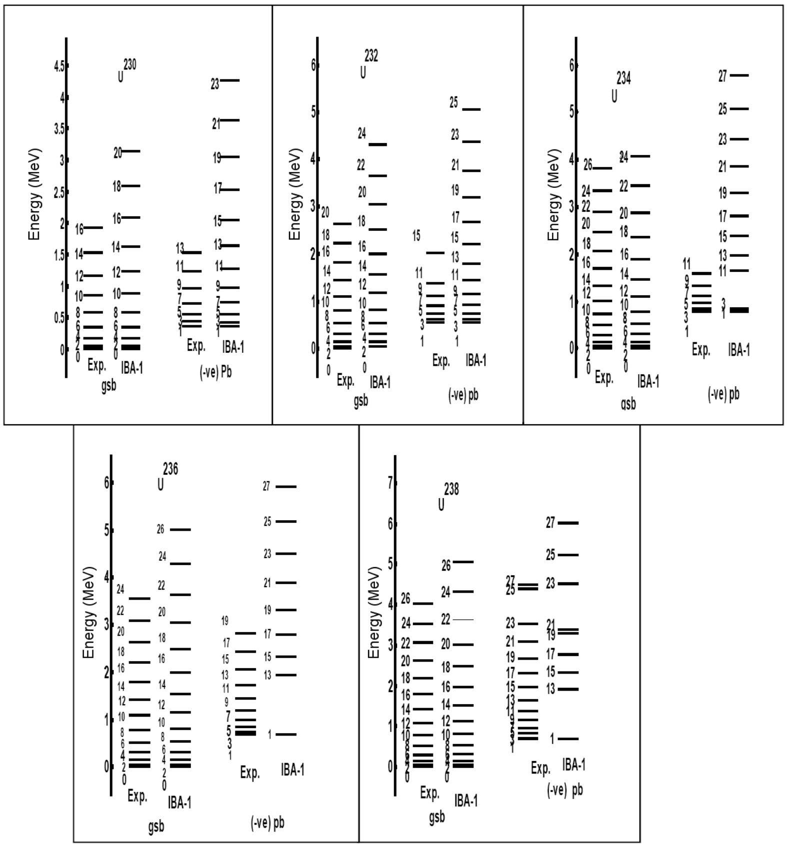

Fig. 3: Comparison between experimental (Exp.) [19–23] and theoretical (IBA-1) energy levels in $^{230\text{-}238}$U.

is due to band crossing and octupole deformation. From γ-bands [24] octupole deformation deformation has observed at $I = 14$ (for ^{232}U), $I = 10$ (for ^{234}U), $I = 15$ (for ^{236}U) and $I = 10$ (for ^{238}U) respectively.

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E2)$ or $B(E1)$ for these series of nuclei. The only measured $B(E2, 0_1^+ \to 2_1^+)$'s are presented, in Table 2 for comparison with the calculated values. The parameters $E2SD$ and $E2DD$ used in the present calculations are displayed in Table 1.

The calculated [equations 12, 13] electric quadrupole moment Q_0 and deformation parameter β_2 are given in Table 4. It is clear that both values are increasing with the increase of the neutron number of uranium isotopes.

$$Q_0 = \left[\frac{16\pi B(E_2)_{exp.}}{5}\right]^{1/2}, \tag{12}$$

$$\beta_2 = \frac{[B(E_2)_{exp.}]^{1/2}}{\frac{3ZR_0^2}{4\pi}} \tag{13}$$

3.3 Electric monopole transitions

The electric monopole transitions, $E0$, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transitions, $X_{if'f}(E0/E2)$, [25] are calculated using equations (14, 15) and presented in Table 5.

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_{'f})}, \tag{14}$$

$$X_{if'f}(E0/E2) = (2.54{\times}10^9)\, A^{3/4} \times$$
$$\times \frac{E_\gamma^5(\text{MeV})}{\Omega_{KL}}\, \alpha(E2)\, \frac{T_e(E0, I_i - I_f)}{T_e(E_2, I_i - I_{'f})}. \tag{15}$$

3.4 Conclusions

The IBA-1 model has been applied successfully to $^{230\text{-}238}$U isotopes and we have got:

1. The ground state and octupole bands are successfully reproduced;
2. The potential energy surfaces are calculated and show rotational behavior to $^{230\text{-}238}$U isotopes where they are mainly prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;

I_i^+	I_f^+	$I_{f'}^+$	^{230}U	^{232}U	^{234}U	^{226}U	^{238}U
0_2	0_1	2_1	0.660	0.560	0.001	0.920	1.470
0_3	0_1	2_1	13.370	1.300	15.910	0.282	—–
0_3	0_1	2_2	2.400	0.410	3.000	1.960	212.500
0_3	0_1	2_3	3.510	0.280	2.420	1.240	0.520
0_3	0_2	2_1	0.620	0.590	0.660	0.500	—–
0_3	0_2	2_2	0.110	0.180	0.120	0.001	1.500
0_3	0_2	2_3	0.180	0.130	0.100	0.001	3.720
0_4	0_1	2_2	1.960	7.750	0.001	0.230	7.250
0_4	0_1	2_3	1.320	0.250	—–	0.250	0.190
0_4	0_1	2_4	32.660	0.330	1.000	0.170	0.460
0_4	0_2	2_2	—–	0.020	0.0000	0.060	3.250
0_4	0_2	2_3	——	0.750	—–	0.070	0.080
0_4	0_2	2_4	—–	—–	0.000	0.100	0.2000
0_4	0_3	2_1	0.330	—–	—–	24.000	19.000
0_4	0_3	2_2	0.020	0.080	0.110	0.330	4.750
0_4	0_3	2_3	0.010	2.750	—–	0.360	0.130
0_4	0_3	2_4	0.330	—–	17.000	0.520	0.300

Table 5: Table 5. Theoretical $X_{if'f}$ ($E0/E2$) ratios for $E0$ transitions in Ra isotopes.

4. Electric quadrupole moment Q_0 are calculated;
5. Deformation parameter β_2 are calculated.

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LETTERS TO PROGRESS IN PHYSICS

A Brief Note on “Un-Particle” Physics

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The possibility of a hidden sector of particle physics that lies beyond the energy range of the Standard Model has been recently advocated by many authors. A bizarre implication of this conjecture is the emergence of a continuous spectrum of massless fields with non-integral scaling dimensions called “un-particles”. The purpose of this Letter is to show that the idea of “un-particles” was considered in at least two previous independent publications, prior to its first claimed disclosure.

The Standard Model (SM) is a highly successful theoretical framework that describes the relationships among all known elementary particles and the attributes of three of the four forces that act on these particles — electromagnetism, the strong force and the weak force. SM covers an energy range upper limited by the weak interaction scale of approx. 300 GeV. Despite the remarkable success of SM, it seems likely that a much deeper understanding of nature will be achieved as physicists continue to probe the fundamental constituents of matter at increasingly higher energies. Both theory and experiments strongly indicate that new phenomena await discovery beyond the SM range and reaching into the Terascale region. The Large Hadron Collider (LHC) at CERN is based on high energy proton beams and is scheduled to begin operation later this year. Moreover, further exploiting the Terascale physics will be possible in the near future with a new accelerator known as the International Linear Collider (ILC). It is believed that running both LHC and ILC will provide clues on how to go about solving many of the open questions challenging the current SM.

The possibility of a yet-unseen sector that lies in the Terascale range and is weakly coupled to SM has been recently advocated by many authors [1–6]. A bizarre implication of this conjecture is the emergence of a continuous spectrum of massless states with non-integral scaling dimensions called “un-particles”. In classical physics, the energy, linear momentum and mass of a free point particle are linked through the relativistic connection ($c = 1$):

$$E^2 = p^2 + m^2. \tag{1}$$

Quantum mechanics converts (1) into a dispersion relation for the corresponding quantum waves, with the mass m fixing the low frequency cut-off ($\hbar = 1$):

$$\omega^2 = k^2 + m^2. \tag{2}$$

Unlike (1) or (2), un-particles are conjectured to emerge as streams of fractional objects, something that has never been either imagined or seen before. A possible signal of un-particles at either LHC or ILC may show up as “missing” energy in certain decay channels [1–6].

The purpose of this Letter is to set the record straight and point out that the idea of “un-particles”, first claimed in [1, 2], was previously considered elsewhere. To the best of our knowledge, there are at least two publications where a similar or identical concept was introduced and discussed:

1. In 2005, Prof. F. Smarandache has launched the term *un-matter* as part of his novel mathematical framework of Neutrosophy and Fuzzy Logic [7, 8];
2. In 2006, the author has formulated the concept of *fractional number of field quanta* in connection with the development of quantum field theory using complex dynamics [9].

It is unfortunate that neither one of [1–6] have referenced these contributions.

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LETTERS TO PROGRESS IN PHYSICS

International Injustice in Science

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In the scientific research, it is important to keep our freedom of thinking and not being yoked by others' theories without checking them, no matter where they come from. *Cogito, ergo sum* (I think, therefore I am), said Descartes (1596–1650), and this Latin aphorism became his first principle in philosophy.

Inspired by D. Rabounski [1] and M. Apostol [2] I read more articles about injustices in science (for example [3]) and in arts and letters occurring in contemporary societies. The poet Plautus (254-184 B.C.) had once exclaimed that *homo homini lupus* (man is a wolf for man), so people make problems to people. In this short letter to the editor, I would like to list some inconvenient cases that manifest today:

There exist reviewing and indexing publications and institutes made just for a propagandistic way, and not reviewing all relevant literature on the topics, but reviewing their people and their ideas while ignoring, boycotting, denigrating, or discrediting other people and ideas. They exercise an international traffic of influence by manipulations and falsifications of information (such as biographies, history of events, etc.), discourage people for working on topics different from theirs, and use subversive techniques in their interest of hegemony in science, arts, and letters.

The science, art, and literature of the powerful are like that: If you don't cite them, it is your fault as if you have not read them. However, if they don't cite you, it's your fault too as if you did not deserve to be cited because you have published in so-called by them "obscure publications", even if these people have "borrowed" your idea without acknowledgement. They categorize as "obscure, unimportant, not by establishment" those journals, publishing houses, cultural centers and researchers or creators that do not obey to them or that dare to be independent thinkers, in order that these people with power positions stigmatize them in the public's eye (because they can not control these publications). While the publications and centers of research they control they proclaim as "the best". The science/art & letters establishments continue to ignore or minimalize the research and creation done outside the establishment. It became a common procedure that people who control the so-called "high" publications abuse their power and they "take" ideas from less circulated publications and publish them in these "high" publications without citation, as their own ideas!

There are journals using hidden peer-reviewers that delay the publication until someone else from their house get credit for your paper's ideas.

Secret groups and services ignore and even boycott personalities who are independent in thinking and don't follow the establishment or don't obey to them; they manipulate national and international awards in science, arts, literature, also they manipulate university positions, high research jobs, funding; they try to confiscate the whole planet's thought by making biased so-called "reference sites" (as the self-called "encyclopedias", "dictionaries", "handbooks", etc.) where they slander independent thinkers, while blocking other sites they don't like; that's why the whole human history of science, arts, letters has to be re-written; the search engines bring these "reference sites" amongst the first pages in a search, even they are not the most relevant to the search topic, and since most of the hurry readers browse only the beginning pages [they don't spend time to look at all of them], it is a high probability that the populace is manipulated according to the biased information of these so-called "free" (just because they are not free!) reference sites; these groups try to confiscate the Internet at the global scale; always, during history, there were and unfortunately there still are intentions from some secret groups or services to dominate others… They try to transform other countries in spiritual colonies by brain washing. Secret groups and services do not only politic, economic, or military espionage, but also scientific, artistic, literary manipulations in the profit of their people.

Unfortunately, big cultures continue to destroy small cultures and to delete the collective memory of small nations. History is written by winners, says the aphorism, but this is not correct, history should be written by all parts. International organisms are created who unfortunately only serve the interests of a few powers, not of the whole world.

There are people believing they detain the **absolute truth**, and if somebody dares to have a different opinion from them, he or she is blacklisted, slandered, banned from various publications, etc.

The public opinion is provoked, manipulated through propaganda, publicity, dissemination by those who detain the power or control the mass media and the national and international awards, and these awards have been created in purpose to impose some people and ideologies.

There exist scientific, artistic, literary, or cultural associations/organizations whose hidden goal is to manipulate people in their propagandistic interest and indoctrinate them. The literature they start to send (after collecting your membership money!) reflects only their ideas and praise only their people, while ignoring or boycotting others'. *Nolens volens* (unwilling or willing) the "member" of such association becomes their spiritual slave. Consequently, you are yoked to this association's propaganda. Better to be independent and not belonging to any association/organization.

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