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Gravity terrain corrections — an overview

D.A.G. Nowell *

St. John's College, Cambridge CB2 1TP, UK

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Abstract

Terrain corrections should normally be standardised out to a distance of 166.735 km and combined with the Bullard B correction for the curvature of the Earth, out to this $1\frac{1}{2}°$ limit, derived from a simple power-series. Particular care must be taken with nearby topography which often forms a significant part of the total correction and changes rapidly with location. This can either be surveyed in situ or calculated as rotational wedge segments of uniform slope out to a variable distance from the point of observation. Increasing vertical separation between adjacent stations requires terrain corrections to be extended further out than would otherwise be needed as they are height dependent. Distant topography beyond about 22 km must be adjusted for the curvature of the Earth and can produce negative terrain corrections. It is important to take into account bodies of water, and marine surveys must be converted to Bouguer gravity if they are to be combined with terrestrial data. Methods for special cases, such as readings on the seabed, towers and under ground, require additional approaches in their computation, especially when trying to detect deviations from Newtonian gravity.

Keywords: Bouguer gravity; Terrain correction; Marine; Height; Earth curvature

1. Introduction

The need to add a correction for the gravitational attraction of the undulations of the terrain about the plane through a gravity station was first recognized by Hayford and Bowie (1912). This terrain correction and a correction for the curvature of the Bouguer slab on the Earth, using tables by Cassinis et al. (1937), were applied out to $1\frac{1}{2}°$ or a distance of 166.735 km from the gravity station by Bullard (1936). Lambert (1930) had discussed the origin of the ideas about terrain corrections leading to the work of Hayford and Bowie (1912), but failed to understand their significance and presented a closed-form solution of the curvature problem which did not address it from the exploration point of view. When Hammer (1939) modified the Hayford and Bowie (1912) system for high precision terrain corrections out to a distance of about 22 km from the gravity station, the method started to be widely applied in gravity surveys.

This paper refines the process of terrain correction for the effects of height, nearby terrain or buildings, the need to correct to the sea or the lake bed instead of to the water surface and for masses of water, as well as locations above and below ground level. Without considering all these factors, errors will occur in the final ter-

* Present address: 2 Tudor Road, New Barnet, Herts EN5 5PA, UK.

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rain corrections for a gravity station which may be much more significant than errors of between 33 and 10 μGal in the measurement of gravity (Rymer, 1989), or the short range precision of 10 μGal (Valliant, 1991) and 3 to 5 μGal accuracy (LaCoste, 1991) for a gravity meter. Whatever the methods that are finally used in producing terrain corrections for a gravity survey, this paper aims to make the reader think carefully about how this is done, along with any likely errors and limitations of their chosen methods.

2. Standard gravity corrections

2.1. Corrections normally applied to gravity data

The following corrections are normally applied to gravity data: Earth tides (with an amplitude of up to 300 μGal {0.3 mGal}); instrumental drift; latitude; free air; topography.

With the free air correction (FAC), LaFehr (1991a) showed that the measured vertical gradients of gravity demonstrate considerable variation. As pointed out by LaFehr (1991a), this does not suggest that the normal free-air gradient of +0.3086 mGal/m (Robbins, 1981) is in error or needs to be locally adjusted. However, a far more precise height (h) and latitude (ϕ) dependent FAC is given by Lambert (1930) as:

$$\mathrm{FAC} = (0.308{,}57 + 0.000{,}21\cos 2\phi)h - 0.072(h/1000)^2 \qquad (1)$$

and Heiskanen and Moritz (1967) as:

$$\mathrm{FAC} = \left(0.308{,}768 - 0.001{,}434\sin^2\phi\right)h + 7.212\times 10^{-8}h^2. \qquad (2)$$

Also, there should ideally be a correction for any departure from the expected level of the geoid, which is calculated from the Bouguer anomaly (Tsuboi, 1983; p. 130) and thus would require a recursive solution to calculate. These anomalies should be taken into account when the geoid height changes rapidly, as in mountainous areas, because the indirect effect (Talwani, 1998) due to the discrepancy between the levels of the geoid and the reference ellipsoid of the International Gravity Formula becomes significant. But, as normally these effects change slowly with distance, it is justified simply to remove a regional field when gravity modelling, since uncertainties about variations in density are far more significant than such very long wavelength anomalies.

As Bullard (1936) recognized, the correction for the gravitational attraction of the topography at a gravity station consists of three parts: the Bouguer correction (Bullard A), which approximates the topography to an infinite horizontal thickness equal to the height of the station above sea level or any other datum plane, the Bouguer slab; the curvature of the Earth (Bullard B), which reduces the infinite Bouguer slab to that of a spherical cap of the same thickness with a surface radius of 166.735 km ($1\frac{1}{2}°$); and the terrain correction (Bullard C), which takes into account the undulations of the topography above and below the curved surface of the Earth at the height of the station (Fig. 1a). As Chapin (1996) pointed out, these corrections to produce the Bouguer anomaly are not gravity reductions, in that the gravity value is not somehow moved or ‘‘reduced’’ to a different location, as station values remain fixed at the point of observation. The Bouguer slab below the gravity station pulls downwards and increases the observed value of gravity: hence this effect has to be subtracted from readings. The formula for the Bouguer Correction BA (Bullard A) is:

$$\mathrm{BA} = 2\pi G\rho h \qquad (3)$$

where G is the gravitational constant, ρ is the density of the surface layer and h is the thickness of the slab. The curvature correction (Bullard B) out to 166.735 km consists of two parts (Fig. 2): the section of the spherical cap directly underlying the infinite slab which dominates up to elevations of 4150 m and pulls downwards

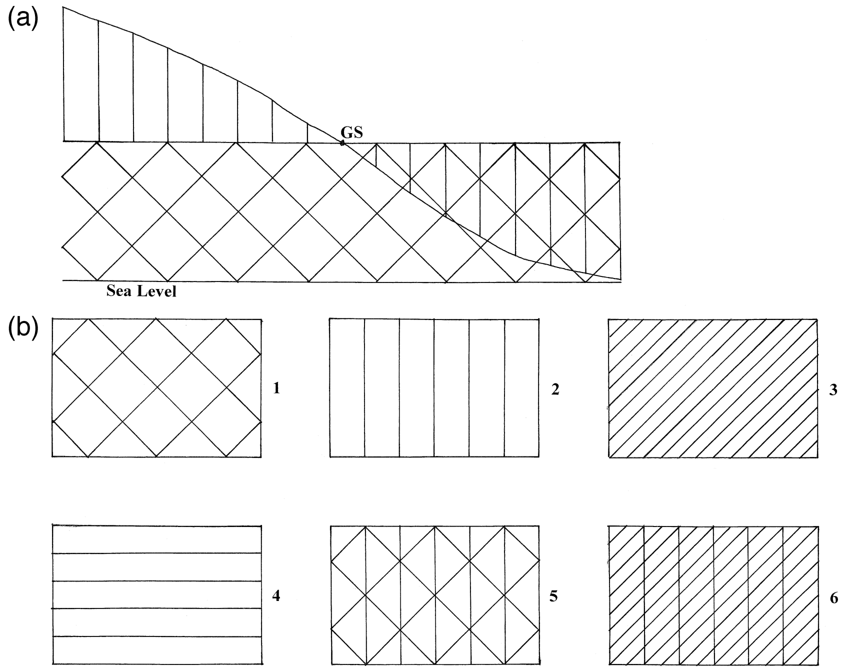

Fig. 1. (a) The corrections for a gravity station (GS) above sea level on land. (b) Key to the corrections for gravity stations in (a) and Figs. 10–14: 1 = the Bouguer slab, which approximates the topography to an infinite horizontal thickness equal to the height of the station above sea level, corrected for by subtracting the Bouguer correction (Bullard A) at the Bouguer density (cross hatching); 2 = the effects of topography above and below the horizontal plane at the height of the station, corrected for by adding the terrain correction (Bullard C) at the Bouguer density (vertical stripes); 3 = the effect of water above the sea bed, corrected for by subtracting a correction at the density of the water (hatching); 4 = the effect of the lack of rock below the station due to the sea, corrected for by adding the marine correction equal to the difference in density between the Bouguer slab and the water (horizontal stripes); 5 = the combined effects of the Bouguer slab (1) and topography (2); 6 = the combined effects of topography (2) and water above the sea bed (3). The curvature of the Earth (Fig. 2) is omitted for simplicity and is corrected for by subtracting the Bullard B curvature correction at the Bouguer density.

increasing the observed value of gravity; and the truncation of the infinite Bouguer slab at 166.735 km which dominates at elevation above 4150 m and decreases the observed value of gravity. Cogbill (1979) developed a more precise approximation than LaFehr (1991b) for the Bullard B correction BB (at a density of 2670 kg/m^3, with an Earth radius of 6371 km),

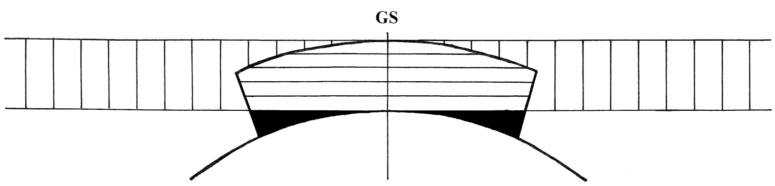

Fig. 2. The Bullard B correction for the curvature of the Earth away from a gravity station (GS): black = the section of the spherical cap directly underlying the infinite slab, which pulls downwards increasing the observed value of gravity; vertical stripes = the truncation of the infinite Bouguer slab at 166.735 km, which decreases the observed value of gravity, after LaFehr (1991b).

which is subtracted from readings, as a power-series:

$$BB = Ah - Bh^2 + Ch^3 + Dh^4 \quad (4)$$

where h is the height of the station, $A = 1.464{,}139 \times 10^{-3}$, $B = 3.533{,}047 \times 10^{-7}$, $C = 1.002{,}709 \times 10^{-13}$ and $D = 3.002{,}407 \times 10^{-18}$ (the B term is not positive as misprinted in the version given by LaFehr, 1991b). This approximation gives corrections to within 0.01 μGal and the exact formula quoted by LaFehr (1991b) can be used if needed. This effect near sea level for high-precision surveys is about 1.4 μGal/m (the typographical error in LaFehr, 1991a,b was corrected in an Erratum, 1992), and must be taken into account. Away from a latitude of 45° the correction varies linearly with height: at 4000 m above sea level on the equator the approximation is only out by 8.5 μGal (Fig. 3). The undulations of topography result in the upwards attraction of hills above the plane of the station and valleys below, which decrease the observed value of gravity, so these effects have to be added to readings.

As Chapin (1996) pointed out, the corrections to produce the Bouguer anomaly are not *gravity reductions*, in that the gravity value is not somehow moved or ''reduced'' to a different location. The corrected value for a station remains fixed at the height and point of observation and is never moved.

2.2. The Hammer method of terrain correction

The method Hammer (1939) used to make terrain corrections (Bullard C) involved dividing the area surrounding the station into circular zones and compartments in which the height above or below the station datum could be estimated and the effect for a given compartment calculated. This is done by using a series of clear overlays showing the limits of the compartments over maps at suitable scales away from the station. If a compartment contains portions which are below and above the station level, the average of the height differences must be found and not the difference between the average height of the compartment and the station level. Manual terrain corrections are only subject to errors from the quality of the topographic maps used (this also applies to digital terrain models which are after all based on them), using too few compartments out to a given distance in an area of extreme topography and the ability of the interpreter correctly to average the height above or below a station in each compartment.

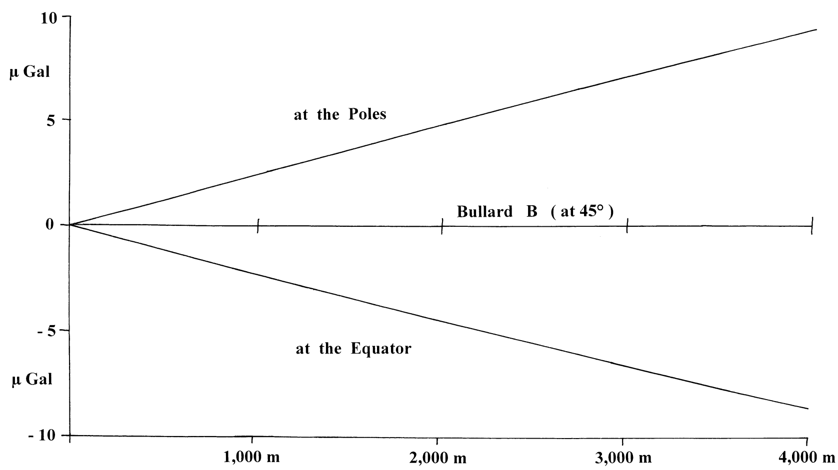

Fig. 3. The effect of latitude with increased height on the Bullard B correction, from LaFehr (1991b).

Hammer (1939) based his correction on the formula for the gravitational attraction (g) in the middle of a vertical hollow cylinder with its base at right angles to the horizontal plane through the station:

$$g = 2\pi G\rho\Big[R_2 - R_1 + \sqrt{(R_1^2 + h^2)} - \sqrt{(R_2^2 + h^2)}\Big] \quad (5)$$

where h is the height of the cylinder and R_1 and R_2 are the inner and outer radii. The cylinder is subdivided into compartments (Table 1) to make finding the height difference between the topography in the cylinder and the station easy. The corrections for the compartments in different cylinder or zones are simply added together to give the terrain correction. To obtain the zone spacing, Hammer (1939) used the following condition:

$$R_2/R_1 = (n + \pi)/(n - \pi) \quad (6)$$

where n is the number of compartments in the zone.

2.3. Computer methods of terrain correction

Bott (1959) showed that computers could be used to speed up large numbers of outer zone corrections with gridded elevation data and Nagy (1966) developed a formula for the gravitational attraction of a right rectangular prism. Blais and Ferland (1984) found that for distances greater than about 12 km, with a 1 km grid, the formula for a prism could be simplified to that for a vertical line mass, height (h), centred at a grid point:

$$g = G\rho s(1/d - 1/d') \quad (7)$$

where s is the cross sectional area, d is the horizontal distance from the station to the middle of the prism and $d' = \sqrt{(d^2 + h^2)}$ the distance between the station and the top of the prism.

With the advent of modern computers, terrain corrections can easily be calculated with a digital terrain model, e.g., Cogbill (1990) and Rollin (1990), or, as Herrera-Barrientos and Fernandez (1991) did, by fitting the topography with a set of Gaussian basis functions. An even more rapid method has been developed by Parker (1996) using Fourier methods, in which the attraction is calculated at points on the terrain surface by converting a power series for the topographic height into a series of convolutions. The cylindrical zone nearest to the station must be computed directly by integration to avoid convergence problems. However, terrain corrections

Table 1
Standard hammer zones with additional zones out to the limit of the Bullard B correction $1\frac{1}{2}°$ (166.735 km) and 10° (1110 km) with the drop below the tangential plane of a station due to the curvature of the Earth

Zone	R_1	R_2	R_e	NC	b
A	0.0	2.0	–	1	–
B	2.0	16.6	4.25	4	–
C	16.6	53.3	25.2	6	–
D	53.3	170.1	83.0	6	–
E	170.1	390.1	238.4	8	–
F	390.1	894.9	546.9	8	–
G	894.9	1530	1131	12	–
H	1530	2615	1933	12	–
I	2615	4469	3303	12	1
J	4469	6653	5349	16	2
K	6653	9903	7962	16	5
L	9903	14,742	11,852	16	11
M	14,742	21,944	17,643	16	24
(N)	21,944	33,000	26,371	20	55
(O)	33,000	50,000	39,777	20	124
(P)	50,000	75,000	60,025	20	283
(Q)	75,000	110,000	89,218	20	625
(R)	110,000	166,735	132,610	20	1380
{S}	166,735	230,000	193,350	24	2930
{T}	230,000	315,000	265,910	24	5550
{U}	315,000	430,000	363,680	24	10,380
{V}	430,000	590,000	497,530	24	19,420
{W}	590,000	810,000	682,820	24	36,560
{X}	810,000	1,110,000	936,700	24	68,740

R_1 = the inner radius of the zone in metres.
R_2 = the outer radius of the zone in metres.
R_e = the approximate radius of equality between the effect in the inner and outer parts of a zone in metres.
NC = the number of compartments in a given zone.
b = the drop below the tangential plane of a station at R_e for a given zone in metres.

made with a digital terrain model are only as good as the data they are based on and depend on how representative the heights used are of the general topography. The digital terrain model is only as good as the original map and the sampling method used to obtain digital data for the terrain correction with a computer terrain model. Also, a digital terrain model has to be set up in the first place, and it may be simpler to make manual corrections for a small number of stations when this will take less time. They can also be done for a few sample stations to give a good idea of the terrain correction distribution and validate the computer calculations.

3. The inner terrain correction

Special consideration has to be given to the area nearest the station which can form a significant part of the total terrain correction, especially within the first 170 m where the correction used often to be approximated to a simple slope through the station (Sandberg, 1958). The latest method, developed by Lyman et al. (1997), uses a reflectorless laser rangefinding system to quickly survey the surface level round the station out to 50–100 m and calculate the terrain correction for this innermost zone (Aiken and Cogbill, 1998). Differences of up to 1.0 mGal were found between this method and calculating the correction from a 40 m terrain grid. The system can cope with grass, but not thicker vegetation or fog, as these block and dissipate the laser signals. A parabolic surface can be fitted to the mean elevations (Rollin, 1990) through octant segments out to 895 m (Turnbull, 1980) and rotation of section through the station out to 895 m (Turnbull, 1984), though these methods can underestimate the effect by cutting out part of the average height of zones E and F with the fitted surface. A sloping wedge technique for calculating inner terrain corrections was developed by Barrows and Fett (1991), where the elevation and range of each change in slope away from a station are picked from a map and calculated for a given arc around the station. Oliver and Simard (1981) developed a method for terrain corrections within $2\frac{1}{2}$ km of the station based on conic prisms, which can smooth out the real topography, instead of Hammer's flat-topped compartments, which are an average of the elevation. Blais and Ferland (1984) found prisms with sloping tops should be used for calculating corrections within 2.5 km of a station, as there is only an average -0.09 mGal difference between this and the template (Hammer) method, compared to an erroneous -2.90 mGal for calculations with the standard flat topped prisms. Granser (1987) approximated the topography in a digital elevation model to a single valued function, to calculate terrain corrections between 50 m and about 1000 m from the station. This method involves the conversion of the volume integral for the gravity effect into a two-dimensional definite integral which is solved using the Gauss–Legendre quadrature formula. But the formulae used to calculate this correction are very complex, necessitate the use of a computer, and assume that the points that form the given model are representative of the general topography around the station: if this is not so, errors will occur. As Cogbill (1990) pointed out, the errors in the location of a station and the digital elevation model relative to the actual topographic surface within 250 m of the station will result in errors. Corrections for this zone with a total value of less than 300 μGal will result in errors of less than 50 μGal, while larger corrections can have errors in excess of 100 μGal. The same effect can be given by using smaller Hammer zones and compartments, though this also requires more calculations than normal. Hammer (1982) pointed out that the estimate of the average elevation in each compartment requires considerably more weight for the inner portion of the topography. This problem is greatest for the innermost zones, and so Hammer (1982) subdivided zones B, C and D for very high precision applications. The radius (r_e) at which the inner and outer halves of the correction in a

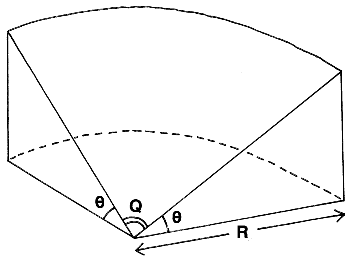

Fig. 4. The correction (g) for a quarter segment (Q) of a wedge with uniform slope θ, above or below the station, with a radius R, rotated around the vertical axis through the observation point. $g = 1/2\pi G\rho R(1 - \cos\ \theta)$ where G is the gravitational constant and π the density of the rock.

Hammer zone are equal varies slightly with height and can only be calculated by iteration. However, this radius (r_e) for any height can be approximated as:

$$r_e = (r_2 + r_1)/2 - 2\left\{\sqrt{(r_2^2 + r_1^2/2)} - (r_2 + r_1)/2\right\} \quad (8)$$

where r_1 and r_2 are the inner and outer radii of the zone.

A simple way of calculating the inner terrain correction (g) is to use the formula for a wedge of uniform slope (θ) above or below the station of a given radius (R), rotated around the vertical axis through the observation point:

$$g = 2\pi G\rho R(1 - \cos\ \theta). \quad (9)$$

This can be adapted to calculate the effect for a quarter (90°) segment (Fig. 4). This new method is more realistic and flexible than the power law approximation for a station on a uniform slope out to zone C (53.3 m) (Campbell, 1980).

4. Buildings

Buildings near a gravity station will have an effect on readings, as their mass results in an upward attraction which decreases the observed value of gravity, and so a correction has to be added to readings. For detailed microgravity the effect of known sources have to be carefully modelled (Patterson et al., 1995) and the method used below can only serve as a rough estimate.

The effect of an average modern building was calculated as a Hammer zone compartment at given distances from the centre, and the resulting data plotted as a graph (Fig. 5). The

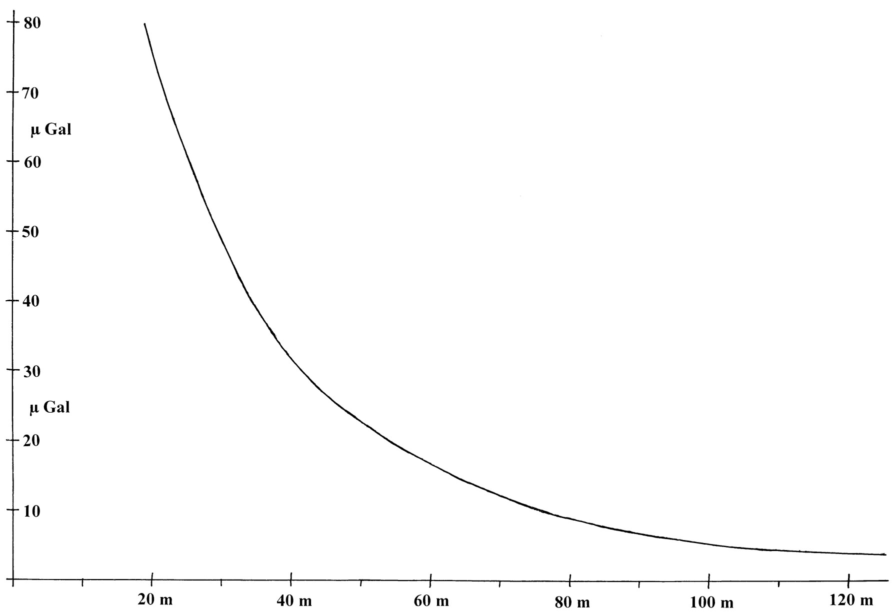

Fig. 5. The effect of a building, 27 m square, 100 m high, 25×10^6 kg mass and average density of 340 kg/m^3, with distance from the centre of the building in μGal, calculated as a Hammer zone compartment of suitable dimensions and density.

building was assumed to be 100 m high and 27 m by 27 m in area with a density for building materials (concrete and steel) of 2300 kg/m^3. Floor decks were 400 mm deep every 3.2 m and columns had sides 300 mm square with a spacing of 9 m between columns. The cladding round the block was assumed to be 0.1 m thick and the inner core of the building was equal to one outer side of cladding. The volume of the building materials was 10,825 m^3 giving a total mass of about 25×10^6 kg which can be used as the basis for finding the effect of any large 20th century building by first calculating its volume. It should be noted that older large buildings often have an average density twice that of modern ones or more. An average 25×10^6 kg building at a distance of 30 m from its centre has a 50 μGal effect while at a distance of 100 m only 5.5 μGal, so a group of nearby buildings could have a significant effect. If the hypothetical 100 m high building had a basement 6.9 m deep, which is not uncommon in modern buildings, this would result an effect equal to half that of the building, given a density of 2670 kg/m^3 for the surrounding rock, which would also have to be added to readings. As basements are out of sight, this could result in small errors, and so for many buildings it may be simpler to add one and a half times the building correction — or some smaller fraction, depending on local knowledge.

5. The outer limit of terrain corrections

There is the problem for a given survey of the distance at which to stop calculating the terrain correction. Hammer (1939) took 21.944 km, while modern surveys, such as the British Geological Survey (BGS), often go out to 50 km or further, and Bullard (1936) went out to 166.735 km. The effect of terrain beyond this distance was found to vary little throughout Czechoslovakia with a range of −112 to −109.5 mGal (Pick, 1987) equivalent to an artificial southwestward gradient of about 7.6 μGal/km. Sprenke (1989) devised a method for optimising the distance to which terrain corrections are made, based on a geostatistical analysis of the topography around a given area. This method cuts out unnecessary terrain corrections beyond a calculated limit based on the accuracy needed. Danes (1982) developed a method to estimate the terrain correction of an outer zone extending to infinity. These methods are based on the elevation of valley floors, summits and the station, and do not take into account the average elevation differences above and below the station. For example, a plain dissected by deep narrow gorges would give odd results, as the elevation of the terrain is not evenly distributed, with the average elevation at a level just below that of the plain and not half way down to the floor of the narrow gorges as these methods would imply. More importantly, Danes (1982) and Sprenke (1989) take no account of the effect of increasing station height, as terrain corrections at large distances are height dependent (Hallinan, 1991) and significant for large height differences within a survey. This was illustrated by LaFehr (1998) for a range of station elevations and topographic relief. In mountainous areas, gravity investigations must not be confined to valleys, as errors of up to 10 mGal can be caused due to fault structures or low density sediments filling the floor of valleys (Steinhauser et al., 1990).

It should be remembered that the complete free-air and Bouguer correction assumes that the data are collected at a common datum. As the data are collected at the level of each observation, the formulae (not the data) are corrected for this incorrect assumption (Chapin, 1996), so that the effects above (or in special cases below) the datum are reduced in a uniform manner. The treatment of variable density should be left for the interpretation and not included as part of the standard data reduction (LaFehr, 1991a). However, bodies of anomalous density outside the limits of the model will be unaccounted for, but these bodies can be estimated in the modelling process and if necessary their effects removed from the data. Thus the resulting anomalies can be modelled using any competent program such as GRAVMAG (Pedley, 1991).

6. The effects of height

For a given height above or below a station, the terrain correction in a Hammer compartment will be less for zones further away from the station (Fig. 6). For a given Hammer zone the increase in terrain correction with height is not linear, and so errors in finding the average height of a compartment will become more significant with an increased height difference between the height of the compartment and the station (Fig. 6). At heights larger than the dimension of the outer radius, the rate of change decreases for a given height difference with greater height, and as the height tends to infinity the correction reaches a limiting value as the zone forms an ever smaller part of the overall Bouguer slab.

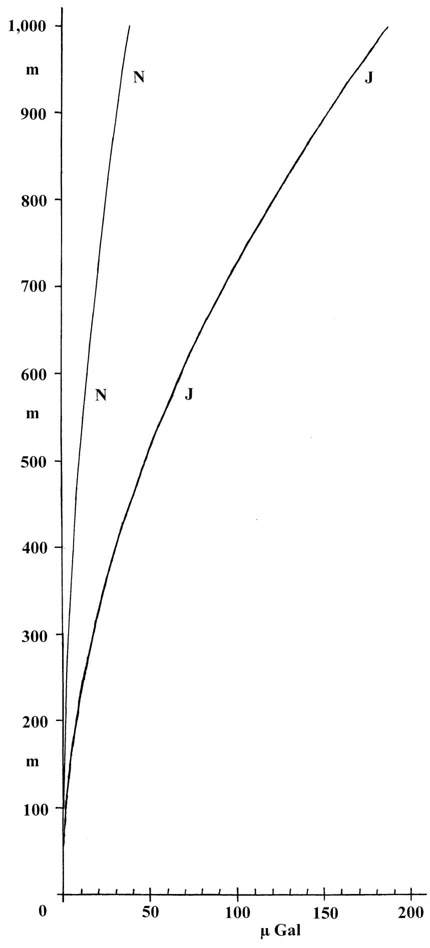

Fig. 6. Increase in terrain correction in μGal with height in metres for a Hammer zone compartment (16th of a zone) in zone J (4469 m to 6653 m) and zone N (21,944 m to 32,667 m), density 2000 kg/m^3.

At distances greater than about 22 km, the amount of terrain correction is height dependent (Hallinan, 1991). For an isolated mountain 2000 m high the Bouguer correction due to the Bouguer slab at a distance greater than 22 km is 10.2 mGal and even at 50 km is 4.5 mGal (Fig. 7). This must be removed by adding terrain corrections out to a large distance of up to 200 km as Hallinan (1991) did.

Extending terrain corrections out to the limit of Bullard B correction for the curvature of the Earth at 166.735 km means that at this point errors from nearby terrain are far more significant than errors resulting from distant topography (LaFehr, 1991a) and have a very long wavelength compared to the high-frequency components of nearby terrain. However, LaFehr (1991a, Fig. 2) concedes that topographic effects beyond 167 km may occasionally be worth taking into account. As this is beyond the Bullard B Bouguer cap, the topography between sea level and the level of the horizontal tangent from the station will result in an additional negative Bouguer correction, while topography above the tangent will result in a normal positive terrain correction, as will bodies of water (corrected for the difference with the Bouguer density) and valleys and voids below sea level. The difference between readings at sea level and a height of 4 km on an isolated island, due to the effect of a 4 km deep ocean beyond the Bullard B limit, is about 6.6 mGal given a density contrast of 1640 kg/m^3. The selection of 166.735 km as the outer limit of the Bullard B correction was based on minimising the difference between the effect of the cap and that of

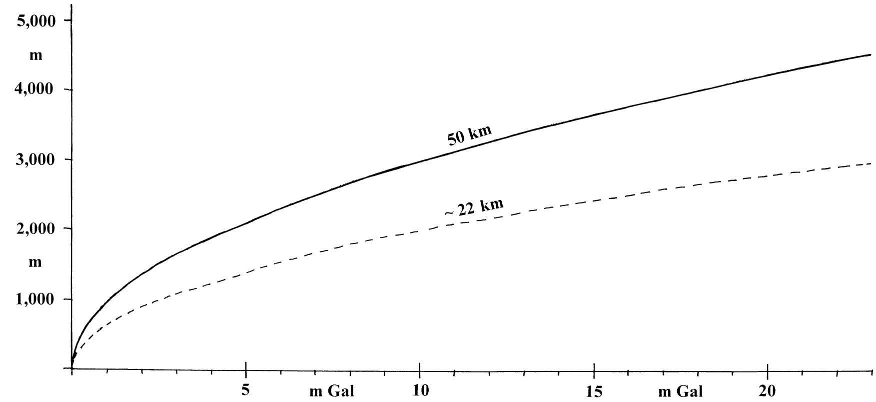

Fig. 7. Difference between Bouguer slab correction and gravitational attraction for a disk out to 21.944 km dashed line and 50 km solid line in mGal with increasing station height, density 2670 kg/m^3.

an infinite horizontal slab for a significant range of elevations (LaFehr, 1991b) (Fig. 8).

7. The effect of the Earth's curvature

A problem with distant terrain corrections for the effects of height would appear to be the curvature of the Earth, which is dealt with in the Hayford–Bowie system. At a distance of 22 km the surface of the Earth drops 38 m below the horizontal tangent of a station and can be approximated to:

$$b = r - \cos\theta r \quad (10)$$

where r is the radius of the Earth (taken as an average 6371 km) and θ the angle at the centre of the Earth due to the distance on the surface (Fig. 9):

$$\theta = 180S/\pi r. \quad (11)$$

This drop (b) can also be approximated more simply by:

$$b = S^2/2r \quad (12)$$

(Parker, 1995). The exact drop at any given point would be difficult to calculate as the radius of the Earth varies with latitude and with the direction away from the station.

If, as the BGS does, a curved Bouguer cap is used for the Bouguer correction (pers. comm. K.E. Rollin), then the curvature of the Earth is taken into account by the BGS. A small positive elevation difference above the station at distances greater than about 20 km can produce a negative terrain correction (Sazhina and Grushinsky, 1971; Rollin, 1990), as can happen with the system devised by Hayford and Bowie (1912). This results from the mass of a hill, below the tangential plane of the station, pulling downwards and increasing the observed value

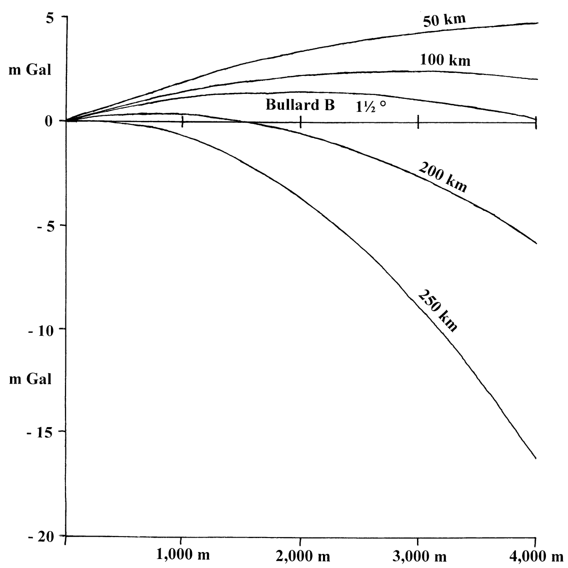

Fig. 8. The varying corrections with height for reducing the Bouguer slab to spherical caps of differing radii, from LaFehr (1991b).

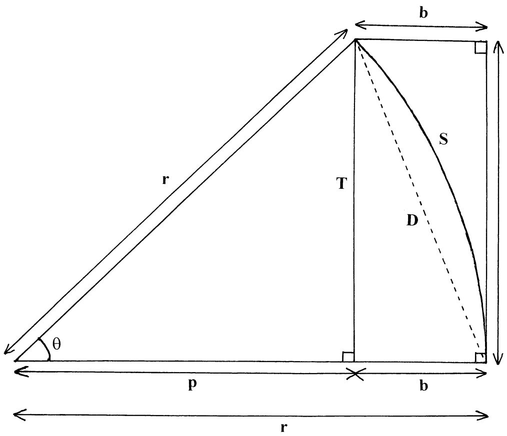

Fig. 9. The geometrical framework for calculating the effects of curvature on the surface of a sphere, where: r = the radius of the Earth; b = the drop below the tangent from the station to a point on the Earth's surface; S = the distance along the surface of the Earth between the station and a given point; D = the direct distance, through the Earth, between the station and a given point; θ = the angle at the centre of a sphere formed by a straight line distance on the surface; T = the distance along the tangent between a given point and the station; p = the difference between the drop b and the radius of the Earth r.

of gravity, so this effect has to be subtracted from the readings, as it was part of the Bouguer slab removed in the Bullard B correction, while the portion of larger hills above the plane of the station will result in a normal positive terrain correction. The combined effects of these two corrections for terrain above the Bouguer cap are set out in tables by Sazhina and Grushinsky (1971). Topography below the height of the station is corrected for in the normal way, after adjusting it for the drop due to the Earth's curvature, as valleys below the height of the station start at the top of the Bouguer cap after the Bullard B correction has been made.

As the drop below the horizontal plane of the station will be different for each of the two radii of a Hammer zone, the correction for the drop (b) would result in a sloping compartment at the correct level relative to the station, which is far more complex to calculate. To get round this the average drop for the compartment can be calculated at the radius of equal weighting (r_e) (Table 1). The curvature adjusted terrain correction for a Hammer compartment can be approximated by calculating the difference between the combined thickness of the height plus drop less the effect of a compartment equal to the drop only. Another effect of increased curvature is that the local vertical of a point at a distance away from the station will be out by θ from that of the vertical at the station, so that the top of a column or radius above this point will be further away and one below nearer to the station. Also, the direct distance (D) between the station and the point will be less than that on the surface of the sphere (Fig. 9) and can be calculated with:

$$D = \sqrt{\left\{(r - \cos\theta r)^2 + (\sin\theta r)^2\right\}}\,. \tag{13}$$

However, apart from the drop (b) below the horizontal tangent from the station, these effects are small, because topographic variations out to distances that influence corrections are small compared with the radius and circumference of the Earth. Even at 10° (1111.95 km), the difference between the surface and direct distances is only 1.41 km or 0.127% and the top of a 5000 m high column is only 868 m further away: this is less than a thousandth of the total distance. With radial line elements (Talwani, 1973), the effect of distant terrain on a spherical Earth can be directly evaluated using a computer program.

8. Terrain corrections to the sea bed

As Bullard (1936) recognized, it is important that terrain corrections are taken down to the sea bed or lake bottom, as has been done on 1:250,000 Bouguer anomaly maps of New Zealand (Reilly, 1972), while Steinhauser et al. (1990) realized that not only lakes had to be taken into account but glaciers with a density of between 790 kg/m^3 and 880 kg/m^3 for the ice. The effect of not including the void below sea level is even more significant than just the difference between the density of the sea water (1030 kg/m^3) and that used for the terrain

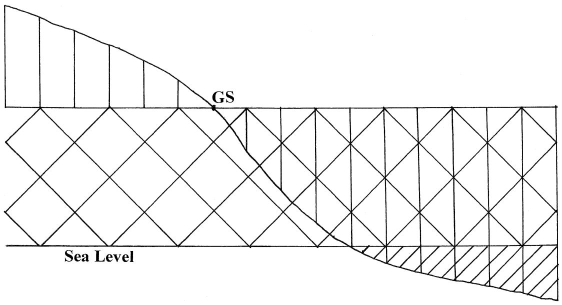

Fig. 10. The corrections for a gravity station (GS) above sea level on land, down to the sea bed, see Fig. 1b for key.

correction (Fig. 10). This arises from the increasing curvature of the Hammer compartment correction graph (Fig. 6) with height, so that the increase in thickness of the compartment will produce a larger correction than that for the layer of water alone, even if it had the same density as that of the terrain correction. This could be overcome by taking the difference in terrain correction in a compartment between the thickness down to sea level and the sea bed and dividing by the ratio of densities to get the correction for the water. But this ignores the fact that a given depth of water does not have that effect in a Hammer compartment. By calculating the effect of the water separately to that of the terrain the terrain correction density can be easily changed as the density of the body of water is fixed.

To show that taking terrain corrections down to the sea bed has a significant effect, gravity data from a survey of a coastal part of eastern Iceland (Nowell, 1994) were examined and set out in Table 2. For a station on the coast, the depth of the sea off Iceland was up to 100 m or more deeper in Zone M. The effect of height can be illustrated by a low level station at the head of a fjord (LJ 20) where the terrain correction down to the sea bed had little effect (4 μGal) while a station further inland but at a height of over 600 m (LJ 11) has a net effect of 25 μGal. Coastal stations near sea level (SW6, SW8, SW10) have small effects of up to 45 μGal, while a fictitious station (M1) at 700 m on the hill just inland above them has a large overall net effect of 309 μGal with the water only accounting for 6 μGal. The importance of taking the terrain corrections down to the sea bed will be much greater for areas where the depth of water increases much faster than off Iceland, such as small mid-ocean islands.

9. Stations on the sea surface

Normally, free air gravity surveys are acquired at sea, but a Bouguer correction can be made for gravity readings taken on the sea

Table 2
Contribution to the total terrain correction from taking the terrain correction down to the sea bed, accounting for the effect of water. Taken from a gravity survey of Eastern Iceland (Nowell, 1994)

Location	Grid Ref	*H*	TC	SB	*W*	NE	PD
SW6	393 754	+15.0	+6195	+36	−6	+30	0.48
SW8	461 778	+16.3	+2422	+57	−12	+45	1.86
SW10	444 825	+15.4	+3047	+8	0	+8	0.26
M1	439 799	+700.0	+11,108	+315	−6	+309	2.78
LJ11	239 883	+615.2	+9058	+26	−1	+25	0.28
LJ	244 860	+33.1	+6162	+5	−1	+4	0.06

Grid Ref = grid reference on 1:50,000 U.S. Army Map of Iceland.
H = height of station in metres above sea level.
TC = terrain correction down to sea bed in μGal given a density of 2600 kg/m^3.
SB = effect due to going down to the sea bed instead of sea level in μGal.
W = effect of water above sea bed given a density of 1030 kg/m^3 in μGal.
NE = net effect of taking the terrain correction down to sea bed in μGal.
PD = percentage of terrain correction due to going down to the sea bed instead of sea level.

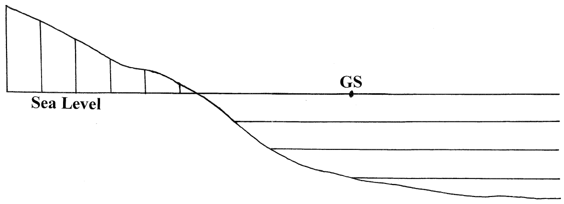

Fig. 11. The corrections for a gravity station (GS) on the sea surface, see Fig. 1b for key.

surface to account for the missing rock between the sea bed and sea level (Fig. 11). This was done by Lefort et al. (1999) to contour the local variations in Moho depth over the continental margin between France and Ireland by processing and filtering the resulting marine Bouguer anomaly map. However, they gave no details of exactly how these Bouguer corrections were made. If marine and land based Bouguer corrected gravity data are to be combined, this marine Bouguer correction must be made in order to avoid errors in the maritime part of any model and especially at the transition between the data sets along the coastline. The lack of mass below the station results in a decrease in the observed value of gravity, so the marine Bouguer correction has to be added to readings. The effects of the terrain of the sea bed and any nearby land also have to be taken into account. The surface of any nearby land is above the station and will result in an upward attraction, which decreases the observed value of gravity, so this effect also has to be added to readings. As it would be complex to add a Bouguer slab and then remove the effects due to sea bed terrain, a combined marine Bouguer correction can be added. The marine Bouguer correction consists of a correction for the disk of water directly below the station and Hammer zones down to the sea bed away from the station. The density used is the difference between the density of the Bouguer slab and the water. The formula for the gravitational attraction (g) of the cylinder around the station on the sea surface down to the sea bed is

$$g = 2\pi G\rho\left[R - \sqrt{(R^2 + h^2)} + h\right] \qquad (14)$$

where h is the depth of water and R the radius of the cylinder. The Hammer zones down to the sea bed are calculated in the normal way with h as the depth of water in each compartment. The effects of the disk and Hammer compartments are added together to get the marine correction. The effect of any nearby land can be accounted for by adding the normal terrain correction with heights relative to sea level and with the density of the Bouguer slab. A rapid and efficient Fourier technique for these corrections for marine gravity surveys in shallow water has been developed by Parker (1995). The depth of water and the height of any land will affect how far out the marine and terrain corrections have to be taken: the deeper the water or the higher the land, the further out corrections will have to go.

10. Stations on the sea bed

The corrections for gravity readings on the sea bed are more complex than those on the sea surface (Fig. 12). As the value of gravity increases towards the centre of the Earth within the crust, the FAC of normally 0.3086 mGal/m below sea level is subtracted from readings. The effect of the water above the station pulls upwards and decreases the observed value of gravity, so these effects have to be added to readings if you are normalising to sea level for a marine Bouguer correction. Sea bed below the level of the station will have a lack of mass due to the missing rock and result in a decrease in the observed value of gravity, and so this effect will

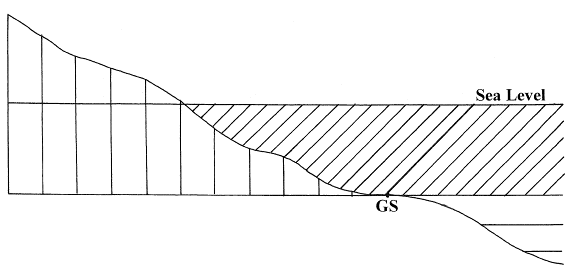

Fig. 12. The corrections for a gravity station (GS) on the sea bed, see Fig. 1b for key.

have to be added to readings. Sea bed and land above the station will also result in an upward attraction from the rock above the station and decrease the observed value of gravity, so these effects have to be added to readings as a terrain correction. As a result the sea bed correction for stations on the sea bed has to be divided into three. First, the effect of the water above the station is taken into account by correcting for a cylinder of water above the station and the water in the Hammer compartments either down to the sea bed, shallower than the station, or to the depth of the station in deeper water. Second, the sea bed below the station is corrected for by calculating the effect in Hammer compartments with a density equal to the difference between that of the Bouguer density and the water, where the thickness of the compartment is the depth between the level of the station and the sea bed below. Third, any sea bed and land above the level of the station are taken into account by finding the height difference between the topography and the station and calculating the effect for each Hammer compartment with the Bouguer density. The effects of all these Hammer compartments and the cylinder of water above the station are added together to give the sea bed correction, which is added to readings. To standardise methods the marine Bouguer correction should be carried out to the same distance away from the station as land surveys, so that they can be compared.

11. Stations above and below ground level

Corrections for gravity stations above ground level, such as up a tower or on an airborne survey, consist of two parts: a Bouguer correction up to the level of the station; and a terrain correction at the Bouguer density, above and below the level of the station (Fig. 13). The free-air correction is added up to the level of the station as normal. The terrain correction, at the Bouguer density and to the level of the station, consists of a correction for the void below the

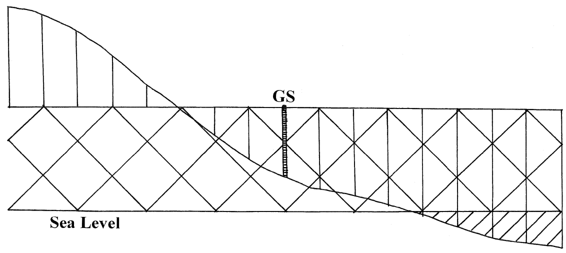

Fig. 13. The corrections for a gravity station (GS) above ground and sea level, see Fig. 1b for key.

station, which is approximated to a cylinder directly below the station, along with Hammer zones down to ground level, as well as Hammer zones for any topography above the level of the station. The terrain correction in effect cancels out that part of the Bouguer slab above the level of the ground, and where applicable it is taken down to the sea or lake bed with any water corrected for. Also, unless the flight line is below the highest topography, the terrain effect in airborne data will be less than those for conventional ground observations (Hammer, 1983) and falls with increasing altitude.

A correction for the tower on which a reading is taken can be based on the gravitational attraction of a right rectangular prism (Nagy, 1966) or cylinder of given length (Formula 14). The correction for material above a station is positive and below a station is negative. With increased height above sea level it may again be necessary to extend the terrain corrections outwards until there is little remaining effect. Kuo et al. (1969), as had Hammer (1938), made these corrections for readings in tall buildings and showed that these vary with height, but failed to extend them beyond Hammer zone M ($\sim$ 22 km) and did not account for the effects of surrounding buildings, even though the effect of the basement still needed a correction for readings on the highest floors.

Corrections for gravity stations below ground level are more complex, depending on whether the station is above or below sea level. The corrections for a gravity station below ground level but above sea level consist of adding a normal FAC and subtracting a Bouguer correction to the level of the station, before adding a terrain correction (Fig. 14). The terrain correction, at the Bouguer density, consists of a correction for the cylinder of rock directly above the station along with Hammer zones, away from the station, down to this level, or up to it in the case of topography below the station.

The correction for a gravity station below the ground and below sea level consists of subtracting a FAC to account for the increase in gravity towards the centre of the Earth, within the crust, before adding a terrain correction, at the Bouguer density, for all the rock above the station which is pulling upwards and decreasing gravity at that point. If the compartments at larger distances are still thick and giving significant corrections, the terrain correction will have to be extended outwards to a greater distance, possibly beyond the limit of the Bullard B correction. In the special case of water being above the plane of the station, an additional correction for the water pulling upwards and decreasing the observed gravity at that point is made using the terrain correction method with the density of the water.

Any voids below ground can be corrected for by calculating the effect of rectangular prisms (Nagy, 1966) with the density of the rock (normally the Bouguer slab density) relative to the station. This can also be done with a vertical line element (Talwani, 1973) of cross sectional area (s) at horizontal distance (r) from the station between two levels (Z_1) near and (Z_2) further above or below the plain of the station:

$$g = G\rho s\left\{1/\sqrt{\left(r^2+Z_1^2\right)} - 1/\sqrt{\left(r^2+Z_2^2\right)}\right\}. \quad (15)$$

The correction for a void above a station is negative and for a void below a station is positive. In a gravity survey of two mines, these corrections for the shaft tended to cancel out (Gibb and Thomas, 1980) but were greatest at the top and bottom of the shafts — up to +70 μGal and −50 μGal, respectively — with a maximum total void correction of −430 μGal midway down a shaft due to the effects of galleries at that level which did not cancel. Gibb and Thomas (1980) found that corrections for the galleries in which the readings were made decayed rapidly with distance from the station: about 85% of the correction occurred within 10 m of a station.

As corrections both above and below ground level vary with height, they should have been made in tower and mine experiments searching for deviations from Newtonian gravity (Fishbach and Talmadge, 1992). Since this seems not to have been done, these results will be erroneous until they are recalculated to take the full terrain correction into account. Even with submarine measurements of gravity in vertical profiles to depths of 5000 m to calculate the gravitational constant, terrain corrections were only made in a 60 km square region (Zumberge et al., 1991).

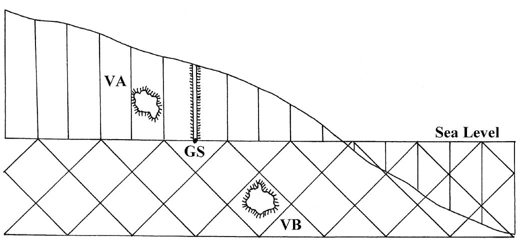

Fig. 14. The corrections for a gravity station (GS) below ground level but above sea level, see Fig. 1b for key. The correction for a void above (VA) a station is negative and for a void below (VB) a station is positive.

12. Summary and conclusions

The effect of topography at a gravity station is removed with a Bouguer correction (Bullard A), a correction for the curvature of the Earth (Bullard B) and a terrain correction (Bullard C). With modern computers, full corrections out to a distance of 166.735 km, the Bullard B limit of $1\frac{1}{2}°$, should become standard. This arbitrary distance can then be used to judge uniformly whether it is worth taking into account the effects of terrain beyond this cutoff. Also, this would eliminate one possible source of mismatch between surveys and reduce the introduction of processing error in the final Bouguer anomalies for little additional cost (LaFehr, 1998). This will be especially true if future gravity instruments (Chapin, 1998) improve and have higher accuracies down to the μGal level or better. In addition, this higher resolution combined with gravity gradiometry (Bell, 1997, 1998) will greatly enhance geological interpretations made with gravity data (Pawlowski, 1998).

In order to get a precise correction, many factors have to be taken into account: otherwise, errors of many mGal or only a few μGal will result, depending on which factors are ignored. The most significant effect is that of increasing station height. This must be accounted for by applying the Bullard B correction for the curvature of the Earth and extending terrain corrections adjusted for Earth curvature outwards, until they have no discernible effect. If this is not done, high stations may be out by many mGal relative to lower stations in an area. The elevations for distant terrain correction can be clustered together being calculated for each station (Hallinan, 1991). Special attention must be given to the ground nearest the station as this can have a significant effect, and a simple correction using the formula

$$g = 1/2\pi G\rho R(1 - \cos\theta) \tag{16}$$

can be applied for the sloping quarter segments around the station. A building near a station can result in effects of a few μGal which can be calculated and removed, assuming that the building has a uniform average density. Also terrain corrections must be applied down to the sea or lake bed, otherwise errors of over 1 mGal could result with stations near deeper water than about 100 m for Hammer zone M off Iceland. Stations in special locations such as the sea surface or bed and above and below ground level can have gravity corrections calculated if all the effects on a station are considered.

It should be hoped that in the future, published Bouguer gravity maps and calculated models will state in much more detail how the corrections to the data (readings) were made so that the final result does not appear to be a geophysical conjuring trick.

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