Conducting a Crucial Experiment of the Constancy of the Speed of Light Using GPS

- Comments on Ashby's "Relativity and the Global Positioning System"

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BIOGRAPHIES

Ruyong Wang graduated from Beijing University of Aeronautics and Aerospace. He worked in the Institute of Mechanics, Chinese Academy of Sciences. Currently he works in St. Cloud State University. He has been interested in Special Relativity, especially its crucial experiments for long time and published several papers on this topic.

Ronald R. Hatch received his Bachelor of Science degree in physics and math in 1962 from Seattle Pacific University. He worked at Johns Hopkins Applied Physics Lab, Boeing and Magnavox before working for a time as a GPS consultant. In 1994 he joined Jim Litton, K.T. Woo, and Jalal Alisobhani in starting what is now NavCom Technology, Inc. He has served a number of roles within the Institute of Navigation (ION), including Chair of the Satellite Division. He is currently finishing a one-year term as President of the ION. Ron has received the Johannes Kepler Award from the Satellite Division and the Colonel Thomas Thurlow Award from the ION. He is also a Fellow of the ION. He has been awarded twelve patents either as inventor or co-inventor. Most of the patents relate to GPS.

ABSTRACT

Contrary to the assertion of Special Relativity, the speed of light is not always constant relative to a moving observer. The Global Positioning System (GPS) shows that the speed of light in the Earth Centered Inertial (ECI) non-rotating frame remains at c relative to the frame—but not relative to an observer or receiver moving in that frame. When a GPS receiver changes its translation speed relative to the ECI frame, the speed of light measured relative to the receiver changes. A crucial experiment of the constancy of the speed of light relative to a moving receiver could be conducted in the following way: Let two GPS satellites and two airplanes be positioned in a straight line. Let the two airplanes travel at the same speed directly toward one of the two satellites and directly away from the other satellite. The travel time differences of GPS signals arriving at the two airplanes is measured and recorded with the airplanes flying first toward one of the satellites and then flying the opposite direction toward the other satellite. The travel time differences obtained as the airplanes fly in opposite directions are compared. If the travel time difference is the same when the velocity of the airplanes is changed, then the speed of light is indeed constant relative to the moving airplanes, otherwise it is not. The calculation using the GPS range equation and the results of a Real-Time Kinematic (RTK) differential GPS test have shown that the constancy of the speed of light relative to moving airplanes is not correct. The change of the time difference could reach about 10 ns for subsonic airplanes and 30 ns for supersonic airplanes. The result of this crucial experiment is not only important scientifically, but also indicates the possibility of a new way to directly measure vehicle speed relative to the ECI frame.

INTRODUCTION

The principle of the constancy of the speed of light asserts that in vacuum light always has a definite speed of propagation that is independent of the motion of the observer [1]. That is, no matter whether the observer is moving or at rest, and no matter how fast the observer is moving, $0.000001c$ or $0.999999c$, the speed of light is always c. This assertion in fact is the most controversial part of Special Relativity. Relativistic physicists claim that people who refuse to accept the constancy of the speed of light simply cannot give up their common sense acquired through slow speed experiences. However, this is not true. Human beings are intelligent and they are flexible as well. Once people have been exposed to solid experimental facts, they are willing to adopt new ideas. The common sense that a falling body descends at a rate that is proportional to its weight has changed to the common sense that, if there is no air resistance, all bodies fall at the same rate. This change in perception is a direct result of the experimental data. The reason some do not believe in the constancy of the speed of light relative to a moving observer is in fact that there are experimental facts which indicate otherwise.

One of the few experiments which directly supports the constancy of the speed of light relative to a moving observer is the Michelson-Morley type of experiment [2]. However, there are some unique features of the Michelson-Morley experiment which do not apply to GPS type experiments. First, the Michelson-Morley experiment is a function of the round-trip (two-way) speed of light. This means that it is a second-order experiment, *i.e.*, the possible time difference in the two light paths is proportional to $(v/c)^2$. Second, the distance the light path travels is determined by a physical structure which is also moving at the same velocity. Clearly, if movement causes a length contraction in the direction of motion as FitzGerald and Lorentz postulated, then the Michelson-Morley experiment will yield null results even when the speed of light is not affected by motion.

Moreover, there are some experiments, such as the Sagnac experiment [3], which yield different results. In the GPS system, when the observer is moving relative to the center of the earth, the speed of light relative to that observer is not equal to *c*.

The truth can only be determined by experiment. Today, the global positioning system provides us with a very big laboratory, a global laboratory, for experiments regarding the speed of light. We do not need the imaginary long Einstein train any more. In this paper, we propose a crucial experiment to examine the constancy of the speed of light relative to a moving observer. However, we wish first to comment on some claims recently presented in a paper by Ashby [4]. According to that paper, the only reason that GPS does not confirm the constancy of the speed of light relative to a moving receiver is that the receiver is moving in a circular path—or that the computation is done in a rotating coordinate frame. We disagree with his claims.

GPS AND THE CONSTANCY OF THE SPEED OF LIGHT

The operations of GPS navigation are based on the propagation delay equation in an earth centered inertial (ECI) non-rotating frame:

$$|r_r(t_r) - r_s(t_s)| = c(t_r - t_s).$$

Here t_s is the instant of transmission of the signal from the source, and t_r is the instant of reception at the receiver; $r_s(t_s)$ is the position of the source at the transmission time, and $r_r(t_r)$ is the position of the receiver at the reception time. Ashby [4] said that the propagation delay equation is a simple application of the principle of the constancy of the speed of light. Wolf and Petit [5] concluded that if the equation is correct, Special Relativity is correct. Are these true?

Let us express the propagation delay equation completely:

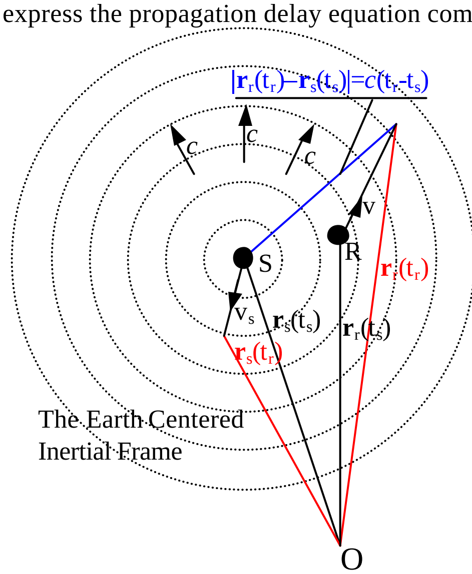

Fig. 1 The propagation of light in the earth centered inertial frame

measured in the ECI frame, the speed of light is isotropic and is equal to *c* (light wave is a spherical wave) no matter whether the source or the receiver is moving relative to the ECI frame or not (fig. 1). On the surface, this looks like the constancy of the speed of light, because only *c* appears in the equation. However, the assertions that the speed of light is constant in just one inertial frame, the ECI frame, really is not what the constancy of the speed of light means.

Let us first consider the propagation of sound in the air. When the disturbance caused by the motion of the source and the receiver can be neglected, measured in a frame stationary in the air, the propagation of sound wave is isotropic with a speed of sound *a* (a spherical wave). We have the propagation delay equation for the sound in the frame of the air (fig. 2):

$$|\mathbf{r}_r(t_r) - \mathbf{r}_s(t_s)| = a(t_r - t_s).$$

Obviously, the equation is the same as the GPS propagation delay equation except there is a constant speed of sound *a* instead of the constant speed of light *c*.

Do we have a principle of the constancy of the speed of sound because of the constant speed of sound *a* appearing in the propagation delay equation? No, we do not. Contrarily, we say that the speed of sound is not independent of the motion of the receiver (observer). Why? In fact, when we judge whether the speed of sound *is* or *is not* independent of the motion of the observer, the speed of sound is measured in the frame of the observer, not the frame of the air. From the physics textbook [6], we can find these:

"The fact that the speed of light is independent of the motion of the source is not at all troublesome, but is it also

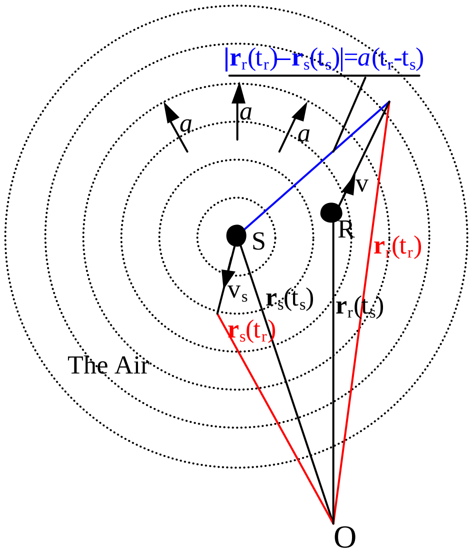

Fig. 2 The propagation of sound in the air

independent of the motion of the detector (i.e., the observer)? Certainly, the speed of sound is not; if the detector rushes toward the source, moving with respect to the air, the measured speed of sound increases. Just imagine two identical ships headed toward a motionless sound-emitting buoy, one steaming along at full speed and the other dead in the water. On both ships, the time it takes a blast of sound to sweep from bow to stern is measured, and the speed of the wave is computed in that inertial system. Clearly, for the ship moving toward the buoy, that time will be shorter (during the interval it takes the sound to traverse the ship, the stern will advance somewhat toward the pulse, shortening the effective length) and the wave speed will be determined to be faster." (fig. 3)

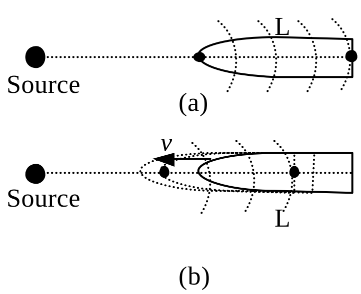

Fig. 3 The propagation of sound

Clearly, measured in the frame of the air, the sound wave propagates with a constant speed of a for both ships, no matter whether the ship is moving or at rest. However, measured in the frame of the observer in the moving ship, the traveling distance of sound wave is still the distance between bow and stern, but the traveling time is shorter. Therefore, measured by this observer, the speed of sound is faster. Hence, we never have a principle of the constancy of the speed of sound. Contrarily, we say that the speed of sound is not independent of the motion of the observer.

When the principle of the constancy of the speed of light asserts that in vacuum, light always has a definite speed of propagation that is independent of the motion of the observer, it means the same as above. That is, not only the speed of light is c in the ECI frame, but also the speed of light is c for any observer, no matter whether the observer is moving or at rest relative to the ECI frame, and no matter how fast the observer is moving relative to the ECI frame. Now let us examine whether these are true, i.e. whether or not they are consistent with GPS observations.

We can still use the similar example, the ship and the wave-emitting buoy, and just modify it in the following way: mount a GPS receiver on each of the two ends of the ship and the two receivers receive the GPS signals from a differential GPS station. To avoid the effect from the rotation of the earth, the DGPS station and the ship are on the same meridian. We compare two cases. First, the ship is dead in water and the speed of the signal is measured by the observer on the ship. Then, the ship steams at full speed and the speed of the signal is measured again. Anyone familiar with GPS can quickly indicate that in the first case, the traveling time of the GPS signal from bow to stern is L/c, where L is the distance from bow to stern. Then in the second case, during the interval it takes the signal to traverse the ship, the stern will advance somewhat, although very short, toward the signal, shortening the effective length. Therefore, the propagation time of GPS signal from bow to stern is shorter than L/c. We can calculate these as follows. Suppose GPS signals are emitted toward a ship located at a distance from the DGPS station. When will the two

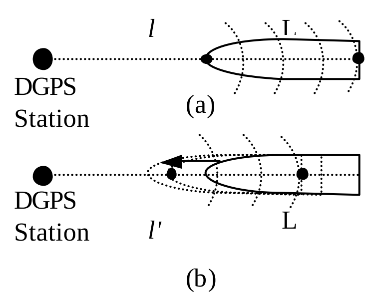

Fig. 4 The propagation of GPS signals

receivers on the ship receive the signal emitted at t_0 from the source in the first case according to the propagation delay equation (fig. 4a)?

For receiver 1:

$$\begin{cases} |x_1 - 0| = c(t_1 - t_0) \\ x_1 = l \end{cases}$$

Hence, $t_1 = t_0 + l/c$.

For receiver 2:

$$\begin{cases} |x_2 - 0| = c(t_2 - t_0) \\ x_2 = l + L \end{cases}$$

Hence, $t_2 = t_0 + (l + L)/c$ and the propagation time from bow to stern is $t_2 - t_1 = L/c$.

When will the two receivers on the ship receive the signal emitted at t'_0 from the source in the second case according to the propagation delay equation (fig. 4b)?

For receiver 1:

$$\begin{cases} |x'_1 - 0| = c(t'_1 - t'_0) \\ x'_1 = l' - v(t'_1 - t'_0) \end{cases}$$

Hence, $t'_1 = t'_0 + l'/(c + v)$.

For receiver 2:

$$\begin{cases} |x'_2 - 0| = c(t'_2 - t'_0) \\ x'_2 = l' + L - v(t'_2 - t'_0) \end{cases}$$

Hence, $t'_2 = t'_0 + (l' + L)/(c + v)$ and the propagation time from bow to stern is $t'_2 - t'_1 = L/(c + v)$.

Therefore, according to the propagation delay equation, the propagation time of GPS signal from bow to stem in case 2 is shorter than that in case 1 and the time difference between two cases, $Dt = (t_2 - t_1) - (t'_2 - t'_1) = L/c - L/(c + v) = vL/c^2$, neglecting the quantities of the second and higher order of v/c. However, for an observer within the system, i.e. the ship, the distance between bow and stern is a constant L. Thus, measured by this observer, the speed of light is not constant. The speed of light is not independent of the motion of the observer relative to the ECI frame. This is what GPS tells us.

The people who are familiar with Special Relativity would argue that the relativity of the simultaneity, an important aspect of Special Relativity, is not considered in the previous analysis. Now, let us consider the relativity of the simultaneity. In fact, the synchronization of the clocks is not needed here. Suppose the GPS receiver on bow has a clock bias of dt_1 and the GPS receiver on stern has a clock bias of dt_2 (neither of the two clocks uses GPS time, nor are they synchronized with each other). Clearly, with these clock biases, the measurement in case 1 will not be $t_2 - t_1 = L/c$, but $t_2 - t_1 = dt_2 - dt_1 + L/c$; the measurement in case 2 will not be $t'_2 - t'_1 = L/(c + v)$, but $t'_2 - t'_1 = dt_2 - dt_1 + L/(c + v)$. Then the time difference with these two clock biases will be $Dt = (t_2 - t_1) - (t'_2 - t'_1) = [dt_2 - dt_1 + L/c] - [dt_2 - dt_1 + L/(c + v)] = vL/c^2$, the same as before. As for the time dilation, the rate change of the clocks caused by the motion, the two clocks have the same rate change, so there will not be a net effect. Thus, the relativity of the simultaneity will not change the conclusion.

Another argument would be the Lorentz contraction related to the motion of the system. However, Lorentz contraction is a second-order effect: $DL = (L/2)(v/c)^2$. The time difference measured here is a first-order effect, $Dt \propto (v/c)^1$. Obviously, the second-order Lorentz contraction could not change the conclusion either.

People who are familiar with the history of Special Relativity know that the constancy of the speed of light has its experimental foundation, as mentioned before, in the null result of the Michelson-Morley experiment. Therefore, they would question why GPS experiences
lol contradict the Michelson-Morley experiment. In fact, the GPS experiences do not contradict the Michelson-Morley experiment. First, the Michelson-Morley experiment has never been conducted in a lab moving relative to the earth. Second, as argued above, the length contraction of the physical structure is all that is needed for the Michelson-Morley experiment to give a null result.

The correct conclusion from the GPS propagation delay equation is that the ECI frame is the preferred frame near the earth. The speed of light measured in the ECI frame is always c whether the receiver is moving relative to the ECI frame or not. When a GPS receiver changes its translational speed relative to the ECI frame, light does not change its speed relative to the ECI frame. Therefore, the speed of light relative to the receiver moving with respect to the ECI frame changes.

THE SAGNAC EFFECT

Background

Georges Sagnac [3] in 1913 published a paper in which he showed that the speed of light relative to a detector on the edge of a spinning disk was a function of whether the light traveled with or against the rotation of the disk. The phenomenon is the basis of all modern laser and fiber optic gyrocompasses. The simplest interpretation of the result is that the speed of light remains constant relative to the center of rotation and, thus, not of constant speed relative to the rotating detector.

Special Relativity (SRT) claims the Sagnac effect is due to the rotation. Since rotation is not relative, the Sagnac effect can be due to non-isotropic light speed and still be consistent with Special Relativity. The effect of the movement of the receiver during the transit time of a GPS :)
signal is referred to in the GPS system as the one-way Sagnac effect.

However, it is not at all evident that the Sagnac effect is due to rotation. Ives [7] claimed in 1938 that the effect is not due to rotation and proposed an experiment in which the light followed a hexagonal path. Ives claimed that the detector could be moved linearly along one side of the hexagonal path and the effect would still be present. Recently, Wang [8, 9] has proposed some first-order interferometric experiments and their results will show that the Sagnac effect exists not only in circular motion, but also in translational motion. :)

GPS, Sagnac Effect, and Ashby Claims

At this point we want to consider a number of claims that Ashby [4] has recently made in regard to GPS and the Sagnac effect. We address three specific comments which Ashby made and respond to each below.

(1) Ashby claimed, "The fundamental principle on which GPS navigation works is an apparently simple application of the second postulate of special relativity—namely, the constancy of *c*, the speed of light." This claim has already been addressed above. Clearly, the GPS range equation does not depend on the constancy of the speed of light *relative to the receiver*, which is the SRT claim. Yes, the GPS equation depends on the constancy of the speed of light relative to the earth-centered inertial (ECI) non-rotating frame—but that is contrary to SRT. A receiver moving in the ECI frame does not see an isotropic light speed of *c*.

(2) Ashby's second claim is: "Observers in the non-rotating ECI inertial frame would not see a Sagnac effect. Instead, they would see that receivers are moving while a signal is propagating." This claim is a bit humorous. It would have been nice if this were the last claim in contention—since Ashby in effect concedes the argument here. Receiver motion during the transit time *is the Sagnac effect*. The only way that Ashby can claim that the Sagnac effect is not seen by a receiver in the ECI frame while admitting that the receiver moves during the transit time is to define the effect of a moving receiver differently depending upon the description of the receivers position—a bit of a sophistry.

(3) The final claim by Ashby, which we contest, is: "Of course if one works entirely in the nonrotating (sic) ECI frame there is no Sagnac effect." The only way this claim can be true is if we adopt the definition sophistry of the prior claim. But we have even more convincing data that Ashby's claim is false. NavCom Technology, Inc. has licensed software developed by the Jet Propulsion Lab (JPL) which, because of historical reasons, does the entire computation in the ECI frame. Because of some discrepancies between our standard earth-centered earth-fixed solution results and the JPL results, we investigated the input parameters to the solution very carefully. The measured and theoretical ranges computed in the two different frames agreed precisely, indicating that the Sagnac correction had been applied in each frame.

The Fundamental Question

As the discussion of the Sagnac effect indicates the fundamental question regarding the speed of light is the following: Is the speed of light constant with respect to the observer (receiver) or is it constant with respect to the chosen inertial (isotropic light speed) ECI frame?

Clearly the GPS range equation indicates the speed of light is constant with respect to the chosen frame. The receiver position in the range equation is its position at the time the signal is received. This means that the pattern of motion of the receiver during the signal transit time is completely immaterial. The receiver could have moved in a huge series of loops during the transit time. It would not matter—it is the receivers position at the time of reception of the signal which matters.

The JPL equations [10], used to track signals from interplanetary space probes, verify that the speed of light is with respect to the chosen frame. In the JPL equations, the chosen frame is the solar system barycentric frame. The motion of receivers during the signal transit time from earth to probe and from probe to earth is taken into account. Even the motion of the earth around the moon/earth center of mass is taken into account. Clearly, the JPL equations treat the speed of light as constant with respect to the frame—not as constant with respect to the receivers. In the GPS nomenclature, the one-way Sagnac effect must be accounted for on all signal paths.

The other question one might ask is at what level curvature is important—if it is circular motion which causes the Sagnac effect as Ashby claims, how much does the path have to deviate from a straight line to cause the effect? At Los Angeles the earth rotates about 27 meters during the nominal 70 millisecond transit time of the signal from satellite to receiver. The deviation of the 27 meter movement from the straight line chord distance is only 35 microns at its largest point. It certainly seems incredible that a 35 micron deviation from a straight line could induce a 27 meter change in the measured range.

As a final proof that it is movement of the receiver which is significant—not whether that movement is in a curved or straight line path—a test was run using the highly precise differential carrier phase solution. The reference site was stationary on the earth and assumed to properly apply the Sagnac effect. However, at the remote site the antenna was moved up and down 32 centimeters (at Los Angeles) over an eight second interval. The result of the height movement was that the remote receiver followed a straight line path with respect to the center of the earth. The Sagnac effect was still applied at the remote receiver. The result was solved for position that simply moved up and down in height the 32 centimeters with rms residuals which were unchanged (i.e. a few millimeters). If a straight line path did not need the Sagnac adjustment to the ranges the rms residuals should have increased to multiple meters. This shows again that it is any motion—not just circular motion which causes the Sagnac effect.

CRUCIAL EXPERIMENT

Crucial experiment of the constancy of the speed of light

To examine whether or not the speed of light measured in the system changes when a system changes its translational speed relative to the ECI frame, a crucial experiment could be conducted. It is crucial especially because in this experiment, simultaneity, or the synchronization of the clocks, is not a concern. This is very important, because in any debate about the speed of light, the problem of simultaneity has always been a focus.

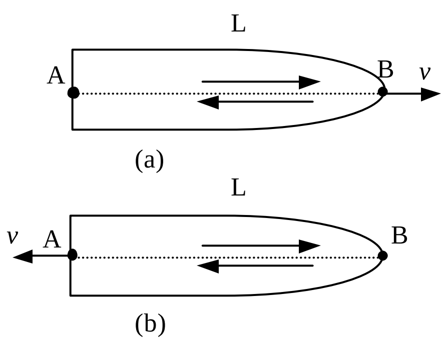

Fig. 5 The crucial experiment

Mount two atomic clocks with the same construction, signal transmitters, reflectors, and receivers on the two ends, points A and B, of a vehicle, *e.g.*, a helicopter (fig. 5). First, the vehicle moves due South (eliminating the effect of the rotation of the earth) with a speed of v. The two clocks are not synchronized with each other. A signal is transmitted from A at $t_1(A)$ (according to clock A) to B (arriving at $t_1(B)$ according to clock B) and reflected back to A (arriving at $t'_1(A)$ according to clock A). By the readings of clocks, we can calculate the difference of the nominal traveling times for two directions, $Dt_1 = [t'_1(A) - t_1(B)] - [t_1(B) - t_1(A)]$. (We say that the traveling times $t_1(B) - t_1(A)$ and $t'_1(A) - t_1(B)$ are nominal because the two clocks are not synchronized. For example, $t_1(B) - t_1(A)$ could be negative if clock B is too much behind clock A.) Then let the vehicle decelerate, stop, and accelerate in the other direction, finally moving due North with the speed v. We repeat the same measurement, and we will obtain $Dt_2 = [t'_2(A) - t_2(B)] - [t_2(B) - t_2(A)]$. If the readings of the clocks show that Δt_1 is different from Δt_2, we think everybody would agree that the experiment refutes the principle of the constancy of the speed of light, especially noting that the relativity of simultaneity is not a problem here, because the synchronization of clocks is not required. If Δt_1 is equal to Δt_2, then the experiment verifies the constancy of the speed of light.

This experiment is crucial also because if the experiment shows that the speed of light in a system moving relative to the ECI frame is different from that in another system, we can invent a new kind of speed detectors that can measure the translational speed of the system relative to the ECI frame directly [11].

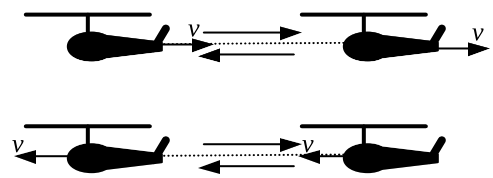

Fig. 6 Crucial experiment with two objects

It is suggested [12] that this experiment can be implemented by mounting the two clocks not in one moving object, but in two separate objects, e.g., two helicopters, that move in a straight line, one after another, with the same velocity (fig. 6). This way, L, the distance between the two clocks can be increased substantially, and hence, the possible time difference can reach several nanoseconds, a value that is relatively easy to detect with current technology. Also, the effect of moving clocks, including time dilation, and the effect resulting from the fact that L is not strictly constant are discussed there in detail, and it has been indicated that these effects will not prevail over the time difference we are trying to detect. In fact, Δt is the time difference of light propagation between two directions and between two cases. Therefore, even if the light paths of the two cases are slightly different, as long as the difference is the same for both directions, the time difference will still exist.

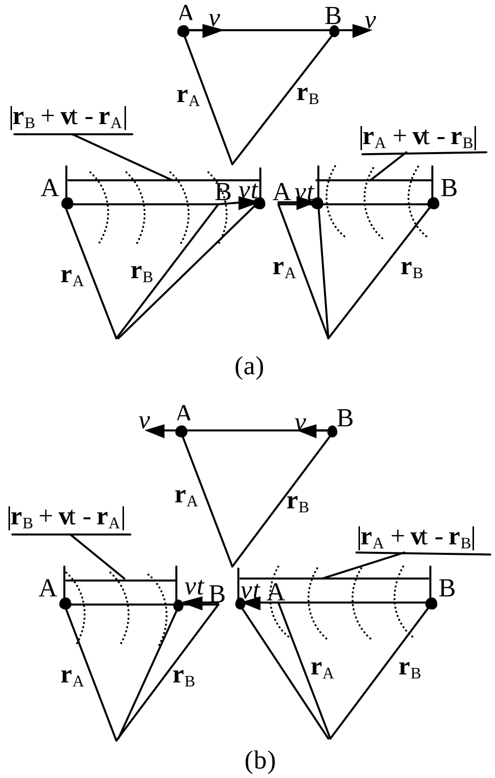

Fig. 7 The Sagnac corrections in the crucial experiment

The Sagnac effect arises because of the motion of the receiver during propagation of the signal from transmitter to receiver as mentioned above. If the Sagnac effect is positive when A is the transmitter and B is the receiver, then it is negative when B is the transmitter and A is the receiver (fig. 7a), and *vise versa* (fig. 7b). Therefore, the propagation times in two opposite directions are not the same, and the total time difference in the experiment is $4vL/c^2$. It is a first-order effect, and the Lorentz contraction, a second-order effect, is not a factor in the experiment.

Simplified crucial experiment using GPS

In the crucial experiment, first, A is a signal source and B is a signal receiver, then B becomes a source and A becomes a receiver. But it does not necessarily mean that we must mount transmitters to A and B. What we need are two signals, a signal that leaves A and arrives B and another signal that leaves B and arrives A. Obviously, powerful airborne transmitters are expensive and not easily available. However in GPS, there are already powerful transmitters in place, the transmitters in GPS satellites and on ground stations. We can utilize the existing powerful transmitters to do the same thing we need for the crucial experiment. As a matter of fact, if the two GPS receivers, A and B, and a GPS satellite, S1, are on the same straight line, the signal propagation time from A to B is the difference between the signal propagation time from S1 to B and the signal propagation time from S1 to A if A and B receive the same signal from the satellite. Therefore, for conducting the crucial experiment, we do not need powerful airborne transmitters on A and B; we just need two GPS satellites, S1 and S2 on the horizon and put two airborne GPS receivers, A and B, on line S1S2. Clearly, the crucial experiment will be greatly simplified by replacing airborne transmitters with airborne GPS receivers when it is conducted in the global lab provided by GPS.

We can conduct the crucial experiment with two airplanes mounted with GPS receivers and two GPS satellites (Fig. 8), all of them are in the same meridian to eliminate the effect of rotation of the earth (one of the satellites could be a geostationary satellite). At first, two airplanes fly due South with a distance of L. Each GPS receiver records the arrival times of signals from S1 and S2. Then, at certain time, two airplanes respectively do a climbing turn or a regular turn, and then both fly North and record the arrival times again. When two airplanes return to ground, we can post-process the recordings and find whether or not there is a time difference between the two moving states, moving North and moving South. We can find $[(t_{S2A2} - t_{S2B2}) - (t_{S1B2} - t_{S1A2})] - [(t_{S2A1} - t_{S2B1}) - (t_{S1B1} - t_{S1A1})]$, and since A and B are located on the straight line S_1S_2, it becomes $(t_{B2A2} - t_{A2B2}) - (t_{B1A1} - t_{A1B1}) \equiv Dt_2 - Dt_1$ in the crucial experiment.

The expected time difference, $4vL/c^2$, is proportional to the speed of airplanes, v, and the distance between them, L. L mainly is restricted by the height of the flight path due to the curvature of the earth. (In meters, $L = 3,572[(h_1)^{1/2} + (h_2)^{1/2}] = 7,144(h)^{1/2}$ if $h_1 = h_2 = h$.) The possible flight heights and the expected time differences are listed as follows:

Height	5 km	10 km	20 km
L	500 km	700 km	1,000 km
$4vL/c^2$ for v = 300 m/s	6.7 ns	9.3 ns	13.3 ns
$4vL/c^2$ for v = 700 m/s	15.6 ns	21.8 ns	31.1 ns

With the current GPS technologies, it is not difficult to decide whether these 10 ns or 30 ns time differences exist.

Calculation of the crucial experiment

We can calculate the expected time difference in the experiment using complete range equation with the biases considered in GPS (See Appendix).

Some practical aspects of the crucial experiment

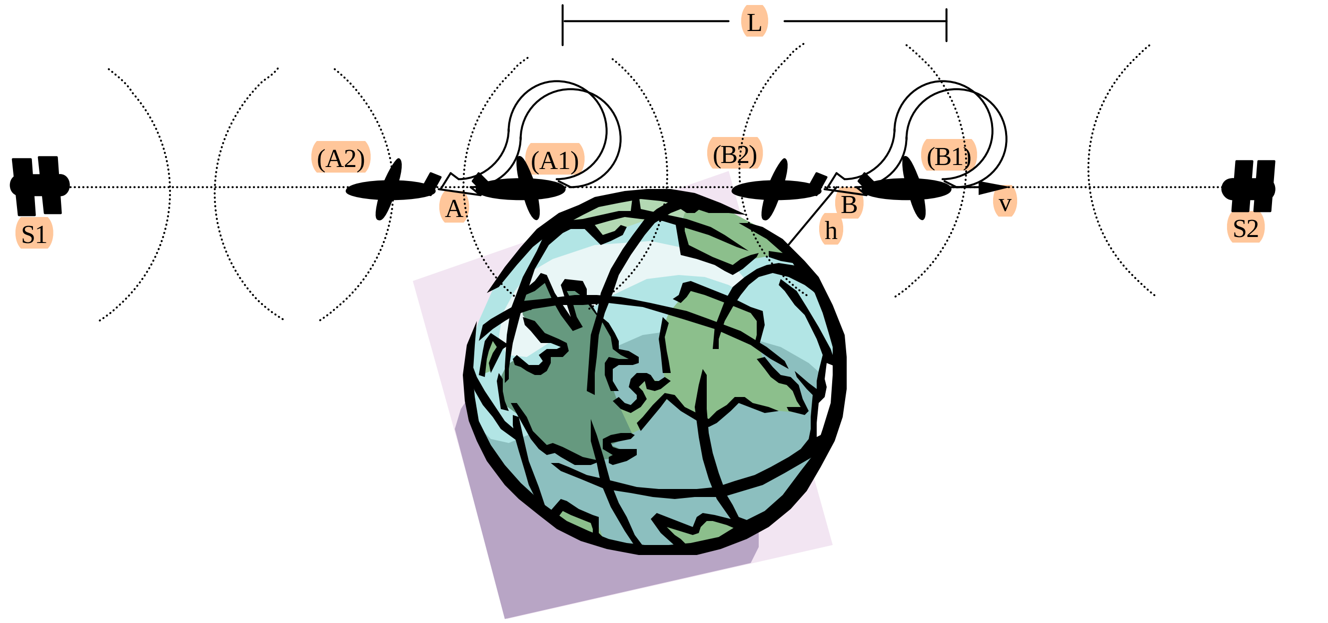

Fig. 8 The simplified crucial experiment

Since it was desired that receiver A, receiver B and S1 (or S2) be on the same straight line, then a proper question will be how much position deviation is allowed, i.e. how much error will be induced if A, B and S1 (or S2) is in fact not on a straight line. This is a critical part of the experiment. On the surface, if A, B and S1 are not on a straight line, it will induce a large error. However, analyzing this problem carefully, one finds that, contrary to what one might expect, any deviation of the position of A from line S1B will cause only a very small difference in the propagation time. (The same for the analysis of relationship between A, B and S2.) Suppose receiver A is not at the ideal position of A, but at the position of A', and *AA'* = *Dh*. (fig. 9) The distance between S1 and A will change from S1A to S1A'. What is the difference between these two distances? We have $S1A' = [S1A^2 + Dh^2]^{1/2}$. Hence, the difference between two distances, $DS1A = SA' - S1A \approx (1/2)(Dh^2 / S1A)$. Since *S1A* is about 26,000 km, we could have the following list:

Δh	100 m	1 km	5 km	10 km
ΔS1A	0.2 mm	2 cm	0.5 m	2 m
ΔS1A/c	0.67 ps	0.067 ns	1.7 ns	6.7 ns

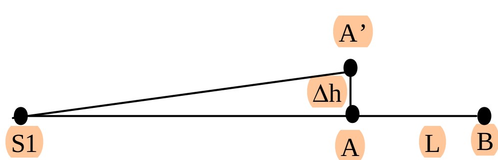

Fig. 9 The effect of the transverse position deviation of one receiver

Actually, since the signal source S1 is far away from the two receivers, the transverse position deviation of the receivers will not cause any noticeable error unless the deviation reaches as much as 5 km.

Another practical problem of the crucial experiment is that GPS satellites are moving and they change their position relative to the receivers during the test. What is the impact of this position change upon the test? A reasonable duration of the experiment is about 30 seconds, and in this short interval the angle of a GPS satellite only changes by about 0.25^0 or moves about 27 km. The change in the position of the GPS satellite will cause another deviation from the straight line between A and S1B. This deviation is $27\ km*700\ km/26{,}000\ km \approx 0.74\ km$. Obviously, it will not cause any noticeable error to the experiment.

We also note that in normal GPS operations, those satellites that are only a few degrees above the horizon are not considered as usable satellites. But here we use the GPS satellites at zero degrees to the horizon. Is this a problem? No, it is not. First, we need a satellite on the horizon because that is required for it to be aligned with the line-of-sight between the two receivers. Second, we are not computing a position. We are only interested in the time difference between two arrival times at two receivers. Therefore, the tropospheric refraction associated with small elevation angles above the horizon is an effect which cancels when the difference is taken. As a matter of fact, this experiment is similar to Differential GPS. Almost all the factors, satellite position bias, satellite clock bias, ionospheric delay and tropospheric delay which affect the accuracy of the GPS measurements will be cancelled when the difference in measurements is taken.

Moreover, it is a multiple-differential GPS experiment. Differences are taken between two receivers, between two propagation directions (from A to B and from B to A), and between two moving states. We can be sure that the experiment will have a high accuracy since most error sources are cancelled in the differencing process.

In the reality, the speed of an airplane will not be the same in two moving states. But this should not be a problem at all. As we mentioned, what we need are two moving states and in fact, any two moving states. Having two states with the same speed, but in opposite directions is just a convenient way to conduct and analyze the experiment. In fact, if two different speeds, V_1 and V_2, are used in the two opposite directions, the expected time difference is $Dt = 2L(V_1+V_2)/c^2$.

In order to avoid the effect of the rotation of the earth, we select a pair of satellites in the direction of North-South and the motions of the airplanes are in the same meridian. Although here the satellite is not necessarily a GPS satellite, it could be a GLONASS satellite or a WAAS geostationary satellite, the selection of only North-South pairs of satellites presents a significant restriction on the selection of the time at which the experiment can be conducted. If this restriction can be eliminated and a pair of satellites in any direction can be used, it will be much easier to find a time at which to conduct the experiment. In fact a pair of satellites in any direction can be chosen.

When a pair of satellites in an arbitrary direction is selected and the motions of the airplanes are in the same direction, the velocity of the airplane relative to ECI frame will have an added velocity caused by the rotation of the earth. This velocity depends primarily on the latitude. This earth spin velocity is no longer necessarily perpendicular to the direction of the flight, therefore it adds a component velocity, $V_1 cosq$ (or $V_2 cosq$), to the airplane's velocity relative to the earth center. (Fig. 10) Generally, the component velocity is not small and cannot be neglected. But in the experiment, we only measure the difference between two motion states. Fortunately, the component of the earth's spin velocity affects both motion states equally as long as the airplane is at the same latitude. Therefore, the net effect of the component velocity caused by the rotation of the earth is zero in the experiment. Thus, while a pair of satellites in North-South direction is preferred, the experiment is not constrained by this restriction and a pair of satellites aligned in any direction can be used.

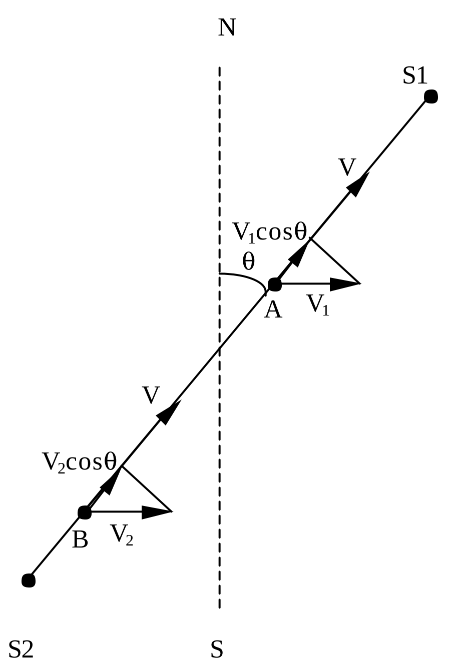

Fig. 10 Simplified crucial experiment utilizing a pair of satellites in any direction

CONCLUSIONS

The strong evidence is that the constancy of the speed of light is wrong. The speed of light is not always *c* relative to a moving observer (receiver). Instead, the speed of light is always *c* relative to the chosen inertial (isotropic light speed) frame. A crucial experiment using GPS has been proposed to verify this claim. This isotropy of light speed relative to the chosen frame is strongly supported by the one-way Sagnac effect. It is clear from the GPS range equation that the motion of the observer during the signal transit time implies that the speed of light relative to a moving observer is not isotropic and clearly differs from *c* due to the receiver motion. This is also evidenced by the JPL space probe equations described by Moyer.

In other words, the Sagnac effect is not due to rotational motion. Contrary to Ashby's claims, the Sagnac effect is caused by any motion of the observer or receiver relative to the chosen inertial frame.

The measurement of the travel time differences between two receivers in motion first one way and then the other can be used as a crucial experiment. Virtually all of the measurement error sources are canceled by the multiple differencing involved in the experiment. This means that the proposed crucial experiment should be capable of easily resolving the fundamental question: "Is the speed of light constant relative to the receiver or is it constant relative to the chosen inertial frame?"

Finally, assuming the crucial test verifies that the speed of light is constant with respect to the chosen inertial frame, it shows that a new method of measuring the velocity is possible. Specifically, measuring the time difference between the signal transit time in the forward and backward direction should give a direct measure of the velocity.

APPENDIX

The calculations of signal propagation time difference

Please notice that here only one satellite is utilized and the result will be doubled when two satellites are utilized. Suppose the satellite S2 is on the origin of the x axis (fig. 11).

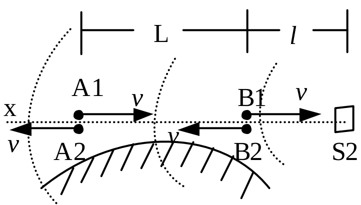

Fig. 11 The calculation using GPS theory

From the GPS theory [13, 14], we know

$c(Tr - Ts) = |\boldsymbol{r}_r(Tr) - \boldsymbol{r}_s(Ts)|$

$r = c(t_r - t_s) = c(Tr - Ts) + cdr - cds + DD + cDI + cDT$

$|\boldsymbol{r}_r(Tr) - \boldsymbol{r}_s(Ts)|$ is the real range from the satellite to the receiver; Ts and Tr are the transmission time and reception time in GPS time.

$r = c(t_r - t_s)$ is the pseudorange; δr and δs are receiver and satellite clock biases; ΔD is satellite position bias and ΔI and ΔT are ionospheric and tropospheric delays.

We investigate the following four cases.

1) Case 1, at t_1, the signal is transmitted from the satellite and then arrives at A2:

$$\begin{cases} c(Tr - t_1) = |x - 0| \\ x = l + L + v(Tr - t_1) \end{cases}$$

We have $c(Tr - t1)(1) = (l + L)c/(c - v)$ and

$r(1) = c(tr - ts)(1) = (l + L)c/(c - v) + cdr(1) - cds(1) + DD(1) + cDI(1) + cDT(1)$

2) Case 2, at t_1, the signal is transmitted from the satellite and then arrives at B2:

$$\begin{cases} c(Tr - t_1) = |x - 0| \\ x = l + v(Tr - t_1) \end{cases}$$

We have $c(Tr - t1)(2) = lc/(c - v)$ and

$r(2) = c(tr - ts)(2) = lc/(c - v) + cdr(2) - cds(2) + DD(2) + cDI(2) + cDT(2)$

3) Case 3, at t_2, the signal is transmitted from the satellite and then arrives at A1:

$$\begin{cases} c(Tr - t_2) = |x - 0| \\ x = l + L - v(Tr - t_2) \end{cases}$$

We have $c(Tr - t2)(3) = (l + L)c/(c + v)$ and

$r(3) = c(tr - ts)(3) = (l + L)c/(c + v) + cdr(3) - cds(3) + DD(3) + cDI(3) + cDT(3)$

4) Case 4, at t_2, the signal is transmitted from the satellite and then arrives at B1:

$$\begin{cases} c(Tr - t_2) = |x - 0| \\ x = l - v(Tr - t_2) \end{cases}$$

We have $c(Tr - t2)(4) = lc/(c + v)$ and

$r(4) = c(tr - ts)(4) = lc/(c + v) + cdr(4) - cds(4) + DD(4) + cDI(4) + cDT(4)$

Therefore, we have

$cDt = [c(tr - ts)(1) - c(tr - ts)(2)] - [c(tr - ts)(3) - c(tr - ts)(4)] =$

$[(l + L)c/(c - v) + cdr(1) - cds(1) + DD(1) + cDI(1) + cDT(1)] -$

$[lc/(c - v) + cdr(2) - cds(2) + DD(2) + cDI(2) + cDT(2)] -$

$[(l + L)c/(c + v) + cdr(3) - cds(3) + DD(3) + cDI(3) + cDT(3)] +$

$[lc/(c + v) + cdr(4) - cds(4) + DD(4) + cDI(4) + cDT(4)]$

Let us check these items. For receiver clock bias δr, we have $dr(1) = dr(3)$ for clock A and $dr(2) = dr(4)$ for clock B because a clock bias will be the same in 30 seconds, a reasonable duration of the experiment.

For satellite clock bias, δs, we have $ds(1) \equiv ds(2)$ and $ds(3) \equiv ds(4)$.

For ΔD, the satellite position bias, we have $DD(1) \equiv DD(2)$ and $DD(3) \equiv DD(4)$.

For ΔI, the ionospheric delay, we have $DI(1) = DI(3)$ and $DI(2) = DI(4)$, and for ΔT, the tropospheric delay, we have $DT(1) = DT(3)$ and $DT(2) = DT(4)$, since ΔI and ΔT will be the same in 30 seconds.

Therefore, finally we have

$c\Delta t = [(l + L)c/(c - v) - lc/(c - v)] - [(l + L)c/(c + v) - lc/(c + v)]$

$= [Lc/(c - v) - Lc/(c + v)]$

$=2Lv/c$, neglecting the quantities of the second and higher order of v/c.

That is,

$Dt = 2Lv/c^2$

As mentioned before, if two satellites, S1 and S2, are utilized, the result will be doubled to $4Lv/c^2$.

ACKNOWLEDGEMENTS

The University of Calgary simulated the scenario described in the section on the crucial experiment. We thank them for their effort in the simulation. Unfortunately, the precision of the transmit time was not sufficient to resolve the measured time differences to the precision needed.

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