# Another Error in Einstein's Calculation of Perihelion for Mercury

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Another error in Albert Einstein's paper, "Erklrung Perihelbewegung des Merkur aus", invalidated his confirmation of the precession of perihelion for Mercury. Einstein included an extra term in an equation to calculate the angle of precession. Unknown to Einstein, the equation contradicts Schwarzschild's equations. Einstein was using the wrong equation.

## I. INTRODUCTION

In 1916, Karl Schwarzschild published a paper[1,2], "ber das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie" in response to Albert Einstein's paper[3], "Erklrung Perihelbewegung des Merkur aus". The response presented an exact solution in comparison with Einstein's approximation solution. Schwarzschild considered the exact solution superior by stating:

"It is always pleasant to have strict, simple form solutions. It is more important that the calculation also gives the unambiguous certainty of the solution, about which Mr. Einstein's treatment still left doubts, and which, according to the way in which it appears below, could hardly be proved by such an approximation procedure.".

Schwarzschild described how a massless point should propagate through an empty space around a point mass at the origin.

Schwarzschild metric was the standard metric for a static mass in an isotropic manifold. Until Einstein's death in 1955, Schwarzschild's metric was the only and unique solution for the orbit of Mercury. Einstein had adopted Schwarzschild's metric but chosen an equation different from Schwarzschild's equations. Einstein was using the wrong equation.

## II. PROOF

### A. Schwarzschild Metric

Schwarzschild derived the metric based on conditions set by Einstein[3]:

1. All the components are independent of the time $x_4$.

2. The equations $g_{\rho 4} = g_{4\rho} = 0$ hold exactly for $\rho =$ 1,2,3.

3. The solution is spatially isotropic.

4. The $g_{\nu\mu}$ vanish at infinity, with the exception of the following four limit values different from zero:

$$g_{44} = 1 \tag{1}$$

$$g_{11} = g_{22} = g_{33} = -1 \tag{2}$$

He emphasized the metric is exclusively for massless object.

The metric in equation 14 of the Schwarzschild's paper[1] is reproduced as

$$ds^2 = (1 - \frac{\alpha}{R})dt^2 - \frac{dR^2}{1-\frac{\alpha}{R}} - R^2(d\theta^2 + sin(\theta)^2 d\phi^2) \tag{3}$$

$$R = (r^3 + \alpha^3)^{1/3} \tag{4}$$

The metric requires all mass at the origin. Any presence of mass on the geodesic will violate the conditions from Einstein and require a new metric.

Schwarzschild applied the isotropic symmetry to the metric by setting

$$\theta = \frac{\pi}{2} \tag{5}$$

and remarked: "If one also restricts himself to the motion in the equatorial plane ($\theta = 90^o, d\theta = 0$)".

He applied the variation principle to the line element to obtain three equations of motion equivalent to geodesic equations.

Equation 15 in his paper[1] is reproduced as

$$(1-\frac{\alpha}{R})(\frac{dt}{ds})^2 - \frac{1}{1-\frac{\alpha}{R}}(\frac{dR}{ds})^2 - R^2(\frac{d\phi}{ds})^2 = const. = h \tag{6}$$

Equation 16 in his paper[1] is reproduced as

$$R^2\frac{d\phi}{ds} = const. = c \tag{7}$$

Equation 17 in his paper[1] is reproduced as

$$(1-\frac{\alpha}{R})\frac{dt}{ds} = const. = 1 \tag{8}$$

Schwarzschild defined $x$ as

$$x = \frac{1}{R} \tag{9}$$

From equations (3,5,6,7,8,9),

$$(\frac{dx}{d\phi})^2 = \frac{\alpha}{c^2}x - x^2 + \alpha x^3 \tag{10}$$

### B. Einstein's Errpr

Equation 11 in Einstein's paper[3] is reproduced as

$$(\frac{dx}{d\phi})^2 = \frac{2A}{B^2} + \frac{\alpha}{B^2}x - x^2 + \alpha x^3 \tag{11}$$

Einstein stated: "That contribution from the radius vector and described angle between the perihelion and the aphelion is obtained from the elliptical integral."

$$\phi = \int_{\alpha_1}^{\alpha_2} \frac{dx}{\sqrt{\frac{2A}{B^2} + \frac{\alpha}{B^2}x - x^2 + \alpha x^3}} \tag{12}$$

With equations (11,12), Einstein claimed to confirm the motion of Mercury. However, unknown to Einstein, there is an obvious error in equation (11).

Equation (11) contradicts equation (10) from Schwarzschild's equations. There is an extra term, $\frac{2A}{B^2}$, in equation (11). $A$ should be zero.

## III. CONCLUSION

The extra term in equation (11) of Einstein's paper renders the rest of the paper invalid. Unknown to Einstein, he was using the wrong equation for calculation.

Einstein's incompetence in mathematics can be clearly illustrated in his research on gravitation. He begged for help from his classmate, Marcel Grossmann, as soon as he encountered Riemannian geometry. Einstein told Grossmann[4]: "You must help me, or else I'll go crazy.". Immediately after Grossmann stopped helping Einstein, Einstein made little progress until 1915 when he met David Hilbert who completed the theory of relativity for Einstein.

As Hilbert once remarked[5], "Every boy in the streets of Gottingen understands more about four-dimensional geometry than Einstein".


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[3] A. Einstein: Erklrung Perihelbewegung des Merkur aus. http://www.relativitycalculator.com/pdfs/einstein_mercury_perihelion_german.pdf

[4] Nature: A helping hand. https://www.nature.com/articles/nphys2885

[5] James Schombert: Hilbert. http://abyss.uoregon.edu/ js/glossary/hilbert.html

[6] Eric Su: Error In Einstein's Calculation Of Perihelion For Mercury, sites.google.com/view/physics-news/gravitation

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