Noon-Midnight Red Shift*

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A terrestrial atomic clock at noon can be some 10^9 cm nearer the sun than an antipodal clock at midnight. The difference in gravitational potential due to the sun corresponds to a difference of time rates corresponding to a red shift $\Delta\nu/\nu = 8\times10^{-13}$. But this red shift is almost exactly cancelled by a violet shift arising from the relativistic Doppler effect, so that the resultant shift is essentially zero. If the earth shielded or focussed the solar gravitational field, the gravitational contribution to the red shift would be altered and one might expect a resultant shift. But the motional contribution to the shift is also altered and, except for unrealistically large shielding or focussing, the resultant shift would still be zero. However, all this is true only if the principle of equivalence is valid. The Pound-Rebka experiment confirms its local validity with a 10% accuracy. A 10% discrepancy could imply a noon-midnight red shift $\Delta\nu/\nu = 8\times10^{-14}$, compared with 5×10^{-16} in the Pound-Rebka experiment. Moreover, since the solar gravitational contribution to the value of g is only $5\times10^{-4}g$, the Pound-Rebka experiment is insensitive to solar effects and would not detect possible anomalies arising from shielding or focussing by the earth of the locally almost uniform solar gravitational field which might nevertheless affect the noon-midnight shift. Detection of a significant noon-midnight shift would be a disproof of the general theory of relativity.

I

With clocks becoming more and more accurate, experiments that only a few years ago would have seemed impossible to perform begin to enter the realm of feasibility. The recent feat of Pound and Rebka[1] of measuring, by means of the Mössbauer effect, a relativistic difference in time rates of one part in 2×10^{14} leads one to examine again some of the relativistic effects that have hitherto lain beyond the range of experimental detection.

A clock on the earth is nearer the sun at noon than at midnight. It is, therefore, at a lower gravitational potential at noon than at midnight, and this would cause it to show a gravitational red shift at noon compared with its rate at midnight.

For simplicity, consider two antipodal clocks, N, M, on the equator at their respective noon and midnight at the time of an equinox. The gravitational red shift depends on the difference of their gravitational potentials. The gravitational potential due to the earth is the same for both clocks (assuming that the earth is a uniform spheroid), but that due to the sun is not. Denote the radius of the earth's orbit by R, the mass of the sun by M, and the Newtonian gravitational constant by G. Then, to a sufficient degree of accuracy, the relative difference in frequency of the two clocks due to the difference in gravitational potential is

$$\Delta\nu_1/\nu_1 = (GM/c^2)\{(R-r)^{-1} - (R+r)^{-1}\} = 2MGr/c^2(R^2-r^2). \quad (1)$$

Since $G = 6.67\times10^{-8}$, $M = 1.98\times10^{33}$, $r = 6.3\times10^8$, $R = 1.5\times10^{13}$, we have

$$\Delta\nu_1/\nu_1 = 8\times10^{-13}, \quad (2)$$

which would be a measurable quantity if the technical difficulties of comparing the rates of such widely separated clocks on a seismic earth could be overcome.

However, the above calculation ignores the Doppler contribution to the red shift. That this is significant can be seen by considering a special case in which the earth is taken to be a test body and to rotate on its axis once a year so that N is perpetually a noon clock and M perpetually a midnight clock; the same effect could be obtained by imagining N and M to be mounted in hypothetical jet planes that kept them in the noon and midnight positions as the earth turned on its axis.

In a Schwarzschild reference frame with the sun at the pole we have, in isotropic coordinates,

$$ds^2 = [(1-\tfrac{1}{2}\Omega)/(1+\tfrac{1}{2}\Omega)]^2 c^2 dt^2 - (1+\tfrac{1}{2}\Omega)^4(dx^2+dy^2+dz^2), \quad (3)$$

where

$$\Omega = MG/c^2(x^2+y^2+z^2)^{\frac{1}{2}}. \quad (4)$$

Since Ω is small, we can write (3) as

$$ds^2 = (1-2\Omega)c^2dt^2 - (1+2\Omega)(dx^2+dy^2+dz^2). \quad (5)$$

A light signal from M to N passing around the equator will follow a certain trajectory relative to the present coordinate system. But the line element is static and spherically symmetric, and we have arranged matters so that M and N are moving in concentric circles around the sun with equal angular velocities. Therefore all such light trajectories will be congruent, and time intervals will be faithfully transmitted from M to N. The same is true for transpolar light signals, or any others following a fixed path relative to the earth.

So, from (5), in the usual manner, we obtain

$$\Delta\nu/\nu = \Delta\Omega + \Delta(v^2)/2c^2, \quad (6)$$

where $\Delta(v^2)$ is the difference in the squares of the speeds of N and M, the term containing it arising from the second-order Doppler effect and corresponding to a change in "centrifugal potential." If ω is the orbital

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[1] R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters **7**, 337 (1960).

angular velocity of the earth, we have

$$\Delta(v^2)/2c^2=\{(R-r)^2-(R+r)^2\}\omega^2/2c^2=-2Rr\omega^2/c^2. \quad (7)$$

But in the Newtonian approximation, which is sufficiently accurate for our purposes here,

$$R\omega^2=MG/R^2; \quad (8)$$

so (7) becomes

$$\Delta(v^2)/2c^2=-2MGr/c^2R^2, \quad (9)$$

which is 8×10^{-13} and thus cancels the gravitational contribution (2). Indeed, by (1) and (9), we find from (6) that $\Delta\nu/\nu$ is of order 10^{-21} (though, of course, our approximations break down before such a small value is reached).

The diurnal rotation of the earth poses a complication. If we replace (7) by

$$\begin{aligned}\Delta(v^2)/2c^2&=\{(R\omega-r\omega_e\cos23°)^2-(R\omega+r\omega_e\cos23°)^2\}/2c^2\\&=(-2Rr\omega\omega_e\cos23°)/c^2, \quad (10)\end{aligned}$$

where ω_e is the diurnal angular velocity of the earth, we find that the term $\Delta(v^2)/2c^2$ is of order 2.6×10^{-10} which is alarmingly large and would have serious implications for terrestrial chronometry and the measurement of artificial satellite red shifts. Adding the $R\omega$ and $r\omega_e$ velocities according to the special relativistic instead of the Galilean formula does not alter the order of magnitude of this second-order Doppler term. However, the rotation of the earth relative to the radial line from the center of the sun to the center of the earth causes the light trajectories between M and N relative to the present coordinate system to be no longer congruent. This means that there will be a "first-order" Doppler effect—which turns out to be of the second order, and, indeed, of the same order of magnitude as the "second-order" Doppler effect in (10). It is not clear, from the present point of view, though, to what order the two terms cancel each other and what the residual noon-midnight red shift will be, because too many dangerous assumptions have to be made in estimating the contributions of various effects to the order of accuracy required. For example, one does not know precisely what *coordinate* shape the rotating, moving earth would have, and so one cannot calculate the crucial "first-order" Doppler shift to the second order with any assurance.

Therefore, a different approach is necessary, and it is given in the Appendix. It is shown there that, so far as the solar field is concerned, to a sufficient degree of accuracy the principle-of-equivalence cancellation of gravitational and centrifugal (second-order Doppler) effects allows one to use Minkowskian coordinates to well beyond distances r from the center of the earth. Essentially this is because, to a sufficient degree of accuracy, the solar gravitational field can be regarded as uniform between N and M, a fact strongly suggested by the null result in the special case considered above.

II

The result of the calculations in the Appendix is that there is essentially zero noon-midnight red shift even when the diurnal rotation of the earth is taken into account. Since the null result can be regarded as due to the cancellation of gravitational and centrifugal effects in accordance with the principle of equivalence, an experimental test of the null result would be a test of that principle. But the principle has already been tested on a local basis by Pound and Rebka, so that a further, and more difficult test would seem superfluous.

However, in all these calculations it is assumed that there is no gravitational shielding, using the term in its broadest possible sense. Yet 8000 massive miles of earth are interposed between the midnight clock and the sun. There is no *a priori* reason why gravitational shielding should be impossible. On the contrary, the nonlinearity of the gravitational equations of the general theory of relativity shows that, according to that theory, there must be gravitational "shielding" effects—either positive or negative—in the sense that the gravitational field of the sun and the earth together is not equal to the sum of their individual fields. Admittedly, these nonlinear effects are extremely small and, in the situation envisaged, well below the present limits of measurability. Also they are of a different nature from electromagnetic shielding effects which arise from the existence of charges of both signs. But they show that there is nothing inherently impossible about the idea of gravitational shielding of some sort.

The success of the Newtonian theory suggests that purely mechanical shielding effects are small. The Eötvös experiment, being a differential experiment that balances centrifugal against gravitational forces at the same place, does not exclude the possibility of weak gravitational shielding. Moreover, it is a purely mechanical experiment, and involves gravitational force, whereas experiments on the red shift are partly optical in nature and involve the gravitational potential. Even if purely mechanical experiments gave no evidence of significant shielding, it would still be worthwhile to see whether semi-optical experiments did too. We shall see that the noon-midnight red shift can give such evidence only if the principle of equivalence is not valid.

III

If the earth shielded the solar gravitational field, it would affect the gravitational contribution to the red shift given in (1), and one would therefore expect a resultant red shift because the gravitational and motional contributions would no longer cancel. But a closer examination of the situation shows that this reasoning ignores an important factor; for the shielding would alter the gravitational pull of the sun on the earth, and this, by altering the radius of the earth's orbit and the value of ω, would affect the motional contribution to the red shift. It is true that the gravitational contribution (1) depends on the potential at one clock minus the potential at the other clock, and not on the manner in which the potential behaves between those two points, while the motional contribution depends on the forces throughout the body of the earth, so that in theory one could have a sizeable difference between the effects of shielding on the gravitational and motional contributions and thus a resultant red shift. But, in practice, no significant resultant shift would occur in any reasonable situation because potential and force are related. For example, consider the rather extreme case in which the solar gravitational *force* is assumed to be unshielded as far as the plane through the center of the earth perpendicular to the radius from the sun, and is completely shielded beyond that plane.

Since the ratio of the maximum to the minimum solar gravitational forces on the earth is

$$(R+r)^2(R-r)^{-2}\sim 1+4r/R\sim 1+1.6\times 10^{-4}, \quad (11)$$

if we denote the mass of the earth by m, we may write the gravitational pull of the sun on the earth in the present situation as

$$GM\{\tfrac{1}{2}m(1+\epsilon)\}/R^2, \quad (12)$$

where ϵ is smaller than 1.6×10^{-4}. Also, since the gravitational potential due to the sun is now constant from the center of the earth to the midnight clock, the difference in potential at N and M will be

$$GM\{(R-r)^{-1}-R^{-1}\}=GMr/R(R-r). \quad (13)$$

So the resultant red shift will be, by (6),

$$\Delta\nu/\nu=(GMr/c^2)\{(R^2-Rr)^{-1}-R^{-2}-\epsilon R^{-2}\}. \quad (14)$$

The first two terms on the right, when their common coefficient is taken into account, are each of order 4×10^{-13} and cancel each other to within 10^{-21} (we are using the old value of R in these estimates since that would anyway be the observed value). The term involving ϵ will therefore yield a residual shift, but it will be of order 6×10^{-17} at most, and thus negligible. We see, then, that a comparison of the rates of noon and midnight clocks would show no significant difference in rates whether there were a realistic amount of shielding or focussing or whether there were none.

IV

However, the above is predicated on the validity of the general theory of relativity, and, in particular, on the validity of the principle of equivalence.

In the Pound-Rebka experiment, the difference in "clock" rates at different heights above the ground at the same geographical location is found to be in accordance with the formula

$$\Delta\nu/\nu=gh/c^2, \quad (15)$$

where h is the difference in heights and g is "the acceleration due to gravity" in the vicinity of the experiment. Pound and Rebka found that

$$(\Delta\nu)_{\rm exp}/(\Delta\nu)_{\rm theor}=1.05\pm 0.10. \quad (16)$$

Let us consider two possibilities: that there is a 10% discrepancy in the principle of equivalence, and that there is no discrepancy detectable by the Pound-Rebka procedure.

A 10% discrepancy could imply a 10% change in the solar gravitational contribution to the noon-midnight red shift. Such a change would yield a resultant shift of 8×10^{-14}, compared with the 5×10^{-16} that is 10% of the Pound-Rebka shift (though the difficulties of the noon-midnight experiment may well nullify this numerical advantage).

Still considering the hypothetical 10% discrepancy, let us suppose that it arises from a shielding or focussing of the earth's radial gravitational field by the earth, its nonmechanical effect differing from its mechanical effect by an amount equal to $gh/10c^2$. Then it would be possible that the shielding or focussing effect of the earth on the locally almost uniform solar gravitational field would be significantly different from that of the earth on its own radial field. If so, the discrepancy in the noon-midnight case could be larger than 10%.

Let us now assume that there is no discrepancy detectable by the Pound-Rebka procedure. This would not settle the question of the validity of the principle of equivalence. It would only show that, so far as that principle is concerned, any shielding or focussing by the earth of its own radial field has approximately equal mechanical and nonmechanical effects. It would not ensure that possible shielding or focussing by the earth of the locally almost uniform solar gravitational field would behave similarly. Therefore there could still be a significant noon-midnight red shift; and it could occur even if the purely mechanical effect of shielding or focussing were zero.

One must ask, though, whether such a shift would be detectable by performing the Pound-Rebka experiment at midnight and comparing the value of $\Delta\nu/\nu g$ so obtained with the value obtained when the experiment was performed at noon. It would not. While g is not really "the acceleration due to gravity" but the resultant of many effects, including the gravitational force of the sun and centrifugal and Coriolis forces, the main contribution to it is that of the earth's gravitational field. The solar gravitational field contributes only an amount $5\times 10^{-4}g$. Therefore even a 100% discrepancy in the solar effect would not be detectable by this technique. In the Pound-Rebka experiment the solar effect is negligible. Since it is a major factor in the noon-midnight experiment, that experiment, if it could be performed with sufficient accuracy, could be a more searching test of the principle of equivalence than the local experiment of Pound and Rebka.

If the calculations in this paper are correct, the general theory of relativity predicts zero noon-midnight red shift whether or not there is small shielding or focussing. Therefore, the detection of a shift different from zero would be significant; for, if not otherwise accounted for, it would not only imply that there was a shielding or focussing effect but would amount to a disproof of the general theory of relativity.

V

The experiment would not be easy to perform. Signalling between antipodal clocks would have to be by coaxial cable, or, failing that, by radio ground wave to avoid time distortion due to reflections of the air wave. Differences in the heights of the antipodal clocks above the geoid—an equipotential surface closely approximating the shape of the earth—could be detected by performing the experiment at 12-hour intervals, and other extraneous effects could be allowed for by comparing results with those for the same clocks in the 6 a.m.–6 p.m. positions, where there would be no significant difference in the terrestrial shielding of the two clocks.

According to the argument in the Appendix, the presence of the moon should not affect the null result of the calculation. If the principle of equivalence were not valid and this was due to shielding or focussing, the lunar contribution would probably be negligible because the expression (1) with lunar instead of solar constants comes to only 5×10^{-3} times the solar value. In general, lunar effects could be recognized by their monthly periodicity.

Vertical tidal distortions of the surface of the earth can be neglected since, apart from being small, they act to raise (or lower) both clocks by substantially the same amounts. Lateral tidal and seismic distortions of the earth's surface could be more important since by introducing a relative velocity (i.e., time rate of change of circumferential distance) between the clocks they could give rise to small "first-order" Doppler effects that would rank as large second-order effects. However, random distortions of this sort would tend to cancel out over an interval of time, and any residual effect during the course of an experiment might be recognized by comparing the shifts obtained when light signals are sent in opposite directions, for a lateral distortion that gave a red shift in the one case would tend to give a violet shift in the other, though the two would not be of equal magnitude.

The great difficulty with experiments on gravity is that the gravitational force is intrinsically extremely weak. But the earth is a massive body that generates relatively large gravitational effects, and the suggested experiment shows how to use it as a piece of experimental apparatus in an investigation of the possible failure of the principle of equivalence through the effects of gravitational shielding or focussing.

APPENDIX

Starting from the line element (5), with Ω given by (4), we shall make a series of transformations that lead to a locally Minkowskian reference frame having the center of the earth, C, as spatial origin.

Let the earth's orbit be in the plane $z=0$. Then we may take the coordinates of the center of the earth to be $(R\cos\omega t, R\sin\omega t, 0, t)$. Since we shall be dealing with values of t that run from zero to approximately $\pi r/c\sim6\times10^{-2}$, so that ωt is at most of order 10^{-8}, we may take the coordinates of C to be

$$(x,y,z,t)=\{R(1-\omega^2t^2/2),\ R\omega t,\ 0,\ t\},\tag{17}$$

despite the fact that R is 1.5×10^{13}.

We now transform to new coordinates, x_1, y_1, z_1, t_1, relative to which C will have the coordinates $(0,0,0,t_1)$ to the desired degree of accuracy, for values of t_1 from zero to beyond 6×10^{-2}. The following transformation accomplishes this:

$$\begin{aligned}&y=\beta(y_1+R\omega t_1),\quad t=\beta(t_1+R\omega y_1/c^2),\\&x=x_1+R(1-\omega^2\beta^2t_1^2/2),\ \ z=z_1,\ \ \beta\equiv(1-R^2\omega^2/c^2)^{-\frac{1}{2}}.\end{aligned}\tag{18}$$

For its inverse is

$$\begin{aligned}&y_1=\beta(y-R\omega t),\quad t_1=\beta(t-R\omega y/c^2),\\&x_1=x-R\{1-\tfrac{1}{2}\omega^2\beta^4(t-R\omega y/c^2)^2\},\quad z_1=z,\end{aligned}\tag{19}$$

and when $y=R\omega t$ we not only have at once that $y_1=0$ but also that the quantity $\frac{1}{2}\omega^2\beta^4(t-R\omega y/c^2)^2$ becomes $\frac{1}{2}\omega^2t^2$ so that

$$x_1=x-R(1-\tfrac{1}{2}\omega^2t^2),$$

and this, by (17), is zero.

Writing the line element ds^2 in (5) in terms of the new coordinates and rejecting terms of order 10^{-16} and terms smaller than this in the coefficients of $c^2dt_1^2$, cdt_1dx_1, cdt_1dy_1, dx_1^2, etc., we have

$$\begin{aligned}ds^2=&(1-2\Omega)c^2dt_1^2-(1+2\Omega)(dx_1^2+dy_1^2+dz_1^2)\\&+(2R\omega^2t_1/c)cdt_1dx_1-(8R\omega\Omega/c)cdt_1dy_1.\end{aligned}\tag{20}$$

Though the coefficient of cdt_1dy_1 is of order 8×10^{-12} and that of cdt_1dx_1 at most of order 2.4×10^{-12}, the former turns out to be negligible but the latter not. This is because of the presence of t_1 in the latter, as will be seen.

Denote $2R\omega\Omega/c$ by μ, so that μ is approximately 2×10^{-12}, and make the transformation

$$ct_1=ct_2+\mu y_2,\quad y_1=-\mu ct_2+y_2,\quad x_1=x_2,\quad z_1=z_2.\tag{21}$$

Then if we ignore μ^2 and $\Omega\mu$, and note that $(2R\omega^2/c)(\mu y_2/c)$ is negligible even when $y_2=r$, we find that

$$\begin{aligned}ds^2=&(1-2\Omega)c^2dt_2^2-(1+2\Omega)(dx_2^2+dy_2^2+dz_2^2)\\&+(2R\omega^2t_2/c)cdt_2dx_2.\end{aligned}\tag{22}$$

We now write

$$ct_2=ct_3(1-R\omega^2x_3/c^2),\quad x_2=x_3,\quad y_2=y_3,\quad z_2=z_3.\tag{23}$$

Then, since even when $x_3=r$ the quantity $R\omega^2x_3/c^2$ is only 4×10^{-13}, we have

$$ds^2=(1-2\Omega-2R\omega^2x_3/c^2)c^2dt_3^2 - (1+2\Omega)(dx_3^2+dy_3^2+dz_3^2). \quad (24)$$

Now $\Omega=MG/c^2(x^2+y^2+z^2)^{\frac{1}{2}}$. We wish to express it in terms of x_3, y_3, and z_3. Consider the quantity $\rho^2\equiv x^2+y^2+z^2$. By (18) we have, to a sufficient degree of accuracy,

$$\rho^2=(x_1+R)^2+(y_1+R\omega t_1)^2+z_1^2 = R^2\{1+2x_1/R+2\omega y_1t_1/R+(x_1^2+y_1^2+z_1^2)/R^2\}.$$

But at most $x_1^2+y_1^2+z_1^2=r^2$, $x_1=r$, and $y_1t_1=10^{-1}r$. So we may ignore the last two terms, and since, by (21) and (23), $x_1=x_3$, we have

$$\rho^2=R^2(1+2x_3/R). \quad (25)$$

Therefore we can take

$$\Omega=MG/c^2R-MGx_3/c^2R^2, \quad (26)$$

and so

$$ds^2=\{1-2MG/c^2R+2MGx_3/c^2R^2-2R\omega^2x_3/c^2\}c^2dt_3^2 - \{1+2MG/c^2R-2MGx_3/c^2R^2\}(dx_3^2+dy_3^2+dz_3^2). \quad (27)$$

We now wish to change the coefficient of the spatial part to minus one. To do this we first write

$$A=1+2MG/c^2R, \quad B=2MG/c^2R^2, \quad (28)$$

so that the coefficient of the spatial part is $-(A-Bx_3)$. We note that A is of order unity, while Bx_3 is at most of order 8×10^{-13}, and we write

$$\begin{aligned} x_3&=A^{-\frac{1}{2}}X+(B/4A^2)(X^2-Y^2-Z^2),\\ y_3&=A^{-\frac{1}{2}}Y+(B/2A^2)XY,\\ z_3&=A^{-\frac{1}{2}}Z+(B/2A^2)XZ, \quad t_3=A^{\frac{1}{2}}T.\end{aligned} \quad (29)$$

Then, on neglecting terms involving B^2, we obtain

$$ds^2=(1+2MGX/c^2R^2-2R\omega^2X/c^2)c^2dT^2 - dX^2-dY^2-dZ^2, \quad (30)$$

and since, by (8), the second and third terms in the parentheses cancel, we see that our coordinate system is Minkowskian.[2]

[2] The fact that this Minkowskian line element cannot account for the tides is puzzling at first, and makes one question the validity of the approximations used in obtaining it. But there is a significant difference between tides and red shifts quite apart from the important $1/c^2$ factor in the expression for the red shift which is absent from expressions for tidal forces—a factor that is partly offset by the precision of modern methods of detecting red shifts. Since a red shift depends on a difference of potential, it can be caused by a uniform force; but a uniform force cannot produce tides because it accelerates everything equally. In expanding ρ^2 we kept the term $2x_1/R$ but rejected the smaller term $(x_1^2+y_1^2+z_1^2)/R^2$. The former contributes to $\Delta\Omega$ between N and M and the latter does not (because N and M are equidistant from C). But since the former yields a uniform force, it would not cause tides no matter how large it might be, while the latter, yielding a nonuniform force, does produce tides. Even if the latter term did contribute to the red shift, its contribution could be neglected since its greatest value is only 10^{-4} times that of $2x_1/R$ which produces a shift of 8×10^{-13}. We see from this that it is not unreasonable that an approximation valid for calculating a noon-midnight red shift should lack tide-producing terms.

We have so far neglected the gravitational field of the earth. But on the surface of the earth it is essentially constant, and can be combined there additively with the solar gravitational field up to the degree of accuracy at which nonlinear effects become important. Since the nonlinear effects will be of order $(2MG/c^2R)(2mG/c^2r)$, where m is the mass of the earth, and since this product is 3×10^{-17}, we may safely include the effect of the earth's gravitational field *on its surface* by adding the *constant* $2mG/c^2r$ to Ω, and since this can be absorbed in A it can be transformed away leaving us with the same Minkowskian line element (30), but now valid only on the earth's surface.[3]

Because $m/M=3\times10^{-6}$, the error in having treated the earth as a test body moving in the sun's gravitational field will not be significant. And the fact that the above calculations show that the solar gravitational field is sufficiently uniform between N and M to permit its effective cancellation by the centrifugal field to extend from the center of the earth (where we knew beforehand that the cancellation must occur) to well beyond the surface of the earth shows that such a cancellation would hold too when the earth was not treated as a test body, for in that case too we know, from the principle of equivalence, that the cancellation occurs at the center of the freely falling earth. The same argument applies when we take account of the effect of the sun and moon combined, provided that the lunar gravitational field is sufficiently uniform across a terrestrial diameter. The maximum nonuniformity will occur at the point on the earth nearest the moon. And the rate of change of the lunar gravitational potential there is only 5.2×10^{-3} times the corresponding rate of change of the solar gravitational potential. Thus the uniformity of the lunar gravitational field is ample.

We now consider the effect of the earth's diurnal rotation. In terms of the Minkowskian coordinates in (30), there is no difficulty about the coordinate shape of the earth (we are assuming the earth to be a sphere, or, at least, the path of the light signal to be a part of a circle). Let the light signal travel in a great circle perpendicular to the axis of the earth, and denote the earth's angular velocity about this axis by ω_e. Using polar coordinates in the plane of this circle, we may take the coordinates of N, M, and the tip, L, of the light signal to be (r,θ_n,T_n), (r,θ_m,T_m), (r,θ_l,T_l), respectively. For L we must have $ds=0$. So, from (24),

$$cdt_l=rd\theta_l,$$

or

$$\theta_l=(c/r)T_l+k. \quad (31)$$

At any time T, we may take $\theta_m=\omega_eT$, $\theta_n=\omega_eT+\pi$. Let the light signal be sent out from M at $T=T_1$ and reach

[3] A Minkowskian line element would, of course, also be valid on any spherical surface having its center at the center of the earth and a radius not too great.

N at $T=T_2$. Then

$$\omega_e T_1 = (c/r)T_1 + k, \tag{32}$$

$$\omega_e T_2 + \pi = (c/r)T_2 + k. \tag{33}$$

So

$$T_2 - T_1 = \pi r/(c - r\omega_e), \tag{34}$$

which shows that the coordinate time interval taken for the transmission is constant, a result that is hardly unexpected in the present simple circumstances. Therefore, since the line element is static, being Minkowskian, time intervals at M are faithfully transmitted to N, with zero first-order Doppler effect. Since the second-order Doppler effects (i.e., the time dilatations) of the two clocks are identical, it follows that there will be zero red shift.

It is worth remarking that if the light signal were sent in the opposite direction, (34) would be replaced by

$$T_2 - T_1 = \pi r/(c + r\omega_e). \tag{35}$$

The time taken is different from before, but it is still constant, and therefore the difference has no effect on the faithful transmission of the *rate* of the clock.

The difference in time, $4\pi r^2\omega/c^2$, between (34) and (35) corresponds to the time difference that produces a displacement of fringes in the Sagnac experiment.[4]

It is clear that a light path along any other great circle would also occupy a constant coordinate time and thus lead to zero red shift.

The quantities $2MGX/c^2R^2$ and $-2R\omega^2X/c^2$ in (30) are essentially the quantities representing the gravitational and motional contributions (1) and (7) when $X=r$. This justifies the use of the simple case in the body of the paper, for the arguments concerning the effects on (1) and (7) can be applied to the above terms in (30) and will lead to the same conclusions, except that when applied to (30) they yield conclusions that are seen to be valid for the general case of a correctly rotating earth, and for every given great-circle light path around the earth.

In connection with the null effect of shielding discussed in Sec. III, we can here argue that if there were such shielding there would still be some point, on the line joining the centers of the sun and the earth and within the earth, at which the solar gravitational and centrifugal forces cancelled. We could have transformed to this point as origin instead of to C, and the same calculations would then have shown that the new reference system was essentially Minkowskian to well beyond the surface of the earth, so that the shielding would not disturb the null shift. Though the center of the rotating earth would not be at the spatial origin of this Minkowskian frame but at some other point having constant coordinates, that would not affect the arguments concerning the addition of the constant terrestrial gravitational potential at the earth's surface and the lack of effect of the earth's diurnal rotation on the noon-midnight shift.

[4] For references see W. Pauli, *Theory of Relativity* (Permagon Press, New York, 1958), p. 18, footnote 51.