On Compatibility of Covariance to the Equivalence Principle

and

Space-Time Coordinate Systems

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Abstract

Einstein makes clear that a space-time coordinate system must satisfy the equivalence principle. He further demonstrated the meaning of this principle by explicit calculations. However, Einstein's equivalence principle is often incorrectly substituted with a mathematical requirement of only an appropriate metric signature. Consequently, the notion of gauge is currently based on diffeomorphism, but ignores the equivalence principle. This inadequacy has its root on Einstein's covariance principle and leads to theoretical inconsistency in physics. It is shown that his covariance principle, in its original form, not only has *never* been established but also disagrees with experiments. Moreover, the assumption that any Gauss system is valid as space-time coordinates, is *incompatible* with the equivalence principle. Thus, defining the light speed in terms of local Minkowski spaces in a manifold is not valid in mathematics and misleading in physics. In conclusion, the general mathematical covariance must be *restricted* with the equivalence principle. Due to this physical restriction, the requirement on light speeds now becomes covariant. It is demonstated also that relativistic causality can be used as a convenient criterion for the validity of the equivalence principle.

"As far as the prepositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." -- A. Einstein (in 'Geometry and Experience', 1921).

1. Introduction.

Einstein [1] forever altered our thinking about space, time and the universe. Nevertheless, not only is the theory of relativity not yet complete [1,2], but also the self-consistency of his assumptions remain to be deliberated [2,3]. In general relativity [1,4], there are three basic assumptions namely: 1) the principle of equivalence; 2) the principle of covariance; and 3) the field equation whose source would be subjected to modification. Being in a tensor form, the field equation is clearly compatible with the principle of covariance. However, it was not clear whether the principle of equivalence is always compatible with the other assumptions. It has been shown that the principle of equivalence may not be consistent with the field equation if its source is inappropriate; and this implies a necessary modification of its source [5].

Moreover, the equivalence principle can be incompatible with a solution of the Einstein equation even if the source is valid in physics [3], i.e., the equivalence principle may not always compatible with the covariance principle. This was somewhat surprising since, it seems, their compatibility had been proven in mathematics. It is well known that the equivalence principle relates a physical space-time to the local Minkowski spaces; and, for a Reimannian space with the proper signature, mathematically there always exist local Minkowski spaces. The reason for the existence of such an incompatibility is due to the fact that in a Reimannian space, a local Minkowski space may not be obtained by free falling [3]. This suggests that the covariance principle should be examined since the equivalence principle is the essence of general relativity [6].

The covariance principle is also related to the current notion of gauge [7-10] which can be defined in terms of the mathematical diffeomorphism [9]. However, such a gauge actually provides no assurance to the physical reality (see §§ 5 & 6) because, unlike in Einstein's calculation (§§ 3 & 4), the equivalence principle is ignored in this notion. Consequently, there are contradictory claims in current theory of relativity.

For example, the exchange of the time coordinate and a space coordinate is a diffeomorphism. According to this, Hawking [11] claims in his book that "In relativity, there is no real distinction between the space and time coordinates just as there is no real difference between any two space coordinates." Nevertheless, from physics he also realizes and writes in the same book that "something that distinguishing the past from the future, giving a direction to time". Another simple example is that the Galilean transformation, being a diffeomorphism, would be considered as "valid" according to this notion of gauge (see also §5). Note that, to establish special relativity, we have proved that such a transformation is not valid according to experiments.

Moreover, this notion of gauge also causes theoretical difficulties. It has been proven recently [2] that this notion of gauge is obstructive to the developments of relativity since such a notion is the major obstacle in resolving the inconsistency in the derivation of Einstein's radiation formula, which has been verified by the famous Taylor-Hulse experiment [12] on binary pulsar PSR 1913+16. In addition, Bonnor et al. [13] discover that solutions such as of Schwarzschild, Curzon and Kerr are endowed with bizarre topologies which are unrealistic and the gauge can lead to inconsistent interpretations. In this paper, it will be shown that such a notion of gauge is related to insufficient understanding of the physics related to the equivalence principle.

To this end, one must distinquish the space-time coordinates of physics from a mathematical coordinate system merely for calculation. Whereas a mathematical coordinate system can be an arbitrary Gauss coordinate system; a space-time coordinate system must be physically realizable (see §§ 3-6) and a space coordinate and the time coordinate must have distinct physical meaning. Moreover, if there is no realizable space-time coordinate system for a solution, this means that the solution or even the equation is not valid in physics. Accepting any solution of the Einstein equation without examining its physical validity is currently a major problem [14]. Thus, this investigation is a crucial step in bring general relativity back to physics.

In this paper, it will be shown that these confusion and inconsistency can be removed in general relativity. To clarify these existing theoretical problems, one must first trace back to the principle of covariance.

2. On Covariance and the Question of Gauge in Relativity

The principle of covariance states [4] that "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." In practice, there are two aspects of the covariance principle: 1) the mathematical formulation in terms of the Riemannian geometry; and 2) the general validity of any Gauss coordinate system as a space-time coordinate system in physics.

To establish the need of Riemannian geometry, Einstein [4] argued "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion." Thus, the inertial systems of reference must be extended. Then, based on a rotating coordinate system, Einstein shows that the rate of a clock can depend upon where the clock may be and that the length of a measuring-rod can depend on the location as well as the orientation of the rod. He thus concludes that "In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial co-ordinates can be directly measured by the unit measuring-rod or differences in the time co-ordinate by a standard clock." But, these arguments are insufficient to conclude the validity of aspect 2) since some Gauss coordinate systems may not be physically realizable, and therefore, no physical measurements can be made with such co-ordinates.

Nevertheless, Einstein believed that any Gauss co-ordinate system is also equivalent to a space-time co-ordinate system in physics. His arguments [4] are as follows:

> "That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

However, to establish the validity of the Gauss systems, his eloquent argument is inadequate. As Eddington [15] pointed out, "space is not a lot of points close together; it is a lot of distances interlocked." For, while all verifications indeed amount to a determination of space-time coincidences, in order to ***predict*** theoretically of such coincidences, one must able to relate events of different locations in a definite manner. Thus, a coördinate system must be related to objective physical measurements. Moreover, as shown by Einstein himself, there are physical properties which imply the invalidity of some Gauss systems (see also §§ 3-6).

For instance, the equivalence principle actually restores the physical objectivity of space and time. Based on Einstein's equivalence principle, the behaviour of rods and clocks in a gravitational field are manifestations of such objectivity. As a result, we observe the gravitational red shifts which relate events of different locations. Such a prediction is possible because the equivalence principle does relate events of different locations. Thus, a space-time coordinate system is not just a Gauss system. On the contrary, ***a Gauss system can be incompatible with experiments*** if the equivalence principle is ***not*** satisfied (see §§ 5-7).

Kretschmann [16] pointed out in 1917 that the postulate of general covariance does not make any assertions about the physical content of the physical laws, but only about their mathematical formulation; and Einstein [17] entirely concurred with this view. As Pauli [6] pointed out "The general covariant formulation of the physical laws acquires a physical content only through the principle of equivalence in consequence of which gravitation is described solely by the (metric) g_{ik} and these latter are not given independently from matter, but are themselves determined by field equations."

In conclusion, while the mathematical formulation can be compatible with the equivalence principle, some coordinate systems and therefore some coordinate transformations are incompatible with the equivalence principle. Thus, the physics of the covariance principle lies not only in the validity of aspect 1) but also the restriction on aspect 2), the conditional validity of a Gauss system in physics (see also §§ 5 & 7).

In Einstein's later year, his viewpoint seems to have changed. Einstein [18] wrote in 1934 "The metrically real is now only given through the combination of the space-time coordinates with the mathematical quantities which describe the gravitational field." and in 1954 [19] "For the functions $g_{\mu\nu}$ describe not only the field, but at the same time also the topological and metrical structural properties of the manifold." In Einstein's calculation [1,4] of the light speeds, a coordinate system clearly provides more than an identification. Moreover, if space and time coordinates really have no physical objectivity, then how can ds^2 ($= g_{ab}dx^a dx^b$) have any physical reality? As pointed out by Weinberg [7], the purely mathematical viewpoint based on geometry alone may not be appropriate.

If the covariance principle should be restricted because of the equivalence principle, the current notion of gauge is inadequate. This notion of gauge can be based on only the mathematical diffeomorphism [9], which is a one-one, onto, C^∞ (infinitely differentiable) map between manifolds and its inverse map is C^∞. It was also claimed that two differeomorphic manifolds have physically identical properties. But, Bonnor et al. [13]

found that such a coordinate freedom leads to difficulties in physical interpretations. Moreover, according to this notion, coordinates cannot have any physical meaning other than identifying space-time points so that neighboring events are associated with neighbouring values of the coordinates. Thus, a space-time coordinate system is incorrectly reduced to just a mathematical Gauss system (see also §§ 3-5).

Historically, as shown by Hilbert [20], the Einstein equation of 1916 version,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - KT_{\mu\nu} , \tag{1}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the space-time metric, $T_{\mu\nu}$ is an energy-stress tensor of matter, and K is the coupling constant, is insufficient to obtain a solution. Because of the identity,

$$\nabla^{\mu} G_{\mu\nu} \equiv 0, \tag{2}$$

four more conditions are needed. These conditions, which result in a choice of coordinates, is called the gauge. But, because a physical space must satisfy the equivalence principle, the choice is actually restricted.

Nevertheless, Einstein considered, as the peers of his time, that such a choice is arbitrary. He wrote [21] in 1950, "Free choice with respect to the coordinate system implies that out of the ten functions of a solution, or components of the field, four can be made to assume prescribed values by a suitible choice of the coordinate system." It should be noted, however, that the full extend of such a freedom of choice has never been demonstrated with examples to illustrate its physical meaning. The reason is simply, as pointed out by Bonnor et al. [13], that such a coordinate freedom is a problem for a consistent physical interpretation.

An often used condition is the harmonic gauge $\Gamma^a = g^{bc}\Gamma^a{}_{bc} = 0$ [7], or equivalently

$$\frac{\partial}{\partial x^a} (|g|^{½} g^{ab}) = 0. \tag{3}$$

Note that this gauge can be incompatible with the equivalence principle [3,5]. However, it should also be noted that Einstein is the first to recognize the need of caution in the usage of a gauge. When applying the linearization of gauge (3), Einstein [1] wrote in his book, ***The Meaning of relativity*** "... there are still four conditions to which the $\gamma_{\mu\nu}$ (= $g_{\mu\nu} - \eta_{\mu\nu}$) may be subjected, provided these condition do not conflict with the condition for the order of magnitude of the $\gamma_{\mu\nu}$." However, this caution is not provided in current text books although such a conflict does exist for some cases [3].

Moreover, it is because of the equivalence principle that the geodesic equation,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \quad (4)$$

where $\Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\alpha g_{\nu\beta})g^{\mu\nu}/2$, can be considered as the equation of motion for a particle. Recently, it is founded that eq. (1) must be modified [2,5], but the required modification does not affect the validity of eq. (4). Thus, the equivalence principle is and remains an integral part of general relativity.

The gauge in electrodynamics is often used as an analogy due to their mathematical similarities. However, such a gauge is chosen in a fixed four-dimensional coordinate system; whereas in general relativity a gauge is a choice of coordinates. Thus, these two kinds of gauge are different in physics (see also [3]).

3. Einstein's Viewpoints on the Space-Time Coordinate Systems

Einstein's viewpoints on the space-time coordinates have changed over years. If we pay more attention to Einstein's subtle changes of view, I believe, the progress of general relativity would be much better than as to-day. A clear example to support the belief of Einstein's change is his view on the equivalence principle. In 'Relativity and Problem of Space (1954)', Einstein [19] added the crucial phrase, "at least to a first approximatioin" on the indistinguishability between gravity and acceleration. In view of these facts, one should take from the viewpoints of his later years as Einstein's viewpoint on a subject.

If Einstein's arguments at the earlier stage of his theory is not so perfect, Einstein seldom allowed such defects be used in his practice, i.e., calculations. In other words, unlike his arguments, his calculations are self-consistent. This is evident in his book, "The Meaning of Relativity" which he edited in 1954. Thus, we can actually obtain his final viewpoints from his calculations in this book. Einstein's viewpoints are:

1) A physical coordinate system must be physically realizable.

Einstein [22] made clear in ***What is the Theory of Relativity***? (1919) that "In physics, the body to which events are spatially referred is called the coordinate system." Furthermore, Einstein wrote "If it is necessary for the purpose of describing nature, to make use of a coordinate system arbitrarily introduced by us, then the choice of its state of motion ought to be subject to no restriction; the laws ought to be entirely independent of this choice (general principle of relativity) [22]. Thus, Einstein's coordinate system has a state of motion and is usually referred to a physical body. Although Einstein presented the abstract Gauss system, the system used in his calculation is alway referred to a body. Moreover, his calculations would not be self-consistent if an arbitrary Gauss system were used (see §5).

2) A physical coordinate system is a Gauss coordinate system in which the equivalence principle satisfies. In current text books of general relativity [7-10], any Gauss system is accepted as a space-time co-ordinate system. One might attempt to justify this viewpoint by pointing out that Einstein also wrote in his book [1] that "In an analogous way (to Gauss curvillear coordinates) we shall introduce in the general theory of relativity arbitrary co-ordinates, x_1, x_2, x_3, x_4, which shall number uniquely the space-time points, so that neighbouring events are associated with neighbouring values of the coordinates; otherwise, the choice of co-ordinate is arbitrary." But, Einstein [1] qualified this with a physical statement that "In the immediate neighbour of an observer, falling freely in a gravitational field, there exists no gravitational field." Later, the usage of the equivalence principle is demonstrated.

3) The equivalence principle requires not only, at each point, the existence of a local Minkowski space

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2, \tag{5}$$

but it must be obtained by free falling (see also §§ 4 & 5).

Einstein [1] wrote, "According to the principle of equivalence, the metrical relation of the Euclidean geometry are valid relative to a Cartesian system of reference of infinitely small dimensions, and in a suitable state of motion (free falling, and without rotation)." Thus, in addition to having the proper metric signature, a solution of eq. (1) (which is approximately valid for the case considered [2]) must satisfy the equivalence principle as an independent physical requirement.

Note that the existence of local Minkowski spaces obtainable by the "free falling" is a physical requirement.

4. Einstein's Illustration of a Physical Space

The above three points are used by Einstein [1]. For instance, he considered the linearization of eq. (1)

$$\frac{\partial^2\gamma_{\mu\nu}}{\partial x_\alpha{}^2} = 2K\ (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}T), \tag{1a}$$

where $\gamma_{\mu\nu}$ (= $g_{\mu\nu} - \eta_{\mu\nu}$) is the deviation from the flat metric $\eta_{\mu\nu}$, with the linearized gauge condition. (Note that equation (1a) can be justified with physical considerations [2,15], which are independent of the harmonic gauge.) Then, he illustrates the meaning of the equivalence principle by considering the metric

$$ds^2 = c^2(1 - \frac{K}{4\pi} \int dV_0 \frac{\sigma}{r})dt^2 - (1 + \frac{K}{4\pi} \int dV_0 \frac{\sigma}{r})(dx^2 + dy^2 + dz^2), \tag{6}$$

which is obtained from eq. (1a). According to eq. (6), since it satisfies the equivalence principle, the "free falling" local Minkowski coordinate system implies that the unit measuring rod has the coordinate length

$$(1 - \frac{K}{8\pi} \int dV_0 \frac{\sigma}{r}) \tag{7a}$$

in respect to the system of coordinate selected and the interval between two beats of the unit clock corresponds to the "time"

$$(1 + \frac{K}{8\pi} \int dV_0 \frac{\sigma}{r}) \tag{7b}$$

in the unit used in the system of co-ordinates. The law of the propagation of light in general co-ordinates is, according to general theory of relativity, characterized, by the light-cone condition,

$$ds^2 = 0. \tag{8}$$

Then, the velocity of light is expressed in our selected coordinates by

$$\frac{[dx^2 + dy^2 + dz^2]^{1/2}}{dt} = c(1 - \frac{K}{4\pi} \int dV_0 \frac{\sigma}{r}). \quad (9)$$

One can therefore draw the conclusion from this, that a ray of light passing near a large mass is deflected [1]. Thus, Einstein has demonstated that a coordinates system must have a physical meaning and that there is a definite physical difference between a space coordinate and a time coordinate.

Moreover, it should be noted that a crucial condition for the above calculation is the satisfaction of the equivalence principle (see eqs. [6] & [7]). If the metric did not satisfy the equivalence principle, $ds^2 = 0$ would lead to an incorrect light velocity because the space is not physically valid. In addition, Einstein's calculational approach would lead to contradictory results. To illustrate these, it will be shown in next section that an arbitrary Gauss system as a space-time coordinate would lead to inconsistency in physics.

5. The Validity of a Metric in Physics and the Equivalence Principle

A major problem in current theory of general relativity is its inability to distinguish a physical space from merely a mathematical manifold [14]. A given metric defines a physical space only if the metric is physically realizable. For clarity we first discuss cases without gravitational forces because eq. (1) must be modified [2,5] when dynamic gravity is present. Let us illustate the above problem by considering the metric,

$$ds^2 = \alpha^2 dt^2 - dx^2 - dy^2 - dz^2, \quad (10)$$

where α ($\geq 2c$) is a constant. Metric (10) is a solution of the Einstein equation $G_{\mu\nu} = 0$. Then, $ds^2 = 0$ implies that the velocity of light is α. One might argue that metrice (10) can be transformed to

$$ds^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2, \quad (11)$$

by the following diffeomorphism,

$$x' = x, \quad y' = y, \quad z' = z, \quad \text{and} \quad t' = t\alpha/c. \qquad (12)$$

Eq. (12) implies, however, that the units of t and t' are distinct and the light speed remains α but ***not*** c.

If one examines the rate of a unit clock, which is arranged to be at rest in the gravitational field (10), then according to Einstein [1,4], for a clock period ds = 1; dx = dy = dz = 0, one has $1 = g_{tt}\, dt^2$. Hence, $dt = 1/\alpha$. It follows that the light speed is α since there is no contraction for the space coordinates.

Eq. (12) is not a recaling since all the physical units remain the same. In a rescaling only the physical units, but not the physics, are changed. For example, the light speed can be expressed as 1 lightyear per year or 186,000 miles per second. However, if $\alpha = 2c$, metric (10) implies that the light speed would be 2c, i.e., 372,000 milles/sec; and metric (11) implies that the light speed is 186,000 miles/half-sec. Thus, if metric (11) were considered as Minkowski, the diffeomorphism (12) would amount to redefining the space.

Now, to see further that the equivalence principle is needed for the self-consistency of general relativity, let us consider another constant metric,

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2, \qquad (13)$$

since any constant metric satisfies the Einstein equation $G_{\mu\nu} = 0$. Then, for light rays in the z'-direction, $ds^2 = 0$ would imply the light speeds were

$$\frac{dz'}{dt'} = c + v, \quad \text{or} \quad \frac{dz'}{dt'} = -c + v. \qquad (14)$$

Clearly, eq. (14) also does not give a correct light speed since (14) violates relativistic causality, i.e. no cause event can propagate faster than the velocity of light in a vacuum.

Moreover, if one examines the rate of a unit clock, which is arranged to be at rest in the static gravitational field (13), then for a clock period ds = 1, one has $1 = g_{tt}\, dt'^2$ and thus $dt' = 1/(c^2 - v^2)^{1/2}$. But, if one examine a uni-measure of lenght laid "parallel to the z'-axis, according to Einstein [1,4] one should have to set $ds^2 = -1$; dx' = dy' = dt' = 0. Therefore, $-1 = g_{zz}\, dz'^2 = -dz'^2$, i.e. there is no coordinate contraction. Then, the light speeds in the z'-direction would be $\pm c(1 - v^2/c^2)^{1/2}$. This is not even in agreement with $ds^2 = 0$. Thus, if metric (13) were valid in physics, Einstein's method would be wrong .

To resolve the above problem, let us check whether the equivalence principle is satisfied. Since (10) and (13) are constant metrices, all their Christoffel symbols $\Gamma^{\mu}{}_{\alpha\beta}$ are zero. Therefore, the manifolds are flat (i.e., $R_{\alpha\beta\mu\nu} = 0$) and, according to the geodesic equation (4), there is no gravitational acceleration. Then, in a non-rotating free falling, the velocity of an observer is a constant. According to special relativity, this observer carries with himself a new coordinate system which must be obtained by a Lorentz transformation. But, a Lorentz transformation cannot transform metric (10) nor (13) to a local Minkowski space. Thus, the equivalence principle is not satisfied. In conclusion, ***the cause of an incorrect light speed and other inconsistancy is identified as due to the failure of satisfying the equivalence principle.***

Moreover, metric (13) is obtained from the flat metric ($ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$) by

$$x' = x, \quad y' = y, \quad z' = z + vt, \quad \text{and} \quad t' = t, \tag{15}$$

where t is the time coordinate and v is a constant. The Galilean transformation (15) is clearly a diffeomorphism. But, one cannot justify transformation (15) as physical since metric (13) is ***not*** physically realizable as verified by experiments. In other words, the prime system (13) is merely a manifold in mathematics whose coordinate system has no physical meaning.

In conclusion, the Galilean transformation (13) is also not valid in general relativity. Due to the requirement of the equivalence principle, a diffeomorphism may not be adequate in physics (see also §§ 6 & 7) although such a diffeomorphism could be useful for the purpose of calculation in mathematics. But, whatever mathematical coordinate systems one may use in calculations, one must first verify that the equivalence principle is valid in a space-time coordinate system on which physical interpretations can be based. Otherwise, as Bonnor [13] has discovered that unrealistic statements would be concluded (see also §7).

Note that the above manifolds are diffeomorphic to physical spaces. Thus, a diffeomorphism could lead to an unphysical four dimensional manifold. Moreover, a diffeomorphism would create an illusion which would lead one to consider incorrectly an unphysical manifold as a physical space. Currently, such an illusion is accomplished in the text books by "defining" the light speed in the local Minkowski space. This is one of the manner that the equivalence principle is practically ignored.

6. An Invalid Mathematical "Definition" of the Light Speed

To circumvent the problem of whether a manifold is a valid physical space (i.e., whether the equivalence principle is valid) and to avoid addressing the issue of light speeds in a coordinate system, some theorists "define" a light speed at any point of a manifold in terms of a local Minkowski space. (According to this, both metrices (10) and (13) would be valid, since the physical essence is taken away from general relativity.) Another seemingly benefit is to have a covariant expression for the physical requirement of light speeds. But, such a "definition" is not valid in mathematics because it not well-defined. For, such a definition would be valid only if the manifold is a physical space. For such a case, it defines a local light speed [10].

However, if the manifold is not a physical space, a light speed of a local Minkowski space is not only meaningless, but misleading. If a manifold is only diffeomorphic to a physical space, such a method would amont to redefining the physical space. As illustrated, manifold (10) is diffeomorphic to a Minkowski space, but its "light speed" is larger than 2c. More important, there are manifolds any of which is not diffeomorphic to a physical space. Then, such a definition would fail to identify an invalid source [3]. (As Kramer et al. [14] pointed out, almost any metric could be considered as a solution if the source is arbitrary.)

An example of such an intrinsically unphysical manifold has a metric as follows:

$$ds^2 = du\,dv + h_{ij}(u)x^i x^j\,du^2 - dx^i\,dx^j , \qquad (16)$$

where $u = t - z$, $v = t + z$ (the light speed in a vacuum is denoted as 1), $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$. Its physical cause can be an electromagnetic plane wave [23]. This metric clearly violates the principle of causality [3] since the gravitational force (related to $\Gamma^z{}_{tt} = (1/2)\partial(h_{ij}x^i x^j)/\partial t$) has arbitrary parameters (the coordinate origin). Metric (16) also does not satisfy relativistic causality (see §7). It has been identified that this failure of satisfying the equivalence principle is due to the source is inadequate [5].

Another problem of such a "definition" is that it would help the failure in identifying the valid range of

Einstein equation (1) (see § 7). Weisskopf [24] pointed out that "The existence of black holes follows from an extrapolation of Einstein's theory of gravity by many orders of magnitude beyond the range for which its validity has not yet been established beyond doubt." If the equivalence principle were well understood, the invalidity of equation (1) for very strong gravity would have been recognized long time ago (see also §7)!

7. Relativistic Causality as a Criterion for a Physical Space

Whereas in a Minkowski space the light speed is a constant c; when gravity is present, the light speed is smaller than c due to the gravitational effects of space constraction and time dilation (see eqs. [7] - [9]). Therefore, *relativistic causality* (i.e., the light speed c is the maximum velocity of propagation for any event in a physical space) can be used as a convenient criterion. This is particularly useful to circumvent the laborious process, if only the necessary condition for the equivalence principle are needed to be considered.

Since the principle of equivalence implies relativistic causality, a physical space must satisfy relativistic causality. Obviously, the requirement that a speed of light is at most c may not be valid for some diffeomorphic manifolds. ***Nevertheless, the requirement of covariance is compatible with relativistic causality since its violation means that the choice of coordinates is not valid in physics.*** This also means that the physics of light speeds is now a covariant requirement which is not possible previously.

Now, let us consider metric (16). For the z-directional light speeds, $ds^2 = 0$ implies

$$(dt - dz)([1 + H]dt + [1 - H]dz) = 0, \quad \text{where} \quad H = h_{ij}x^i x^j \tag{17a}$$

Thus

$$dz/dt = 1, \quad \text{or} \quad dz/dt = -(1 + H)/(1 - H). \tag{17b}$$

Then, relativistic causality requires $H < 0$. This is impossible.

For an isolated static system of matter, the principle of causality requires the flat metric to be the asymptotic limit. Relativistic causality would be satisfied by the exterior Schwarzschild solution [6-10,22],

$$ds^2 = (1 - \frac{C}{r})dt^2 - (1 - \frac{C}{r})^{-1}dr^2 - r^2 d\Omega^2, \tag{18}$$

where C ($= KM/4\pi$) is a positive constant, $d\Omega^2 = (d\theta^2 + \sin^2\theta \, d\phi^2)$, and (r,θ,ϕ) are spherical coordinates. Thus, for $r > C$, the light speeds in the r–direction and θ–direction are respectively,

$$\frac{dr}{dt} = \pm \left(1 - \frac{C}{r}\right), \quad \text{and} \quad \frac{r d\theta}{dt} = \pm \left(1 - \frac{C}{r}\right)^{1/2} . \tag{19}$$

Eq. (19) shows that light speeds are slower. For $r \leq C$, this is usually the interior region of a star.

If the star were so compact that the exterior included $r<C$, then $rd\theta/dt$ is imaginary, and $|dr/dt|>1$ would be possible. One may argue that eq. (19) is no longer valid since both g_{tt} and g_{rr} change sign; t becomes space–like and r becomes time–like. However, this also mean that the metric (18) is neither static nor spherically symmetric. Moreover, Bonnor [13] pointed out, "This is the region of final approach to the black hole which is supposed to exist at $r = 0$; notice that this (physical) singularity is space–like." Thus, solution (18) actually supports Einstein's remark that his equation may not be valid for very compact objects [1].

When $r = C$, since $g_{tt} = 0$ and $g_{rr} = \infty$, these points are called the "Schwarzschild Singularity" [7–9]. Mathematically, it is obvious that a diffeomorphic coordinate transformation cannot remove a singularity. Nevertheless, some theorists claimed that these singularities were "removable" by a coordinate transformation. Now, let us examine the validity of such a claim through analyzing the commonly accepted transformation,

$$\left(\frac{r}{C} - 1\right)e^{r/C} = X^2 - T^2, \qquad \frac{t}{C} = \ln\left(\frac{T+X}{X-T}\right) = 2\tanh^{-1}(T/X) \tag{20}$$

which gives

$$ds^2 = \frac{4C^3}{r} e^{-r/C} (dT^2 - dX^2) - r^2 d\Omega^2. \tag{21}$$

The Kruskal extension (21) has no singularity. But, (20) is not one–one at ($X = 0$ and $T = 0$), and it is not diffeomorphic in the set ($r=C$, and $t=3$). From eq. (20), the set of points ($T^2-X^2 = 0$; $X \neq 0$) corresponds to $r = C$, and $t = \pm \infty$. So, the singularity has not been eliminated, but is actually hidden by the transformation. Thus, the Schwarzschild singularity is real [25,26] and the Kruskal extension is unrealistic [13].

It seems, these symptoms manifest that the Schwarzschild singularity is not realizable as Einstein advocated and a limitation for eq. (1), is the exterior $r > C$. (According to the perfect fluid model, an interior solution requires the radius of a star, $R > 9C/8$ [9]. The proponents of black holes argued, however, when the interior pressure p is insufficient to resist gravity, a star would collapse to zero size. But, such an argument actually depends on many unverified assumptions [27]. Among these assumptions, that mass and energy are unconditionally equivalent and that eq. (1) is valid for dynamic problems, are incorrect [2,28].) Note that all the extra labors would have been saved if one uses the relativistic causality to examine metric (18).

let us examine, in terms of relativistic causality and the principle of causality, a slightly different metric,

$$ds^2 = \alpha^2(1 - \frac{C}{r})dt^2 - (1 - \frac{C}{r})^{-1}dr^2 - r^2d\Omega^2, \tag{22}$$

where α ($\neq 1$) is a positive constant. Then, for this static metric, the light speeds in the r-direction are

$$\frac{dr}{dt} = \pm\, \alpha(1 - \frac{C}{r}). \tag{23}$$

If $\alpha > 1$, for a sufficiently large r, the light speeds would be faster than 1, and relativistic causality is violated. If $\alpha < 1$, although the light speeds are smaller than 1, the flat metric is not the asymptotic limit.

8. Conclusions and Discussions

In conclusion, the Einstein equation, the equivalence principle, and covariance are independent physical requirements. Although a diffeomorphism is compatible with covariance, the equivalence principle may be violated. If a coordinate transformation results in a manifold not satisfying the equivalence principle, theoretical inconsistency and disagreements with experiments would follow [2,3]. As Einstein [1] illustrated, a satisfaction of the equivalence principle is necessary such that the light speeds can be calculated with the light-cone condition. Eddington [15] also did not accept the gauge notion related to an arbitrary Gauss system.

A consequence of the clarification of this paper is that, as already mentioned in § 7, the current notion of black holes and singularity is clearly in violation of the principle of equivalence and is therefore not valid in general relativity. This notion is associated with the concept that everything is attracted by gravity. Such a concept is based on the misunderstanding [28] that $E = mc^2$ implies the unconditional equivalence of mass and energy. Historically, this notion was originated from the dark star [29] based on Newtonian theory.

Nevertheless, theorists such as Synge [30] advocated explicitly that the basis of general relativity should be the Einstein field equation alone rather than the equivalence principle. However, theoretically, as pointed out by Klein [31], there is no satisfactory proof of rigorous validity of Einstein's field equation; and experimentally, as pointed out by Weisskopf [25], the validity of Einstein's equation has not yet been established beyond doubt. In fact, the invalidity of Einstein's equation for two-body problems was conjectured by Hogarth [32] in 1953; and Einstein himself had pointed out that his equation may not be valid for matter of very high density [1]. Moreover, it has been proven experimentally that Einstein's equation must be modified. But, the equivalence principle remains indispensible because of its solid experimental foundation [2,33].

Some theorists believe the issue of whether a manifold is physical and a coordinate system is realizable could be circumvented. They instead "define" the light speed for any point with a local Minkowski space. However, such a "definition" is not valid since there are manifolds with proper signature, which cannot be transformed to physical spaces [3]. Thus, such a "circumvention" is also misleading. Note that the prevailing belief that a gauge is arbitrarily applicable [34], has been proven to be incorrect in general relativity [2].

A Galilean transformation is not valid in physics because it results in disagreement with experiments and theoretical incompatibility with the equivalence principle. In general, the equivalence principle restricts the general covariance and valid space-time coordinates. Thus, from the standpoint of physics, it is necessary to reduces the covariance to among valid physical coordinate systems only [35]. This ***restricted*** mathematical covariance should be called the ***physical-covariance*** to distinguish the mathematical general covariance.

Due to this physical restriction, the light speed requirement now becomes covariant (see § 7). Only then, general relativity becomes completely and truely a covariant theory. Moreover, just as in special relativity, once the space coordinates are chosen, the time coordinate is fixed. Thus, without mentioning the time coordinate, Einstein [22] can identify the coordinate system with a referring body only.

From the viewpoint of physics, however, there is actually no restiction since the equivalence principle is satisfied by all physical spaces. Therefore, the theory remains rightfully as general relativity. In conclusion, the mathematical restriction due to the equivalence principle reduces the excess unphysical transformations; and brings general relativity back, as Einstein originally intended, a true covariant physical theory.

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