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# Farewell to General Relativity

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**Abstract**

The kinematical successes of general relativity are legendary: the perihelion precession, the gravitational red-shift, the bending of light. However, at the level of dynamics, relativity is faced with insurmountable difficulties. It has failed to define the energy, momentum, and stress of the gravitational field. Moreover, it offers no expression of energy-momentum transfer to or from the gravitational field. These are symptoms of a far graver malady: general relativity violates the principle of energy conservation.

In general relativity, the equation of planetary motion is derived by means of the geometric variation

$$\delta \int ds = \delta \int \sqrt{g_{\mu\nu} dx^\mu dx^\nu} = 0 \tag{1}$$

This yields the geodesic equation

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0 \tag{2}$$

The equation of motion can be recast in terms of kinematics, by introducing a vector basis $\mathbf{e}_\mu$ and the velocity four-vector $\mathbf{u} = \mathbf{e}_\mu u^\mu$. The infinitesimal change of the basis is expressed in terms of connection coefficients

$$d\mathbf{e}_\mu = \mathbf{e}_\lambda \, \Gamma^\lambda_{\mu\nu} \, dx^\nu \tag{3}$$

which enables the calculation

$$\begin{aligned} \frac{d\mathbf{u}}{ds} &= \left\{ \mathbf{e}_\mu \frac{du^\mu}{ds} + \frac{d\mathbf{e}_\mu}{ds} u^\mu \right\} \\ &= \mathbf{e}_\mu \left\{ \frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda \right\} \end{aligned} \tag{4}$$

This shows that $d\mathbf{u}/ds = 0$ along any geodesic path.

How do these formulae relate to the observed planetary motion? Expanding $\mathbf{u} = \mathbf{e}_0 u^0 + \mathbf{e}_i u^i$ we have

$$\frac{d(\mathbf{e}_0 u^0)}{ds} + \frac{d(\mathbf{e}_i u^i)}{ds} = 0 \tag{5}$$

This shows that, during geodesic motion, the rate of change of three-velocity $\mathbf{e}_i u^i$ is equal and opposite to that of speed $\mathbf{e}_0 u^0$. The rates are determined by (2) and (3)

$$\frac{d(\mathbf{e}_0 u^0)}{ds} = -\frac{d(\mathbf{e}_i u^i)}{ds} = -\mathbf{e}_0 \Gamma^0_{i\nu} u^i u^\nu + \mathbf{e}_i \Gamma^i_{0\nu} u^0 u^\nu \tag{6}$$

In flat spherical coordinates, the right-hand side of this equation is zero: both speed and velocity are constant. In Schwarzschild coordinates, the right-hand side is not zero: the speed and velocity continually change. Thus, according to the geodesic hypothesis, the changes which we observe in a planet's speed and velocity are due to the curved geometry of space-time.

The success of the geodesic formula was one of the great triumphs of twentieth-century physics. Yet, we know that a planet possesses dynamical properties of energy and momentum. What is taking place at this, the dynamical level? The energy-momentum vector of a planet is

$$\begin{aligned} \mathbf{p} &= m\mathbf{u} \\ &= \mathbf{e}_\mu p^\mu = \mathbf{e}_0 p^0 + \mathbf{e}_i p^i \end{aligned} \tag{7}$$

The first term is the (rest + kinetic) energy $\mathbf{e}_0 p^0$ while the second term is the momentum $\mathbf{e}_i p^i$. During geodesic motion, the energy-momentum of the planet is conserved

$$\frac{d\mathbf{p}}{ds} = m\frac{d\mathbf{u}}{ds} = m\mathbf{e}_\mu \left\{ \frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda \right\} = 0 \tag{8}$$

Therefore, the rates of change of energy and momentum are equal and opposite

$$\frac{d(\mathbf{e}_0 p^0)}{ds} = -\frac{d(\mathbf{e}_i p^i)}{ds} \tag{9}$$

Neither the planet's energy nor its momentum is conserved; rather, one continually transforms into the other. During orbital motion, the non-conservation of linear momentum is to be expected. What surprises is that energy conservation is violated. The energy principle forces us to abandon the geodesic hypothesis of planetary motion.

The treatment of light-rays is similar to that of particle motion, in that the bending of light and the gravitational red-shift are determined by the kinematics of curved space-time. The red-shift can be expressed in terms of the null four-vector

$$\mathbf{k} = \mathbf{e}_0 k^0 + \mathbf{e}_i k^i \tag{10}$$

where $\mathbf{e}_0 k^0$ is the frequency of light, and $\mathbf{e}_i k^i$ is its wave vector. Along any light ray $d\mathbf{k}/d\lambda = 0$ and we obtain

$$\frac{d(\mathbf{e}_0 k^0)}{d\lambda} = -\frac{d(\mathbf{e}_i k^i)}{d\lambda} = -\mathbf{e}_0 \Gamma^0_{i\nu} k^i u^\nu + \mathbf{e}_i \Gamma^i_{0\nu} k^0 u^\nu \tag{11}$$

Therefore, in the presence of space-time curvature, the frequency and wavelength will vary from point to point along the light ray. This is the gravitational red-shift.

The energy-momentum vector of a light complex is given by the quantum formula

$$\mathbf{p} = \hbar\mathbf{k} \tag{12}$$

Once again, energy-momentum is conserved $d\mathbf{p}/d\lambda = 0$ and

$$\frac{d(\mathbf{e}_0 p^0)}{d\lambda} = -\frac{d(\mathbf{e}_i p^i)}{d\lambda} \tag{13}$$

Thus, as frequency and wavelength change, the energy and momentum transform into one another; neither is conserved.

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The above examples illustrate the violation of energy conservation during geodesic motion. We will now make use of the field equations to show that this problem is intrinsic to the theory, and stems from the fact that the gravitational field is incapable of exchanging energy-momentum with any physical field. The field equations are given by

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\kappa T^{\mu\nu} \tag{14}$$

where $T^{\mu\nu}$ is the stress-energy-momentum tensor of matter and electromagnetism. The covariant divergence of the left-hand side is identically zero, therefore

$$T^{\mu\nu}_{;\nu} = \frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}\,T^{\mu\nu}}{\partial x^\nu} + \Gamma^\mu_{\nu\lambda}T^{\nu\lambda} = 0 \tag{15}$$

Let us investigate this equation, by way of three examples from classical physics.

Consider a free electromagnetic field, with the energy tensor

$$T^{\mu\nu}_{e-m} = F^\mu_{\ \alpha}F^{\alpha\nu} + \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{16}$$

A lengthy but straightforward calculation yields

$$T^{\mu\nu}_{;\nu\ e-m} = \frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}\,F^{\alpha\nu}}{\partial x^\nu}F^\mu_{\ \alpha} + \frac{1}{2}g^{\mu\nu}F^{\alpha\beta}\left\{\frac{\partial F_{\beta\nu}}{\partial x^\alpha} + \frac{\partial F_{\nu\alpha}}{\partial x^\beta} + \frac{\partial F_{\alpha\beta}}{\partial x^\nu}\right\} \tag{17}$$

We note that the term $\Gamma^\mu_{\nu\lambda}T^{\nu\lambda}$ must be included, in order to obtain this covariant expression. Maxwell's equations for charge-free space are

$$\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}\,F^{\alpha\nu}}{\partial x^{\nu}} = 0 \tag{18}$$

$$\frac{\partial F_{\beta\nu}}{\partial x^{\alpha}} + \frac{\partial F_{\nu\alpha}}{\partial x^{\beta}} + \frac{\partial F_{\alpha\beta}}{\partial x^{\nu}} = 0 \tag{19}$$

and we obtain

$$T^{\mu\nu}_{;\nu\ \ e-m} = 0 \tag{20}$$

This result is especially significant, because it shows that there is no mechanism whatsoever for the exchange of energy-momentum between the electromagnetic and gravitational fields. The energy-momentum of electromagnetism alone is conserved.

Secondly, consider the matter tensor

$$T^{\mu\nu}_{m} = \rho u^{\mu}u^{\nu} \tag{21}$$

A simple calculation yields the covariant expression

$$T^{\mu\nu}_{;\nu\ \ m} = \rho u^{\nu}\frac{\partial u^{\mu}}{\partial x^{\nu}} + u^{\mu}\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}\,\rho u^{\nu}}{\partial x^{\nu}} + \Gamma^{\mu}_{\nu\lambda}\,\rho u^{\nu}u^{\lambda} \tag{22}$$

The second term is zero, if rest mass is conserved, leaving

$$T^{\mu\nu}_{;\nu\ \ m} = \rho\left\{u^{\nu}\frac{\partial u^{\mu}}{\partial x^{\nu}} + \Gamma^{\mu}_{\nu\lambda}\,u^{\nu}u^{\lambda}\right\} \tag{23}$$

The hydrodynamical form of the geodesic equation then gives

$$T^{\mu\nu}_{;\nu\ \ m} = 0 \tag{24}$$

The energy-momentum of matter alone is conserved.

Finally, consider the case of charged matter together with electromagnetism

$$T^{\mu\nu} = T^{\mu\nu}_{m} + T^{\mu\nu}_{e-m} \tag{25}$$

Here, coupling occurs via Maxwell's equation

$$\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}F^{\alpha\nu}}{\partial x^{\nu}} = -j^{\alpha} \tag{26}$$

and we obtain

$$T^{\mu\nu}_{;\nu} = \rho \left\{ u^\nu \frac{\partial u^\mu}{\partial x^\nu} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda \right\} - j^\alpha F^\mu_{\ \alpha} = 0 \tag{27}$$

The Lorentz force is covariant and describes the exchange of energy-momentum between matter and the electromagnetic field.

These examples show that whether space-time is curved or not, i.e., whether a gravitational field exists or not, the energy-momentum of matter and electromagnetism is conserved. It follows that any change wrought by curvature—in speed, velocity, frequency, and wavelength—will violate the principle of energy conservation. A gravitational exchange term is needed in order to account for the changes in energy and momentum. The theory of relativity neither provides such a term nor defines the energy, momentum, and stress of the gravitational field. If we adhere to the energy principle, then general relativity cannot be the answer to the question of gravitation.