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From the triangle Sagnac experiment to a practical, crucial experiment of the constancy of the speed of light using atomic clocks on moving objects

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Abstract. - According to the triangle Sagnac experiment, between point A and point B that are moving in a circular motion, the travel times for light or radio signals from A to B and from B to A are different. The difference Δt, i.e. the Sagnac effect, equals $2V_D L/c^2$, where D is the foot of the altitude to AB, V_D is the speed of point D and L is the distance from A to B. The Sagnac effect exists whether the radius of the circle is as small as only few centimeters, e.g., in fiber-optic gyroscopes, or as big as twenty thousand kilometers, e.g., in GPS. Therefore, if we mount an atomic clock and signal transmitter and receiver on each of two objects moving at the same speed in a circular motion (it is not necessary to synchronize the two clocks beforehand), we will find such a time difference. Practically, using sufficiently large L and V_D, this time difference can reach around 1 ns, which is relatively easy to detect with current technology. This experiment would yield both practical applications and theoretical implications. First, it can be used as a verification of Sagnac corrections in GPS. Second, a theoretical problem arises when these two objects change their paths to a straight line. Would the time difference still exist (then it contradicts the principle of the constancy of the speed of light) or does the time difference "jump" to zero? The result of the experiment will be of great interest.

Introduction. - In 1913, Sagnac [1] conducted an experiment which is now named after him. This experiment consists of a beam splitter and several mirrors mounted on a disk (fig. 1). The beam splitter divides the light beam into two portions; one traverses clockwise along the quadrilateral formed by the mirrors, the other counterclockwise. An interference pattern is formed when the beams unite. When the disk rotates clockwise with an angular velocity ω around its axis, there is a shift of fringes $\Delta N = 4\,\omega\, S_{ABCD}/c\lambda$ (S_{ABCD} is the area of the quadrilateral ABCD and λ the wavelength of light). If the disk rotates counterclockwise, the shift is in the opposite direction. The general expression of the Sagnac effect is $\Delta N = 4\,\omega\, S/c\lambda$, where S is the area enclosed by the light path and can be of any shape.

Since then, the Sagnac experiment has been conducted in many different ways. For example, the Michelson-Gale experiment [2] examined the effect of the rotation of the earth instead of

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the rotation of the disk, and the around-the-world Sagnac experiment [3] recorded signal arrival times with atomic clocks instead of the interferometers. The Sagnac effect has been applied to many systems ranging in size from a few centimeters, e.g., in fiber-optic gyroscopes [4], to twenty thousand kilometers, e.g., in GPS [5], but the interpretation of the Sagnac effect is still a very controversial topic. The simplest way to interpret the Sagnac effect is by the classical physics viewpoint as Sagnac himself did. Another popular way is to use General Relativity because rotation is involved [6]. Besides these two, there are several other ways to interpret it [7]. To emphasize the essence of the controversy, let us now examine two examples

First, a triangle Sagnac experiment. An observer at A will find a drift of the fringes $\Delta N = 4\omega S_{ABC}/c\lambda$ in the Sagnac experiment where the light path is an isosceles triangle (shown in fig. 2) when the disk rotates (the apex C being the center of rotation) comparing to when there is no rotation. That means, for the observer at A, the two light beams traveling in opposite directions do not return to the starting point at the same instant when the disk rotates and the time difference is $\Delta t = (t_{AC} + t_{CB} + t_{BA}) - (t_{AB} + t_{BC} + t_{CA}) = \Delta N\,\lambda/c = 4\omega S_{ABC}/c^2$. But the travel times along the radial legs AC and BC cannot differ. Hence we have $t_{BA} - t_{AB} = 4\omega S_{ABC}/c^2 = 4\omega(hL/2)/c^2 = 2V_D L/c^2$, where h is the height of the triangle L is the distance from A to B, D is the foot of the altitude to AB and V_D is the speed of point D. (In fact, this is true for any triangle, because the rotation will not change the travel time along its radial legs.) This means, for the observer at A, the light's travel times from A to B and from B to A are different when the disk rotates.

We can repeat the experiment while increasing the triangle's height (and probably with a slower angular velocity ω), the observer at A will always find the difference between the light travel times from B to A and from A to B, $t_{BA} - t_{AB} = 2V_D L/c^2$. However, when the height increases, the motion of AB gradually approaches a translational motion. According to Special Relativity, there must be no time difference between light beams traveling in opposite directions when AB makes a "purely" translational motion. So it seems there is a "jump" of the time difference from a "quasi"-translational motion to a "purely" translational motion

Second, the Sagnac corrections in GPS. The Sagnac effect in GPS (the signal travel time from a transmitter to a receiver changes due to their motions) is called the Sagnac delay or Sagnac correction [8]. It is thought that the need for Sagnac correction arises because of the motion of the receiver during propagation of the signal. Let there be a transmitter at position $\mathbf{r}_A$, a receiver at position $\mathbf{r}_B$ and the receiver has velocity $\mathbf{v}$, the Sagnac delay used is

$\Delta t_{Sagnac} = (\mathbf{r}_B - \mathbf{r}_A) \bullet \mathbf{v}/c^2 = \Delta\mathbf{r} \bullet \mathbf{v}/c^2$. (This is for one-way propagation, otherwise it should be doubled.) Suppose that the transmitter and the receiver are two satellites in the same orbit, and the distance between them, Δr, is 1000 km, then there will be a substantial Sagnac delay. As the distance between them decreases to 10 km, 100 m, and so on, the Sagnac delay also decreases, but it is always of a finite value. Now let the distance be 1 m, there should still be a finite Sagnac delay. In this case, would this phenomenon conflict with the principle of the constancy of the speed of light?

Recently, in a paper to interpret the Sagnac effect with non-zero photon mass, J. P. Vigier [9] stated that the Sagnac effect is an unsolved fundamental problem in physics. The best way to solve this problem is by conducting experiments. The following is a practical, crucial experiment based on the triangle Sagnac experiment mentioned above.

The clock-type experiment on a rotating disk. - Let us utilize clocks instead of the interferometer in the triangle Sagnac experiment. Mount an atomic clock and light pulse or radio signal transmitter and receiver on both A and B, and also a reflector on B to reflect the signal back to A. We will show later that it is not necessary to synchronize these two clocks beforehand.

Case 1: the disk does not rotate. Let a pulse be sent from A to B and reflected back to A. Because the speed of light is the same in the two different directions, we should have the following if the two clocks are synchronized:

$$[t'_1(A) - t_1(B)] - [t_1(B) - t_1(A)] = 0, \text{ or } [t'_1(A) + t_1(A)] - 2t_1(B) = 0,$$

where $t_1(A)$ is the time when the pulse leaves A, $t_1(B)$ is the time when the pulse arrives at B and $t'_1(A)$ is the time when the pulse returns back to A.

However, we will record a ΔT because the two clocks are not synchronized. That is, we will have

$$[t'_1(A) + t_1(A)] - 2t_1(B) = \Delta T. \quad (\Delta T \text{ can be positive or negative or zero.})$$

Case 2: let the disk rotate clockwise with an angular velocity ω. (The acceleration process can be very short if the angular acceleration is big.) The rates of the two clocks may change when the disk rotates, but the changes are the same for both clocks and therefore, the net result of the measurements will not be affected. Because of the Sagnac effect, we should record

$$[t'_2(A) - t_2(B)] - [t_2(B) - t_2(A)] - \Delta T = 2V_D L/c^2 \text{ or } [t'_2(A) + t_2(A)] - 2t_2(B) - \Delta T = 2V_D L/c^2,$$

where $t_2(A)$ is the time when the pulse leaves A, $t_2(B)$ is the time when the pulse arrives at B and $t'_2(A)$ is the time when the pulse returns back to A.

The process where we obtain ΔT when the disk does not rotate really is a process to synchronize the clocks: the two clocks will be synchronized if the recordings on clock B are added by $\Delta T/2$. In fact, this is the same as the way Einstein suggested to synchronize two separated clocks in a reference frame [10].

To practically measure the Sagnac effect, we must have a fast enough speed of the circular motion, V, and a long enough distance between the two clocks, L. To increase V, we can utilize moving objects in circular motions, such as cars, trains, and airplanes. The range of their speeds is from 10 m/s to 300 m/s. To increase L, we can put the clocks at the front and back ends of the moving objects. The range can be from 50 m to 400 m. For example, in an airplane, V = 300 m/s, L = 50 m, we can have $2VL/c^2 = 0.3 \times 10^{-12}$ s. This is detectable by today's instruments, but it might be difficult to implement. Can we increase this effect more so it could be easily detected? We can mount the two clocks not in one moving object, but in two separate objects moving at the same speed. This way, L will not be limited by the length of the object, so we can increase L to several kilometers, even several hundred kilometers. Of course, now L, the distance between the two clocks, will not be exactly constant. Will the measurement still be valid? We will show, later in this proposal, that the change of the distance between the clocks on two moving objects will not seriously influence the effect we are trying to detect. Let us first examine the way to conduct this experiment.

The experiment with two objects in a circular motion. - Let us utilize two helicopters because the helicopters will facilitate the propagation of the signals when they are at rest. We mount two atomic clocks with the same construction, signal transmitters, reflectors and receivers on the helicopters. First, the two helicopters stay stationary in the air with a distance L. A signal is sent from A to B, and returned to A by a reflector on B. As mentioned above, we will have

$$[t'_1(A) + t_1(A)] - 2t_1(B) = \Delta T.$$

Now let the helicopters fly at the same speed forward and keep the same distance with the earth surface (say, a kilometer). That means that both of them move in a circular motion with a radius of about R_e, although any radius will be sufficient (fig. 3). As indicated before, the Sagnac effect exists and if a signal is sent from A to B and returned from B to A again, we will find

$$[t'_2(A) + t_2(A)] - 2t_2(B) - \Delta T = 2V_D L/c^2.$$

(The Sagnac effect caused by the rotation of the Earth is included in this ΔT. However, it will not affect the final result, since it will affect both measurements in the same way. Or we can eliminate the effect of the rotation of the earth by choosing a direction along a meridian.)

The speed of a helicopter can reach up to 100 m/s and when the distance between two helicopters is 400 km; this Sagnac effect will be about 1 ns. Modern atomic clocks have fractional frequency stability of the order of 10^{-12}, 10^{-13}, or even better [5]. The whole process only takes less than a minute. Therefore only a short-term stability is required. The detection of a time difference of 1 ns is relatively easy.

Furthermore, helicopters can fly backward. If we compare the result when they are moving forward with the result when they are moving backward, we will find a bigger time difference (or choose a smaller distance L). As we mentioned above, to utilize helicopters is to facilitate the propagation of signals when both objects are at rest. In fact, we can use two airplanes with distance L between them. The only complication here is the curvature of the Earth. We may put clock A on a tower first to synchronize clock A with clock B, then move clock A to the airplane in the same place. The speed of airplanes can reach 300 m/s and the Sagnac effect will be 1 ns with the distance L of 150 km.

The influence of the distance change between two moving objects. - Since the two clocks are mounted on two separate objects traveling in a circular motion, the distance L between them is not constant. We should, therefore, ask whether this change in L would influence the effect we are trying to detect. Changes in L exist because two objects cannot always maintain the same speed, resulting in δV, the difference between the speeds of these objects. Now let us examine how δV influences the results in different stages of the experiment.

First measurement. We conduct the first measurement when the two helicopters are stationary in the air. However, the helicopters are not absolutely stationary. How much error will their motions bring to the measurements? Let δV_1 represent the drift speed of the helicopters. The total time duration of the measurement is $2L/c$. During this period, the maximum change of the distance will be $\Delta L_1 = (2L/c)2\delta V_1$. The error that it may cause is $\Delta L_1/c = 4\delta V_1 L/c^2$. It is much less than the quantity being measured, $2V_D L/c^2$, since $2\delta V_1 << V_D$.

Between the two measurements. It may take 30 s for a helicopter to fly from zero speed to the maximum speed. During this time, because of the difference between the speeds of the two helicopters, the distance between them will change. This change will influence the quantity of the Sagnac effect, $2V_D L/c^2$, increasing or decreasing its value. But, what is important for us is not the quantity itself, but the existence of this quantity [11].

Second measurement. Let us find out how much error the difference between the speeds of the two helicopters, δV_2, will bring to the second measurement. The total time duration of the measurement is $2L/c$. During this period, the maximum change of the distance will be $\Delta L_2 = (2L/c)\delta V_2$. The error that it may cause is $\Delta L_1/c = 2\delta V_2 L/c^2$. It is much less than the quantity being measured, $2V_D L/c^2$, since $\delta V_2 << V_D$.

Hence, we can conclude that it is possible to mount two clocks on two objects moving separately in a circular motion. This way, the Sagnac effect becomes easily measurable with current technology.

The verification of Sagnac corrections in GPS. - The Sagnac effect has been found in electromagnetic signals propagating from GPS satellites to ground stations [3]. As we mentioned before, it is suggested that the Sagnac correction should be made when the GPS receiver is mounted on a low-Earth orbit satellite or a Space Shuttle. A numerical example given in [8] uses $\Delta t_{\mathrm{Sagnac}} = v|\mathbf{r}_B - \mathbf{r}_A|/c^2$ for the Sagnac correction. It shows that between a GPS satellite ($r_A = 2.66 \times 10^7$ m) and a satellite at about 800 km altitude ($r_B = 7.178 \times 10^6$ m and $v = 7.5 \times 10^3$ m/s), $\Delta t_{\mathrm{Sagnac}} == 2500$ ns. Needless to say, similar corrections would be made on signals propagating between two satellites in the same orbit or between a satellite and a Space Shuttle. So far, these corrections have not been verified by experiments or measurements. The proposed experiment can act as a verification of such corrections.

The crucial experiment. - The experiment with two helicopters mentioned above not only can be used as an analogy of two satellites, it also has the advantage that, although you cannot readily change the orbit of a satellite, you can easily change the path of a helicopter. Therefore, it can be conducted as a crucial experiment of the principle of the constancy of the speed of light.

When we make the second measurement, let the two helicopters fly in one straight line with the same speed (fig. 4). Only one of the following two can happen: either the time difference still exists, or the time difference disappears. If the former happens, that means, in a reference frame moving translationally relative to the Earth (moving helicopters), the speeds of light are different in different directions, therefore, it will contradict the principle of the constancy of the speed of light. If the latter happens, it will still be very interesting. The two helicopters can fly in a circular motion with a bigger radius. If the time difference still exists, that means, we will find a new kind of "jump" in the nature: when two objects are in a circular motion with a radius of 6380 km or even bigger, there is a difference of the signal propagation times in different directions between two objects; when these two objects make a "purely" translational motion, the difference "jumps" to zero. So no matter what happens, the result of the experiment will be of great interest.

Conclusion. - Based on the triangle Sagnac experiment, we proposed a practical experiment. First, it can be used as a verification of Sagnac corrections in GPS. Second, it will reveal whether or not the speed of light is constant in a reference frame moving translationally relative to the Earth. Because of the practical and theoretical significance and its relatively easy implementation, we hope the proposal will become a reality in the near future.

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