

Relativistic Doppler Effect and the Principle of Relativity

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Abstract

The frequency shifts predicted by the ‘relativistic’ Doppler effect are derived in the photon picture of light. It turns out that, in general, the results do not depend exclusively on the relative velocity between observer and light source. In this respect, the relativistic Doppler effect is not distinct from the classical one, where the shifts are also different depending on whether the source or the observer moves. The ‘relativistic’ formulae for these two cases have been confirmed by experiment and are described in many textbooks. It was, however, not recognized that they are at variance with Einstein’s relativity principle extended to electromagnetic waves.

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I Introduction

When a wave with angular frequency ω and phase velocity c propagates in a medium, an observer moving with velocity v parallel to c experiences a shifted frequency $\omega_1 = \omega (1 - v/c)$. When the source oscillating with frequency ω moves with velocity v away from the observer, who is at rest in the medium, he will also measure a shifted frequency $\omega_2 = \omega / (1 + v/c)$ which is, however, different from ω_1 . These frequency shifts are known as ‘classical Doppler effect’ and may be observed in sound waves, for example. Because of the intervening medium, they do not merely depend on the relative velocity between observer and source, in contrast to other mechanical phenomena such as collision processes between material bodies which, in Newtonian mechanics, depend only on relative motion.

Electromagnetic waves share with sound the property that their propagation velocity is independent of the motion of the source, as Einstein postulated in 1905 [1]. He formulated in the same paper a second postulate, his ‘extended principle of relativity’, namely that all physical phenomena, including the electromagnetic ones, should only depend on relative velocities, as far as constant motion is concerned. From these postulates he deduced the Lorentz transformation, which led to the consequence that the velocity of light is constant in all inertial systems, regardless how they move relatively to each other.

It is obvious that Einstein’s postulates are not compatible with a medium (‘ether’) theory of light. If both postulates are realized in nature, the Doppler effect of light must distinguish itself from that of sound, insofar as the frequency shift of light can only depend on the relative velocity between source and

observer. Amazingly, the new ‘relativistic’ Doppler formulae, which are derived from the Lorentz transformation in textbooks, do not have this property: As in the case of sound, there is still a difference depending on whether the observer moves and the source is at rest, or whether the source moves and the observer is at rest. Even more surprising, the formulae applying to the different situations have been checked by experiment to some extent and were found to be correct within experimental error. As far as we can see, it was not recognized that these results cast doubt on the validity of Einstein’s relativity principle with regard to the physics of electromagnetic waves.

In this paper the formulae for the relativistic Doppler effect are derived without using the Lorentz transformation (Section II). This can be done in the photon picture of light in combination with the relativistic modification of Newtonian mechanics applied to detector and source. We find indeed different frequency shifts depending on whether the observer moves or the source. In Section III these results are compared with those deduced by Einstein from the Lorentz transformation. It turns out that there is practical agreement with the formulae derived in Section II. In Section IV the conclusion drawn from the Lorentz transformation that a wave front ‘appears’ tilted, if there is a relative velocity between observer and source, is scrutinized and found to be untenable. In Section V consequences of our analysis for the principle of relativity and for the Lorentz transformation are discussed.

II Doppler effect derived in the photon picture of light

Without using the Lorentz transformation, the Doppler effect can be explained in the photon picture which was established by the Planck-Einstein quantum theory of light. According to this model the energy of a photon is:

$$E = \hbar \omega \quad (1)$$

and its momentum:

$$p = \hbar \omega / c \quad (2)$$

For a body with finite mass the relativistically modified formulae of Newtonian mechanics apply:

$$E = m c^2 \quad (3)$$

$$\vec{p} = m \vec{v} \quad (4)$$

$$m = \frac{m_0}{\sqrt{1 - |\vec{v}|^2 / c^2}} \quad (5)$$

They are valid as long as the De Broglie wavelength of a moving body is small compared to its dimensions. The rest-mass m_0 takes into account any internal energy which may be of kinetic or potential form.

Let us assume that a detector with mass m moves with velocity v in x-direction and absorbs a photon with total momentum p , as depicted in Figure 1. The momentum balance before and after absorption yields:

$$p \cos \alpha + m v = m' v'_x \quad (6)$$

$$p \sin \alpha = m' v'_y \quad (7)$$

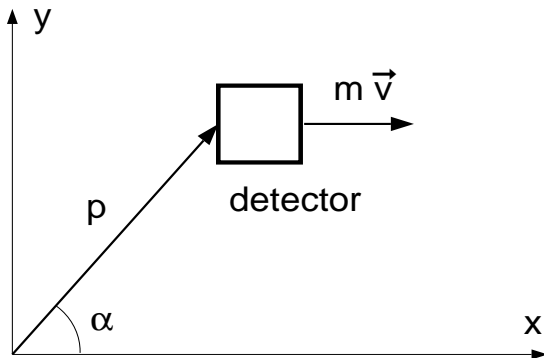


Figure 1: Absorption of a photon in a moving detector

where $m' = m'_0 / \sqrt{1 - (v'_x{}^2 + v'_y{}^2) / c^2}$ is the mass of the detector after absorption, and $v'_{x,y}$ are its velocity components. The energy balance requires:

$$c p + m c^2 = m' c^2 \quad (8)$$

The rest-mass of the detector is increased when it absorbs a certain amount of energy W :

$$m'_0 = m_0 + \frac{W}{c^2} \quad (9)$$

With (1) this energy may be expressed by a frequency $\hbar \omega_1 = W$, and can be determined experimentally by measuring the excitation energy of the detector. Eliminating from (6) and (7) the velocity components after absorption one obtains together with (9) and (5):

$$m'^2 = \left(m_0 + \frac{W}{c^2} \right)^2 + \frac{1}{c^2} (p^2 + m^2 v^2 + 2 p m v \cos \alpha) \quad (10)$$

Inserting this into (8) yields:

$$W = \hbar \omega_1 = p c \frac{1 - (v/c) \cos \alpha}{\sqrt{1 - v^2/c^2} (1 + W/(2 m_0 c^2))} \quad (11)$$

The very small ratio of photon energy to rest-energy of the detector may be neglected, and one obtains with (2) and the abbreviation $\beta = v/c$ the final result:

$$\omega_1 = \omega \frac{1 - \beta \cos \alpha}{\sqrt{1 - \beta^2}} \quad (12)$$

It is distinguished from the classical formula for sound by the denominator which describes the ‘quadratic’ part of the Doppler effect for light and leads to an *increase* of the measured frequency. The necessary energy is, of course, taken from the kinetic energy of the detector.

Let us assume now that a source with mass m moves with velocity v in x-direction and emits a photon, as depicted in Figure 2. The momentum balance before and after emission yields in this case:

$$m v = p \cos \alpha + m' v'_x \quad (13)$$

$$0 = p \sin \alpha + m' v'_y \quad (14)$$

and the energy balance requires:

$$m c^2 = c p + m' c^2 \quad (15)$$

The excitation energy of the emitting atom may be expressed by a frequency: $W = \hbar \omega$. After de-excitation the rest-mass of the source is decreased by the amount W/c^2 :

$$m'_0 = m_0 - \frac{W}{c^2} \quad (16)$$

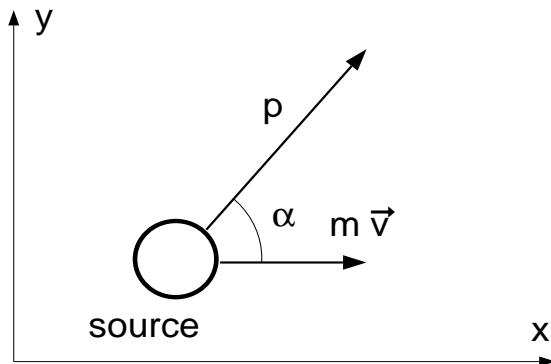


Figure 2: Emission of a photon by a moving source

Eliminating from (13-16) the mass m' and the velocity components of the source after emission one obtains with (5):

$$pc = \frac{W \sqrt{1 - \beta^2} (1 - W / (2 m_0 c^2))}{1 - \beta \cos \alpha} \quad (17)$$

Ignoring again the ratio $W / (2 m_0 c^2)$ one finds with (1) and (2) the frequency of the emitted photon:

$$\omega_2 = \omega \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \alpha} \quad (18)$$

This is the classical Doppler formula for a moving source, multiplied with the 'dilation' factor $\sqrt{1 - v^2/c^2}$, which leads to a relative *decrease* of the emitted frequency.

If both source and detector move with the same velocity, the motion dependent factors in (12) and (18) cancel out so that just the frequency $\hbar\omega = W$ is measured. The cancellation occurs

also when detector and source move in opposite directions, but α is 90 degrees. The null result of the experiment published in [2], where source and detector are placed at the periphery of a rotating disk, confirms this expectation.

The relativistic Doppler formulae contain - in contrast to the classical formulae - the dilation factor. From the above derivation it is evident that the dilation factor originates from the fact that inelastic scattering processes were considered. If the photons are neither emitted nor absorbed, but just elastically scattered at moving mirrors, the rest-mass of the mirrors remains unchanged and the dilation factor does not play a role. This may be shown explicitly by replacing the absorbing detector in Figure 1 by a mirror which reflects the impinging photon into the y-direction. Momentum and energy balance become now at constant rest-mass:

$$\begin{aligned} m v + p_1 \cos \alpha &= m' v'_x, & p_1 \sin \alpha &= m' v'_y + p_2 \\ p_1 - p_2 &= (m' - m) c \end{aligned} \quad (19)$$

Eliminating the velocity components after reflection yields with (5) the relationship:

$$(m'^2 - m^2) c^2 = 2 m v p_1 \cos \alpha + p_1^2 + p_2^2 - 2 p_1 p_2 \sin \alpha \quad (20)$$

Together with the energy balance in (19) one obtains the result:

$$\begin{aligned} p_2 &= p_1 - \frac{2 m v p_1 \cos \alpha + p_1^2 + p_2^2 - 2 p_1 p_2 \sin \alpha}{(m' + m) c} \\ &= p_1 (1 - \beta \cos \alpha), \text{ for } m_0 \rightarrow \infty \end{aligned} \quad (21)$$

For an infinitely large rest-mass of the mirror, this is the classical Doppler formula without the dilation factor, in contrast to equation (12).

Just as in the classical description, the frequencies ω_1 and ω_2 in the relativistic formulae (12) and (18) are not equal at $\alpha \neq 0$, if one identifies in (18) β , the velocity of the source, with $-\beta$ in (12), the velocity of the detector. This means that the relativistic Doppler effect does not depend exclusively on the relative velocity between observer and source, which clearly contradicts Einstein's extended relativity principle. On the other hand, formula (12) has been confirmed in a Mössbauer experiment with a moving detector [3], and formula (18) was early verified by measuring the emission from moving atoms [4]. The outcome of these experiments and the null result reported in [2] refute Einstein's relativity postulate with respect to its application on electromagnetic waves. This was apparently not recognized in the literature, but it was erroneously claimed that the experiments confirm the extended relativity principle. The reason for this misinterpretation must be sought in the fact that the relativistic Doppler formulae have also been derived from the Lorentz transformation which rests on Einstein's relativity principle. The connection between Lorentz transformation and the relativistic Doppler effect will be discussed in the next Section.

III Doppler effect and Lorentz transformation

Einstein derived on page 911 of his paper [1] from the Lorentz transformation a new formula for the Doppler effect which applies to a moving observer. His deduction may be delineated as follows. Let us assume that there is a coordinate system (x, y) and a second system (x', y') which coincides with the unprimed system at $t = 0$, but moves with velocity v in positive x-direction. The Lorentz transformation and its inverse are then

given by the expressions:

$$\begin{aligned} t' &= \frac{t - v x/c^2}{\sqrt{1 - v^2/c^2}} , & x' &= \frac{x - v t}{\sqrt{1 - v^2/c^2}} , & y' &= y \\ t &= \frac{t' + v x'/c^2}{\sqrt{1 - v^2/c^2}} , & x &= \frac{x' + v t}{\sqrt{1 - v^2/c^2}} , & y &= y' \end{aligned} \quad (22)$$

A plane light wave travelling in vacuo in the direction of the wave vector \vec{k} with phase velocity $\omega/k = c$ in the unprimed system is described by:

$$\cos(\omega t - k x \cos \alpha - k y \sin \alpha) \quad (23)$$

In the primed system it is also a plane wave described by:

$$\cos(\omega' t' - k' x' \cos \alpha' - k' y' \sin \alpha') \quad (24)$$

When the amplitude of the wave vanishes at a certain space-time point, it must vanish in both systems which requires that the phase in (23) at (x, y, t) must be equal to the phase in (24) at (x', y', t') :

$$\omega t - k x \cos \alpha - k y \sin \alpha = \omega' t' - k' x' \cos \alpha' - k' y' \sin \alpha' \quad (25)$$

The propagation velocity is the same in both systems:

$$\omega/k = \omega'/k' = c \quad (26)$$

which follows from differentiation of the phases in (25) with respect to time in the two systems. Inserting the second line of (22) into (23) one obtains:

$$\cos \left(t' \frac{\omega - k v \cos \alpha}{\sqrt{1 - v^2/c^2}} - x' \frac{k \cos \alpha - \omega v/c^2}{\sqrt{1 - v^2/c^2}} - y' k \sin \alpha \right) \quad (27)$$

Since, in view of (25), the expression (27) should be equal to (24) for all values of (x', y', t') , we find with (26) the identifications:

$$\omega' = \frac{\omega (1 - \beta \cos \alpha)}{\sqrt{1 - \beta^2}} \quad (28)$$

$$k' \cos \alpha' = \frac{k (\cos \alpha - \beta)}{\sqrt{1 - \beta^2}} \quad , \quad k' \sin \alpha' = k \sin \alpha \quad (29)$$

These transformation formulae are independent of where the source or where the detector is located. If one assumes with Einstein that the detector is at rest in the moving primed system, the measured frequency ω_1 is according to (28):

$$\omega_1 = \frac{\omega_S (1 - \beta \cos \alpha)}{\sqrt{1 - \beta^2}} \quad (30)$$

in agreement with (12) and the formula given by Einstein in [1]. It can also be found in textbooks, for example [5, 6]. The angle α is, in principle, a function of time, if the source is located at a finite distance R from the detector:

$$\alpha (t) = \alpha + \Delta \alpha (t) \simeq \alpha + \frac{v t}{R} \sin \alpha \quad (31)$$

Choosing the distance sufficiently large, however, the temporal change can be made arbitrarily small for finite observation times.

If one assumes that the source is at rest in the moving primed system, it has the velocity $-v$ with respect to the detector. Equation (28) yields in this case:

$$\omega_S = \frac{\omega_2 (1 + \beta \cos \alpha)}{\sqrt{1 - \beta^2}} \quad (32)$$

Resolved with respect to the detector frequency ω_2 one obtains:

$$\omega_2 = \frac{\omega_S \sqrt{1 - \beta^2}}{1 + \beta \cos \alpha} \quad (33)$$

in agreement with (18) for $v \rightarrow -v$. Again, the angle α changes slowly in time as in (31), but for $R \rightarrow \infty$ it stays constant. Formula (33), which was first confirmed experimentally by Ives and Stilwell [4], can be found in all textbooks dealing with the subject.

As demonstrated above, both formulae of Section II can also be derived from the Lorentz transformation, but they are not equal, since:

$$\omega_1 - \omega_2 = \frac{\omega_S \beta^2 \sin^2 \alpha}{\sqrt{1 - \beta^2} (1 + \beta \cos \alpha)} \quad (34)$$

With the exception of the case $\alpha = 0$, which, exclusively, is considered by Feynman [5], the frequencies (30) and (33) have the same property as the classical ones: They do not depend on the relative velocity alone, in contradiction to the relativity principle.

It should be noted that the derivation of (33) did not make any use of equations (29), which predict a change of the angle α when the source starts to move. From equation (31) we rather concluded that α is neither influenced by the velocity of the observer, nor by the velocity of an infinitely distant source. This was in perfect agreement with the postulate that the propagation of the wave is independent of the motion of the source. The apparent inconsistency between (31) and (29) requires to take a closer look at the phenomenon of aberration, which will be discussed in the next Section.

IV Aberration

Einstein has used the Lorentz transformation to give a new interpretation of the astronomically observed phenomenon of aberration. From the two equations (29) one can eliminate k and obtain the transformation law for the angle between the velocity and the wave vector:

$$\cos \alpha' = \frac{\cos \alpha - \beta}{1 - \beta \cos \alpha} \quad (35)$$

This equation can be found on page 912 of Einstein's paper [1]. The interpretation of (35) is that a wave travelling at the angle α in the unprimed system with respect to the x-axis will 'appear' to travel at the angle α' with respect to the x'-axis in the moving primed system where the observer is at rest. For small β one obtains:

$$\alpha - \alpha' = \alpha_A = \frac{\beta \sin \alpha}{1 - \beta \cos \alpha} \quad (36)$$

Since the earth changes annually its velocity with respect to the fixed stars, one should observe an aberration angle α_A given by (36) in the focal plane of a telescope. This is indeed the case and was already discovered in the early eighteenth century by Bradley [7]. He gave an explanation on the basis of the ether theory and the finite velocity of light. Aberration is a first order effect and follows also from the Galilei transformation. Hence, the explanation did not have to wait for the construction of the Lorentz transformation.

Similarly as in (28), it is not specified in (35) in which system the source or the observer are located. If we assume now that the observer is at rest and the source moves, we have to interchange

the primes in (35) and put $v \rightarrow -v$ as before. The result is:

$$\cos \alpha = \frac{\cos \alpha' + \beta}{1 + \beta \cos \alpha'} \quad (37)$$

which is not distinguished from (35). This means that aberration is a symmetric phenomenon according to the Lorentz transformation which depends only on the relative velocity between source and observer.

In Figure 3 four cases are selected with different velocities of a telescope and a star. In the upper row the predictions of the Lorentz transformation concerning the occurrence of aberration are sketched on the basis of equations (35) and (37). In case I a telescope is at rest and a star - also at rest - is located on its optical axis at infinite distance. The plane wave fronts of the light entering the telescope are imaged by the lens to a diffraction spot which lies in the focal plane on the optical axis. When the telescope moves parallel to the wave fronts (case II) equation (35) predicts a tilting of the wave fronts so that the image of the star is shifted to the left of the optical axis. In case III the star moves with the same velocity as the telescope to the right. This case is equivalent to case I, as the relative velocity vanishes. Equations (35) and (37) predict no aberration of the image in this case, since $\alpha = \alpha' = 90$ degrees. In case IV the telescope is at rest and the star moves to the left. The relative velocity is not distinguished from that of case II and equation (37) predicts, consequently, the same aberration of the image as in case II.

In the lower row of Figure 3 the aberration shifts are sketched which can be inferred from astronomical observations. As the

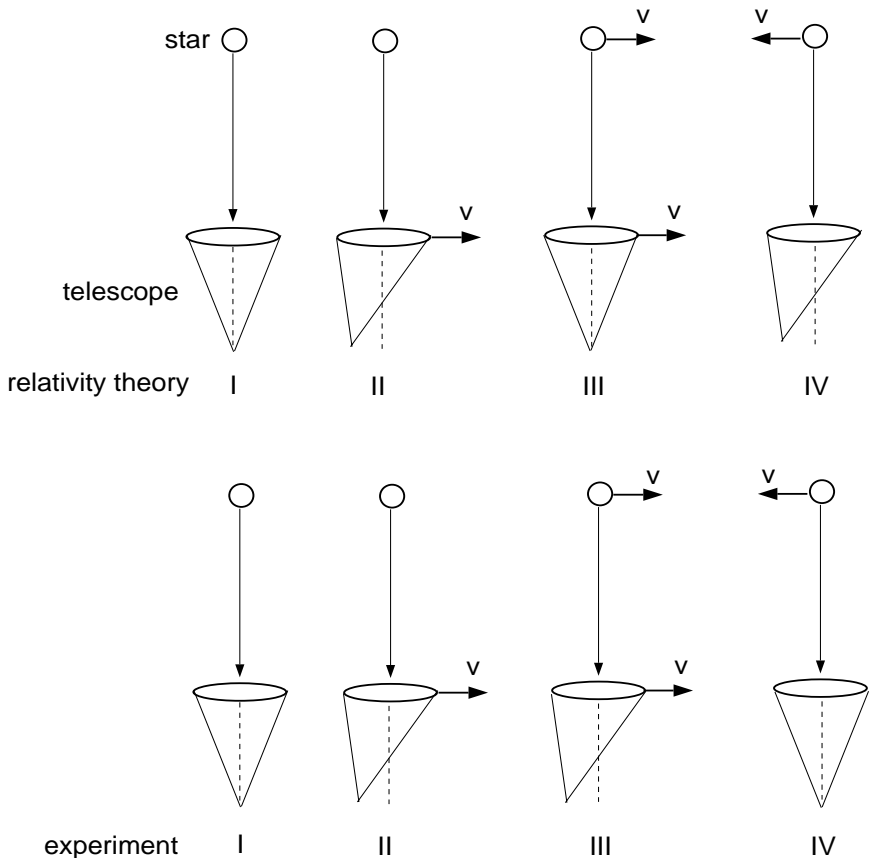


Figure 3: Aberration shift as predicted by the Lorentz transformation (upper row). Aberration shift as inferred from astronomical observations (lower row)

earth reverses its velocity within half a year, one observes aberration for all fixed stars by the same amount in agreement with equation (35). Case II applies and the aberration angle is given by the ratio of the earth's velocity to the velocity of light according to equation (36). This is also in agreement with Bradley's ether theory. If the motion of the stars would not change during terrestrial observation periods, the relevant relative velocity would be exclusively determined by the annual changes of the earth's velocity. Any motion of the stars, as in case III or IV, would have no influence on the actual observation, since the true position of the stars is unknown.

There are, however, double star systems [8] the components of which change their velocity on a time scale ranging from days to years. Spectroscopic binaries, in particular, which are too close to be resolved by a telescope, have velocities exceeding the earth's velocity round the sun. They revolve around their common center of gravity within days, a period during which the motion of the earth is practically constant. In this case equation (37) predicts an aberration shift which should have a magnitude given by the ratio of the star's velocity to the velocity of light. The components of the binary system should be easily separable, when their changing velocities are comparable to the earth's velocity round the sun. This is, however, not observed. The binary components remain unresolved which means that their velocity has no influence on aberration, as depicted in case IV in the lower row, in agreement with equation (31). Rotating binary systems follow the same pattern as all fixed stars and are observed within a period of a year under the same universal aberration angle, i.e. their apparent position changes with an

annual period common to all distant stars, as depicted in case III, lower row.

The astronomical observations on double star systems favor apparently equation (31) rather than equation (37), as (31) predicts no detectable tilting of the wave front when the star moves, provided it is sufficiently far away. If a star would rotate on an orbit which has the same diameter as the earth's orbit round the sun, it would appear under the same parallax angle under which a star at rest is normally observed from the orbiting earth. It is well known that the parallax angle even of the closest stars is an order of magnitude smaller than the universal aberration angle due to the earth's velocity. The parallax angle of spectroscopic double star systems is so small that it cannot be resolved by a telescope. Their large velocity does not change this in any way.

The correct prediction of the terrestrially observed aberration angle by equation (36) must be considered as fortuitous. The Lorentz transformation fails completely to predict correctly the observations made on double star systems, when equation (37) is applied. It is, of course, a contradiction to say on the one-hand-side that two coordinate systems move such that the y' - and the y -axis remain parallel during the motion, as follows from the Lorentz transformation (22), and say on the other hand, one and the same wave front spans an angle α with the y -axis, but a different angle α' with the y' -axis. The usual terminological escape from the discrepancy is to say that the wave fronts *appear* tilted rather than they *are* tilted. This wording only covers the problem, but it does not solve it.

V Discussion and conclusion

The Lorentz transformation may be seen as a formalism of merely kinematic nature to ensure that the velocity of light is the same in all inertial systems, independent of their motion. It ignores completely the dynamics involved in the interaction of light with matter. Hence, it would be surprising, if it could predict the exact formulae of the relativistic Doppler effect, which were rigorously derived in Section II on the basis of the dynamic formulae (1-5). The ratio of photon energy to rest-energy in (11) and (17), although negligible in practice, did not appear in the derivations of Section III. Under this aspect the agreements with the approximate formulae in Section II must be considered as more or less fortuitous.

Surprisingly, it was not recognized in the literature that the correct Doppler formulae of Section II, which appear to be confirmed by the Lorentz transformation, are at variance with its underlying idea: Extension of the principle of relativity from mechanics to light. Given now the fact that the relativistic Doppler formulae are confirmed by experiment, Einstein's extended relativity principle must be rejected. His conclusion drawn from his two postulates, namely that the velocity of light is constant in all inertial systems, appears untenable. Only Einstein's postulate that the velocity of light is independent of the motion of the source, which holds also for a medium theory, has been confirmed by experiment. Observations on the cosmic background radiation, as measured with earthbound detectors, show that there is a unique reference system in which the background radiation is isotropic and electromagnetic waves propagate with velocity c in all directions. The motion of the earth relative

to this system can clearly be determined by using the Doppler effect.

The Lorentz transformation predicts an apparent tilting of the angle spanned by the wave vector and the relative velocity between observer and source. This claim is refuted by astronomical observations on moving light sources, as pointed out in Section IV. Neither the velocity of an infinitely distant source nor the velocity of an observer influences the direction of the wave vector. This is in agreement with the postulate that the propagation of light in free space is independent of the motion of the source. As aberration is of first order in β , the observed asymmetry of the effect proves even more convincingly than the comparison of the different Doppler formulae that the Lorentz transformation contradicts experimental facts. The terrestrial observation of aberration, which depends only on the velocity of the earth, has nothing to do with a hypothetical tilting of the wave fronts, but can be explained satisfactorily by the finite travel time of light between the imaging element and the focal plane of a telescope.

Apart from the relativistic Doppler effect, the null result of the Michelson-Morley experiment has been taken as experimental proof for the validity of the Lorentz transformation. In recent work by the present author [9] it has been shown in detail that the outcome of the Michelson-Morley experiment proves the opposite: The Lorentz-Fitzgerald contraction, which is derived from the Lorentz transformation, cannot be real. If this were the case, Michelson should have measured a fringe shift. The misinterpretation of this experiment in the literature is due to a wrong assumption concerning the reflection angle of photons at a moving mirror. Adopting the correct physical law of

reflection, a fringe-shift is not expected.

Taking the experimental evidence together, sufficient proof has been presented that the Lorentz transformation does not represent a physical law which is realized in nature. Its underlying principle, that optical phenomena depend only on relative velocities, cannot be maintained.

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