Einstein's Violation of General Covariance

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Abstract

Einstein rejected the differential law of energy-momentum conservation $T^{\mu\nu}_{;\nu} = 0$. In doing so, he violated the principle of general covariance. Here, we prove the conservation law $T^{\mu\nu}_{;\nu} = 0$ and discuss its significance for general relativity.

In his founding paper on general relativity, Einstein stated the principle of general covariance:

> "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)." [1]

In that same paper, he rejected the differential law of energy-momentum conservation

$$T^{\mu\nu}_{;\nu} = \frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}\, T^{\mu\nu}}{\partial x^\nu} + \Gamma^\mu_{\nu\lambda} T^{\nu\lambda} = 0 \tag{1}$$

He writes as follows: the term $\Gamma^\mu_{\nu\lambda} T^{\nu\lambda}$ "shows that laws of conservation of momentum and energy do not apply in the strict sense for matter alone."[2] This statement constitutes a direct violation of general covariance. All textbook authors to date have followed Einstein's erroneous lead in this matter. Here are two exceptionally forthright quotations:

(a) the equations $T^{\mu\nu}_{;\nu} = 0$ "are not what can properly be called conservation laws" [3];

(b) the equation $T^{\mu\nu}_{;\nu} = 0$ "does not generally express any conservation law whatever." [4]

It is not difficult to prove the conservation law (1). In flat rectangular coordinates $x^\mu = (x^0, x, y, z)$ conservation is expressed by the Lorentz covariant equation

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0 \tag{2}$$

Suppose, instead, that we choose ordinary flat spherical coordinates $x^{\mu\prime} = (x^0, r, \theta, \phi)$. What will be the correct equation in the new coordinate system? To answer this question, we begin with equation (2) and substitute the transformed quantities

$$T^{\mu\nu} = \frac{\partial x^\mu}{\partial x^{\alpha\prime}} \frac{\partial x^\nu}{\partial x^{\beta\prime}} T^{\alpha\beta\prime} \tag{3}$$

$$\frac{\partial}{\partial x^\nu} = \frac{\partial x^{\gamma\prime}}{\partial x^\nu} \frac{\partial}{\partial x^{\gamma\prime}} \tag{4}$$

We then make use of

$$\Gamma^{\mu}_{\nu\lambda}{}' = \frac{\partial x^{\mu\prime}}{\partial x^{\alpha}}\frac{\partial x^{\beta}}{\partial x^{\nu\prime}}\frac{\partial x^{\gamma}}{\partial x^{\lambda\prime}}\Gamma^{\alpha}_{\beta\gamma} + \frac{\partial x^{\mu\prime}}{\partial x^{\alpha}}\frac{\partial^2 x^{\alpha}}{\partial x^{\nu\prime}\partial x^{\lambda\prime}} \tag{5}$$

$$\frac{1}{\sqrt{-g}'}\frac{\partial\sqrt{-g}'}{\partial x^{\nu\prime}} = \Gamma^{\alpha}_{\alpha\nu}{}' \tag{6}$$

and arrive at the equation

$$\frac{1}{\sqrt{-g}'}\frac{\partial\sqrt{-g}'T^{\mu\nu\prime}}{\partial x^{\nu\prime}} + \Gamma^{\mu}_{\nu\lambda}{}'T^{\nu\lambda\prime} = 0 \tag{7}$$

This proves the differential law of energy-momentum conservation in the spherical coordinate system. Because this law is generally covariant, it must hold true for all systems of coordinates, flat or curved (principle of general covariance).

The equation $T^{\mu\nu}_{;\nu} = 0$ is not open to interpretation, any more than Maxwell's equations are open to interpretation. They are generally covariant laws of nature, all of which belong to four-dimensional tensor analysis. These laws are *beyond personal choice*; it is this fact which demonstrates the power of general covariance. To reject such an equation is simply to make a mistake. Einstein made such a mistake in rejecting the law of energy-momentum conservation $T^{\mu\nu}_{;\nu} = 0$.

This law has profound physical consequences for general relativity. The gravitational field equations are

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\kappa T^{\mu\nu} \tag{8}$$

$T^{\mu\nu}$ is the stress-energy-momentum tensor of matter and electromagnetism. The covariant divergence of the left-hand side is identically zero, therefore

$$T^{\mu\nu}_{;\nu} = 0 \tag{9}$$

This equation means that the energy-momentum of matter and electromagnetism is conserved, at all space-time points. In other words, there is no exchange of energy-momentum with the gravitational field. Conclusion: Einstein's gravitational field has no energy, momentum, or stress [5, 6].

References

1. A. Einstein, "The Foundation of the General Theory of Relativity" in *The Principle of Relativity* (Dover, New York, 1952) section 3.

2. A. Einstein, Ref. 1, section 18.

3. P. Bergmann, *Introduction to the Theory of Relativity* (Dover, New York, 1976) page 194.

4. L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Pergamon, Elmsford, 1975) 4th ed., section 96.

5. K. Dalton, "Energy and Momentum in General Relativity", *Gen. Rel. Grav.* **21**, 533-544 (1989).

6. K. Dalton, "Manifestly Covariant Relativity", *Hadronic J.* **17**, 139-142 (1994); also, www.arxiv.org/physics/0608030.