The Doppler Effect Considered in Relation to the Michelson-Morley Experiment

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(Received September 9, 1937)

In an earlier paper[1] I have shown that a null result in the Michelson-Morley experiment can be explained on the assumption of contractions of the apparatus in the ratio

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{n+1} : 1$$

in the direction of motion and

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{n} : 1$$

at right angles to the direction of motion. The results of the Kennedy-Thorndyke experiment are then explained if the clock at the origin is altered in frequency in the ratio

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{1-n} : 1.$$

The value of n in these expressions is not determinable from the Michelson-Morley type of experiment.

I consider in this paper an optical experiment which is competent to decide the value of n. This experiment is the observation of the frequencies of the displaced lines in the Doppler effect, as obtained for instance with canal rays.

We start with the "classical" formula for the Doppler effect, which assumes frequencies to be independent of motion. In its complete form it is

$$\nu'=\nu\frac{1-(v_0/c)\cos\varphi}{1-(v_s/c)\cos\varphi}, \tag{1}$$

where (Fig. 1), ν' is the observed frequency for the observer on particle o, ν is the natural frequency of the light sources; v_s and v_0 are the velocities of source and observer respectively, with respect to the ether, c the velocity of light, and φ the angle between the axis of motion of the (parallel) moving bodies, and the direction of the light ray between them.[2]

We now modify the formula (1) in the manner called for by the results of the Michelson-Morley and Kennedy-Thorndyke experiments. We note that ν becomes, in terms of ν_0, the frequency of the source when stationary in the ether,

$$\nu_s=\nu_0[(1-v_s^2/c^2)^{\frac{1}{2}}]^{1-n}, \tag{2}$$

and that the observed frequency is re-evaluated in terms of the frequency of the observer's standard in the ratio

$$1/[(1-v_0^2/c^2)^{\frac{1}{2}}]^{1-n}, \tag{3}$$

so that we have for our general Doppler effect formula:

$$\nu'=\nu_0\left[\frac{(1-v_s^2/c^2)^{\frac{1}{2}}}{(1-v_0^2/c^2)^{\frac{1}{2}}}\right]^{1-n}\frac{1-(v_0/c)\cos\varphi}{1-(v_s/c)\cos\varphi}. \tag{4}$$

We shall now study the phenomena to be expected for different values of n in the above formula. This we shall do by taking certain selected values, namely $n=1$, which corresponds to a contraction of the Michelson-Morley apparatus in the ratio $1-(v^2/c^2) : 1$ in the direction of motion; and $n=0$, which corresponds to the contraction of the Michelson-Morley apparatus proposed by Fitzgerald, namely in the ratio $(1-v^2/c^2)^{\frac{1}{2}} : 1$ in the direction of motion.

The case, $n=1$, to be first considered, obviously corresponds to the "classical" formula (1). Taking that formula let us consider first the phenomena to be expected by end-on observation of a moving light source, that is where $\varphi=0$ and Π. For the sake of concreteness, we shall imagine we are observing a canal-ray tube, exhibiting in the spectroscope an undisplaced line and a displaced line; v_s is then the absolute velocity of the canal rays, v_0 that of observer and the "stationary" particles; v_s and v_0 being in the same direction.

Putting $\varphi=0$ we get, for the frequency of the

[1] "Graphical Exposition of the Michelson-Morley Experiment," J. Op. Soc. Am. 27, 177 (1937).
[2] See Born, *Einstein's Theory of Relativity*, p. 109, except for the factor cos φ.

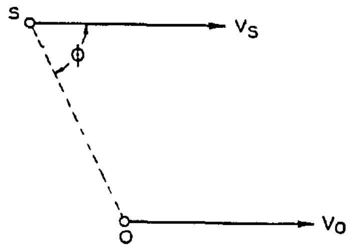

FIG. 1. Representation of parallel moving source (s) and observer (o) for obtaining formula for Doppler effect.

displaced line,

$$\begin{aligned}\nu' &= \nu_0 \frac{1-(v_0/c)}{1-(v_s/c)} \\ &= \nu_0\left(1-\frac{v_0}{c}\right)\left(1+\frac{v_s}{c}+\frac{v_s^2}{c^2}+\cdots\right) \qquad (5)\\ &= \nu_0\left[1+\frac{(v_s-v_0)}{c}+\frac{v_s}{c}\frac{(v_s-v_0)}{c}+\cdots\right].\end{aligned}$$

Putting v_c for the velocity of the canal rays relative to the observer, i.e. v_c is the velocity as calculated from the applied field, we get

$$v_s - v_0 = v_c$$
$$v_s = v_c + v_0,$$

giving for (5)

$$\begin{aligned}\nu' &= \nu_0\left[1+\frac{v_c}{c}+\frac{(v_c+v_0)}{c}\frac{v_c}{c}+\cdots\right] \\ &= \nu_0\left[1+\frac{v_c}{c}+\frac{(v_c+v_0)}{c}\cdot\frac{v_c}{c}+\cdots\right] \qquad (6)\\ &= \nu_0\left[1+\frac{v_c}{c}+\frac{v_c^2}{c^2}+\frac{v_0 v_c}{c^2}+\cdots\right]\end{aligned}$$

and for $\varphi = \Pi$

$$\nu' = \nu_0\left[1-\frac{v_c}{c}+\frac{v_c^2}{c^2}+\frac{v_0 v_c}{c^2}\right]. \qquad (7)$$

We therefore have as a first-order effect, displaced lines at $+$ and $-\nu_0(v_c/c)$, that is symmetrical displacements for the two directions of observation; to these are added second-order displacements, which introduce a lack of symmetry which is a function of the velocity of the system as a whole.

Consider next the case of observation at right angles to the direction of motion of the canal rays. Here, due to aberration, our actual angle of observation is $\cos^{-1}(v_0/c)$, which put in formula (1), gives

$$\begin{aligned}\nu' &= \nu_0\left[\frac{1-(v_0^2/c^2)}{1-(v_s v_0/c^2)}\right] \\ &= \nu_0\left[1-\frac{v_0^2}{c^2}+\frac{v_s v_0}{c^2}\cdots\right] \\ &= \nu_0\left[1+\frac{v_0}{c}\left(\frac{v_s-v_0}{c}\right)\cdots\right] \\ &= \nu_0\left[1+\frac{v_0 v_c}{c^2}\cdots\right].\end{aligned} \qquad (8)$$

We should therefore expect to observe a displaced line, whose direction of displacement depends on the direction of motion of the system or of the canal rays, as either is changed.

In order to illustrate these conclusions on the Doppler effect when $n=1$ the content of the above equations is exhibited in Fig. 2, for arbitrarily chosen values, $v_c/c=0.1$, $v_0/c=0.2$, and the canal-ray tube turned end for end (changing the sign of v_c). From this figure it is evident that unsymmetrical shifts of the displaced canal ray lines are to be expected, from which the motion of the observing platform with

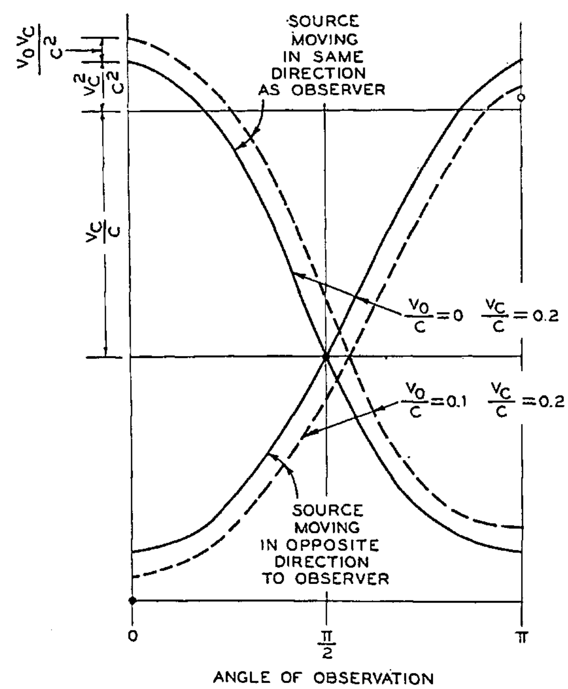

FIG. 2. Frequency of displaced canal-ray line as function of angle of observation and direction of rays, for the case of $n=1$.

respect to the ether could be deduced. The center of gravity of the two displaced lines, obtained by observations at 0° and 180°, will be displaced toward higher frequencies for canal rays moving in the same direction as the system, and to lower frequencies for the opposite direction.

For lateral observation, no displacement is observed when the canal rays are viewed at the aberration angle associated with the velocity of the observer. Part of the unsymmetrical character of the curves in Fig. 2 is due, it will be observed, to the second-order terms in v_c/c, which arise from plotting in terms of *frequencies*. The behaviors characteristic of the motion of the system are more clearly differentiated if the plot is made in terms of wave-length as is done in Fig. 3. This plot shows that for the case of $v_0/c=0$, the displacements are entirely symmetrical. For finite values of v_0/c the center of gravity of the displaced lines is toward the blue for one orientation of the canal-ray tube, toward the red for the opposite orientation, by the amount $\lambda_0(v_0v_c/c^2)$.

We now proceed to consider the same problem as it is modified by the choice of the value 0 for n. Formula (4) then becomes

$$\nu'=\frac{\nu_0(1-(v_s^2/c^2))^{\frac{1}{2}}}{(1-(v_0^2/c^2))^{\frac{1}{2}}}\cdot\frac{1-(v_0/c)\cos\varphi}{1-(v_s/c)\cos\varphi}. \qquad (9)$$

We will now consider in turn the cases discussed above by the classical treatment:

For $\cos\varphi=1$ we have

$$\nu'=\nu_0\frac{\left[\left(1-\frac{v_s}{c}\right)\left(1+\frac{v_s}{c}\right)\left(1-\frac{v_0}{c}\right)\left(1-\frac{v_0}{c}\right)\right]^{\frac{1}{2}}}{\left[\left(1-\frac{v_0}{c}\right)\left(1+\frac{v_0}{c}\right)\left(1-\frac{v_s}{c}\right)\left(1-\frac{v_s}{c}\right)\right]^{\frac{1}{2}}}$$

$$=\nu_0\frac{\left[1+\frac{(v_s-v_0)}{c(1-(v_0v_s/c^2))}\right]^{\frac{1}{2}}}{\left[1+\frac{(v_0-v_s)}{(1-(v_0v_s)/c^2))}\right]^{\frac{1}{2}}} \qquad (10)$$

$$=\nu_0\frac{1+\frac{(v_s-v_0)}{c(1-(v_0v_s/c^2))}}{\left[1-\frac{(v_0-v_2)^2}{c^2(1-(v_0v_s/c^2))^2}\right]^{\frac{1}{2}}}.$$

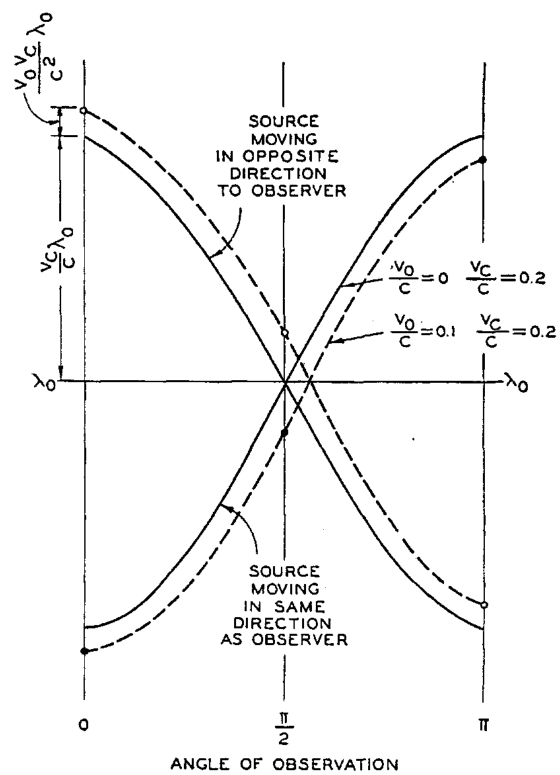

FIG. 3. Wave-length of displaced canal-ray line as function of angle of observation and direction of rays, for the case of $n=1$.

It becomes necessary at this point to evaluate the term

$$\frac{v_s-v_0}{(1-(v_0v_s/c^2))}. \qquad (11)$$

From a previous paper[3] we take the result that in a system where clock rates vary in the manner now under consideration, this expression is a function of the *observed* difference of velocities, which becomes identical with this observed difference, when the clocks used in observing the passage of one body past the other are moved infinitely slowly; (11) is thus for the latter case simply the observed velocity of the canal rays, as obtained for instance by the measurement of field strength in the tube. We designate this observed velocity by the symbol used in the classical case, v_c. We therefore have

$$\nu'=\nu_0\frac{1+(v_c/c)}{[1-(v_c^2/c^2)]^{\frac{1}{2}}}. \qquad (12)$$

[3] "Light Signals on Moving Bodies as Measured by Transported Rods and Clocks," J. Op. Soc. Am. **27**, 263 (1937).

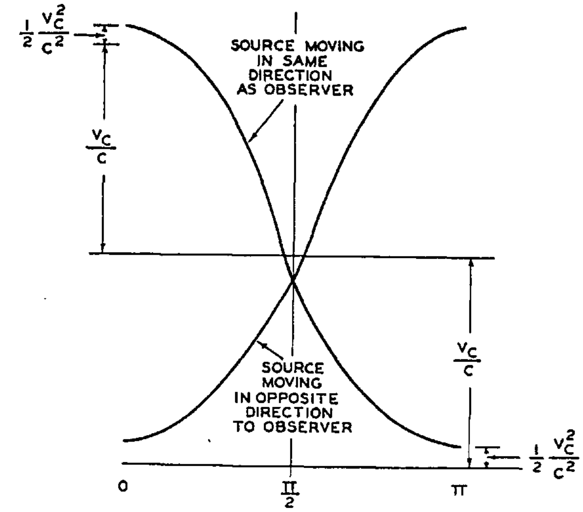

FIG. 4. Frequency of displaced canal-ray line as function of angle of observation and direction of rays for the case of $n=0$

We obtain similarly for $\cos\varphi=-1$

$$\nu'=\nu_0\frac{1-(v_c/c)}{[1-(v_c^2/c^2)]^{\frac{1}{2}}}. \qquad (13)$$

Let us next turn our attention to observation perpendicular to the direction of motion of the bodies. For this case, taking into account aberration, which makes the actual angle of observation $\cos^{-1}(v_0/c)$, we have

$$\nu'=\nu_0\frac{(1-(v_s^2/c^2))^{\frac{1}{2}}}{(1-(v_0^2/c^2))^{\frac{1}{2}}}\cdot\frac{1-(v_0^2/c^2)}{1-(v_0v_s/c^2)}$$

$$=\nu_0\left(1-\frac{(v_s-v_0)^2}{c^2(1-(v_0v_s/c^2))^2}\right)^{\frac{1}{2}} \qquad (14)$$

$$=\nu_0\left(1-\frac{v_c^2}{c^2}\right)^{\frac{1}{2}}.$$

The formula indicates that for lateral observation at apparent 90° from the direction of the discharge the frequency is *reduced*.

Summarizing, we find that the frequencies of the canal ray lines are symmetrically displaced, except for the second-order term $(1-(v_c^2/c^2))^{\frac{1}{2}}$, which acts to *increase* the frequency as given by the first-order terms for "end on" observation in either direction, and to *decrease* it for lateral observation. These relations are exhibited in Fig. 4. Compared with the case for $n=1$, the changes in frequency are seen to be simply a *decrease* for all conditions of observation by the factor $\frac{1}{2}(v_c^2/c^2)$, from what would occur if, with $n=1$, the observer were stationary in the ether. When expressed in terms of wave-length instead of frequency, as was done for the case of $n=1$ in Fig. 3, these relations reduce to a simple increase of wave-length for all conditions of observation in the ratio $1/(1-(v^2/c^2))^{\frac{1}{2}}:1$, over the stationary classical case.

Certain important general conclusions are embodied in formulae (13) and (14). We note that the phenomena are described entirely in terms of *observed relative motions* of light source and observer. Velocities with respect to the ether have dropped out and the phenomena are thus invariant with motion of the system through the ether. (It is evident, without formal proof, that this is the case only for the value 0 for the exponent n.) In this invariance the Doppler effect phenomena thus become similar to the Michelson-Morley phenomena. There is however a vital difference in the character of this invariance. In the case of the Michelson-Morley experiment the function of the term $[(1-(v^2/c^2))^{\frac{1}{2}}]^{n+1}$ is to make the effect not only invariant but *null*, that is, the predicted behavior corresponds to the "classical" case for the apparatus stationary in the ether. In the Doppler effect the result predicted is invariant, but *different* from the classical case for the observer stationary in the ether, as shown by comparing Eqs. (12), (13) and (14), with Eqs. (6), (7) and (8), with v_0 put equal to zero. Experiments on canal rays, of the sort here assumed, should therefore be capable of furnishing positive optical evidence for the validity of the contractions postulated by Fitzgerald, Larmor and Lorentz.[4]

[4] The crucial character of such experiments for deciding between the several possibilities for explaining the Michelson-Morley experiment (entrained ether, ballistic character of light emission, contractions of matter in traversing the universal pattern of radiant energy) was pointed out by Einstein and by Ritz thirty years ago.