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Relativity and real length

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RELATIVITY AND REAL LENGTH.

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IT is a rash thing sometimes to ask an innocent question. This is particularly the case where one has to answer the question oneself, and in such a way as to satisfy the relativist. I glance at a ruler lying on my study table, and turning to my relativist friend, I ask him to tell me, if he can, how long the ruler really is. I soon discover that the whole relativist universe has been set vibrating by the question, and that its import is well-nigh unfathomable.

The very first step towards a solution brings the difficulties full into view. The ruler's real length, we are told, is a matter that concerns not the ruler only, but oneself also as observer. Length, we are assured, is not a quality of the ruler, but a relation between ruler and observer. Sitting opposite to the ruler I may assess its length at one foot, but flashing past it at a speed approaching that of light, I must judge it to be six inches or less. Length, then, is a relation, not a quality, and the same holds good of duration. They "are not things inherent in the external world; they are relations of things in the external world to some specified observer." (Eddington, "Space, Time and Gravitation," p. 34.)

For simplicity's sake we will assume that the ruler is a perfectly rigid rod, and not liable, therefore, to change its length with fluctuations of temperature, or its straightness with conditions favouring flexure.* Still, despite its rigidity, the relativist rod will show a length that varies with the rate at which the observer increases or decreases his distance from it.

Common-sense is apt to stumble over this fundamental requirement of Relativity Theory. The ruler we handle seems to us so manifestly to possess a length of its own, a length perfectly constant if the bar be a rigid one. And our first rejoinder to the relativist may very well be that he is confusing the ruler's own natural length with the observer's measure of that length. The measured length may vary according to relativist requirements, but the rod's own length, surely, remains steadfast throughout. But to this objection the relativist replies: "I cannot conceive of any 'length' in nature

*The discrepancy between the ideally rigid body and the body as it occurs in nature is covered in practice through the mediation of accurately tested measuring-rods. "It is not a difficult task," says Einstein (Sidelights on Relativity, pp. 36-37), "to determine the physical state of a measuring-rod so accurately that its behaviour relatively to other measuring bodies shall be sufficiently free from ambiguity to allow it to be substituted for the 'rigid body.' It is to measuring-bodies of this kind that statements as to rigid bodies must be referred."

independent of a definition of the way of measuring length. And, if there is, we may disregard it in physics, because it is beyond the range of experiment." (Eddington, id. p. 8). It is indeed only as measured that facts have any relevancy for physics, and even our common-sense perception, we are told, is "a kind of crude physical measurement" (id. p. 15). If we insist on accepting the rod's own extendedness as a real length that remains constant through all variations in the measuring, we shall have to explain how we know the unmeasured length to be constant, or, in default, retain on our hands the old spectral thing in itself, about whose properties. as we know, it is wisest to say nothing.

A more promising line of criticism might be the following. We might argue that though the measured length of a rigid rod might vary with its motion relatively to an observer (according to the Restricted Theory), or with its place and position in the gravitational field (as the General Theory requires), this would not compel us to admit that the rod had no objective length or extendedness at all. Might it not have as its intrinsic property an "indeterminate length," relation to the observer having the effect not indeed of first introducing the length relation, but only of rendering an indeterminate quality of the rod determinate?

The suggestion has a certain plausibility, and, no doubt, some share of truth. It seems prima facie reasonable that the real length of the rod should be its algebraical length x, and that the measurable arithmetical lengths should record its various appearances according to circumstance and in accordance with law. We recall the Boundless of Anaximander, and the Primary Matter of Aristotle. We recall the view that space and time are, in themselves, not actualities, but possibilities, real possibilities of figure, measure, duration. In language familiar to philosophy we would now say that the rod's real length is its universal length, and that its particular lengths are its lengths as they appear according to changing conditions of movement and position. Moreover, this view of real length would fit in quite well with geometrical requirements. Consider the real length of the hypotenuse AB of the Euclidean right-angled triangle ABC. This mathematically real length is precisely any length. What is determinate here is simply the relation of the length of the hypotenuse C to the lengths of the sides A B containing the right angle, as given by the equation $c^2=a^2+b^2$ but a and b may have any arithmetical values we please to give them. So the radius of a Euclidean circle has a purely indeterminate length. Its proper length is any length.

But despite the simplicity of the view that the rod's real length is indeterminate, it is not scientifically satisfactory. And this for two main reasons. First, not being measurable or countable, its reality is not strictly a physical reality, which must rest on number and measure as a basis; and second, being intrinsically indeterminate, it cannot satisfy the requirement that the real shall be something permanent and abiding, something that abides through change. Instead of being an invariant, the indeterminate length is intrinsically a variable.

It is not easy to see how these two main requirements of measurability and of invariancy can be simultaneously met. If we return to the view that the rigid ruler's real length, in so far as it is measurable, is indeed its natural length as an objective fact controlling the empirical measurings of the individual observer, and that in no other sense is it both measurable and real—real, that is, in the sense of being just what it is, and not what the measurer would wish it or think it to be—we seem driven to add that it cannot then be an invariant, the same for all fields and observers, for has not Einstein shown that length is a function of the mobile gravitational field as well as of the relative movement of the particular observer?

Certainly the requirements conflict. But may not the conflict arise from a tendency to simplify the matter overmuch?

The Logic of Relativist Reality can be understood, I would venture to say, only if we bear in mind the complexity of the requirements of physical theory as an organised system: the need for direct contact with measurable fact, on the one hand; the equally imperative need for organised unity of scientific grasp, on the other; and thirdly, the need for keeping these two fundamental requirements in working harmony through the binding force of mathematics. Now, Einstein has succeeded in adequately meeting these three main needs. "Whatever can be measured," says Planck, "is real."* Einstein's whole procedure is controlled by respect for this dictum, and to this extent and in this sense he is a radical empiricist. One fundamental feature, at any rate, of the reality of the rigid ruler, is, in his eyes, its measurability through the help of sense and muscle, through the sensori-motor mechanism of the individual human body. And yet this respect for fact is dominated by respect for law, and above all by the basic recognition that it is only as an element in a natural order, only as conforming to law and measure, that

*Quoted by Moritz Schlick, "Space and Time in Contemporary Physics," p. 23.

fact has any meaning for physics at all. It is because Einstein accepts a fact's conformity to law as essential to its relevancy for physics that he can logically unite his empirical convictions with an absorbing quest for the conditions which shall justify the reign of law in the world of relativity. When Einstein discovers his general formula for the fundamental laws of nature, a formula which holds good for all natural events, whatever be the special point of view, or special axes of reference adopted by the observer, he connects in his achievement the requirements of measurability and of invariancy alike. Starting from the conception of a measurable Euclidean straight line, relativistic theory takes us through its two levels—those of its restricted and generalised formulations—to the weird, but fascinating conception of a line-element, ds, the differential interval between two point-events in a spatio-temporal continuum, four-dimensional, non-Euclidean, dependent for its geometrical form on the conditions of the gravitational field within which it figures. This "interval," as the fundamental invariant of the relativist doctrine is the supremely real thing for it. It is, in its general form, non-intuitable, non-measurable, and yet is no inaccessible thing-in-itself, but intelligibly connected through appropriate gradings with what is intuitable and measurable, as its limiting cases.* Hence to grasp Einstein's idea of physical reality we must include in one scheme both its empiricist and its rationalist elements, the measurable fact accessible to sense and muscle, and the invariant accessible only to the mathematical mind. The latter connects continuously with the former. Like the Platonic Good, it is the abiding source of all that is intelligible in the physical world, but, unlike it, overflows into all the sub-worlds of apprehension without break of continuity, and under clear limitations which can be mathematically controlled and defined.

*Expressed in the light of the six equations in which Einstein's Gravitational Theory is embodied, it takes the form: $ds^2 = -\frac{dr^2}{j} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + j dt^2$, where $j = 1 - \frac{2m}{r}$, m being the mass of the attractive particle to which the gravitational field is due; and r, θ, ϕ the polar co-ordinates corresponding to the x, y, z in the standard expression for the interval element. Now let $dt = 0$ and we get a formula for measured length which shows that in this "real" universe it is a function of the gravitational field. For any given value of m, the rod's real geodetic length (for it can no longer be straight, the geometry being non-Euclidean) will be a fixed amount, the same for all values of the co-ordinates r, θ, ϕ. And if $m = 0$ and $\therefore j = 1$ the invariant takes the form proper to the restricted theory, and the measured length is Euclidean. There is therefore a continuous mathematical connection between the actual measure of the ruler taken under ordinary physical conditions and the leading invariant of the General Theory, the real matrix of all physical reality. (Vide: Bolton, "An Introduction to the Theory of Relativity," ch. xviii).

Hans Reichenbach, in his "Relativitaets-theorie and Erkenntnis Apriori," Section viii. (Julius Springer, 1920), draws attention to the change in the object-concept brought about by the Theory of Relativity, and illustrates by reference to the concept of length. What we measure as length must be stated in terms of some system of co-ordinates. So stated it is a clearly-defined magnitude. But it is also subjective since the system of co-ordinates is arbitrarily chosen by the observer. "It is when, in addition, we supply the transformation formulae for every other system that our statement first wins a really independent meaning. The new method of the Theory of Relativity consists precisely in this, that through the transformation formulae the subjective assertion wins an objective meaning. . . . What is ascertainable is only the length as measured in some one system. And this is only one expression of the real relation. . . . It is only when we supply in addition the transformation formulae that we eliminate the subjective, arbitrary influence of the reference system, and also reach at last a genuinely objective determination of the real" (id., pp. 92-93).

On this view the physically objective is reached only through mental considerations whereby a purely subjective starting-point is adequately compensated, and its subjectivity sifted away. The very meaning of the General Theory of Relativity is that metrics are much more than a mathematical measuring scheme for bodies: that they are in fact the form for conceptually presenting a body as an element in the material world (cf., id., pp. 97-98).

Now, it seems to me that this view, fine and suggestive as it is, does not take sufficient note of the fact that there is a privileged, though not an absolute position from which the length relationship can be stated, namely, that in which the observer's view-point coincides with that of the object he proposes to measure, so that, for that position, variations due to differences of time and rate of relative motion do not arise: it is the position of the measuring-rod itself. Objectivity of the empirical kind is here secured through the sole agency of measurement, and apart from the accessory assistance of the transformation equations through which the findings of different observers may be interconnected, and results at one point of observation reduced to corresponding results obtainable at any other observation-point. It is true that measurement itself is a metrical relation, so that the relativist view that metrics is the key to matter is not hereby impugned, but objectivity and measurement are more intimately connected. The measure taken of a ruler by means of a measuring-rod reveals the natural length of the ruler for all the purposes of physical science. And the measure includes no reference to a clock. It is true that the length will vary with movement and in the direction of motion, and to ascertain the law according to which this variation takes place is of the utmost importance for reductions of measurements from one observer's viewpoint to another's. But the standard measurement will always be that obtained through the direct application of the measuring-rod, or, where such superposition is not possible, the nearest indirect equivalent, whatever that may be.

Further, the relativist concept of objectivity is incomplete apart from a reference to the "interval" or line-element as the fundamental invariant. It is only in relation to this invariant that the deeper objectivity and the part played by mathematics in discovering and defining it can be made clear. The measured length, on the relativist view, is necessarily an abstraction. For the measure is spatial only, and we are none of us allowed to forget Minkowski's dictum, uttered in 1908: "From henceforth space in itself, and time in itself, sink to mere shadows, and only a kind of union of the two preserves an independent existence."*

Summing up on the matter of the ruler's real length, we would suggest: first, that for all purposes of empirical application the real length is the natural length as discoverable through direct measurement, or its nearest mathematical substitute; second, that this natural length is none the less abstract and derivative: the space-shadow projected by a "world-line" whose law of being and of movement is given by an ideal invariant which, with mathematical rigour, controls the metrics of the relativist world; and third, that through the principle of continuity the natural length as directly measured is connected with the ideal length as a world-line through a wonderful system of mathematical workmanship, so that the one participates in the other, the natural in the ideal, as its projection or partial manifestation. To grasp the natural length as a mathematically controlled projection from an unpicturable universe of world-lines is no easy challenge for the mind to meet, but owing mainly to the rigorous mathematical character of the invariants involved, the challenge opens up a view of intelligibly organized physical reality which should be an inspiration to truth-seekers for many years to come.

*Moritz Schlick ("Space and Time in Contemporary Physics," p. 66) points out that Minkowski's synthesis is itself "a mere shadow, an abstraction," and "only the oneness of space time and things has an independent existence."